

Molecular Quarkonium

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I discuss topics related to four-quark states of the ‘molecular quarkonium’ type, i.e. resonances that could be considered as (dominantly) made from a heavy meson and antimeson. Of the so far observed resonances such picture is very likely applicable to the state X(3872), and I also discuss its possible relevance to the peak near the $D^*\bar{D}^*$ threshold in e^+e^- annihilation.

1. Introduction

It has been pointed out long time ago [1] that there should exist bound and/or resonant states of heavy hadrons, dubbed ‘molecular’ and arising due to the strong force between the light constituents of the heavy hadrons. Indeed, considering the interaction between a heavy meson, consisting of a heavy quark Q and a light antiquark \bar{q} , and an antimeson ($\bar{Q}q$), it is clear that there is a normal strong force between such objects arising from an exchange of a light $\bar{q}q$ pair, which can be thought of as an exchange of light mesons. The range and the strength of the resulting strong interaction does not depend on the mass m_Q of the heavy quark Q in the limit $m_Q \rightarrow \infty$. If the interaction corresponds to an attraction in some states of the heavy meson-antimeson pair, it should inevitably give rise to bound states in those channels at sufficiently large m_Q . However, given the lack of detailed knowledge of the strong interaction between heavy hadrons, it has not been clear, whether the charmed quark is heavy enough for the reasoning based on the heavy quark limit to be applicable, and thus whether resonances of such type with hidden charm do exist. Another difficulty of identifying states with hidden charm as molecular, or more generically as four-quark systems, is that in most models of light meson exchange between charmed mesons [1–3] the interaction with the strongest attraction arises in channels with zero isospin and with non-exotic quantum numbers J^{PC} , which can be also realized as a conventional $c\bar{c}$ charmonium. For this reason it could be a typical situation that some dominantly four-quark states may be indistinguishable from states of charmonium, unless some other properties would favor a molecular interpretation. One such interpretation was argued in the past [4] for the resonance in the e^+e^- annihilation at the threshold of the $D^*\bar{D}^*$ production, then called $\psi(4028)$, and presently referred to [5] as $\psi(4040)$. It was noted that the cross section for production of the $D^*\bar{D}^*$ pairs accounted for more than one half of the total production of charmed meson pairs in spite of the very strong suppression by the P wave kinematical factor p^3 at energy close to the threshold. Thus it was suggested that there is a resonance very close to the threshold actually made from $D^*\bar{D}^*$ meson pairs.

The idea of molecular states was revived in a modern context by the observation [6] in the Belle experiment of the resonance X(3872) decaying into $\pi^+\pi^-J/\psi$, and produced in the decays $B \rightarrow XK$ of the B mesons. The resonance is quite narrow, and its width is not yet resolved ($\Gamma_X < 2.3$ MeV at 90% CL), and the updated value of its mass [7] $m_X = (3871.2 \pm 0.6)$ MeV coincides within the errors with the mass of the $D^0\bar{D}^{*0}$ meson pair: $M(D^0) + M(D^{*0}) = 3871.2 \pm 1$ MeV. The extreme closeness of the newly observed resonance to the meson pair threshold is very suggestive of an interpretation [8–11] of this resonance as a ‘molecule’ dominantly consisting of neutral charmed meson pairs: $D^0\bar{D}^{*0} \pm D^{*0}\bar{D}^0$.

2. X(3872)

The discovery [6] and observation [12–14] of X(3872) through the decay $X \rightarrow \pi^+\pi^-J/\psi$ was followed by finding [15] also the decays $X \rightarrow \pi^+\pi^-\pi^0J/\psi$ and $X \rightarrow \gamma J/\psi$. Clearly, the co-existence of the latter decays with the discovery mode $X \rightarrow \pi^+\pi^-J/\psi$ implies an isospin breaking in X(3872). Indeed the two-pion state and the three-pion one have opposite G parity, and one or the other would have to be forbidden if the isospin symmetry were applicable in the decays of X(3872). Furthermore, the existence of the radiative decay $X \rightarrow \gamma J/\psi$ requires X to have positive C parity, and the two-pion system emitted in the decay $X \rightarrow \pi^+\pi^-J/\psi$ has to be in a C-odd state, which corresponds to the isospin of this system equal to one: $I_{\pi\pi} = 1$. Thus the X(3872) resonance is required to have a substantial $I = 1$ component, which is impossible for a pure $c\bar{c}$ charmonium. In other words, the resonance has to contain light $u\bar{u}$ and/or $d\bar{d}$ quark pairs in addition to hidden charm, which thus qualifies it as a four-quark state. Moreover, the decays $X \rightarrow \pi^+\pi^-J/\psi$ and $X \rightarrow \pi^+\pi^-\pi^0J/\psi$ have comparable rates [15], which implies that the X(3872) resonance does not have a definite isospin, and is mixture of $I = 1$ and $I = 0$ states. In addition, a subsequent study [16] of the angular correlations and the dipion mass spectrum in the decay $X \rightarrow \pi^+\pi^-J/\psi$ strongly favors the spin-parity assignment $J = 1^+$ for the X(3872).

Combined with the extreme proximity of its mass to $M(D^0) + M(D^{*0})$ the known properties of X(3872) most strongly suggest that a sizeable part of the wave function of this resonance is contributed by the C-even S -wave state of charmed meson pair $D^0\bar{D}^{*0} + D^{*0}\bar{D}^0$, which thus has the quantum numbers $J^{PC} = 1^{++}$.

The binding energy w of the neutral meson pair in X(3872) is small: likely $w \lesssim 1$ MeV. In this scale the mass gap δ between the corresponding threshold for the pair of charged mesons and that for the neutral ones $\delta = M(D^+) + M(D^{*+}) - M(D^0) - M(D^{*0}) \approx 8$ MeV is large enough to suppress the weight of the state $D^+\bar{D}^{*-} + D^-\bar{D}^{*+}$ in the wave function of X(3872), thus explaining the unusual isotopic properties of the resonance.

Generally, the wave function of the resonance X(3872) can be written in terms of a Fock decomposition:

$$|X\rangle = a_0 |D^0\bar{D}^{*0} + D^{*0}\bar{D}^0\rangle + a_1 |D^+\bar{D}^{*-} + D^{*+}\bar{D}^- \rangle + \sum |\text{other}\rangle, \quad (1)$$

where first two terms correspond to the ‘molecular’ states of charmed meson pairs, and the ‘other’ states in the sum describe the admixture of non-molecular states, such as e.g. a 3P_1 charmonium $c\bar{c}$ pair. The latter states are localized at shorter distances within the range of the strong interaction, while the ‘molecular’ part refers to the motion of the charmed mesons at (mostly) the distances beyond the range of strong interaction [10, 17]. At the latter distances the coordinate wave function $\phi(r)$ for each pair is that of a free motion:

$$\phi_n(r) = \sqrt{\frac{\kappa_n}{2\pi}} \frac{\exp(-\kappa_n r)}{r}, \quad (2)$$

for the meson combination $(D^0\bar{D}^{*0} + D^{*0}\bar{D}^0)/\sqrt{2}$ and

$$\phi_c(r) = \sqrt{\frac{\kappa_c}{2\pi}} \frac{\exp(-\kappa_c r)}{r}, \quad (3)$$

for $(D^+\bar{D}^{*-} + D^{*+}\bar{D}^-)/\sqrt{2}$. The virtual momenta κ_n and κ_c are related to the binding energy w and the reduced mass of the two-meson system $m_r \approx 970$ MeV as $\kappa_n = \sqrt{2m_r w} \approx 44$ MeV $\sqrt{w/\text{MeV}}$ and $\kappa_c = \sqrt{2m_r(\delta + w)} \approx 125$ MeV.

The dominance of the $D^0\bar{D}^{*0} + D^{*0}\bar{D}^0$ at long distances translates into a substantial isospin violation in the processes determined by the ‘peripheral’ dynamics, examples of which are apparently the observed decays $X \rightarrow \pi^+\pi^-J/\psi$ and $X \rightarrow \pi^+\pi^-\pi^0J/\psi$. It is quite likely however that this isospin-breaking behavior is only a result of the ‘accidentally’ large mass difference $\delta \approx 8$ MeV between D^+D^{*-} and $D^{*0}\bar{D}^0$. Therefore it is natural to expect that at shorter distances within the range of the strong interaction the isospin symmetry is restored and at those

distances the wave function of X(3872) is dominated by $I = 0$, so that the ‘core’ of the wave function of X is an isotopic scalar. Then the wave functions of the $D^+D^{*-} + D^-D^{*+}$ and $D^0\bar{D}^{*0} + D^{*0}\bar{D}^0$ should coincide at short distances, which allows to estimate the relative statistical weight of these two states in the X(3872) resonance (i.e. the ratio of the corresponding coefficients in Eq. 1):

$$\lambda \equiv \frac{|a_1|^2}{|a_0|^2} = \frac{\kappa_n}{\kappa_c}. \quad (4)$$

Strictly speaking, the wave functions in Eq. 2 and Eq. 3 cannot be applied at short distances in the region of strong interaction, where the mesons overlap with each other and cannot be considered as individual particles. In order to take into account this behavior an ‘ultraviolet’ cutoff should be introduced. One widely used method for introducing such cutoff is to consider the meson wave functions only down to a finite distance r_0 , at which distance the boundary condition of the state being that with $I = 0$ is imposed (a discussion of a similar situation can be found e.g. in Ref.[18]). An alternative, somewhat more gradual cutoff, described by parameter Λ , can be introduced[19] by subtracting from the wave functions (2) and (3) an expression $c e^{-\Lambda r}/r$. One can also readily see, that an introduction of any such cutoff eliminates relatively more of the charged meson wave function than of the neutral one, thus reducing the estimate of the relative statistical weight as compared to that in Eq. 4, so that Eq. 4 gives in fact the upper bound for the ratio.

3. Peripheral Decays of X(3872): $X \rightarrow D^0\bar{D}^0\pi^0$ and $X \rightarrow D^0\bar{D}^0\gamma$

The peripheral $D^0\bar{D}^{*0} + D^{*0}\bar{D}^0$ component with the quasi-free mesons should give rise to the decays $X \rightarrow D^0\bar{D}^0\pi^0$ and $X \rightarrow D^0\bar{D}^0\gamma$ due to the underlying decays of the D^{*0} and \bar{D}^{*0} mesons. The widths of the corresponding underlying decays can be deduced from the available data [5]: $\Gamma_\pi \equiv \Gamma(D^{*0} \rightarrow D^0\pi^0) = 43 \pm 10$ KeV and $\Gamma_\gamma \equiv \Gamma(D^{*0} \rightarrow D^0\gamma) = 26 \pm 6$ KeV. In the C-even state X(3872) the amplitudes of the decays $D^{*0} \rightarrow D^0\pi^0$ and $\bar{D}^{*0} \rightarrow \bar{D}^0\pi^0$ interfere constructively and the result for the rate of the decay of X can be written as [10]

$$\Gamma(X \rightarrow D^0\bar{D}^0\pi^0) = 2|a_0|^2 \Gamma_\pi [A(w) + \eta B(w)], \quad (5)$$

with $A(w)$ describing the non coherent contribution of the decays from D^{*0} and \bar{D}^{*0} and $B(w)$ being the interference term. The dependence of these coefficients on the binding energy w is shown in Figure 1. As can be seen from the plot the interference is quite important at w as small as 0.1 MeV and its relative

importance increases with w , since at stronger binding the distance between the mesons in X(3872) becomes shorter. The overall fall-off of both A and B at larger w is the result of the decrease of the phase space for the decay.

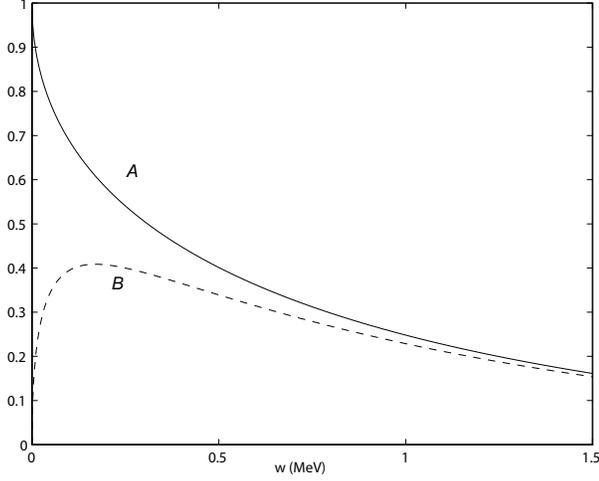


Figure 1: The dependence of the coefficients A and B in Eq. 5 on the binding energy w .

The amplitudes of the radiative transitions $D^{*0} \rightarrow D^0\gamma$ and $\bar{D}^{*0} \rightarrow \bar{D}^0\gamma$ interfere destructively in the decay $X(3872) \rightarrow D^0\bar{D}^0\gamma$.¹ The expression for the rate has the form [10, 17]:

$$\Gamma(X \rightarrow D^0\bar{D}^0\gamma) = \Gamma_\gamma |a_0|^2 \left[1 - \frac{2\kappa_n}{\omega} \arctan \frac{\omega}{2\kappa_n} \right], \quad (6)$$

where the photon energy ω can be approximated by that in the free meson decay $D^{*0} \rightarrow D^0\gamma$: $\omega \approx \omega_0 = 137$ MeV, due to the fact that the weak binding only slightly spreads the photon spectrum in the decay $X \rightarrow D^0\bar{D}^0\gamma$ near ω_0 , as can be seen from the plots shown in Figure 2. The shape of the spectrum is determined by the wave function of the meson pair in the X(3872) and thus a measurement of this shape is a direct probe of the motion of the mesons in the ‘molecule’.

It can be noted that the other kinematically allowed radiative decay channel $X \rightarrow D^+D^-\gamma$ should be quite suppressed [17]. The suppression of the direct peripheral contribution due to the transitions $D^{*\pm} \rightarrow D^\pm\gamma$ arises from the discussed small overall statistical weight of the $D^+\bar{D}^{*-} + D^{*+}\bar{D}^-$ state in the wave function (1) and from a strong destructive interference between the amplitudes of the processes $D^{*+} \rightarrow D^+\gamma$ and $D^{*-} \rightarrow D^-\gamma$ due to a significantly shorter spatial separation of the charged

¹Such effect of destructive interference in the photon emission is known as subradiance [20, 21].

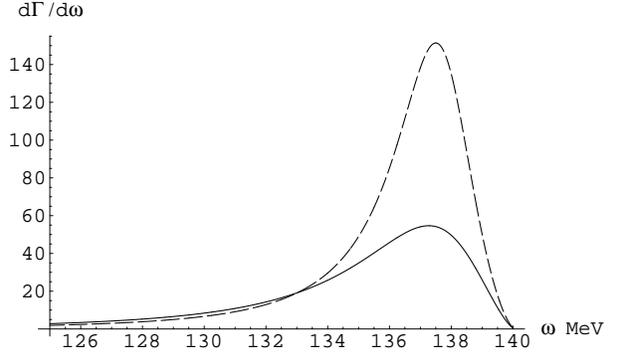


Figure 2: The photon energy spectrum in the peripheral decay $X(3872) \rightarrow D^0\bar{D}^0\gamma$, at $w = 0.3$ MeV (dashed) and at $w = 1$ MeV (solid)

mesons in X(3872). The contribution due to the final-state rescattering $X \rightarrow D^0D^0\gamma \rightarrow D^+D^-\gamma$ is also very small [17] due to the p^3 behavior of the scattering $D^0D^0 \rightarrow D^+D^-$ very near the threshold below the $\psi(3770)$ resonance. The most efficient way of producing the $D^+D^-\gamma$ final state in the decay of X(3872) appears to be due to the transition $X(3872) \rightarrow \psi(3770)\gamma$ from the ‘core’ of the X resonance. Notice however that such transitions result in the photon energy $\omega \approx 100$ MeV i.e. well below the peak from the peripheral process (cf. Figure 2).

4. The Core of X(3872)

4.1. Spin Selection Rule

The structure of the ‘core’ of the X(3872) resonance represented by the ‘other’ states in the wave function (1) is largely unknown and in discussing this part we have to resort to general considerations and quite approximate estimates. One such general property based on the heavy quark limit for charmed quarks is the spin-selection rule for the $c\bar{c}$ pair in X(3872) [22]. Namely, unlike a generic four quark state, the S wave C-even ($D^0\bar{D}^{*0} + \bar{D}^0D^{*0}$) state is unique in the sense that the total spin of the heavy $c\bar{c}$ pair is fixed: $S_{c\bar{c}} = 1$. Indeed, for the S wave there is no orbital motion, and the total spin of the state is given by the vector sum of the spins of the $c\bar{c}$ and $u\bar{u}$ quark pairs: $\vec{J} = \vec{S}_{c\bar{c}} + \vec{S}_{u\bar{u}}$. However the states of the type $S_{c\bar{c}} = 0 \otimes S_{u\bar{u}} = 1$ and $S_{c\bar{c}} = 1 \otimes S_{u\bar{u}} = 0$ have negative C parity: $C=-1$. Only the state $S_{c\bar{c}} = 1 \otimes S_{u\bar{u}} = 1$ results in $C=+1$. The total spin of the light quark pair is not ‘traceable’, i.e. can be flipped in the mixing with the ‘core’ states. The heavy quark spin however is conserved in the limit $m_Q \rightarrow \infty$. Therefore one arrives at the spin selection rule (valid up to $O(\Lambda_{QCD}/m_c)$): the states with $S_{c\bar{c}} = 1$ dominate in the Fock sum in Eq. 1 for X(3872).

As one application of the spin selection rule one can consider hadronic transitions from X(3872) to the charmonium states. The spin selection rule implies that the transitions to the spin-singlet charmonium levels such as $X(3872) \rightarrow \pi\pi\eta_c$ should be suppressed while the transitions to spin-triplet states should be allowed. The latter include the observed decays $X \rightarrow \pi^+\pi^-J/\psi$, $X \rightarrow \pi^+\pi^-\pi^0J/\psi$ and $X \rightarrow \gamma J/\psi$, and also the yet unobserved decays to the spin-triplet χ_{cJ} levels: $X \rightarrow \pi^0\chi_{cJ}$. The conservation of the spin of the $c\bar{c}$ pair implies the statistical behavior of the decay rates:

$$\Gamma(X \rightarrow \pi^0\chi_{cJ}) \propto (2J+1), \quad (7)$$

and an approximate estimate of the absolute rate [22] suggests that the width of the decay $X \rightarrow \pi^0\chi_{c1}$ can be as large as about $0.3\Gamma(X \rightarrow \pi^+\pi^-J/\psi)$.

4.2. Production of X(3872)

The core of the wave function of the X(3872) resonance is located at short distances. Thus it is natural to expect that the production of the resonance in hard processes proceeds through the production of the core. Furthermore, the quantum numbers $J^{PC} = 1^{++}$ can be realized in a 3P_1 $c\bar{c}$ state, so that one can expect that the production properties of the X resonance should be similar to those of charmonium with the rates scaled by the statistical weight of the core $|a_c|^2$. The hard processes where X(3872) has been observed so far are the decays $B \rightarrow XK$ and the production in the proton-antiproton annihilation. The analysis by CDF [12] suggests that the production of the X(3872) in the $p\bar{p}$ collisions is indeed similar to that of the ψ' charmonium resonance. The similarity between the production of X(3872) and of the pure charmonium states allows to approximately estimate the weight factor of the core $|a_c|^2$. Indeed, it is an experimental fact that the known charmonium states are produced in B decays in association with a single Kaon with approximately the same rate (within a factor of two). One might expect then that the core states of the X(3872) are produced in similar decays at approximately the same rate, so that the only suppression factor in the observed [6] rate of decays $B \rightarrow XK$ is that of the probability weight $|a_c|^2$, i.e. few percent.

Furthermore, as discussed, the core part of the wave function (1) is expected to be dominantly an isotopic scalar, thus within such production mechanism one should expect that the decays $B^0 \rightarrow XK^0$ and $B^+ \rightarrow XK^+$ should have approximately equal rates [22]. This conclusion is significantly different from the calculation [23] of the ratio of these decay rates, based on the mechanism where the resonance is produced through its peripheral meson component. The latter mechanism predicts $\mathcal{B}(B^0 \rightarrow XK^0) \ll$

$\mathcal{B}(B^+ \rightarrow XK^+)$. The current data [24] on these decays

$$\frac{\mathcal{B}(B^0 \rightarrow XK^0)}{\mathcal{B}(B^+ \rightarrow XK^+)} = 0.50 \pm 0.30 \pm 0.05 \quad (8)$$

tend to favor the core production mechanism, although an improvement of the experimental accuracy would be certainly helpful.

5. Models

The finding of X(3872) and the realization that this is likely a dominantly molecular-type state were quite unexpected, and the problem of understanding the dynamics of such states and predicting other possible instances of ‘molecules’ still remains. Although early papers [1–3] have provided the general idea of molecular hadronic systems, a detailed description of the strong interaction involving heavy and light quarks is still to be developed. In this section I briefly review some models which have been suggested after the discovery of the X(3872).

The model suggested by Swanson [25] involves a calculation of the dynamics of coupled channels including the $D\bar{D}^*$, $\bar{D}D^*$ pairs and also the pairs $\rho J/\psi$ and $\omega J/\psi$, assuming a certain specific form of the Hamiltonian for rearrangement of quarks between the hadrons. The model produced a successful prediction of the decay $X \rightarrow \omega J/\psi$ ($X \rightarrow \pi^+\pi^-\pi^0 J/\psi$) with the rate comparable to that of the discovery decay mode $X \rightarrow \rho J/\psi$ ($X \rightarrow \pi^+\pi^- J/\psi$). The model provides a detailed description of the decays of X(3872) to these channels as well as to $D\bar{D}\pi$, $D\bar{D}\gamma$, and $\pi^0\gamma J/\psi$. I am not aware of predictions of this model for other resonances of this type with hidden charm, however the model predicts a 1^{++} resonance in the $B\bar{B}^*$ system at 10562 MeV and a 0^{-+} state at 10545 MeV.

In the scheme of Maiani *et al.* [26] the resonance X(3872) belongs to a large class of four-quark states for which the building blocks are color-antisymmetric diquarks and antidiquarks. In this model this resonance is identified as a 1^{++} state of the structure $X_u = [cu]_{S=1} \times [\bar{c}\bar{u}]_{S=0} + [cu]_{S=0} \times [\bar{c}\bar{u}]_{S=1}$. The model predicts also $X_d = [cd][\bar{c}\bar{d}]$ resonance with the mass splitting $M(X_d) - M(X_{(u)}) \approx 8 \pm 3$ MeV, and also charged partners X^\pm . None of the predicted resonances with masses very close to that of X(3872) were found so far. It can be mentioned in connection with a possible resonance of the X_d type, that even if it exists, it may be quite difficult to observe it, since it is likely to be broad due to the allowed decay into pairs of neutral mesons: $D^0\bar{D}^{*0}$ and $D^{*0}\bar{D}$. This remark however does not apply to the charged states X^\pm .

Karliner and Lipkin [27] have recently considered a nonrelativistic quark model with pairwise interactions between the constituent quarks and antiquarks described by an oscillator potential and proportional to

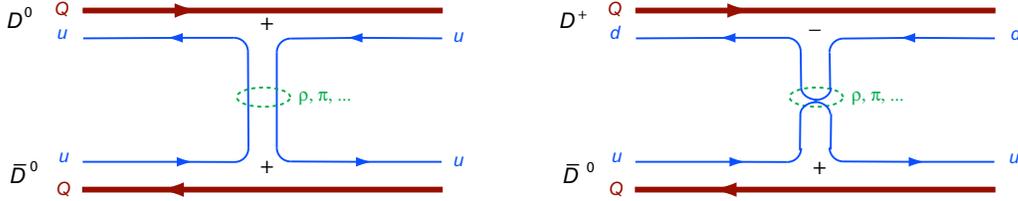


Figure 3: The interaction through an isovector exchange between the neutral mesons has opposite sign compared to that between a neutral and a charged meson

the gluon-exchange-type color correlation within each pair. The Hamiltonian in the model has the form

$$H = \sum_i \frac{p_i^2}{2m_i} + \sum_{i \neq j} \frac{V_0}{4} \cdot \lambda_c^i \cdot \lambda_c^j \cdot r_{ij}^2, \quad (9)$$

with λ_c being the color λ matrices. The authors find that the model predicts four-quark bound states of the $b\bar{c}q\bar{q}$ type, but not of the $c\bar{c}q\bar{q}$ one. They suggest that the resonance X(3872) can still be explained within this model, once the spin-dependent forces are taken into account.

The scheme considered by Braaten and Kusunoki [28] phenomenologically treats the near-threshold dynamics of the $D^0\bar{D}^{*0} + \bar{D}D^{*0}$ pair as being exclusively and universally determined by the pole in the scattering amplitude, corresponding to X(3872), and conversely, the properties of the resonance as being totally determined by the meson pairs. This approach corresponds to an ‘extreme’ molecular treatment of the resonance. In other words, it assumes the limit, where any other states in the Fock decomposition in Eq. 1 are neglected in comparison with the first term, whose coordinate wave function is given by Eq. 2. Clearly, such approach would miss the properties of X(3872) determined by its ‘core’, in particular, as discussed above, it appears that the core dominates the production of X in hard processes.

As a general remark, it can be mentioned that existence of the X(3872) as a dominantly $D^0\bar{D}^{*0} + \bar{D}D^{*0}$ shallow bound state, does not necessarily imply existence of nearby resonances, in particular of the charged partners X^\pm . Indeed, if a strong force due to exchange of isovector light-meson states (ρ, π, \dots) contributes to the binding in the $D^0\bar{D}^{*0} + \bar{D}D^{*0}$ channel, the sign of such force would be reversed in the charged states, as can be readily seen from the Figure 3: the relative sign of the two vertices for the exchanged isovector state is different between the two shown systems.

6. $D^*\bar{D}^*$ Threshold in e^+e^- Annihilation

The very strong enhancement of the e^+e^- production of the $D^*\bar{D}^*$ meson pairs very near the threshold

is known for a long time, and it served as motivation for the suggestion [4] that this is an effect of a ‘molecular’ type P -wave $J^{PC} = 1^{--}$ state of $D^*\bar{D}^*$. Indeed according to the new CLEO-c measurements [29] (which also agree with the old data [5]) the cross section for $e^+e^- \rightarrow D^*\bar{D}^*$ at $E_{c.m.} = 4030$ correspond in units of R to $R_{D^*\bar{D}^*} \approx 0.75$ even though the velocity v of the D^* mesons at this energy is $v \approx 0.1$ and value of $R_{D^*\bar{D}^*}$ is proportional to the P wave factor $v^3 \approx 10^{-3}$. It appears quite unlikely that such a strong enhancement of the production is possible without a singularity near the $D^*\bar{D}^*$ threshold. Furthermore, the features in the cross section of the e^+e^- annihilation into channels with pairs of lighter mesons: $D\bar{D}$, $D^*\bar{D}$ and $D_s\bar{D}_s$ around the same energy, although present, are still of moderate amplitude [29]. This behavior points to an existence of a near threshold resonance, which is dominantly coupled to the $D^*\bar{D}^*$ channel and has only moderate coupling to pairs of lighter mesons.

It should be mentioned that the strong p^3 growth of the cross section for production of the $D^*\bar{D}^*$ pairs above the threshold leads to the behavior where the local maximum of either the total cross section of the e^+e^- annihilation (located close to 4040 MeV, hence the notation $\psi(4040)$ [5]) or the cross section for the $D^*\bar{D}^*$ production, generally does not coincide with the actual position of the resonance. This position should be below the maximum of the $D^*\bar{D}^*$ production cross section, and can be either below, or above the threshold, or even between the thresholds for the neutral $D^{*0}\bar{D}^{*0}$ and charged $D^{*+}\bar{D}^{*-}$ vector mesons, which thresholds are split by about 6.6 MeV.

The possible existence of a resonance dominantly coupled to $D^*\bar{D}^*$ generally leads to interesting features in the cross section for production of lighter meson pairs due to channel coupling [31]. A detailed experimental study of these features can potentially provide a very interesting information on the details of strong interaction between charmed mesons, and might shed some new light on understanding ‘molecular’-type states. The phenomenon of reflection of the threshold onset in one channel on another coupled channel is well known (see e.g. in the textbook [30], Sect. 147). However the specifics of the pure P wave dynamics in the meson production amplitudes in the e^+e^- annihilation and the apparent presence of resonance near the threshold bring in in-

interesting features, which justify a separate study of this phenomenon in the discussed context.

In order to illustrate the reflection of a resonance near the $D^*\bar{D}^*$ threshold we neglect here the isotopic mass splitting between the D^* mesons and note that the resonance couples to a specific coherent mixture of the two possible production P -wave amplitudes for the $D^*\bar{D}^*$ channel: one with the total spin of the vector mesons $S = 0$ and the other with $S = 2$.² Thus the final state of the $D^*\bar{D}^*$ pair can be considered as one channel, referred here as H (heavy). A channel with a lighter meson pair, i.e. $D\bar{D}$, $D^*\bar{D}$, or $D_s\bar{D}_s$, is denoted here as L (light). The c.m. momentum in either of the L channels is already large and varies only slightly across the considered narrow energy range near the $D^*\bar{D}^*$ threshold, and this variation is neglected here.

Let W denote the total c.m. energy E in the e^+e^- annihilation relative to the threshold of production of the two vector mesons, each with the mass M : $W = E - 2M$, and let the ‘nominal’ position of the resonance be at the complex energy corresponding to $W = W_0 - i\Gamma_0/2$. According to the general quantum-mechanical consideration, based on the unitarity and analyticity of the production and scattering amplitudes (see e.g. in the textbook [30], Sects. 133 and 145) at energy above the ‘heavy’ threshold, the resonant production amplitude A_H has the form

$$A_H = \frac{b_H k}{W - W_0 + \frac{i}{2}(\Gamma_0 + g_H^2 k^3)} \quad , \quad (10)$$

and the production amplitudes for each of the light channels are parametrized as

$$A_L = a_L + \frac{b_L}{W - W_0 + \frac{i}{2}(\Gamma_0 + g_H^2 k^3)} \quad , \quad (11)$$

where $k = \sqrt{MW}$ is the c.m. momentum of the $D^*\bar{D}^*$ pair. The coefficients a_L , b_L , and b_H are complex parameters, which are also taken to be constant in the present consideration. The magnitudes and the complex phases of these coefficients result from the rescattering among the ‘light’ channels and the corresponding absorptive parts associated with these channels. It is worth noting that there is no reason to expect the phase of the coefficient b_H to vanish at $k = 0$, due to the existence of ‘light’ inelastic channels strongly coupled to the ‘heavy’ meson pair. The normalization convention used here for the amplitudes corresponds to the cross section $\sigma(e^+e^- \rightarrow H) = |A_H|^2 k$ for the ‘heavy’ channel and $\sigma(e^+e^- \rightarrow L) = |A_L|^2 p_L$ for each

of the ‘light’ channels with p_L being the c.m. momentum in the corresponding ‘light’ channel at the ‘heavy’ threshold, which momentum is taken as a constant across the considered energy range.

The expressions (10) and (11) can be readily found from the standard summation of the ‘blobs’ in the graphs of the type shown in Figure 4, where the ultraviolet part of the loop is absorbed into the definition of the overall normalization and of the position W_0 of the resonance. However the general treatment of the threshold behavior of the amplitudes[30] guarantees that these expressions are quite independent of assumptions about the form factors used in the graphs of Fig. 4.

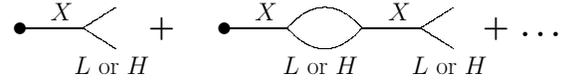


Figure 4: The graphs for the self energy of the resonance X near the threshold for the ‘heavy’ channel (H). The absorptive and dispersive parts of the self energy arise from loops with the ‘light’ (L) and heavy meson pairs. The filled circle stands for the electromagnetic vertex.

Furthermore, the ‘nominal’ width Γ_0 is due to the coupling to the lighter channels, and the term $g_H^2 k^3$ in the denominator in equations (10) and (11) represents the additional contribution to the resonance width associated with the decay into the ‘heavy’ meson pair, and g_H is the coupling constant with the dimension of length. This extra contribution to the width can be written in terms of the excitation energy W as $g_H^2 k^3 = W \sqrt{W/\epsilon}$, where

$$\epsilon = g_H^{-4} M^{-3} \quad (12)$$

is a parameter with dimension of energy. The critical point of the present consideration is that the amplitudes A_L as functions of the energy W are analytical functions of W (with a cut at positive W) and the expression (11) can be analytically continued to negative values of W . Using the notation $\Delta = -W$ at $W < 0$, the amplitudes A_L below the ‘heavy’ threshold take the form

$$A_L = a_L + \frac{b_L}{\frac{1}{2}\Delta\sqrt{\frac{\Delta}{\epsilon}} - \Delta - W_0 + \frac{i}{2}\Gamma_0} \quad . \quad (13)$$

The term in the denominator in the latter formula, proportional to $\Delta^{3/2}$, is clearly the analytical continuation of the $g_H^2 k^3$ term in Eq. 11, and the proper sign is as indicated in Eq. 13³. This term becomes

²Another generally allowed amplitude, corresponding to $S = 2$ and an F -wave, has to be small near the threshold and is neglected here.

³This sign is due to the P wave motion, and is opposite to the one that would arise in a similar term for an S wave ($\propto -\Delta^{1/2}$).

more essential than the one linear in Δ , starting from $\Delta \approx \epsilon$, and gives rise to the discussed channel coupling effects. In particular, the expression $\frac{1}{2}\Delta\sqrt{\frac{\Delta}{\epsilon}} - \Delta$ cannot be arbitrarily negative and reaches its minimum, equal to $-16\epsilon/27$ at $\Delta = \Delta_m = 16\epsilon/9$. Thus in the case where the resonance is below the $D^*\bar{D}^*$ threshold, i.e. at negative $W_0 = -\Delta_0$, the real part of the resonance denominator has either two zeros if $\Delta_0 < 16\epsilon/27$, or no zero at all otherwise. In the latter case the position of the minimum of the absolute value of the denominator, i.e. of the maximal effect on the cross section, is at $\Delta = \Delta_m$ and does not depend on Δ_0 at all, but rather is determined by the parameter ϵ . The energy corresponding to $\Delta = \Delta_m$ in fact also determines the position of the maximum of the resonance amplitude in the case where the ‘nominal’ position of the resonance is *above* the threshold, i.e. at positive W_0 . This is due to the fact that at sufficiently small ϵ the damping of the ‘light’ channel amplitude A_L due to the absorptive term $g_H^2 k^3$ in the denominator in Eq. 11 suppresses the ‘light’ channel above the ‘heavy’ threshold.

Clearly, the significance of the discussed threshold effect critically depends on the value of the parameter ϵ for the near-threshold resonance. At least for two known resonances, $\psi(3770)$ and $\Upsilon(4S)$, at the thresholds of pairs of pseudoscalar heavy mesons the value of ϵ can be found from the data: $\epsilon \approx 60$ MeV for $\psi(3770)$, and $\epsilon \approx 20$ MeV for $\Upsilon(4S)$. There is every reason to expect, from the observed onset of the $D^*\bar{D}^*$ production, that for the discussed possible resonance the coupling to the vector meson pair should be stronger than for the case of pseudoscalar mesons, and the parameter ϵ should be significantly smaller than for $\psi(3770)$. The parameter ϵ is proportional to g_H^{-4} (cf. Eq. 12). Therefore, it is quite likely that for the $D^*\bar{D}^*$ threshold resonance this parameter can be smaller than that for the $\psi(3770)$ by a factor of 10 or more. It would come as no surprise if the value of ϵ will be eventually found between 1 and 10 MeV.

The possible types of behavior of the production cross section in a ‘light’ channel in the narrow energy range around the $D^*\bar{D}^*$ threshold is illustrated in Figure 5 for some representative values of the parameters, and where the isotopic splitting of the threshold is also taken into account. It is worth mentioning that the interference of the resonant amplitude with the non-resonant background is generally different for different channels, which may result in a significantly different behavior of the observed shape of the cross section for each channel. It is important to notice however that the features in the behavior of the cross section generally do not coincide with the position of the resonance. The shape of the cross section of the e^+e^- annihilation into the vector meson pairs $D^*\bar{D}^*$ above the threshold is only weakly sensitive to the parameters of the resonance and is dominated by the very

rapid rise from the threshold due to the P wave factor k^3 .

7. Angular Correlations in $D^*\bar{D}^*$ Production Near the Threshold

As previously mentioned the threshold resonance should couple to a specific mixture of the states of the $D^*\bar{D}^*$ pair with the total spin $S = 0$ and $S = 2$. The coefficients in this mixture and their relative phase can be studied by measuring angular correlations in the production in e^+e^- annihilation near the threshold [32]. It would also be of interest to study the behavior of these coefficients with increasing energy in order to possibly untangle the resonant and the non-resonant production amplitudes.

Let A_0 and A_2 denote the production amplitudes for correspondingly the $S = 0$ and $S = 2$ states, normalized in such a way that the total cross section is given by $\sigma(e^+e^- \rightarrow D^*\bar{D}^*) = |A_0|^2 + |A_2|^2$. Then the distribution of the production rate in the angle θ between the direction of the momentum of either of the mesons and the e^+e^- beams has the form

$$\frac{d\sigma}{d\cos\theta} \propto 1 - \frac{|A_2|^2 + 10|A_0|^2}{7|A_2|^2 + 10|A_0|^2} \cos^2\theta. \quad (14)$$

Clearly, this angular distribution is not sensitive to the relative phase φ of the amplitudes A_0 and A_2 . Such sensitivity however arises in the distribution over the angle ϑ_π between the momentum of the parent D^* (\bar{D}^*) and the D (\bar{D}) emerging from the decay $D^* \rightarrow D\pi$ ($\bar{D}^* \rightarrow \bar{D}\pi$), or the similar correlation over the angle ϑ_γ for the events with the decays $D^* \rightarrow D\gamma$ ($\bar{D}^* \rightarrow \bar{D}\gamma$). The angular distributions are expressed through the two discussed amplitudes as

$$\begin{aligned} \frac{d\sigma}{d\cos\vartheta_\pi} &\propto |A_0|^2 + \frac{1}{20}|A_2|^2(13 + 21\cos^2\vartheta_\pi) \\ &+ \frac{2}{\sqrt{5}}|A_0||A_2|\cos\varphi(3\cos^2\vartheta_\pi - 1), \end{aligned} \quad (15)$$

and

$$\begin{aligned} \frac{d\sigma}{d\cos\vartheta_\gamma} &\propto |A_0|^2 + \frac{1}{20}|A_2|^2(13 + \frac{21}{2}\sin^2\vartheta_\gamma) \\ &+ \frac{2}{\sqrt{5}}|A_0||A_2|\cos\varphi(\frac{3}{2}\sin^2\vartheta_\gamma - 1). \end{aligned} \quad (16)$$

The latter two angular distributions however are not independent: one is found from the other by a simple substitution $\cos^2\vartheta_\pi \leftrightarrow \frac{1}{2}\sin^2\vartheta_\gamma$. The reason of course is that they both contain the same information about the vector meson polarization density matrix. Thus measuring one or the other, or both, depends on the specific experimental setup.

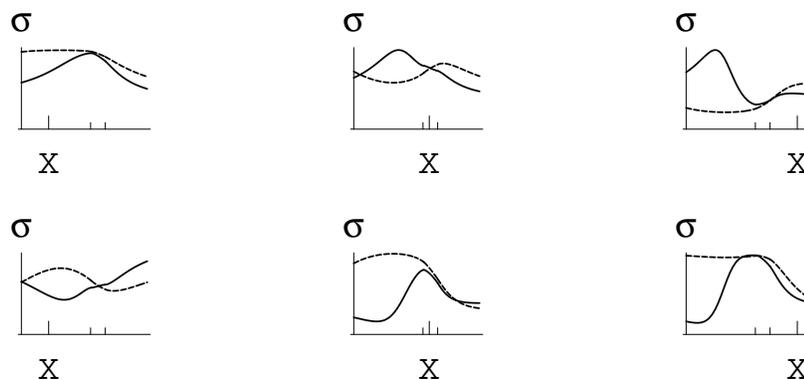


Figure 5: The types of behavior of the e^+e^- annihilation cross section (in arbitrary units) into one of the ‘light’ channels near the $D^*\bar{D}^*$ threshold. The horizontal axis in each plot spans the c.m. energy range from 3980 MeV to 4040 MeV. The $D^{*0}\bar{D}^{*0}$ and $D^{*+}\bar{D}^{*-}$ thresholds are shown with shorter vertical tick marks. The plots are shown for three assumed values of the ‘nominal’ position W_0 of the resonance X relative to the $D^{*0}\bar{D}^{*0}$ threshold: -20, 3, and 20 MeV, and the assumed value for this position is indicated in each plot by the longer vertical tick mark. In the plots in the upper row it is assumed that the coefficients a_L and b_L are relatively real and have the same sign. In the lower row of plots the relative sign of these coefficients is assumed to be negative. The solid lines in the plots correspond to $\epsilon = 1$ MeV, and the dashed one are for $\epsilon = 10$ MeV. The width parameter Γ_0 is fixed at $\Gamma_0 = 50$ MeV.

8. Summary

It is quite likely that in the resonance $X(3872)$ we have come across a ‘molecular’ state made out of dominantly D^0 and D^{*0} . Although the existence of such states for “sufficiently heavy” heavy hadrons was not unexpected theoretically, it was not clear whether the charmed mesons were heavy enough. Given the present uncertainty in understanding the strong dynamics of systems of heavy hadrons it is by far not clear yet whether the $X(3872)$ is a result of coincidence of several factors facilitating the binding, and no other such states in the charmonium mass region exist, or there are other similar states with hidden charm. One such candidate for a molecular state is the apparent strong threshold singularity in e^+e^- annihilation to $D^*\bar{D}^*$. Further theoretical understanding of the strong interaction between heavy mesons can be (hopefully) facilitated by a more detailed study of the resonance $X(3872)$ and by a detailed scan of the e^+e^- annihilation into exclusive channels with charmed mesons in the vicinity of the $D^*\bar{D}^*$ threshold. In this respect the interesting studies which look feasible include the measurement of the decays $X(3872) \rightarrow D^0\bar{D}^0\pi$, $X(3872) \rightarrow D^0\bar{D}^0\gamma$ (the photon spectrum in this decay is directly related to the motion of the mesons inside the X resonance), a search for the decay $X(3872) \rightarrow D^+D^-\gamma$, which according to the presented discussion is sensitive to the transition $X(3872) \rightarrow \psi(3770)\gamma$, a search for the decays $X(3872) \rightarrow \chi_{cJ}\pi^0$. For understanding the structure of the resonance very near the $D^*\bar{D}^*$ threshold in the e^+e^- annihilation it is most interesting to have a detailed scan of the production cross section for the

lighter pairs: DD , $D^*\bar{D}$ and $D_s\bar{D}_s$, since these channels may exhibit a quite intricate behavior sensitive to detailed parameters of the resonance. Also a study of the angular correlations in $e^+e^- \rightarrow D^*\bar{D}^*$ near the threshold would provide an information on the ‘hyperfine’ structure of the possible molecular state.

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