Physics of Neutrino Mass

Rabindra N. Mohapatra
Department of Physics, University of Maryland, College Park, MD-20742, USA

Recent discoveries in the field of neutrino oscillations have provided a unique window into physics beyond the standard model. In this lecture, I summarize how well we understand the various observations, what they tell us about the nature of new physics and what we are likely to learn as some of the planned experiments are carried out.

1. Introduction

For a long time, it was believed that neutrinos are massless, spin half particles, making them drastically different from their other standard model spin half cousins such as the charged leptons (e, μ, τ) and the quarks (u, d, s, c, t, b), which are known to have mass. In fact the masslessness of the neutrino was considered so sacred in the 1950s and 1960s that the fundamental law of weak interaction physics, the successful V-A theory for charged current weak processes was considered to be intimately linked to this fact.

During the past decade, however, there have been a number of very exciting observations involving neutrinos emitted in the process of solar burning, produced during collision of cosmic rays in the atmosphere as well as those produced in terrestrial sources such as reactors and accelerators that have conclusively established that neutrinos not only have mass but they also mix among themselves, like their counterparts (e, μ, τ) and quarks, leading to the phenomenon of neutrino oscillations. The detailed results of these experiments and their interpretation have led to quantitative conclusions about the masses and the mixings, that have been discussed in other lectures[1]. They have also been summarized in many recent reviews[2]. I will start with a brief summary of the results. I use the notation, where the flavor or weak eigenstates are denoted by ν_α (with $\alpha = e, \mu, \tau, \cdots$), that are expressed in terms of the mass eigenstates ν_i $(i = 1, 2, 3, \cdots)$ as follows: $\nu_\alpha = \sum_i U_{\alpha i}\nu_i$. The $U_{\alpha i}$ can be also be expressed in terms of mixing angles and phases as follows:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & c_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} K \tag{1}$$

where $K = diag(1, e^{i\phi_1}, e^{i\phi_2})$. This matrix characterizes the weak charged current for leptons:

$$\mathcal{L}_{wk} = \frac{g}{2\sqrt{2}}\bar{e}_\alpha U_{\alpha i}\gamma_\mu(1+\gamma_5)\nu_i W^{\mu,-} + h.c. \tag{2}$$

We denote the neutrino masses by m_i $(i = 1, 2, 3)$.

1.1. What we know about masses and mixings

Analysis of present neutrino data tells us that (at the 3σ level of confidence):

$$\begin{aligned} sin^2 2\theta_{23} &\geq 0.89 \\ \Delta m^2_A &\simeq 1.4\times 10^{-3}\ eV^2 - 3.3\times 10^{-3}\ eV^2 \\ sin^2\theta_{12} &\simeq 0.23 - 0.37 \\ \Delta m^2_\odot &\simeq 7.3\times 10^{-5}\ eV^2 - 9.1\times 10^{-5}\ eV^2 \\ sin^2\theta_{13} &\leq 0.047 \end{aligned} \tag{3}$$

While the mass differences that go into the discussion of oscillation rate are fairly well determined (at least within the assumption of three neutrinos and no exotic interactions), the situation with respect to absolute values of masses is much less certain. There are three possibilities:

- (i) Normal hierarchy i.e. $m_1 \ll m_2 \ll m_3$. In this case, we can deduce the value of $m_3 \simeq$

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$\sqrt{\Delta m^2_{23}} \equiv \sqrt{\Delta m^2_A} \simeq 0.03 - 0.07$ eV. In this case $\Delta m^2_{23} \equiv m^2_3 - m^2_2 > 0$. The solar neutrino oscillation involves the two lighter levels. The mass of the lightest neutrino is unconstrained. If $m_1 \ll m_2$, then we get the value of $m_2 \simeq\simeq 0.008$ eV.

- (ii) Inverted hierarchy i.e. $m_1 \simeq m_2 \gg m_3$ with $m_{1,2} \simeq \sqrt{\Delta m^2_{23}} \simeq 0.03 - 0.07$ eV. In this case, solar neutrino oscillation takes place between the heavier levels and we have $\Delta m^2_{23} \equiv m^2_3 - m^2_2 < 0$.

- (iii) Degenerate neutrinos i.e. $m_1 \simeq m_2 \simeq m_3$.

The above conclusions do not depend on whether the neutrinos are Dirac or Majorana fermions (Majorana fermions are their own anti-particles).

If neutrinos are Majorana fermions, they break lepton number by two units and nuclear decay processes such as $(A, Z) \rightarrow (A, Z+2) + e^- + e^-$ if allowed by kinematics can proceed. These are called $\beta\beta_{0\nu}$ process. The $\beta\beta_{0\nu}$ decay rate is directly proportional to the neutrino mass since it is the neutrino mass term in the Hamiltonian that breaks the lepton number symmetry. Present upper limits on the $\beta\beta_{0\nu}$ decay rate puts an an upper limit on a particular combination of masses and mixings (see the talk by G. Gratta at this school[3]):

$$m_{eff} \;=\; \sum_i \left[U^2_{ei} m_i\right] \leq 0.3\ eV \tag{4}$$

An important point here is that converting the neutrinoless double beta upper limit to information about neutrino mass depends on the type of spectrum[4]. Fig. 1 gives the values of the effective neutrino mass m_{eff} predicted for the allowed range of mass differences and mixings given by the present oscillation data. It is clear that for the case of inverted hierarchy, one expects a lower bound on m_{eff} in the range 30 to 50 meV, whereas strictly speaking for the normal hierarchy, this value can be zero due to CP violating phases that can lead to possible cancellations. At present there is also a claim of a positive signal for $\beta\beta_{0\nu}$ decay at the level of few tenths of an eV that needs to be confirmed[5]. There are also limits from tritium beta decay end point search for neutrino mass. In this case one gets a limit on the combination of masses[6]:

$$\sum_i |U_{ei}|^2 m^2_i \leq (2.2\ eV)^2 \tag{5}$$

From the above results, one can safely conclude that all known neutrinos have masses in the eV to sub-eV range.

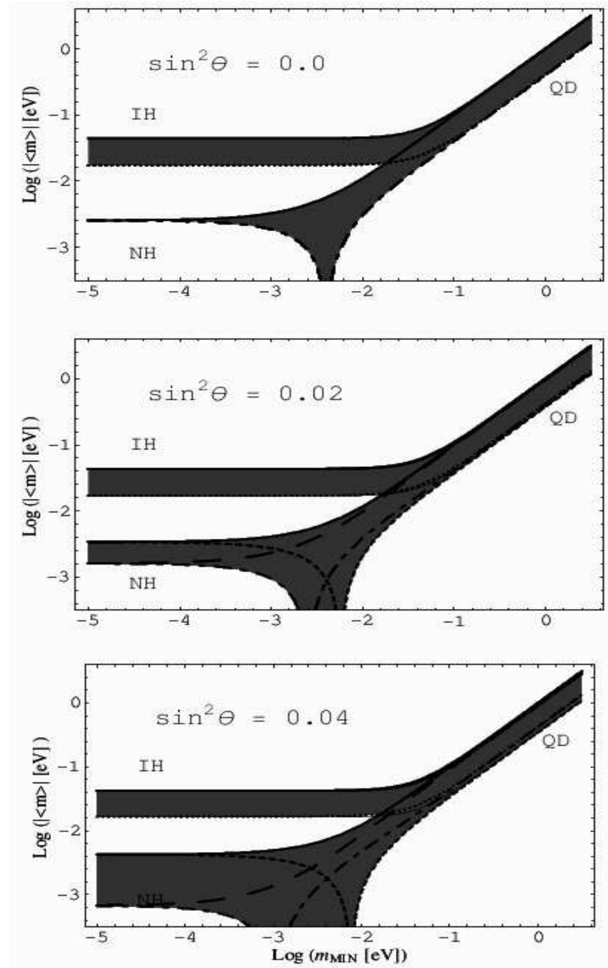

Figure 1: The dependence of m_{eff} on $\langle m \rangle_{min}$ in the case of the LMA-I solution, for normal and inverted hierarchy and for the best fit values of the neutrino oscillation parameters. Figure supplied by the authors of the last reference in [4].

There are also limits from cosmological observations such as WMAP and SDSS observations which put the limits in less than an eV range.

$$\sum_i m_i \leq 0.4\ eV \tag{6}$$

It is important to point out that a number of experiments are either approved or planning or ongoing stage in the arena of tritium beta decay (KATRIN), neutrinoless double beta decay (CUORE, MAJORANA, EXO etc.)[3] that will improve the above limits. From the domain of cosmology, the PLANCK experiment will also tighten the upper limits on neutrino masses.

1.2. Number of neutrinos

In the above discussion we have assumed that there are only three neutrino species. The question that arises is "How well do we know this ?". What we know from laboratory experiments is that measurement of the Z-width at LEP and SLC allows only three species of light neutrinos that couple to the Z-boson. It is however quite plausible to have additional neutrinos that are light and do not couple (or couple very, very weakly) to the Z-boson or the W boson. We will call them sterile neutrinos ν_s or ν'. They are therefore unconstrained by the Z-width data. However if they mix with known neutrinos they can manifest themselves in the early universe since the active neutrinos which are present in abundance in the early universe can oscillate into the sterile neutrinos giving rise to a density of ν_s's same as that of $\nu_{e,\mu,\tau}$. This will effect the synthesis of Helium and Deuterium by enhancing the expansion rate of the Universe. Thus our knowledge of the primordial Helium and Deuterium abundance will then provide constraints on the total number of neutrinos (active and sterile). The limit on the number of sterile neutrinos from BBN depends on several inputs: the baryon to photon ratio $\eta \equiv \frac{n_B}{n_\gamma}$ and the value of the He^4 fraction Y_p. The first (i.e. η) is now very well determined by the WMAP observation of the angular power spectrum[7]. The He^4 fraction Y_p has however been uncertain.

There have been new developments in this field. This has to do with our knowledge of primordial Helium abundance, which is derived from the analysis of low metallicity HII regions. It is now believed[8] that there are more systematic uncertainties in the estimates of Helium abundance from these analyses than was previously thought. The latest conclusion about the number of neutrinos depends on which observations (He^4, D^2 or WMAP) are taken into consideration. For example, He^4, D^2 and η_{CMB} together seem to give[8] $N_\nu \leq 4.44$ (compared to 3.3 before). One can therefore allow more than one sterile neutrino mixing with the active neutrinos without conflicting with cosmological observations. This has important implications for interpretation of the positive neutrino oscillation signals observed in the LSND experiment and now being tested by the Mini Boone experiment. We discuss this at a later section of this paper.

2. Neutrino Mass: Dirac vrs Majorana

In this section, I give a brief explanation of how to understand a Majorana neutrino. Let us write down the Dirac equation for an electron[1]:

$$i\gamma^\lambda \partial_\lambda \psi - m\psi = 0 \tag{7}$$

This equation follows from a free Lagrangian

$$\mathcal{L} = i\bar{\psi}\gamma^\lambda \partial_\lambda \psi - m\bar{\psi}\psi \tag{8}$$

The second term in the Lagrangian is the mass of the electron. However, Lorentz invariance allows another bilinear for fermions that could also act as a mass term i.e. $\psi^T C^{-1}\psi$, where C is the charge conjugation matrix. The difference between these two mass terms is that the first one is invariant under a transformation of the form $\psi \to e^{i\alpha}\psi$, whereas the second one is not. To discriminate between the two kinds of mass terms, we need to know the meaning of such a transformation: invariance under a phase transformation implies the existence of a charge which is conserved (e.g. the elctric charge, baryonic charge, leptonic charge etc.). Thus the presence of the second kind of mass term means the theory breaks all symmetries. Further note that if ψ satisfies the condition of being self charge conjugate, i.e.

$$\psi = \psi^c \equiv C\bar{\psi}^T, \tag{9}$$

then the mass term $\bar{\psi}\psi$ reduces to the mass term $\psi^T C^{-1}\psi$. Thus, the second mass term really implies that the neutrinos are their own anti-particles. Furthermore, this constraint reduces the number of independent components of the spinor by a factor of two, since the particle and the antiparticle are now the same particle. This mass term is called the Majorana mass in contrast to the form $\bar{\psi}\psi$ which will be called Dirac mass term.

Thus given a number of arbitrary spinors describing spin 1/2 particles, one can write either only Dirac type mass terms or Majorana type mass terms or both. Note that when a particle has a conserved quantum number (e.g. electric charge for the electron), one cannot write a Majorana mass term since it will break electric charge conservation. However for particles such as the neutrino which are electrically neutral, both mass terms are allowed in a theory. In fact one can stretch this argument even further to say that if for an electrically neutral particle, the Majorana mass term is not included, there must be an extra symmetry in the theory to guarantee that it does not get generated in higher orders. In general therefore, one would expect the neutrinos to be Majorana fermions. That is what most extensions of the standard model seem to predict. For a detailed discussion of this see [9].

At the moment we do not know if neutrinos are Dirac or Majorana fermions. A crucial experiment that will determine this is the neutrinoless double beta decay experiment[3]. A positive signal in this experiment conclusively establishes that neutrinos are Majorana fermions[10], although contrary to popular belief, it will not be easy without further experiments to determine the mass of the neutrino. The main reason for this is that there could be heavy beyond the standard model particles that could lead to $\beta\beta_{0\nu}$ decay withour at the same time giving a "large" enough neutrino mass[9].

An interesting question is: can we ever tell whether the neutrino is a Dirac fermion ? One can of course never say whether a very tiny Majorana mass term is present in the neutrino mass. This is in fact true for all symmetries in Nature that we assume are exact e.g. Lorentz invariance, electric charge conservation etc. What we can however say is whether the Dirac mass term dominates over the Majorana mass term overwhelmingly. This can be done by a combination of the three experiments: (i) $\beta\beta_{0\nu}$ decay experiments which are supposed to reach the level of sensitivity of 30-50 milli eV, (ii) tritium beta decay experiment KATRIN which is expected to push down the mass limit to the level of 0.2 eV and (iii) a long base line experiment that can presumably determine the sign of the atmospheric mass difference square. In Table I we give the situations when one can conclude that the neutrino is a Dirac particle[11] and when not.

Table I Conditions under which one can determine when neutrino is a Dirac particle. Normal, inverted and degenerate refer to the various mass patterns already discussed.

$\beta\beta_{0\nu}$	Δm^2_{23}	KATRIN	Conclusion
yes	> 0	yes	Degenerate, Majorana
yes	> 0	No	Degenerate, Majorana
			or normal or heavy exchange
yes	< 0	no	Inverted, Majorana
yes	< 0	yes	Degenerate, Majorana
no	> 0	no	Normal, Dirac or Majorana
no	< 0	no	Dirac
no	< 0	yes	Dirac
no	> 0	yes	Dirac

Before closing this section, let us again summarize the open questions raised by present data which need to be addressed by future experiments:

- Are neutrinos Dirac or Majorana?
- What is the absolute mass scale of neutrinos?
- How small is θ_{13}?
- How "maximal" is θ_{23}?
- Is there CP Violation in the neutrino sector?
- Is the mass hierarchy inverted or normal?
- Is the LSND evidence for oscillation true? Are there sterile neutrino(s)?

It is important to emphasize that if the full menu of experiments being proposed currently such as searches for neutrinoless double beta decay, searches for θ_{13}, precision measurements of θ_{12} and θ_{23} using reactor and long baseline experiments, all these questions would recieve answers.

3. Implications for Physics Beyond the Standard Model

These discoveries involving neutrinos, which have provided the first evidence for physics beyond the standard model, have raised a number of challenges for theoretical physics. Foremost among them are, (i) an understanding of the smallness of neutrino masses and (ii) understanding the vastly different pattern of mixings among neutrinos from the quarks. Specifically, a key question is whether it is possible to reconcile the large neutrino mixings with small quark mixings in grand unified frameworks suggested by supersymmetric gauge coupling unifications that unify quarks and leptons.

3.1. Why neutrino mass requires physics beyond the standard model ?

We will now show that in the standard model, the neutrino mass vanishes to all orders in perturbation theory as well as nonperturbatively. The standard model is based on the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ group under which the quarks and leptons transform as described in Table II.

The electroweak symmetry $SU(2)_L \times U(1)_Y$ is broken by the vacuum expectation of the Higgs doublet

Table II The assignment of particles to the standard model gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$.

Field	gauge transformation
Quarks Q_L	$(3, 2, \frac{1}{3})$
Righthanded up quarks u_R	$(3, 1, \frac{4}{3})$
Righthanded down quarks d_R	$(3, 1, -\frac{2}{3})$
Lefthanded Leptons L	$(1, 2-1)$
Righthanded leptons e_R	$(1, 1, -2)$
Higgs Boson $\mathbf{H}$	$(1, 2, +1)$
Color Gauge Fields G_a	$(8, 1, 0)$
Weak Gauge Fields $W^{\pm}$, Z, γ	$(1, 3+1, 0)$

$< H^0 >= v_{wk} \simeq 246$ GeV, which gives mass to the gauge bosons and the fermions, all fermions except the neutrino. Thus the neutrino is massless in the standard model, at the tree level. There are several questions that arise at this stage. What happens when one goes beyond the above simple tree level approximation ? Secondly, do nonperturbative effects change this tree level result ? Finally, how to judge how this result will be modified when the quantum gravity effects are included ?

The first and second questions are easily answered by using the B-L symmetry of the standard model. The point is that since the standard model has no $SU(2)_L$ singlet neutrino-like field, the only possible mass terms that are allowed by Lorentz invariance are of the form $\nu^T_{iL} C^{-1} \nu_{jL}$, where i, j stand for the generation index and C is the Lorentz charge conjugation matrix. Since the ν_{iL} is part of the $SU(2)_L$ doublet field and has lepton number +1, the above neutrino mass term transforms as an $SU(2)_L$ triplet and furthermore, it violates total lepton number (defined as $L \equiv L_e + L_\mu + L_\tau$) by two units. However, a quick look at the standard model Lagrangian convinces one that the model has exact lepton number symmetry after symmetry breaking; therefore such terms can never arise in perturbation theory. Thus to all orders in perturbation theory, the neutrinos are massless. As far as the nonperturbative effects go, the only known source is the weak instanton effects. Such effects could effect the result if they broke the lepton number symmetry. One way to see if such breaking weak instanton effects. Such effects could effect the result if they broke the lepton number symmetry. One way to see if such breaking occurs is to look for anomalies in lepton number current conservation from triangle diagrams. Indeed $\partial_\mu j^\mu_\ell = cW\tilde{W} + c' B\tilde{B}$ due to the contribution of the leptons to the triangle involving the lepton number current and W's or B's. Luckily, it turns out that the anomaly contribution to the baryon number current nonconservation has also an identical form, so that the $B-L$ current j^μ_{B-L} is conserved to all orders in the gauge couplings. As a consequence, nonperturbative effects from the gauge sector cannot induce $B-L$ violation. Since the neutrino mass operator described above violates also $B-L$, this proves that neutrino masses remain zero even in the presence of nonperturbative effects.

Let us now turn to the effect of gravity. Clearly as long as we treat gravity in perturbation theory, the above symmetry arguments hold since all gravity coupling respect $B-L$ symmetry. However, once nonperturbative gravitational effects e.g black holes and worm holes are included, there is no guarantee that global symmetries will be respected in the low energy theory. The intuitive way to appreciate the argument is to note that throwing baryons into a black hole does not lead to any detectable consequence except thru a net change in the baryon number of the universe. Since one can throw in an arbitrary numnber of baryons into the black hole, an arbitrary information loss about the net number of missing baryons would prevent us from defining a baryon number of the visible universe- thus baryon number in the presence of a black hole can not be an exact symmetry. Similar arguments can be made for any global charge such as lepton number in the standard model. A field theoretic parameterization of this statement is that the effective low energy Lagrangian for the standard model in the presence of black holes and worm holes etc must contain baryon and lepton number violating terms. In the context of the standard model, the only such terms that one can construct are nonrenormalizable terms of the form $LHLH/M_{P\ell}$. After gauge symmetry breaking, they lead to neutrino masses; however these masses are at most of order $v^2_{wk}/M_{P\ell} \simeq 10^{-5}$ eV. But as we discussed in the previous section, in order to solve the atmospheric neutrino problem, one needs masses at least three orders of magnitude higher.

Thus one must seek physics beyond the standard model to explain observed evidences for neutrino masses. While there are many possibilities that lead to small neutrino masses of both Majorana as well as Dirac kind, here we focus on the possibility that there is a heavy right handed neutrino (or neutrinos) that lead to a small neutrino mass. The resulting mechanism is known as the seesaw mechanism [12] and leads to neutrino being a

Majorana particle.

3.2. Seesaw mechanism

The basic idea of seesaw mechanism is to have a minimal extension of the standard model that add one heavy right handed neutrino per family. In this case ν_L and ν_R can form a mass term; but apriori, this mass term is like the mass terms for charged leptons or quark masses and will therefore involve the weak scale. If we call the corresponding Yukawa coupling to be Y_ν, then the neutrino mass is $m_D = Y_\nu v/\sqrt{2}$. For a neutrino mass in the eV range requires that $Y_\nu \simeq 10^{-11}$ or less. Introduction of such small coupling constants into a theory is generally considered unnatural and a sound theory must find a symmetry reason for such smallness. As already already alluded to before, seesaw mechanism[12], where we introduce a singlet Majorana mass term for the right handed neutrino is one way to achieve this goal. What we have in this case is a (ν_L, ν_R) mass matrix which has the form:

$$M = \begin{pmatrix} 0 & M^D_\nu \\ M^{T,D}_\nu & M_R \end{pmatrix} \tag{10}$$

The light neutrino mass matrix obtained by integrating out heavy right-handed neutrinos is given by

$$M_\nu = -M^D_\nu M^{-1}_R (M^D_\nu)^T, \tag{11}$$

where M^D_ν is the Dirac neutrino mass matrix and M_R is the right-handed Majorana mass matrix. Since M_R is not constrained by the standard model symmetries, it is natural to choose it to be at a scale much higher than the weak scale, leading to a small mass for the neutrino. This provides a natural way to understand a small neutrino mass without any unnatural adjustment of parameters of a theory. A question that now arises is: what is the meaning of the new scale M_R ?

4. Physics of the Seesaw Mechanism

Inclusion of the right handed neutrino to the standard model open up a whole new way of looking at physics beyond the standard model and transforms the dynamics of the standard model in a profound way. To clarify what we mean, note that in the standard model (that does not contain a ν_R) the $B-L$ symmetry is only linearly anomaly free i.e. $Tr[(B-L)Q^2_a] = 0$ where Q_a are the gauge generators of the standard model but $Tr(B-L)^3 \neq 0$. This means that $B-L$ is only a global symmetry and cannot be gauged. However as soon as the ν_R is added to the standard model, one gets $Tr[(B-L)^3] = 0$ implying that the B-L symmetry is now gaugeable and one could choose the gauge group of nature to be either $SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L}$ or $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, the latter being the gauge group of the left-right symmetric models[13]. Furthermore the presence of the ν_R makes the model quark lepton symmetric and leads to a Gell-Mann-Nishijima like formula for the elctric charges[14] i.e.

$$Q = I_{3L} + I_{3R} + \frac{B-L}{2} \tag{12}$$

The advantage of this formula over the charge formula in the standard model charge formula is that in this case all entries have a physical meaning. Furthermore, it leads naturally to Majorana nature of neutrinos as can be seen by looking at the distance scale where the $SU(2)_L \times U(1)_Y$ symmetry is valid but the left-right gauge group is broken. In that case, one gets

$$\begin{aligned} \Delta Q &= 0 = \Delta I_{3L} : \\ \Delta I_{3R} &= -\Delta \frac{B-L}{2} \end{aligned} \tag{13}$$

We see that if the Higgs fields that break the left-right gauge group carry righthanded isospin of one, one must have $|\Delta L| = 2$ which means that the neutrino mass must be Majorana type and the theory will break lepton number by two units. As we see below this Majorana mass arises via the seesaw mechanism as was first shwon in the last reference in [12]. It also further connects the nonzero neutrino mass to the maximal V-A character of the weak interaction forces. To show this, we discuss the left-right models and show how neutrino small neutrino mass arises in this model via the seesaw mechanism and how it is connected to the scale of parity violation. This may provide one answer to the raised in C. Quigg's lecture at this institute regarding why weak interactions are maximally parity violating unlike any other force in Nature.

4.1. Left-right symmetry, neutrino mass and origin of V-A weak interactions

The left-right symmetric theory is basrd on the gauge group $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ with quarks and leptons transforming as doublets under $\mathrm{SU}(2)_{L,R}$. In Table III, we denote the quark, lepton and Higgs fields in the theory along with their transformation properties under the gauge group.

Table III Assignment of the fermion and Higgs fields to the representation of the left-right symmetry group.

Fields	$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ representation
Q_L	$(2,1,+\frac{1}{3})$
Q_R	$(1,2,\frac{1}{3})$
L_L	$(2,1,-1)$
L_R	$(1,2,-1)$
ϕ	$(2,2,0)$
Δ_L	$(3,1,+\ 2)$
Δ_R	$(1,3,+\ 2)$

The first task is to specify how the left-right symmetry group breaks to the standard model i.e. how one breaks the $SU(2)_R \times U(1)_{B-L}$ symmetry so that the successes of the standard model including the observed predominant V-A structure of weak interactions at low energies is reproduced. Another question of naturalness that also arises simultaneously is that since the charged fermions and the neutrinos are treated completely symmetrically (quark-lepton symmetry) in this model, how does one understand the smallness of the neutrino masses compared to the other fermion masses.

It turns out that both the above problems of the LR model have a common solution. The process of spontaneous breaking of the $SU(2)_R$ symmetry that suppresses the V+A currents at low energies also solves the problem of ultralight neutrino masses. To see this let us write the Higgs fields explicitly:

$$\begin{aligned} \Delta &= \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}; \\ \phi &= \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \end{aligned} \qquad (14)$$

All these Higgs fields have Yukawa couplings to the fermions given symbolically as below.

$$\begin{aligned} \mathcal{L_Y} &= h_1 \bar{L}_L \phi L_R + h_2 \bar{L}_L \tilde{\phi} L_R \\ &\quad + h_1' \bar{Q}_L \phi Q_R + h_2' \bar{Q}_L \tilde{\phi} Q_R \\ &\quad + f(L_L L_L \Delta_L + L_R L_R \Delta_R) + \ h.c. \end{aligned} \qquad (15)$$

The $SU(2)_R \times U(1)_{B-L}$ is broken down to the standard model hypercharge $U(1)_Y$ by choosing $< \Delta_R^0 >= v_R \neq 0$ since this carries both $SU(2)_R$ and $U(1)_{B-L}$ quantum numbers. It gives mass to the charged and neutral righthanded gauge bosons i.e. $M_{W_R} = g v_R$ and $M_{Z'} = \sqrt{2} g v_R cos\theta_W / \sqrt{cos 2\theta_W}$. Thus by adjusting the value of v_R one can suppress the right handed current effects in both neutral and charged current interactions arbitrarily leading to an effective near maximal left-handed form for the charged current weak interactions.

The fact that at the same time the neutrino masses also become small can be seen by looking at the form of the Yukawa couplings. Note that the f-term leads to a mass for the right handed neutrinos only at the scale v_R. Next as we break the standard model symmetry by turning on the vev's for the ϕ fields as $Diag < \phi >= (\kappa, \kappa')$, we not only give masses to the W_L and the Z bosons but also to the quarks and the leptons. In the neutrino sector the above Yukawa couplings after $SU(2)_L$ breaking by $< \phi > \neq 0$ lead to the so called Dirac masses forthe neutrino connecting the left and right handed neutrinos. In the two component neutrino language, this leads to the following mass matrix for the ν, N (where we have denoted the left handed neutrino by ν and the right handed component by N).

$$M = \begin{pmatrix} 0 & h\kappa \\ h\kappa & f v_R \end{pmatrix} \qquad (16)$$

Note that m_D in previous discussions of the seesaw formula (see Eq. ()) is given by $m_D = h\kappa$, which links it to the weak scale and the mass of the RH neutrinos is given by $M_R = f v_R$, which is linked to the local B-L symmetry. This justifies keeping RH neutrino mass at a scale lower than the Planck mass. It is therefore fair to assume that seesaw mechanism coupled with observations of neutrino oscillations are a strong indication of the existence of a local B-L symmetry far below the Planck scale.

4.2. Parity symmetry and type II seesaw

In deriving the above seesaw formula for neutrino masses, it has been assumed that the vev of the left-handed triplet is zero so that the $\nu_L \nu_L$ entry of the neutrino mass matrix is zero. However, in the left-right model which provide an explicit derivation of this formula, there is an induced ve for the Δ_L^0 of order $< \Delta_L^0 >= v_T \simeq \frac{v_{wk}^2}{v_R}$. In the left-right models, this this arises from the presence of a coupling in the Higgs potential of the form $\Delta_L \phi \Delta_R^\dagger \phi^\dagger$. In the presence of the

Δ_L vev, the seesaw formula undergoes a fundamental change. One can have two types of seesaw formulae depending on whether the Δ_L has vev or not. The new seesaw formula now becomes:

$$M_\nu^{\rm II} = M_L - M_\nu^D M_R^{-1} (M_\nu^D)^T, \tag{17}$$

where $M_L = f v_L$ and $M_R = f v_R$, where $v_{L,R}$ are the vacuum expectation values of Higgs fields that couple to the right and lefthanded neutrinos. This formula for the neutrino mass matrix is called type II seesaw formula [15]. In Fig. 2, we give the diagrams that in a parity symmetric theory lead to the type II seesaw formula.

One may perhaps get some hint as to which type of seesaw formula is valid in Nature once the neutrino spectrum is determined. In the type I seesaw formula, what appears is the square of the Dirac neutrino mass matrix which in general expected to have the same hierarchical structure as the corresponding charged fermion mass matrix. In fact in some specific GUT models such as SO(10), $M_D = M_u$ which validates this conjecture leading to the common statement that neutrino masses given by the seesaw formula are hierarchical i.e. $m_{\nu_e} \ll m_{\nu_\mu} \ll m_{\nu_\tau}$ and even a more model dependent statement that $m_{\nu_e} : m_{\nu_\mu} : m_{\nu_\tau} = m_u^2 : m_c^2 : m_t^2$.

On the other hand in the type II seesaw formula, there is no reason to expect a hierarchy and in fact if the neutrino masses turn out to be degenerate (as discussed before as one possibility), one possible way to understand this may be to use the type II seesaw formula. Since the type II seesaw formula is a reflection of the parity invariance of the theory at high energies, evidence for it would point very strongly towards left-right symmetry at high energies. It also must be stated that a hierarchical mass spectrum could result in either type of seesaw formula. For an example of type II seesaw formula with hierarchical spectrum, see the SO(10) model below.

The generic seesaw models lead to a number interesting phenomenological and cosmological consequence that we will not discuss in this talk:

- Seesaw mechanism embedded into a supersymmetric framework with supersymmetry broken at the weak scale leads to nonvanishing lepton flavor violation. The detailed predictions for flavor violation depends on specific assumptions. But still one would generally expect in these models that the branching ratio for $\mu \to e + \gamma$ in these models is expected to be above 10^{-14}, which is the current goal of the PSI MEG experiment.[16].

- The decay of right handed neutrinos in conjunction with CP violation in the right handed neutrino sector has been a very viable mechanism for origin of matter via lkeptogenesis[17].

- The above hig scale CP violation could lead to measurable electric dipole moments for leptons[18].

5. Understanding Large Mixings

While the seesaw formula provides an elegant way to understand the small neutrino masses, it throws no light on the nature of the neutrino mixings. The reason essentially is that for three active neutrinos, the seesaw formula involves 18 unknown parameters whereas thye number of observables for neutrinos is nine including all three phases. One must therefore make specific assumptions or models in order to understand mixings[19].

The neutrino mixing angles get contributions from the mass matrices for the charged leptons as well as neutrinos. Since we can choose an arbitrary basis for either the charged leptons or the neutrinos without effecting weak interactions, it is often convenient to work in a basis where charged lepton mass matrix is diagonal. A fundamental theory can of course determine the structure of both the charged lepton and the neutrino mass matrices and therefore will lead to predictions about lepton mixings. However, in the absence of such a theory, if one wants to adopt a model independent approach and look for symmetries that may explain say the maximal value of θ_{23} or large θ_{12} etc., it is useful to work in a basis where charged leptons are mass eigenstates and hope that any symmetries for leptons revealed in this basis are true or approximate symmetries of Nature.

It could of course be that the large mixings are the result of some dynamical mechanism e.g. radiative corrections or grand unification and not a symmetry. In such a case, there is no need to start with a particular basis. However, we must then find some characxteristic experimental signatures that could point towards such a theory.

In any case, it is necessary to look for signatures of the two approaches to mixing angles i.e. whether it is the symmetry that is responsible for large mixings or dynamics. Below, we describe, one give several examples where either a symmetry or some dynamical reason leads to large mixings.

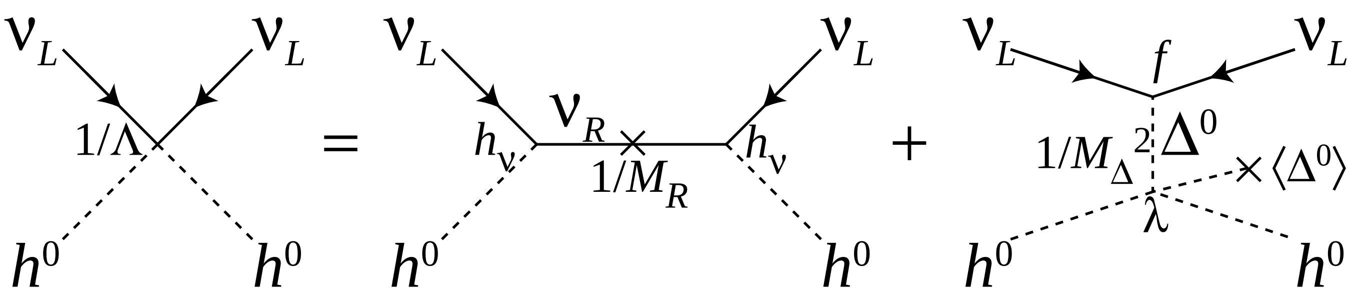

Figure 2: Seesaw mechanism: the first diagram involves the exchange of the heavy right handed neutrino and by itself leads to type I seesaw whereas the second figure gives the extra contribution to seesaw formula in parity symmetric theories and leads to type II seesaw. Of the two Δ fields in this figure, the one that has a vev is the Δ_R field of Table III and the one that connects the two vertices is the Δ_L of Table III.

5.1. $\mu - \tau$ symmetry and large atmospheric mixings

In the basis where charged leptons are mass eigenstates, a symmetry that has proved useful in understanding maximal atmospheric neutrino mixing is $\mu \leftrightarrow \tau$ interchange symmetry[20]. The mass difference between the muon and the tau lepton of course breaks this symmetry. So we expect this symmetry to be an approximate one. It may however happen that the symmetry is truly exact at a very high scale; but at low mass scales, the effective theory only has the $\mu - \tau$ symmetry in the neutrino couplings but not in the charged lepton sector so that we have $m_\tau \gg m_\mu$[21].

To see how the symmetry of the mass matrix affects the mixing matrix, let us consider the case of only two neutrino generations i.e. that of μ and τ. Experiments indicate that the atmospheric mixing angle is very nearly maximal i.e. $\theta_A = \pi/4$. Working in the basis where the charged lepton mass matrix is diagonal, it is obvious that the nautrino Majorana mass matrix that gives maximal mixing is:

$$\mathcal{M}^{(2)}_\nu = \begin{pmatrix} a & b \\ b & a \end{pmatrix}. \qquad (18)$$

This mass matrix has $\mu - \tau$ interchange symmetry. Smallness of solar neutrino mass difference implies that we can write $b = -1$ and $a = 1 + \epsilon$. Clearly, if such a symmetry is responsible for maximal atmospheric mixing angle, it will be against the spirit of quark lepton unification that is a fundamental part of the idea of grand unification. Since there also grand unified models that can lead to near maximal mixing, an important question is: how to distringuish a lepton specific symmetry approach from a general quark-lepton unified GUT approach.

To answer this question let us extend the above symmetry discussion to the case of three neutrinos. We then have

$$\mathcal{M}_\nu = \frac{\sqrt{\Delta m^2_A}}{2} \begin{pmatrix} c\epsilon & d\epsilon & b\epsilon \\ d\epsilon & 1 + a\epsilon & -1 \\ b\epsilon & -1 & 1 + \epsilon \end{pmatrix} \qquad (19)$$

Note that if $a = 1$ and $b = d$, this mass matrix has $\mu - \tau$ symmetry and leads to large solar mixing. It also predicts $\theta_{13} = 0$. However as (i) $a \neq 1$ or (ii) $b \neq d$, we get nonzero θ_{13} and for case (ii) $\theta_{13} \sim \sqrt{\Delta m^2_\odot/\Delta m^2_A}$ and $\theta_{13} \sim \Delta m^2_\odot/\Delta m^2_A$ in case (i)[22].

In comparision, in a dynamical approach such as those based on grand unified theories, we would have to have a mass matrix of type in Eq. (19) but since there is no symmetry, we would expect both $a \neq 1$ and $b \neq d$. So that we would expect $\theta_{13} \geq \sqrt{\Delta m^2_\odot/\Delta m^2_A}$. Since the next generation of neutrino experiments are expected to push the limit on θ_{13} down to the level of 0.04 or so[23], it should provide a hint as to whether the GUT approach or the symmetry approach is more promising.

5.2. Inverted hierarchy and $L_e - L_\mu - L_\tau$ symmetry and large solar mixing

Another very natural way to understand large mixings is to assume the symmetry $L_e - L_\mu - L_\tau$ for neutrinos. This symmetry as we see below, leads to an inverted mass hierarchy for neutrinos, which is therefore a clear experimental prediction of this approach. Consider the mass matrix

$$M_\nu = m_0 \begin{pmatrix} \epsilon & c & s \\ c & \epsilon & \epsilon \\ s & \epsilon & \epsilon \end{pmatrix}. \quad (20)$$

where $c = cos\theta_A$ and $s = sin\theta_A$. This mass matrix leads to mixing angles that are completely consistent with all data. The mass pattern in this case is inverted i. e. the two mass eigenstates responsible for solar neutrino oscillation are nearly degenerate in mass and the third neutrino mass is much smaller and could be zero. In the limit of $\epsilon \to 0$, this mass matrix has $L_e - L_\mu - L_\tau$ symmetry. One therefore might hope that if inverted hierarchy structure is confirmed, it may provide evidence for this leptonic symmetry which can be an important clue to new physics beyond the standard model. In fact large departure of the solar mixing angle from its maximal value means that $L_e - L_\mu - L_\tau$ symmetry must be badly broken[24].

As in the above example, in the mass matrix in Eq. (), when we set $cos\theta_A = sin\theta_A = \frac{1}{sqrt2}$, the theory becomes $\mu - \tau$ symmetric and we get $\theta_{13} = 0$. Therefore there is a correlation between θ_A and θ_{13} in the case for this case.

5.3. Quark-lepton complementarity and large solar mixing

There has been a recent suggestion[25] that perhaps the large but not maximal solar mixing angle is related to physics of the quark sector. According to this, the deviation from maximality of the solar mixing may be related to the quark mixing angle $\theta_C \equiv \theta^q_{12}$ and is based on the observation that the mixing angle responsible for solar neutrino oscillations, $\theta_\odot \equiv \theta^\nu_{12}$ satisfies an interesting complementarity relation with the corresponding angle in the quark sector $\theta_{Cabibbo} \equiv \theta^q_{12}$ i.e. $\theta^\nu_{12} + \theta^q_{12} \simeq \pi/4$. While it is quite possible that this relation is purely accidental or due to some other dynamical effects, it is interesting to pursue the possibility that there is a deep meaning behind it and see where it leads. It has been shown in a recent paper that if Nature is quark lepton unified at high scale, then a relation between θ^ν_{12} and θ^q_{12} can be obtained in a natural manner provided the neutrinos obey the inverse hierarchy[26]. It predicts $sin^2\theta_\odot \simeq 0.34$ which agrees with present data at the 2σ level. It also predicts a large $\theta_{13} \sim 0.18$, both of which are predictions that can be tested experimentally in the near future.

5.4. Large mixing for Degenerate neutrinos

In this case, there are two ways to proceed: one may add the unit matrix to either of the above mass matrices to understand large mixings or look for some dynamical ways by which large mixings can arise. It turns that in this case, one can generate large mixings out of small mixings[27, 28] purely as a consequence of radiative corrections. We will call this possibility radiative magnification.

Let us illustrate the basic mechanism for the case of two generations. The mass matrix in the $\nu_\mu - \nu_\tau$ sector[28] cab written in the flavor basis as:

$$M_F(M_R) = U(\theta) \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} U(\theta)^\dagger \quad (21)$$

where $U(\theta) = \begin{pmatrix} C_\theta & S_\theta \\ -S_\theta & C_\theta \end{pmatrix}$. This mass matrix is defined at the seesaw (GUT) scale, where we assume the mixing angles to be small. As we extrapolate this mass matrix down to the weak scale, radiative corrections modify it to the form[27]

$$\mathcal{M}_\mathcal{F}(\mathcal{M}_\mathcal{Z}) = \mathcal{R}\mathcal{M}_\mathcal{F}(\mathcal{M}_\mathcal{R})\mathcal{R} \quad (22)$$

where $\mathcal{R} = \begin{pmatrix} 1+\delta_\mu & 0 \\ 0 & 1+\delta_\tau \end{pmatrix}$. Note that $\delta_\mu \ll \delta_\tau$. So if we ignore δ_μ, we find that the $\tau\tau$ entry of the $\mathcal{M}_\mathcal{F}(\mathcal{M}_\mathcal{Z})$ is changed compared to its value at the seesaw scale. If the seesaw scale mass eigenvalues are sufficiently close to each other, then the two eigenvalues of the neutrino mass matrix at the M_Z scale can be same leading to maximal mixing (much like MSW matter resonance effect) regardless what the values of the mixing angles at the seesaw scale are. Thus at the seesaw scale, lepton mixing angles can even be same as the quark mixing angles as a quark-lepton symmetric theory would require. We call this phenomenon radiative magnification of mixing angles. It requires no assumption other than the near degeneracy of neutrino mass eigenvalues and that all neutrinos have same CP (or all mass terms have same sign). This provides a new dynamical mechanism to understand large mixings.

This radiative magnification mechanism has recently been generalized to the case of three neutrinos[29], where assuming the neutrino mixing angles at the seesaw scale to be same as the quark mixing angles renormalization group extrapolation alone leads to large solar and atmospheric as well as small θ_{13} at the weak scale provided the common mass of the neutrinos $m_0 \geq 0.1$ eV. The values of the mixing angles at the weak scale are in agreement with observations i.e. while both the solar and atmospheric mixing angles become large, the θ_{13} parameter remains small (0.08).

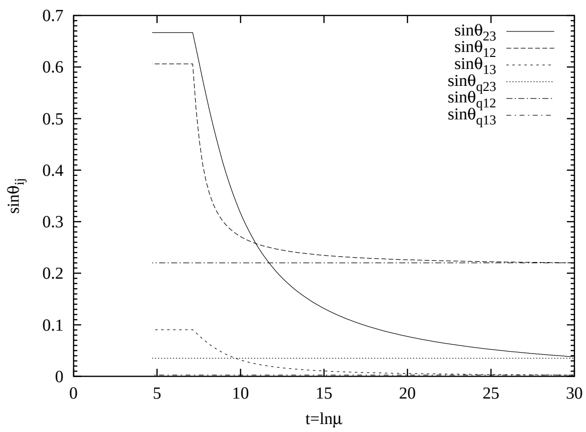

Figure 3: High scale mixing unification and Radiative magnification of mixing angles for degenerate neutrinos. Note that while the lepton mixing angles get magnified, the quark mixings do not due essentially to the hierarchical pattern of masses.

As already noted, an important prediction of this model is that the common mass of the neutrinos must be bigger than 0.1 eV, a prediction that can be tested in the proposed neutrinoless double beta decay experiments.

There are many other proposals to understand large neutrino mixings; see for instance [30] as one class of models and others summarized in [19]. An important physical insight one gains from the various ways (models) of ensuring large θ_A and large $\theta_\odot$ is that each have their characetristic predictions for θ_{13} as well as deviation from solar as well as atmospheric neutrino mixing from maximality. For a sample of these predictions, see [31]. As further high precision neutrino experiments are carried out, they can be used to test the various ideas hopefully leading to new insight into the nature of new physics.

6. Seesaw Mechanism and Grand Unification

A naive estimate of the seesaw scale (or the scale of B-L symmetry) can be obtained by using the $\Delta m^2_A \sim 2.5\times 10^{-3}$ eV^2 and the seesaw formula $m_3 \simeq\simeq \sqrt{\Delta m^2_A} \simeq \frac{m^2_{33,D}}{M_R}$. The value of $m^2_{33,D}$ is of course unknown however in the context of specific models that unifiy quarks and leptons, one expects this to be of order 100 GeV or so. Using this, one can conclude that $M_R \simeq 10^{14} - 10^{15}$ GeV. This value is tantalizingly close to the scale of coupling unification in supersymmetric theories, which is around 10^{16} GeV[32]. A natural possibility is therefore to discuss the seesaw mechnism within the framework of grand unified theories.

As a simple possibility, one may consider the supersymmetric grand unified theories. In this class of models, one assigns matter and Higgs to the representations as follows: matter per generation are assigned to $\bar{5} \equiv \bar{F}$ and $10 \equiv 10$ dimensional representations whereas the Higgs fields are assigned to $\Phi \equiv 45$, $H \equiv 5$ and $\bar{H} \equiv \bar{5}$ representations.

Matter Superfields:

$$\bar{F} = \begin{pmatrix} d^c_1 \\ d^c_2 \\ d^c_3 \\ e^- \\ \nu \end{pmatrix};$$

$$T\{10\} = \begin{pmatrix} 0 & u^c_3 & -u^c_2 & u_1 & d_1 \\ -u^c_3 & 0 & u^c_1 & u_2 & d_2 \\ u^c_2 & -u^c_1 & 0 & u_3 & d_3 \\ -u_1 & -u_2 & u_3 & 0 & e^+ \\ -d_1 & -d_2 & -d_3 & -e^+ & 0 \end{pmatrix} \quad (23)$$

In the following discussion, we will choose the group indices as α, β for SU(5); (e.g.$H^\alpha, \bar{H}_\alpha, \bar{F}_\alpha T^{\alpha\beta} = -T^{\beta\alpha}$); $i, j, k..$ will be used for $SU(3)_c$ indices and p, q for $SU(2)_L$ indices.

To discuss symmetry breaking and other dynamical aspects of the model, we choose the superpotential to be:

$$W = W_Y + W_G + W_h + W\prime \quad (24)$$

where

$$W_Y = h^{ab}_u \epsilon_{\alpha\beta\gamma\delta\sigma} T^\alpha_a \beta T^{\gamma\delta}_b H^\sigma + h^a_d b T^{\alpha\beta} \bar{F}_\alpha \bar{H}_\beta \quad (25)$$

(a, b are generation indices). This part of the superpotential is resposible for giving mass to the fermions. Effective superpotential for matter sector at low energies then looks like:

$$\begin{aligned} W_{matter} &= h_u Q H_u u^c + h_d Q H_d d^c \\ &\quad + h_l L H_d e^c + \mu H_u H_d \end{aligned} \quad (26)$$

Note that h_d and h_l arise from the $T\bar{F}\bar{H}$ coupling and this satisfy the relation $h_d = h_l$. This relation leads to mass equalities at the GUT scale of the form: $m_e = m_d$; $m_\mu = m_s$ and $m_\tau = m_b$. These relations have to be extrapolated to the weak scale to compared with observations. While the extrapolation for the third generation is in very good agreement with data, it is far from observations for the first and the second generations. This is of course a problem for minimal SUSY SU(5) GUT. This model of course has the problem of R-parity breaking by dimension 4 operators, which can lead to very rapid proton decay.

Ignoring the fermion mass and R-parity problems, we can proceed to see how it accomodates light neutrino masses. Again as in the case of standard model, one can add three right handed neutrinos as singlets to the SU(5) theory and use the seesaw mechanism to generate small neutrino masses. The problem however is that we have no reason to choose the mass scvale of the RH neutrinos to be at the GUT scale. In fact a natural choice would be the Planck scale. Thus while as a practical model for neutrino masses, SU(5) GUT theory may be OK, it faces the naturalness problem with respect to the seesaw scale.

7. SO(10) Grand Unification and Neutrino Mixings

We will now consider the SO(10) model, which as wel will see has a number of virtues that make it just the right GUT model for neutrino masses.

First point to note is that the **16** dimensional spinor representation of SO(10) consists of all fifteen standard model fermions plus the right handed neutrino arranged according to the it $SU(2)_L \times SU(2)_R \times SU(4)_c$ subgroup as follows:

$$\mathbf{\Psi} = \begin{pmatrix} u_1 & u_2 & u_3 & \nu \\ d_1 & d_2 & d_3 & e \end{pmatrix} \tag{27}$$

We can take three such spinors for three fermion families. The presence of the Pati-Salam subgroup $SU(4)_c$ allows relations between the neutrino couplings and the quark couplings thereby raising the possibility there will be fewer parameters in the model and more predictivity in the neutrino sector, compared to the simple seesaw formula.

Secondly, SO(10) contains the B-L symmetry as a gauge symmetry. since the mass of the righthanded neutrino breaks B-L symmetry, it has to be constrained from above by the GUT scale, thus eliminating the hierarchy problem that emerged in the SUSY SU(5) case.

In order to implement the seesaw mechanism, one must break the B-L symmetry, since the right handed neutrino mass breaks this symmetry. One implication of this is that the seesaw scale is at or below the GUT scale. Secondly in the context of supersymmetric SO(10) models, the way B-L breaks has profound consequences for low energy physics. There are two ways to break B-L in SUSY SO(10) models: (i) by **16** Higgs or (ii) by **126** Higgs. Below we give a comparision between the two ways and the present recent results that follow from the second way.

7.1. Breaking B-L: 16 vrs 126

If B-L is broken by a Higgs field belonging to the **16** dimensional Higgs field (to be denoted by Ψ_H), then the field that acquires a nonzero vev has the quantum numbers of the ν_R field i.e. B-L breaks by one unit. If we recall the definition of R-parity i.e. $R_p = (-1)^{3(B-L)+2S}$, we see that this vev hav has $R_p = -1$. This implies that the effective MSSM below the GUT scale in such theories will break R-parity. To see how dangerous these operators can be, note that in this case higher dimensional operators of the form $\Psi\Psi\Psi\Psi_H$ are the ones that lead to R-parity violating operators in the effective low energy MSSM theory. They then lead to operators such as $QLd^c, u^c d^c d^c$ etc. Together these two can lead to large breaking of lepton and baryon number symmetry with a strength of $\left(\frac{v_{B-L}}{M_P M_{\tilde{q}}}\right)^2$. They lead to unacceptable rates for proton decay (e.g. $\tau_p \leq$ sec.). This theory also has no dark matter candidate without making additional assumptions.

Secondly, in this class of theories, the right handed neutrino mass is assumed to arise out of operators of the form $\lambda\Psi\Psi\Psi_H\Psi_H/M_P$. To get M_R of order 10^{14} GeV, we would need to assume $\lambda \simeq 1$. However, it is well known that similar dimension 5 operators $\lambda'\Psi\Psi\Psi\Psi/M_P$ can also lead tp proton decay rate in contradiction with observations unless $\lambda' \leq 10^{-6}$. This raises a naturalness question which is why some operators have coefficients of order one whereas others have coefficients of order $1o^{-6}$.

On the other hand, if one break B-L by a **126** dimensional Higgs field, none of these problems arise.To see this note that the member of this **126** multiplet that acquires vev has $B-L=2$ and therefore it leaves R-parity as an automatic symmetry of the low energy Lagrangian. There is a naturally stable dark matter in this case. Secondly, in this case, all fermion masses (including the right handed neutrinos) arise from dimension four operators e.g. $\psi\psi\mathbf{1\bar{2}6}$ gives rise to right handed neutrino masses. Therefore we can safely put all dimension five operators to have couplings less than 10^{-6} without any problem.

A further point is that, any theory with asymptotic parity symmetry leads to type II seesaw formula. It turns out that if the B-L symmetry is broken by **16** Higgs fields, the first term in the type II seesaw (effective triplet vev induced term) becomes very small compared to the type I term. On the other hand, if B-L is broken by a **126** field, then the first term in the type II seesaw formula is not necessarily small and can in principle dominate in the seesaw formula. We will discuss a model of this type below.

7.2. Minimal SO(10) with a single 126 as a predictive model for neutrinos

The basic ingredients of this model are that one considers only two Higgs multiplets that contribute to fermion masses i.e. one **10** and one **126**. A unique property of the **126** multiplet is that it not only breaks the B-L symmetry and therefore contributes to right handed neutrino masses, but it also contributes to charged fermion masses by virtue of the fact that it contains MSSM doublets which mix with those from the **10** dimensional multiplets and survive down to the MSSM scale. This leads to a tremendous reduction of the number of arbitrary parameters, as we will see below.

There are only two Yukawa coupling matrices in this model: (i) h for the **10** Higgs and (ii) f for the **126** Higgs. SO(10) has the property that the Yukawa couplings involving the **10** and **126** Higgs representations are symmetric. Therefore if we assume that CP violation arises from other sectors of the theory (e.g. squark masses) and work in a basis where one of these two sets of Yukawa coupling matrices is diagonal, then it will have only nine parameters. Noting the fact that the (2,2,15) submultiplet of **126** has a pair of standard model doublets that contributes to charged fermion masses. In SO(10) models of this type, the **126** multiplet contains two parity partner Higgs submultiplets (called $\Delta_{L,R}$) which couple to $\nu_L\nu_L$ and N_RN_R respectively and after spontaneous symmetry breaking lead to the type II seesaw formula for neutrinos, which plays an important role in magnifying the neutrino mixings despite quark-lepton unification[34, 35].

As we will see a further advantage of using **126** multiplet is that it unifies the charged fermion Yukawa couplings with the couplings that contribute to righthanded as well as lefthanded neutrino masses, as long as we do not include nonrenormalizable couplings in the superpotential. This can be seen as follows[33]: it is the set $\mathbf{10}+\overline{\mathbf{126}}$ out of which the MSSM Higgs doublets emerge; the later also contains the multiplets $(3,1,10)+(1,3,\overline{10})$ which are responsible for not only lefthanded but also the right handed neutrino masses in the type II seesaw formula. Therefore all fermion masses in the model are arising from only two sets of 3×3 Yukawa matrices one denoting the **10** coupling and the other denoting $\overline{\mathbf{126}}$ couplings. The SO(10) invariant superpotential giving the Yukawa couplings of the **16** dimensional matter spinor ψ_i (where i, j denote generations) with the Higgs fields $H_{10}\equiv\mathbf{10}$ and $\Delta\equiv\overline{\mathbf{126}}$.

$$W_Y \;=\; h_{ij}\psi_i\psi_j H_{10} + f_{ij}\psi_i\psi_j\Delta \tag{28}$$

In terms of the GUT scale Yukawa couplings, one can write the fermion mass matrices (defined as $L_m \;=\; \bar{\psi}_L M\psi_R$) at the seesaw scale as:

$$\begin{aligned} M_u &= h\kappa_u + fv_u \\ M_d &= h\kappa_d + fv_d \\ M_\ell &= h\kappa_d - 3fv_d \\ M_{\nu_D} &= h\kappa_u - 3fv_u \end{aligned} \tag{29}$$

where $\kappa_{u,d}$ are the vev's of the up and down standard model type Higgs fields in the **10** multiplet and $v_{u,d}$ are the corresponding vevs for the same doublets in **126**. Note that there are 13 parameters in the above equations and there are 13 inputs (six quark masses, three lepton masses and three quark mixing angles and weak scale). Thus all parameters of the model that go into fermion masses are determined.

These mass sumrules provide the first important ingredient in discussing the neutrino sector. To see this let us note that they lead to the following sumrule involving the charged lepton, up and down quark masses:

$$k\tilde{M}_l = r\tilde{M}_d + \tilde{M}_u \tag{30}$$

where k and r are functions of the symmetry breaking parameters of the model. It is clear from the above equation that smallquark mixings imply that the contribution the charged leptons to the neutrino mixing matrix i.e. U_ℓ

in the formula $U_{PMNS} = U_\ell^\dagger U_\nu$ is close to identity and the entire contribution therefore comes from U_ν. Below we show that $U_n u$ has the desired form with θ_{12} and θ_{23} large and θ_{13} small.

7.3. Maximal neutrino mixings from type II seesaw

In order to see how the type II seesaw formula provides a simple way to understand large neutrino mixings in this model, note that in certain domains of the parameter space of the model, the second matrix in the type II seesaw formula can much smaller than the first term. This can happen for instance when V_{B-L} scale is much higher than 10^{16} GeV. When this happens, one can derive the sumrule

$$M_\nu \;=\; a(M_\ell - M_d) \tag{31}$$

This equation is key to our discussion of the neutrino masses and mixings.

Using Eq. (31) in second and third generation sector, one can understand how large mixing angle emerges.

Let us first consider the two generation case [34]. The known hierarchical structure of quark and lepton masses as well as the known small mixings for quarks suggest that the matrices $M_{\ell,d}$ for the second and third generation have

$$M_\ell \;\approx\; m_\tau \begin{pmatrix} \lambda^2 & \lambda^2 \\ \lambda^2 & 1 \end{pmatrix} \tag{32}$$
$$M_q \;\approx\; m_b \begin{pmatrix} \lambda^2 & \lambda^2 \\ \lambda^2 & 1 \end{pmatrix}$$

where $\lambda \sim 0.22$ (the Cabibbo angle). It is well known that in supersymmetric theories, when low energy quark and lepton masses are extrapolated to the GUT scale, one gets approximately that $m_b \simeq m_\tau$. One then sees from the above sumrule for neutrino masses Eq. (31) that there is a cancellation in the (33) entry of the neutrinomass matrix and all entries are of same order λ^2 leading very naturally to the atmospheric mixing angle to be large. Thus one has a natural understanding of the large atmospheric neutrino mixing angle. No extra symmetries are assumed for this purpose.

For this model to be a viable one for three generations, one must show that the same $b-\tau$ mass convergence at GUT scale also explains the large solar angle θ_{12} and a small θ_{13}. This has been demonstrated in a recent paper[35].

To see how this comes about, note that in the basis where the down quark mass matrix is diagonal, all the quark mixing effects are then in the up quark mass matrix i.e. $M_u = U^T_{CKM} M^d_u U_{CKM}$. Using the Wolfenstein parametrization for quark mixings, we can conclude that that we have

$$M_d \;\approx\; m_b \begin{pmatrix} \lambda^4 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \tag{33}$$

and M_ℓ and M_d have roughly similar pattern due to the sum rule . In the above equation, the matrix elements are supposed to give only the approximate order of magnitude. As we extrapolate the quark masses to the GUT scale, due to the fact just noted i.e. $m_b - m_\tau \approx m_\tau \lambda^2$, the neutrino mass matrix $M_\nu = c(M_d - M_\ell)$ takes roughly the form:

$$M_\nu \;=\; c(M_d - M_\ell) \approx\; m_0 \begin{pmatrix} \lambda^4 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{pmatrix} \tag{34}$$

It is then easy to see from this mass matrix that both the θ_{12} (solar angle) and θ_{23} (the atmospheric angle) are large. It also turns out that the ratio of masses $m_2/m_3 \approx \lambda$ which explains the milder hierarchy among neutrinos compared to that among quarks. Furthermore, $\theta_{13} \sim \lambda$. A detailed numerical analysis for this modelhas been carried out in [35] and it substantiates the above analytical reasoning and makes detailed predictions for the mixing angles[35]. We find that the predictions for $sin^2 2\theta_\odot \simeq 0.9 - 0.94$, $sin^2 2\theta_A \leq 0.92$, $\theta_{13} \sim 0.16$ and $\Delta m^2_\odot / \Delta m^2_A \simeq 0.025 - 0.05$ are all in agreement with data. Furthermore the prediction for θ_{13} is in a range that can be tested partly in the MINOS experiment but more completely in the proposed long baseline experiments.

This model has been the subject of many further investigation including such questions as to how to include CP violation, its predictions for proton decay etc.[36].

We have not discussed the SO(10) models with **16** Higgs[37] or multiple **126** models[38].

8. LSND and Sterile Neutrino

The first need for sterile neutrinos came from attempts to explain[39] apparent observations in the Los Alamos

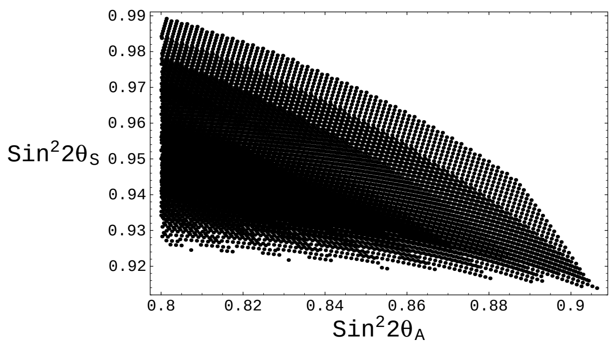

Figure 4: $sin^2 2\theta_{12}$ vrs $sin^2 2\theta_{23}$; scatter corresponds to different allowed quark mass values.

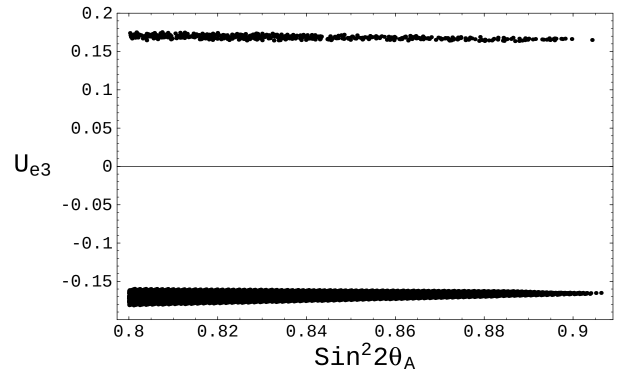

Figure 6: $U_{e3} \equiv \theta_{13}$ and just below the present upper limit: "high" value due to no $\mu \leftrightarrow \tau$ symmetry (see before).

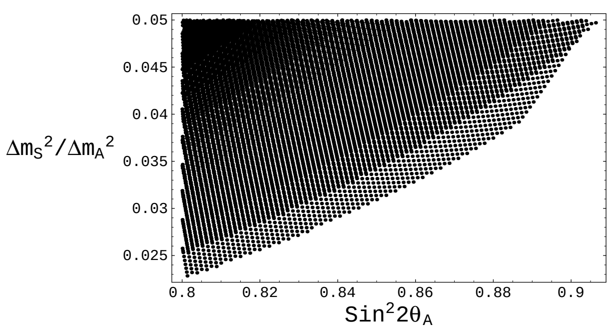

Figure 5: scatter corresponds to uncertainty in quark mass values.

Liquid Scintillation Detector (LSND) experiment[40] , of oscillations of $\bar{\nu}_\mu$'s from a stopped muon (DAR) as well as of the ν_μ's accompanying the muon in pion decay (known as the decay in flight or DIF neutrinos) have apparently been observed. The evidence from the DAR is statistically more significant and is an oscillation from $\bar{\nu}_\mu$ to $\bar{\nu}_e$. The mass and mixing parameter range that fits data is:

$$\Delta m^2 \simeq 0.2 - 2eV^2; sin^2 2\theta \simeq 0.003 - 0.03 \qquad (35)$$

There are points at higher masses specifically at 6 eV^2 which are also allowed by the present LSND data for small mixings. KARMEN experiment at the Rutherford laboratory has very strongly constrained the allowed parameter range of the LSND data[41]. Currently the Miniboone experiment at Fermilab is under way to probe the LSND parameter region[42] using ν_μ beam.

Since the Δm^2_{LSND} is so different from that $\Delta m^2_{\odot,A}$, the simplest way to explain these results is to add one[39] or two[43] sterile neutrinos. For the case of one extra sterile neutrino, there are two scenarios: (i) 2+2 and (ii) 3+1. In the first case, solar neutrino oscillation is supposed to be from ν_e to ν_s. This is ruled out by SNO neutral current data. In the second case, one needs a two step process where ν_μ undergoes indirect oscillation to ν_e due to a combined effect of $\nu_\mu - \nu_s$ and $\nu_e - \nu_s$ mixings (denoted by $U_{\mu,s}$ and U_{es} respectively, rather than direct $\nu_\mu - \nu_e$ mixing. As a result, the effective mixing angle in LSND for the 3+1 case is given by $4U^2_{es}U^2_{\mu s}$ and the measured mass difference is given by that between $\nu_{\mu,e} - \nu_s$ rather than $\nu_\mu - \nu_e$. This scenario is constrained by the fact that sterile neutrino mixings are constrained by two sets of observations: one from the accelerator searches for nu_μ and ν_e disappearance and the second from big bang nucleosynthesis.

The bounds on U_{es} and $U_{\mu s}$ from accelerator experiments such as Bugey, CCFR and CDHS are of course dependent on particular value of $\Delta m^2_{\alpha s}$ but for a rough order of magnitude, we have $U^2_{es} \leq 0.04$ for $\Delta m^2 \geq 0.1$ eV^2 and $U^2_{\mu s} \leq 0.2$ for $\Delta m^2 \geq 0.4$ eV^2[44].

It is worth pointing out that SNO neutral current data has ruled out pure $\nu_e - \nu_s$ transition as an explanation of solar neutrino puzzle by 8σ's; however, it still allows as much as 40% admixture of sterile neutrinos and as we will see below, the sterile neutrinos could very well be present at a subdominant level. Thus the 2+2 scenario seems to be highly disfavored by observations, whereas the 3+1 scenario is barely acceptable.

8.1. Theoretical Implications of a confirmation of LSND

If LSND results are confirmed by the Mini Boone experiment, it will require substantial revision of our thinking about neutrinos. For one thing one should expect deviation from the unitarity constraint on the three active neutrino mixings. But a much more fundamental alteration in our thinking about neutrinos may be called for.

One such interpretation is in terms of breakdown of CPT invariance resulting in different spectra for neutrinos compared to anti-neutrinos[46]. This hypothesis is now pretty much in conflict with observations after the KamLand experiment[47].

Another possibility is that there may be one or more sterile neutrinos in Nature. The immediate challenge for theory then is why a sterile neutrino which is a standard model singlet (since it does not couple to the W and Z bosons) has a mass which is so light. A priori one would expect it to be of order of the Planck scale.

A model that very cleverly solves this problem is the mirror universe model where it is postulated that coexisting with the standard model particles and forces is an exact duplicate of it, the mirror sector to our universe[48]. The forces and matter in the mirror are different but mirror duplicates of what we are familiar with. We will not see the mirror particles or forces because they do not couple to our forces or matter. Gravity of course couples to both sectors.

In this models there will be analogs of $\nu_{e,\mu,\tau}$ in the mirror sector ($\nu'_{e,\mu,\tau}$). They will play the role of the sterile neutrinos. It is then clear that whatever mechanism keeps our neutrinos light will keep the mirror neutrinos light too, thereby solving the most vexing problem with sterile neutrinos. In Table IV, we present the particle assignment for the mirror model.

Table IV

visible sector $SU(3)_c \times SU(2)_L \times U(1)_Y$	mirror sector $SU(3)'_c \times SU(2)'_L \times U(1)'_Y$
W, Z, γ, gluons	W', Z', γ', gluons'
$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} u'_L \\ d'_L \end{pmatrix}$
u_R, d_R	u'_R, d'_R
$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu'_L \\ e'_L \end{pmatrix}$
e_R, N_R	e'_R, N'_R

The next question is how to understand why the sterile neutrinos required to understand the LSND experiment so much heavier ($\sim$ eV) than the acive neutrinos. This is explained in the mirror model by postulating that the weak scale in the mirror sector is about 10 to 20 times heavier than the familiar weak scale. We of course do not know the reason for this. But it is possible that it is tied to another feature of mirror models required for their viability i.e. asymmetric inflation which says that the reheat temperature of the mirror sector after inflation is smaller than the visible sector so that the number density of relativistic particles at the epoch of inflation is very small and does not affect the success of BBN. This is the so called asymmetric mirror model[49].

This model has all the ingredients needed to understand the LSND results.

9. Conclusion

Neutrino physics right now is at a crossroad. Enough important discoveries have been made so that the knowledge of the masses and mixings are playing a significant role in influencing the direction of new physics beyond the standard model; on the other hand, to raise our knowledge about neutrinos to the same level as quarks as well as to decide more precisely the direction of new physics, we need more precise information about masses and mixings than we currently have.

For instance, on the theoretical side, the seesaw mechanism for understanding the scale of neutrino masses is regarded as the prime candidate not only due to its simplicity but also its theoretical appeal. It is perhaps hinting at the new physics beyond the standard model to be left-right symmetric (due to the introduction of the right handed neutrinos) and possibly also quark-lepton $SU(4)_c$ symmetric[50] as well as grand unification. On the last point of grand inification, there are large variety of models based on the simple SO(10) group. A generic prediction of SO(10) models is the normal hierarchy or quasi-degeneracy for neutrinos. So evidence for inverted hierarchy would be point strongly away from the SO(10) route. Similarly, evidence for Dirac rather than Majorana neutrino (see Table I) would be a strong blow to the simple seesaw mechanism.

On the hand our understanding of mixing angles is far from complete. No clear consensus seems to have emerged about any particular idea. An exhaustive list of scenarios have been suggested to understand the new and unusual pattern of intra-family mixing among leptons with their characteristic predictions for observable parameters such as $\theta_A - \pi/4$, θ_{13} and m_{eff} in $\beta\beta_{0\nu}$ decay. Experiments will play a crucial role in clarifying the picture here. It is therefore important to implement the proposals for measuring these observables in the next decade and one will then not only have a better understanding of the neutrinos but also a more definite direction in the nature of new physics beyond the standard model.

Thus, there remain enough important things about neutrinos that are unknown so that a healthy investment in the field will definitely broaden the frontier of our overall understanding of forces, matter and the Universe. For example, it is very likely going to throw light on such important cosmological issues as the origin of matter and formation of structure in the Universe.

Neutrino physics has been full of surprises and there may yet be some more waiting. For instance, confirmation by Mini Boone of the LSND results will be one such major branch point. While, our general discussion of mixings will receive a small perturbation, the impact on the theoretical side will be very major, raising a completely new set of questions and opening a brand new frontier in particle physics.

Exciting times are ahead in neutrino physics !!

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