

# CKM Physics from Lattice QCD

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I discuss the lattice calculations relevant to recent advances in CKM phenomenology, focusing on those relevant to new experimental results reported at this workshop.

## 1. Introduction

Lattice QCD calculations make possible some of the most important results in CKM phenomenology. For example, with lattice QCD, new constraints on the CKM matrix can be obtained from the two most important new experimental results reported at this conference: the observation of the purely leptonic decay of the  $B$  meson by Belle [1, 2], and the measurement of  $B_s\bar{B}_s$  mixing by CDF [3] and D0 [4]. These quantities are examples of golden quantities for lattice QCD, processes with stable particles and a maximum of one hadron in the lattice box at a time. Other such quantities relevant to CKM phenomenology include other meson leptonic and semileptonic decays and  $M\bar{M}$  mixings. Good prototype calculations exist in the quenched approximation (ignoring light quark loops), and many unquenched. We know of no impediment to making them more and more accurate as methods and computers continue to improve.

There are several different methods for putting quarks on a lattice that have widely differing virtues and defects. Wilson and clover (or improved Wilson) fermions have been the most widely used. They employ a strong breaking of chiral symmetry at the lattice spacing scale for technical reasons (to solve the fermion doubling problem). This has led to difficulties in reducing the light quark masses to close to their physical values. Staggered fermions have a chiral symmetry. This makes possible closer approach to the physical, light quark mass limit, smaller uncertainties from chiral extrapolation and statistics than is possible with other methods. However, they also have residual fermion doubling, which in unquenched calculations is cured by taking a root of the fermion determinant. This procedure has been shown to make sense in every calculational test to it has been put, but more work is on-going to investigate it. Two new related methods, domain wall fermions and overlap fermions, suffer from neither of these defects, but are computationally much more expensive. Practical calculations with these last two methods are in their infancy. There are therefore multiple opinions among lattice theorists about the optimal fermion method for phenomenological calculations at present. Several methods may have to be pushed to much higher accuracy to test that all methods give the same results and resolve the question.

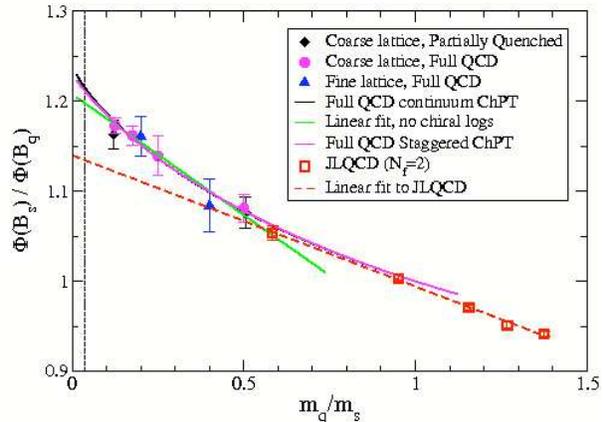


Figure 1: The light quark mass dependence of  $f_{B_s} \sqrt{M_{B_s}} / (f_B \sqrt{M_B})$  with improved staggered fermions (HPQCD,  $n_f = 2 + 1$ , black symbols) and clover fermions (JOQCD,  $n_f = 2$ , red symbols). Black and red lines are extrapolations using the two sets of data. The vertical dashed line is the physical light quark mass. [5]

In this talk, I will discuss mainly lattice results of direct interest to this workshop. For recent more complete reviews, see [5–7].

## 2. Meson Decay Constants

The practical trade-offs between Wilson and staggered fermions are illustrated in Fig. 1 [5]. It shows  $f_{B_s} \sqrt{M_{B_s}} / (f_B \sqrt{M_B})$  as a function of light quark mass. The vertical dashed line shows the physical quark mass. The Wilson data [8] are shown in red, and the staggered data [9] in blue. The two methods agree where their data overlap, at  $m_l \geq m_s/2$ , but curvature at light quark masses yields a significantly higher value at the physical quark mass than is obtained from a naive extrapolation of the Wilson data from the region  $m_l \geq m_s/2$ . The staggered data yield the result

$$f_B = 0.216(9)(19)(4)(6) \text{ GeV}. \quad (1)$$

The uncertainties in  $f_B$  and  $f_{B_s}$ , other than those arising from chiral extrapolation and statistics largely, cancel out in the ratio. HPQCD finds, with staggered

fermions [9] in blue.,

$$f_{B_s}/f_B = 1.20(3)(1), \quad (2)$$

where the first error is from chiral extrapolation and statistics and the second is from everything else. This can be compared with the result that JLQCD found earlier, chirally extrapolating from a much larger quark mass:  $f_{B_s}/f_B = 1.13(3)(^{+13}_{-2})$ . [8]

For  $f_{B_s}$ , HPQCD quotes

$$f_{B_s} = 0.260(7)(26)(9) \text{ GeV}. \quad (3)$$

The uncertainties in  $f_{B_s}/f_B$  and  $f_{B_s}$  are relatively independent of each other, since the uncertainties in the former are dominated by statistics and chiral extrapolation, while the uncertainties in the latter are dominated by everything else.  $f_{B_s}/f_B$  is more independent of  $f_{B_s}$  than of  $f_B$ .

$D$  and  $D_s$  meson decay constants are also of great interest because of the improvements to the experimental numbers coming from CLEO-c. The issues for these calculations are very similar to those for the  $B$  mesons. With staggered fermions and 2 + 1 unquenched light quark flavors (that is, degenerate up and down quarks, and a nondegenerate strange), Fermilab/MILC obtain [10]

$$f_D = 201(03)_{\text{stat}}(17)_{\text{sys}} \text{ MeV} \quad (4)$$

$$f_{D_s} = 249(03)_{\text{stat}}(16)_{\text{sys}} \text{ MeV}. \quad (5)$$

With clover fermions and 2 light flavors, CP-PACS obtains [5]

$$f_D = 202(12)_{\text{stat}}(^{+20}_{-25})_{\text{sys}} \text{ MeV} \quad (6)$$

$$f_{D_s} = 238(11)_{\text{stat}}(^{+07}_{-27})_{\text{sys}} \text{ MeV}. \quad (7)$$

This may be compared with the recent result from CLEO-c [11]

$$f_D = 223(17)(3) \text{ MeV}, \quad (8)$$

assuming the canonical value of  $V_{ud}$ . Alternatively, the comparison could be used to obtain an independent determination of  $V_{ud}$ .

### 3. Semileptonic Decays

The shape of the form factor for the decay  $D \rightarrow Kl\nu$  was obtained in 2004 by the Fermilab/MILC collaboration using unquenched staggered fermions [12]. It has been confirmed by several experiments with increasing accuracy, most recently by Belle [13] and BaBar. (See Fig. 2.) CLEO-c is expected to determine this shape with even greater precision, yielding an even more stringent test.

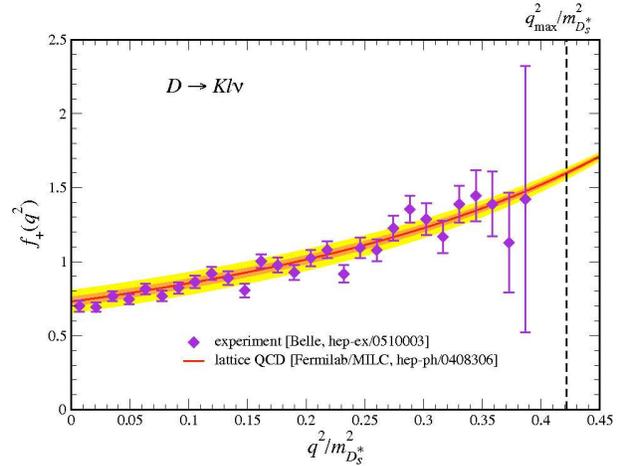


Figure 2: The predicted shape of the form factor for  $D \rightarrow Kl\nu$  has been tested with increasing accuracy by experiment, most recently by Belle [13].

The decay  $B \rightarrow \pi l\nu$  is a measure of  $V_{ub}$ , and provides a measurement of the height of the unitarity triangle that is competitive with  $b \rightarrow u$  inclusive decays. For  $B \rightarrow \pi l\nu$ , the shape of the form factors is a critical issue, even for extracting the CKM matrix element. The reason is that the uncertainties in the form factors are highly  $q^2$  dependent for both theory and experiment. In lattice calculations, predictions do not yet exist in the high recoil region.

It has long been known that analyticity and unitarity can be used to constrain the shape of form factors. Arnesen *et al.* [16] have recently applied these ideas to  $B$  decay. They show that when the variable  $q^2$  is mapped into a well chosen new variable,  $z$ , unitarity constrains the coefficients in the  $z$  expansion of the form factors so that only five or six terms suffice to describe the form factors to 1% accuracy. They propose to use various theoretical methods to determine the coefficients, including lattice QCD.

Becher and Hill have recently pointed out that heavy quark theory leads to the expectation that in the heavy quark limit, the coefficients fall as  $(\Lambda_{\text{QCD}}/M_B)^{1.5}$ , an even tighter constraints on the form factors. [14] They argue that this leads to expectation that only two or three terms are required to describe the data to high accuracy, and show that the BaBar data so far conform to this expectation. (See Fig. 3.)

This expectation is also confirmed in lattice data re-analyzed the same way [15]. Therefore, in this expansion, to the accuracy of the present data, comparing theory and experiment means testing theory against experiment with the slope, comparing the overall normalizations to obtain  $V_{ub}$ , and searching for evidence of the curvature term.

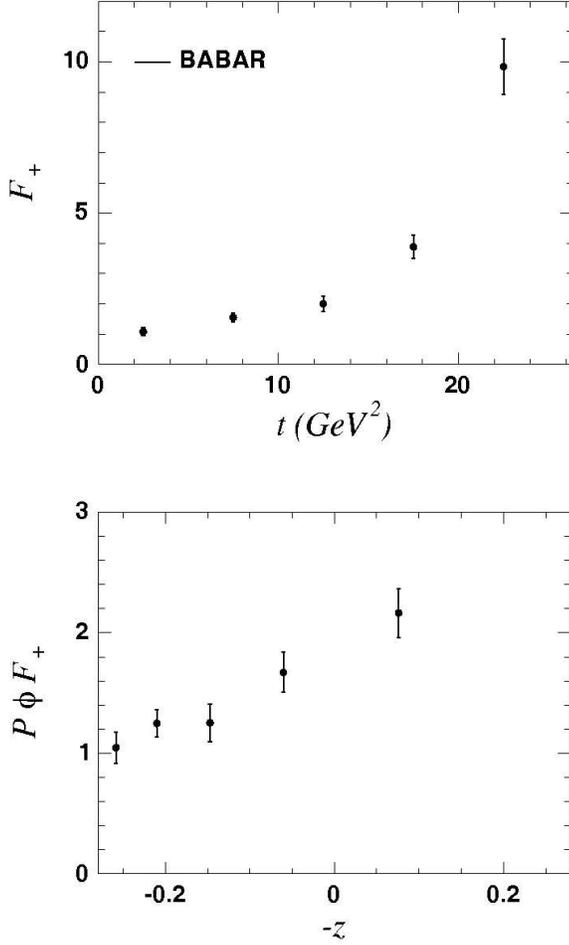


Figure 3: Plotting the form factor shape for  $B \rightarrow \pi l \nu$  decay versus a well chosen new variable  $z$  (instead of  $t \equiv q^2$ ) produces a function that is linear to the accuracy of existing data, and can thus be describe with only two parameters.

## 4. Meson Mixings

Matrix elements for  $B\bar{B}$  and  $B_s\bar{B}_s$  mixing are conventionally parameterized in terms of the meson decay constants and bag parameters, as in the kaon system. What is actually calculated on the lattice, however, are the combinations  $f_B\sqrt{\hat{B}_B}$  and  $f_{B_s}\sqrt{\hat{B}_{B_s}}$ . Since, these combinations are what is needed for phenomenology, these are what lattice theorists should be reporting.

Unquenched results were reviewed at *Lattice 2005* by Okamoto [5]. Since unquenched results using staggered fermions had not yet appeared, he took results for bag parameters from JLQCD using two flavors of clover quarks,

$$B(m_b) = 0.836(27) \begin{pmatrix} +56 \\ -62 \end{pmatrix} \quad (9)$$

and

$$\hat{B}_s/\hat{B} = 1.017(16) \begin{pmatrix} +56 \\ -17 \end{pmatrix}. \quad (10)$$

For his best estimate of the combined matrix elements, he combined these with the HPQCD, staggered fermion results for the decay constants already discussed, and obtained

$$f_B\sqrt{\hat{B}_B} = 244(26) \text{ MeV} \quad (11)$$

and

$$\xi = f_{B_s}/f_B\sqrt{\hat{B}_s/\hat{B}} = 1.210 \begin{pmatrix} +47 \\ -35 \end{pmatrix}. \quad (12)$$

Following this same procedure, one can obtain for the  $B_s$

$$f_{B_s}\sqrt{\hat{B}_{B_s}} = 294(33) \text{ MeV}. \quad (13)$$

This quantity and  $\xi$  are the combination of  $B\bar{B}$  mixing quantities with uncertainties that are most independent of each other.

Lifetime differences require a different operator than the one for the mass difference. Unquenched calculations of matrix elements for this operator do not yet exist. The required operator has been given as part of the complete set of four-quark operators, including those arising in supersymmetric theories, and initial results have been reported in the quenched approximation [17].

## 5. Effect on the $\rho - \eta$ plane

### 5.1. Effects of Lattice Fermion Methods

In this talk, I have emphasized staggered fermion results because they are for the most part fully unquenched, with the right number of light quark flavors, and with light quarks masses much closer to their physical values. Other lattice theorists have a preference for other fermion methods, and the lattice community is not yet settled on the optimum approach. Another widely used set of numbers for  $B\bar{B}$  mixing parameters comes from the CERN CKM study of 2003 [18] (based on the 2002 review of Lellouch [7])

$$f_B\sqrt{\hat{B}_B} = 235(33) \begin{pmatrix} 0 \\ 24 \end{pmatrix} \quad (14)$$

$$f_{B_s}\sqrt{\hat{B}_{B_s}} = 276(38) \quad (15)$$

$$\xi = 1.18(4) \begin{pmatrix} 12 \\ 0 \end{pmatrix}. \quad (16)$$

These were based on a set of mostly clover lattice results, most in the quenched approximation but

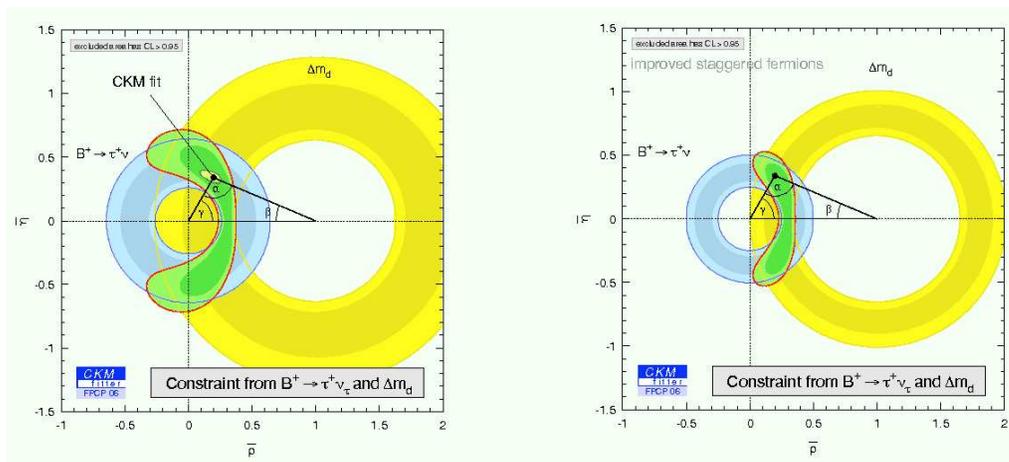


Figure 4: Constraints on the  $\rho - \eta$  plane derived from  $B$  meson leptonic decay and  $B\bar{B}$  mixing. The bounds in the left figure employ clover fermions, those on the right employ staggered.

with some two flavor results used to extrapolate to the three-flavor theory. The results are compatible with the staggered results already discussed, but with larger uncertainties.

The effects of the smaller uncertainties of staggered fermions were shown by J. Charles at this workshop [19]. Fig. 4 shows the bounds on the  $\rho - \eta$  plane derived solely from  $B$  meson leptonic decay and  $B\bar{B}$  mixing. The bounds in the left figure employ the clover fermion results just discussed, those on the right employ the staggered fermion results discussed earlier.

## 5.2. CKMfitter and UFit

There are two groups producing widely cited global fits to CKM data: CKMfitter [20] and UFit [21]. The groups employ different statistical methods, and obtain somewhat different results. For example, for  $\Delta m_s$  predicted without incorporating the experimental  $B_s\bar{B}_s$  mixing results, UFit obtains  $21.5(2.6) \text{ ps}^{-1}$ , while CKMfitter reports  $21.7^{(5.9)}_{(4.2)} \text{ ps}^{-1}$ . This is puzzling, since one would expect this quantity to be sensitive mainly to lattice uncertainties, and both groups take the lattice results of Ref. [18] as the starting point. One possible difference is that UFit lists  $f_{B_s}\sqrt{\hat{B}_{B_s}}$  and  $\xi$  as inputs, while CKMfitter lists  $f_B\sqrt{\hat{B}_B}$  and  $\xi$ . The latter combination contains highly correlated uncertainties that must be treated with care. However, a more significant difference seems to arise from statistical methods and treatments of combinations of uncertainties, rather than differences of lattice inputs. These differences remain yet to be resolved.

## 6. Outlook

Lattice calculations are playing an essential role in enabling some of the most important results in the CKM experimental program. Most of the key lattice calculations for CKM physics involve single-hadron processes with hadronically stable meson, which are among the most solid current lattice calculations. Significant issues are currently being worked through, for example, the best fermion method for phenomenological calculations, the best way to incorporate lattice results into Standard Model global fits, and the best way compare theory and experiment in semileptonic decays. Lattice methods are currently in a state of productive ferment, with several different methods for unquenched lattice fermions under active investigation by various groups. Lots of progress is being made in algorithms for these various methods [22]. The computing power being applied to lattice phenomenology is rising exponentially [23]. There are excellent prospects for further progress in lattice CKM phenomenology.

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