

# Constraints on the CKM Matrix

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We update the analyses of the Cabibbo-Kobayashi-Maskawa matrix, both within the Standard Model and for arbitrary New Physics contributions to the mixing amplitudes, using new inputs from the Winter 2006 conferences.

## 1. Introduction

The most important observables that were part of the planned  $B$ -factory program have now been measured. With the addition of the new  $\Delta m_s$  constraint from the Tevatron experiments, the three main flavor-changing neutral current transitions ( $s \rightarrow d$ ,  $b \rightarrow d$  and  $b \rightarrow s$ ) are tested to different precision levels and compared to Standard Model predictions. In these proceedings we update the analyses of the Cabibbo-Kobayashi-Maskawa (CKM) matrix of Ref. [1] (where all notations and technical details can be found) with the recently measured relevant observables.

## 2. Inputs to the global CKM fit

The inputs are listed in Tables I and II. In the following we discuss to some detail the status of the angles  $\alpha$  and  $\gamma$ , and of the  $B_s \bar{B}_s$  oscillation frequency  $\Delta m_s$ .

### 2.1. The angle $\alpha$

The current direct constraint on  $\alpha$  comes from mixing-induced CP-violating measurements, through the combination of the two-body isospin analyses of  $B \rightarrow \pi\pi$  and  $B \rightarrow \rho\rho$ , and the Dalitz plot analysis of  $B \rightarrow \rho\pi$  (see, e.g., Ref. [2] and references therein). Among the three channels  $B \rightarrow \pi\pi$  plays only a minor rôle because of the pattern of discrete ambiguities. In  $B \rightarrow \rho\rho$  a missing observable, namely  $\text{BR}(B \rightarrow \rho^0 \rho^0)$ , prevents to perform a full Gronau-London [3] analysis. The current upper bound on this mode actually implies bounds à la Grossman-Quinn [4] on the difference  $|\alpha - \alpha_{\text{eff}}|$ . Before the update on the  $B \rightarrow \rho^+ \rho^0$  branching fraction that was presented by the *BABAR* collaboration at the winter conferences [5], the world average data were in slight disagreement with the existence of an isospin triangle [1]. This somewhat “lucky” fluctuation was reflected in the fit result by a sharp peak, that has now evolved to a plateau and a bit larger error on  $\alpha$ , in agreement with what is expected from theoretical bounds. We find at 68% CL (Fig. 1)

$$\alpha = (100_{-9}^{+15})^\circ. \quad (1)$$

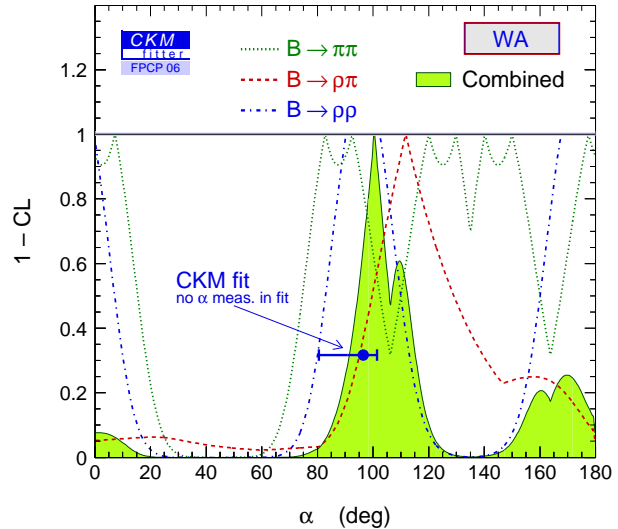


Figure 1: Confidence level for the angle  $\alpha$  from  $B \rightarrow \pi\pi$ ,  $B \rightarrow \rho\pi$ ,  $B \rightarrow \rho\rho$  and combination (world average data).

### 2.2. The angle $\gamma$

The extraction of  $\gamma$  stems from direct CP-violation measurements in  $B \rightarrow DK$  modes. Although it is theoretically simpler than that of  $\alpha$ , because the formulae are more compact and do not involve the use of flavor symmetry [6, 7], the statistical interpretation of present data requires advanced techniques. Of crucial importance for the performance of the analysis is the size of the  $r_B$  parameter (where there is one for each decay channel), the ratio of  $b \rightarrow u\bar{c}s$  to  $b \rightarrow c\bar{u}s$  amplitudes: the larger  $r_B$ , the smaller the error on  $\gamma$ . With current statistics however,  $r_B$  remains not too far from zero, which in turn implies that the minimum  $\chi^2$  result for  $r_B$  (resp. the error on  $\gamma$ ) is biased towards larger values (resp. smaller values). In order to evaluate this bias and to correct for this unwanted effect, one must perform a full frequentist analysis by means of toy Monte-Carlo studies as a function of the true parameter values. Both *BABAR* [8] and *Belle* [9] use a Neyman-type construction of the confidence level in the full parameter space, with different choices of the ordering function [10] (*Belle*'s choice is equivalent to the subtraction of the global minimum from the

$\chi^2$  function). Then the next question is how to get rid of the nuisance parameters ( $r_B$  and the associated strong phase) in order to determine the confidence level for the desired parameter  $\gamma$ . Both *BABAR* and Belle make a gaussian-like assumption by using the known correspondence between confidence levels of different dimensionality, through the specification of the number of degrees of freedom [11]. While this is presumably a very good approximation for the case of  $\gamma$ , this method is not completely general as in some situations the number of degrees of freedom of the likelihood function can be ill-defined. For the sake of generality the CKMfitter group has decided to use a new method that avoids this assumption. Its application to the present case leads to a slightly larger error on  $\gamma$ : for the *BABAR* data on  $B \rightarrow DK$  in the Dalitz plot analysis (statistical errors only) CKMfitter finds a 68% CL interval of  $[35^\circ, 102^\circ]$  instead of  $[39^\circ, 101^\circ]$  as quoted by *BABAR*. The bad news is that even when one averages over all channels and all data one finds the rather loose determination <sup>1</sup> (Fig.2)

$$\gamma = (62_{-25}^{+35})^\circ. \quad (2)$$

More detailed studies on the origin of this error (pre-

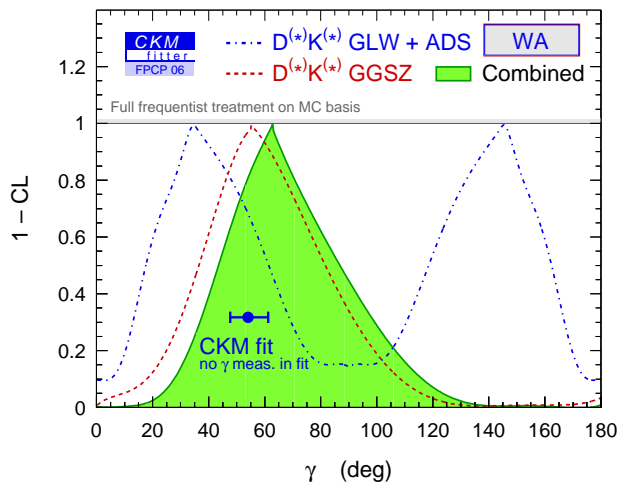


Figure 2: Confidence level for the angle  $\gamma$  from the GLW, ADS and GGSZ methods [6] (world average data).

sumably a combination of statistical effects and preferred  $r_B$  values), as well as coverage tests (preliminary results show a reasonably good behavior) will be published elsewhere.

<sup>1</sup>With the frequentist method that was used by CKMfitter for the summer 2005 conferences (with the result  $\gamma = (63_{-12}^{+15})^\circ$  [12]), but with the present winter 2006 data, we find  $\gamma = (62_{-19}^{+28})^\circ$ . This method is being generalized.

### 2.3. The oscillation frequency $\Delta m_s$

The most important news concerning flavor physics at the 2006 winter conferences is of course the first two-sided bound on  $\Delta m_s$  by D0 [13] followed by the 99.5% SL measurement by CDF [14]. The 68% CL range found by CDF,  $\Delta m_s = 17.33_{-0.21}^{+0.42} \pm 0.07 \text{ ps}^{-1}$ , is in excellent agreement with the indirect prediction of the Standard Model global fit,  $\Delta m_s = 21.7_{-4.2}^{+5.9} \text{ ps}^{-1}$ , as can be seen in Fig. 3. Taken as an input, the combination of  $\Delta m_s$  and  $\Delta m_d$  has a strong impact on the  $(\bar{\rho}, \bar{\eta})$  plane (Fig. 5), but the corresponding constraint is now completely dominated by the theoretical uncertainty (from lattice simulations) on the ratio  $\xi$  of the relevant matrix elements.

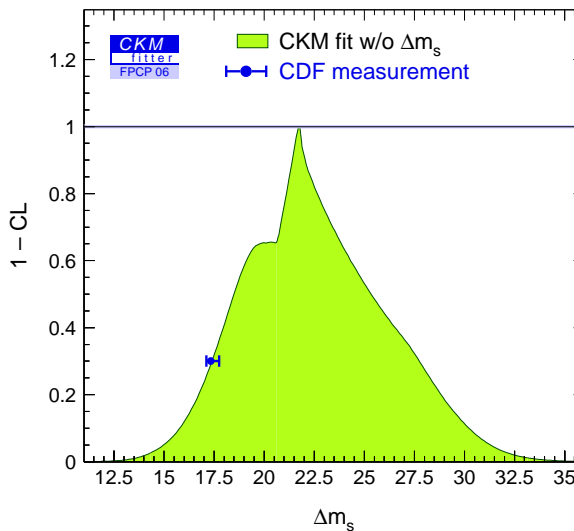


Figure 3: Comparison of the direct measurement of  $\Delta m_s$  by the CDF collaboration with the indirect prediction from the standard global fit.

### 2.4. Theoretical uncertainties

All non-angle measurements are now dominated by theoretical uncertainties, coming from the evaluation of non perturbative hadronic effects by lattice simulations, QCD sum rules or possibly other methods. Nevertheless these errors have decreased and further significant progress is expected. Fig. 4 shows the constraints on  $(\bar{\rho}, \bar{\eta})$  coming from  $\Delta m_d$  and the recent Belle's measurement of the leptonic decay [15]  $\text{BR}(B \rightarrow \tau \nu) = (1.06_{-0.28}^{+0.34}(\text{stat})_{-0.25}^{+0.22}(\text{syst})) \times 10^{-4}$ , and of the combination of the two. The left plot corresponds to our default input values, while for the right plot the results obtained with the improved staggered fermion action have been used. The latter method aims at the efficient estimate of unquenched effects coming from the light sea quark masses [16]. While

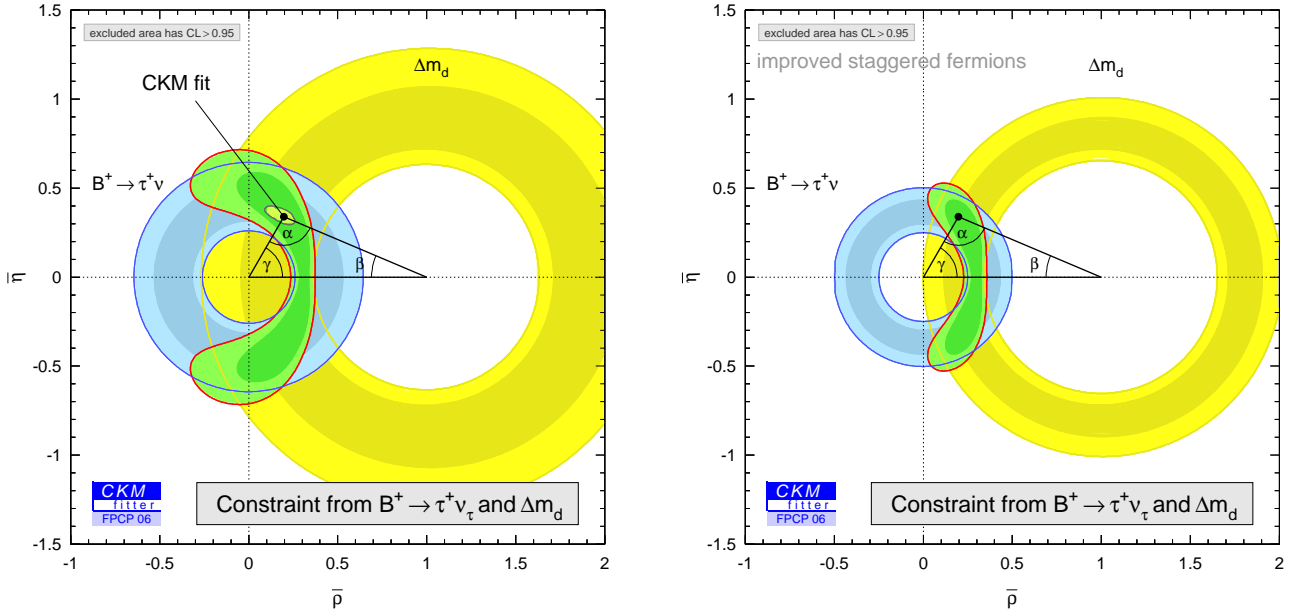


Figure 4: 95% CL constraints in the  $(\bar{\rho}, \bar{\eta})$  plane from  $B \rightarrow \tau\nu$ ,  $\Delta m_d$  and combination. In the combination the decay constant  $f_{B_d}$  cancels, so that the only source of theoretical uncertainty comes from the bag factor  $B_{B_d}$ . Left: our default input values (Tables I and II); right: inputs from improved staggered fermions formalism [17].

the outcome of this approach is impressive and has passed several non trivial tests, the precise relation to full continuum QCD is not completely understood and there is no consensus on the accuracy of this formalism.

As a side remark to this comparison, we would like to stress that there are significant correlations between the evaluation of the various theoretical parameters, that have a rôle when the errors decrease. For example the two choices  $(f_{B_d}, f_{B_s}/f_{B_d}, B_{B_s}, B_{B_d})$  and  $(f_{B_d}, f_{B_s}/f_{B_d}, B_{B_s}, B_{B_s}/B_{B_d})$  for the decay constants and bag parameters are mathematically equivalent, but the corresponding predictions for the relevant observables are different if one uses the inputs of Ref. [17]. This is because we have neglected correlations between the parameters. We then encourage our colleagues working on the lattice to publish the correlation matrices found in the simulations, at least for the purely statistical part of the error.

### 3. Fit results

The global CKM reference fit is defined as the combination of constraints on the CKM matrix elements on which we think we have a sufficiently good theoretical control. This correspond to the charged current couplings  $|V_{ud}|$ ,  $|V_{us}|$ ,  $|V_{cb}|$  as well as the following quantities that are specifically sensitive to  $(\bar{\rho}, \bar{\eta})$ :  $|V_{ub}|$ ,  $|\epsilon_K|$ ,  $\Delta m_d$ ,  $\Delta m_s$ ,  $\alpha$ ,  $\beta$  and  $\gamma$ . The individual constraints as well as the combination are shown in Fig. 5, all of them are in good or even excellent agree-

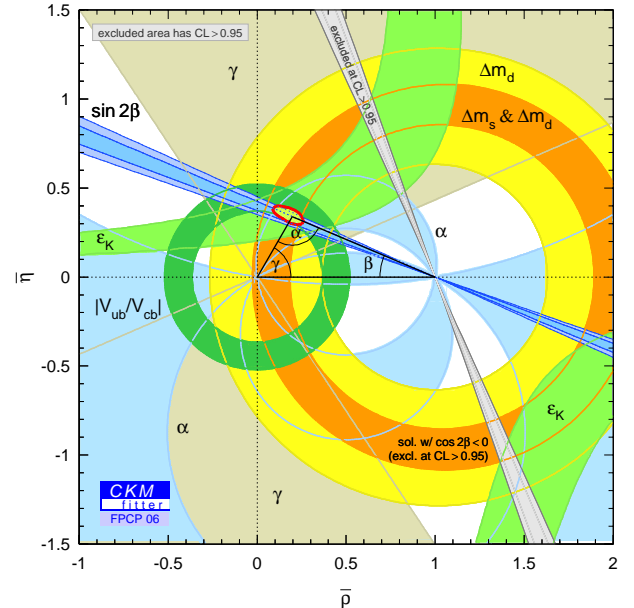


Figure 5: Confidence level in the  $(\bar{\rho}, \bar{\eta})$  plane for the global CKM fit. The shaded area indicate the regions of at least 95% CL.

ment with each other. Tables III and IV show the  $1\sigma$ ,  $2\sigma$  and  $3\sigma$  allowed ranges for several particularly interesting parameters and observables.

The seven constraints in the  $(\bar{\rho}, \bar{\eta})$  plane somewhat dilutes the information. We have thus done partial analyses in order to compare, respectively, CP-violating observables with CP-conserving

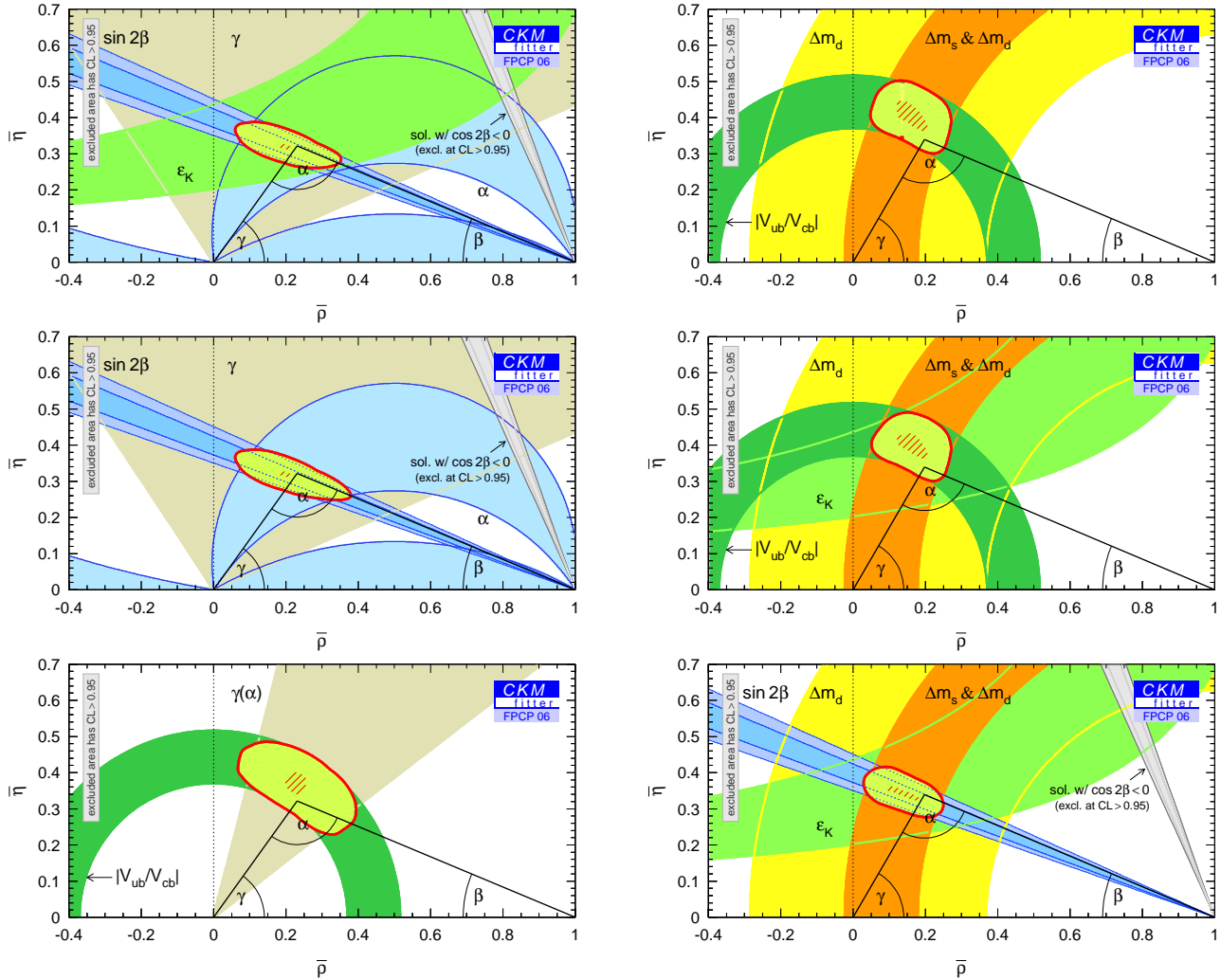


Figure 6: Confidence level in the  $(\bar{\rho}, \bar{\eta})$  plane for the global CKM fit. From top to bottom and left to right: CP-violating observables vs. CP-conserving ones, theory-free constraints vs. QCD-based ones, and tree-dominated observables vs. loop-dominated ones. In the bottom left plot the constraint on  $\alpha$  has been used assuming there is no New Physics contribution to the  $\Delta I = 3/2$   $b \rightarrow d$  electroweak penguin amplitudes. In the combination of this constraint with  $\beta$  from  $B \rightarrow M_{c\bar{c}}K_S$  modes the New Physics mixing phase cancels, so that it gives a New Physics-free determination of  $\gamma = \pi - \beta - \alpha$ .

ones, theory-free constraints with QCD-based ones, and tree-dominated observables with loop-dominated ones. The overall consistency of these fits is striking, and the small deviations that can be seen here and there are well compatible with what is expected from the convolution of statistical fluctuations with theoretical uncertainties.

#### 4. New Physics in mixing amplitudes

We have updated the model-independent analysis of possible New Physics effects in meson mixing amplitudes, as described in Ref. [1] For the  $B_d\bar{B}_d$  case (Fig. 7), the data are well compatible with Standard Model values for the parameters, and the region in-

volving new contributions has decreased. For  $B_s\bar{B}_s$  the constraint is weak, despite the new  $\Delta m_s$  input: this is because of the theoretical uncertainties. On Fig 4 a tentative extrapolation of the situation after a few weeks of nominal LHCb running is shown, when the  $B_s\bar{B}_s$  mixing phase will have been probably measured.

#### 5. Conclusion

Since the last two years, CKM physics has entered its precision era. Experimental data become more precise and systematics and specific statistical effects require careful treatment. On the other hand theory of hadronic matrix elements has made progress and

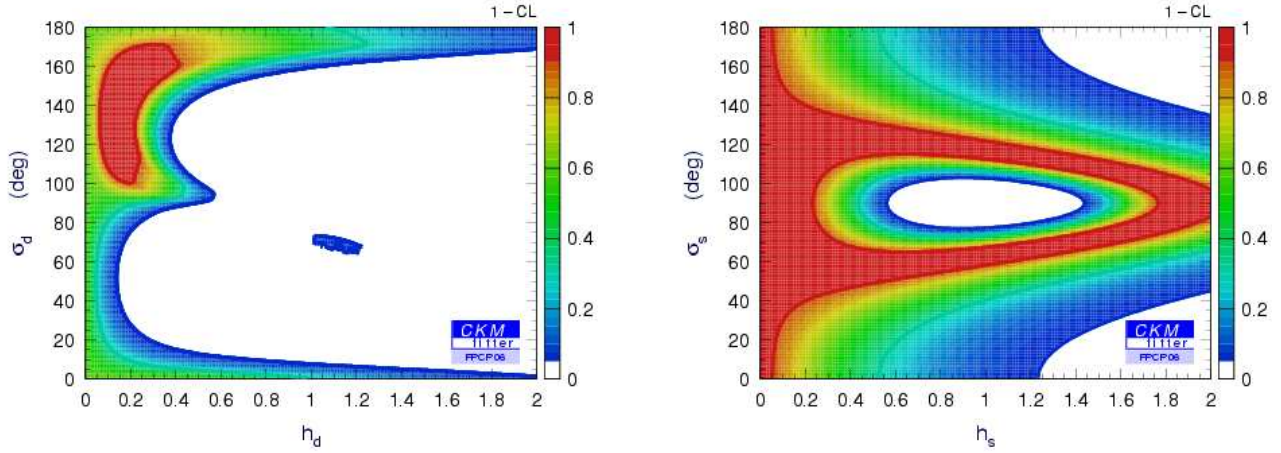


Figure 7: Confidence level in the  $(h, \sigma)$  plane for the global CKM fit, assuming possible New Physics contributions to the mixing amplitudes.  $(h, \sigma)$  parametrizes the deviation of the matrix elements with respect to their Standard Model values [18], so that SM corresponds to  $h = 0$ . In addition to the standard inputs we use the semileptonic asymmetry  $\mathcal{A}_{SL}(B_d) = -0.0005 \pm 0.0055$ . Left:  $B_d \bar{B}_d$  system; right:  $B_s \bar{B}_s$  system.

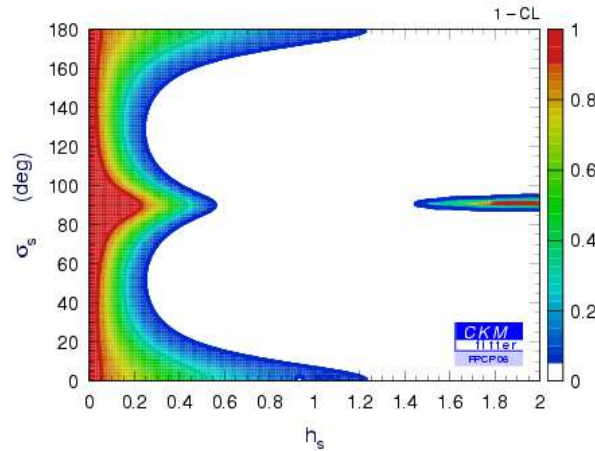


Figure 8: Confidence level in the  $(h_s, \sigma_s)$  plane for the global CKM fit, assuming possible New Physics contributions to the mixing amplitudes, and a measurement of  $B_s \bar{B}_s$  mixing at  $\Delta m_s = 20.000 \pm 0.035 \text{ ps}^{-1}$ ,  $\sin 2\beta_s = 0.036 \pm 0.100$  that approximately corresponds a few weeks of LHCb running ( $0.2 \text{ fb}^{-1}$ ).

is expected to continue on the way. For the “cleanest” observables the Standard Model does not show any sizable anomaly with respect to the data. Still, important observables, such as very rare kaon or  $B$  decays, are missing from the overall pattern and future experiments and theoretical methods will bring up new information on quark mixing and CP violation.

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Table I *Inputs to the standard CKM fit. If not stated otherwise: for two errors given, the first is statistical and accountable systematic and the second stands for systematic theoretical uncertainties. The last two columns indicate the treatment of the input parameters: measurements or parameters that have statistical errors (we include here experimental systematics) are marked in the “GS” column by an asterisk; measurements or parameters that have systematic theoretical errors are marked in the “TH” column by an asterisk. Upper part: experimental determinations of the CKM matrix elements. Middle upper part: CP-violation and mixing observables. Middle lower part: parameters used in SM predictions that are obtained from experiment. Lower part: parameters of the SM predictions obtained from theory.*

Parameter	Value $\pm$ Error(s)	Reference	Errors	
			GS	TH
$ V_{ud} $ (nuclei)	$0.97377 \pm 0.00027$	[19]	*	-
$ V_{us} $ ( $K_{\ell 3}$ and $K_{\mu 2}$ )	$0.2257 \pm 0.0021$	[28]	*	-
$ V_{ub} $ (incl.)	$(4.45 \pm 0.23 \pm 0.39) \times 10^{-3}$	[20, 21]	*	*
$ V_{ub} $ (excl.)	$(3.94 \pm 0.28 \pm 0.51) \times 10^{-3}$	[28]	*	*
$ V_{cb} $ (incl.)	$(41.70 \pm 0.70) \times 10^{-3}$	[28]	*	-
$ V_{cb} $ (excl.)	$(41.18 \pm 1.71) \times 10^{-3}$	[20]	*	-
$ \varepsilon_K $	$(2.221 \pm 0.008) \times 10^{-3}$	[23]	*	-
$\Delta m_d$	$(0.507 \pm 0.004) \text{ ps}^{-1}$	[20]	*	-
$\Delta m_s$	Amplitude spectrum+CDF -LogL	[24]	*	-
$\sin(2\beta)_{[c\bar{c}]}$	$0.687 \pm 0.032$	[20]	*	-
$S_{\pi\pi}^{+-}$	$-0.50 \pm 0.12$	[20]	*	-
$C_{\pi\pi}^{+-}$	$-0.37 \pm 0.10$	[20]	*	-
$C_{\pi\pi}^{00}$	$-0.28 \pm 0.39$	[20]	*	-
$\mathcal{B}_{\pi\pi}$ all charges	Inputs to isospin analysis	[20]	*	-
$S_{\rho\rho,L}^{+-}$	$-0.22 \pm 0.22$	[20]	*	-
$C_{\rho\rho,L}^{+-}$	$-0.02 \pm 0.17$	[20]	*	-
$\mathcal{B}_{\rho\rho,L}$ all charges	Inputs to isospin analysis	[20]	*	-
$B^0 \rightarrow (\rho\pi)^0 \rightarrow 3\pi$	Time-dependent Dalitz analysis	[25]	*	-
$B^- \rightarrow D^{(*)} K^{(*)-}$	Inputs to GLW analysis	[20]	*	-
$B^- \rightarrow D^{(*)} K^{(*)-}$	Inputs to ADS analysis	[20]	*	-
$B^- \rightarrow D^{(*)} K^{(*)-}$	GGSZ Dalitz analyses	[20]	*	-
$\mathcal{B}(B^- \rightarrow \tau^- \bar{\nu}_\tau)$	Experimental likelihoods	[27]	*	-
$\bar{m}_c(m_c)$	$(1.24 \pm 0.037 \pm 0.095) \text{ GeV}$	[22]	*	*
$\bar{m}_t(m_t)$	$(162.3 \pm 2.2) \text{ GeV}$	[26]	*	-
$m_{K^+}$	$(493.677 \pm 0.016) \text{ MeV}$	[28]	-	-
$\Delta m_K$	$(3.4833 \pm 0.0066) \times 10^{-12} \text{ MeV}$	[28]	-	-
$m_{B_d}$	$(5.2794 \pm 0.0005) \text{ GeV}$	[28]	-	-
$m_{B_s}$	$(5.3696 \pm 0.0024) \text{ GeV}$	[28]	-	-
$m_W$	$(80.423 \pm 0.039) \text{ GeV}$	[28]	-	-
$G_F$	$1.16639 \times 10^{-5} \text{ GeV}^{-2}$	[28]	-	-
$f_K$	$(159.8 \pm 1.5) \text{ MeV}$	[28]	-	-

Table II Inputs to the standard CKM fit (continued)

Parameter	Value $\pm$ Error(s)	Reference	Errors	
			GS	TH
$B_K$	$0.79 \pm 0.04 \pm 0.09$	[19]	*	*
$\alpha_S(m_Z^2)$	$0.1176 \pm 0.0020$	[28]	-	*
$\eta_{ct}$	$0.47 \pm 0.04$	[29]	-	*
$\eta_{tt}$	$0.5765 \pm 0.0065$	[29, 30]	-	*
$\eta_B(\overline{\text{MS}})$	$0.551 \pm 0.007$	[31]	-	*
$f_{B_d}$	$(191 \pm 27) \text{ MeV}$	[19]	*	-
$B_d$	$1.37 \pm 0.14$	[19]	*	-
$\xi^{(a)}$	$1.24 \pm 0.04 \pm 0.06$	[19]	*	*

<sup>(a)</sup>anticorrelated theory error with  $f_{B_d}\sqrt{B_d}$ .

Table III Fit results and errors using the standard input observables. For results marked with “meas. not in fit”, the measurement of the corresponding observable has not been included in the fit.

Observable	central $\pm$ CL $\equiv$ 1 $\sigma$	$\pm$ CL $\equiv$ 2 $\sigma$	$\pm$ CL $\equiv$ 3 $\sigma$
$\lambda$	0.2272 <sup>+0.0010</sup> <sub>-0.0010</sub>	+0.0020 -0.0020	+0.0030 -0.0030
$A$	0.809 <sup>+0.014</sup> <sub>-0.014</sub>	+0.029 -0.028	+0.044 -0.042
$\bar{\rho}$	0.197 <sup>+0.026</sup> <sub>-0.030</sub>	+0.050 -0.087	+0.074 -0.133
$\bar{\eta}$	0.339 <sup>+0.019</sup> <sub>-0.018</sub>	+0.047 -0.037	+0.075 -0.057
$J$ [10 <sup>-5</sup> ]	3.05 <sup>+0.18</sup> <sub>-0.18</sub>	+0.45 -0.36	+0.69 -0.54
$\sin(2\alpha)$	-0.25 <sup>+0.17</sup> <sub>-0.15</sub>	+0.49 -0.28	+0.71 -0.42
$\sin(2\alpha)$ (meas. not in fit)	-0.23 <sup>+0.55</sup> <sub>-0.16</sub>	+0.72 -0.32	+0.83 -0.45
$\sin(2\beta)$	0.716 <sup>+0.024</sup> <sub>-0.024</sub>	+0.048 -0.049	+0.074 -0.075
$\sin(2\beta)$ (meas. not in fit)	0.752 <sup>+0.057</sup> <sub>-0.035</sub>	+0.105 -0.073	+0.135 -0.112
$\alpha$ (deg)	97.3 <sup>+4.5</sup> <sub>-5.0</sub>	+8.7 -14.0	+13.7 -20.7
$\alpha$ (deg) (meas. not in fit)	96.5 <sup>+4.9</sup> <sub>-16.0</sub>	+9.9 -21.2	+14.6 -25.3
$\alpha$ (deg) (dir. meas.)	100.2 <sup>+15.0</sup> <sub>-8.8</sub>	+22.7 -18.2	+32.0 -28.1
$\beta$ (deg)	22.86 <sup>+1.00</sup> <sub>-1.00</sub>	+2.03 -1.97	+3.22 -2.93
$\beta$ (deg) (meas. not in fit)	24.4 <sup>+2.6</sup> <sub>-1.5</sub>	+5.1 -3.0	+6.9 -4.5
$\beta$ (deg) (dir. meas.)	21.7 <sup>+1.3</sup> <sub>-1.2</sub>	+2.6 -2.4	+4.1 -3.6
$\gamma \simeq \delta$ (deg)	59.8 <sup>+4.9</sup> <sub>-4.1</sub>	+13.9 -7.9	+20.8 -12.1
$\gamma \simeq \delta$ (deg) (meas. not in fit)	59.8 <sup>+4.9</sup> <sub>-4.2</sub>	+14.1 -8.0	+21.0 -12.3
$\gamma \simeq \delta$ (deg) (dir. meas.)	63 <sup>+35</sup> <sub>-25</sub>	+62 -40	+100 -54
$\beta_s$ (deg)	1.045 <sup>+0.061</sup> <sub>-0.057</sub>	+0.151 -0.114	+0.238 -0.177
$\sin(2\beta_s)$	0.0365 <sup>+0.0021</sup> <sub>-0.0020</sub>	+0.0053 -0.0040	+0.0083 -0.0062
$R_u$	0.391 <sup>+0.015</sup> <sub>-0.015</sub>	+0.031 -0.029	+0.049 -0.044
$R_t$	0.872 <sup>+0.033</sup> <sub>-0.028</sub>	+0.095 -0.054	+0.143 -0.082
$\Delta m_d$ (ps <sup>-1</sup> ) (meas. not in fit)	0.394 <sup>+0.097</sup> <sub>-0.097</sub>	+0.219 -0.132	+0.361 -0.162
$\Delta m_s$ (ps <sup>-1</sup> )	17.34 <sup>+0.49</sup> <sub>-0.20</sub>	+0.65 -0.35	+0.78 -0.49
$\Delta m_s$ (ps <sup>-1</sup> ) (meas. not in fit)	21.7 <sup>+5.9</sup> <sub>-4.2</sub>	+9.7 -6.8	+13.1 -9.1
$\epsilon_K$ [10 <sup>-3</sup> ] (meas. not in fit)	2.46 <sup>+0.63</sup> <sub>-0.88</sub>	+1.05 -1.05	+1.50 -1.20
$f_{B_d}$ (MeV) (lattice value not in fit)	183 <sup>+10</sup> <sub>-10</sub>	+21 -20	+34 -28
$\xi$ (lattice value not in fit)	1.061 <sup>+0.122</sup> <sub>-0.047</sub>	+0.213 -0.083	+0.324 -0.119
$B_K$ (lattice value not in fit)	0.722 <sup>+0.251</sup> <sub>-0.084</sub>	+0.348 -0.157	+0.461 -0.216
$m_c$ (GeV/ $c^2$ ) (meas. not in fit)	0.81 <sup>+0.93</sup> <sub>-0.36</sub>	+1.08 -0.36	+1.23 -0.81
$m_t$ (GeV/ $c^2$ ) (meas. not in fit)	150 <sup>+27</sup> <sub>-21</sub>	+57 -35	+79 -46

Table IV Fit results (continued)

Quantity	central $\pm$ CL $\equiv$ 1 $\sigma$	$\pm$ CL $\equiv$ 2 $\sigma$	$\pm$ CL $\equiv$ 3 $\sigma$
$\mathcal{B}(B^+ \rightarrow \tau^+ \nu_\mu)$ [10 <sup>-5</sup> ]	9.6 <sup>+1.5</sup> <sub>-1.5</sub>	+3.3 -2.9	+5.4 -4.0
$\mathcal{B}(B^+ \rightarrow \mu^+ \nu_\mu)$ [10 <sup>-7</sup> ]	4.32 <sup>+0.58</sup> <sub>-0.57</sub>	+1.27 -1.12	+2.05 -1.62
$\mathcal{B}(K_L^0 \rightarrow \pi^0 \nu \bar{\nu})$ [10 <sup>-11</sup> ]	2.58 <sup>+0.48</sup> <sub>-0.40</sub>	+1.01 -0.68	+1.53 -0.93
$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ [10 <sup>-11</sup> ]	7.5 <sup>+1.8</sup> <sub>-2.0</sub>	+2.5 -2.4	+3.2 -2.7
$ V_{ud} $	0.97383 <sup>+0.00024</sup> <sub>-0.00023</sub>	+0.00047 -0.00047	+0.00071 -0.00071
$ V_{us} $	0.2272 <sup>+0.0010</sup> <sub>-0.0010</sub>	+0.0020 -0.0020	+0.0030 -0.0030
$ V_{ub} $ [10 <sup>-3</sup> ]	3.82 <sup>+0.15</sup> <sub>-0.15</sub>	+0.31 -0.29	+0.49 -0.44
$ V_{ub} $ [10 <sup>-3</sup> ] (meas. not in fit)	3.64 <sup>+0.19</sup> <sub>-0.18</sub>	+0.39 -0.36	+0.60 -0.55
$ V_{cd} $	0.22712 <sup>+0.00099</sup> <sub>-0.00103</sub>	+0.00199 -0.00205	+0.00300 -0.00307
$ V_{cs} $	0.97297 <sup>+0.00024</sup> <sub>-0.00023</sub>	+0.00048 -0.00047	+0.00071 -0.00071
$ V_{cb} $ [10 <sup>-3</sup> ]	41.79 <sup>+0.63</sup> <sub>-0.63</sub>	+1.26 -1.27	+1.89 -1.90
$ V_{cb} $ [10 <sup>-3</sup> ] (meas. not in fit)	44.9 <sup>+1.2</sup> <sub>-2.8</sub>	+2.4 -5.7	+3.8 -7.7
$ V_{td} $ [10 <sup>-3</sup> ]	8.28 <sup>+0.33</sup> <sub>-0.29</sub>	+0.92 -0.57	+1.38 -0.86
$ V_{ts} $ [10 <sup>-3</sup> ]	41.13 <sup>+0.63</sup> <sub>-0.62</sub>	+1.25 -1.24	+1.87 -1.86
$ V_{tb} $	0.999119 <sup>+0.000026</sup> <sub>-0.000027</sub>	+0.000052 -0.000054	+0.000078 -0.000082
$ V_{td}/V_{ts} $	0.2011 <sup>+0.0081</sup> <sub>-0.0065</sub>	+0.0230 -0.0127	+0.0345 -0.0195
$ V_{ud}V_{ub}^* $ [10 <sup>-3</sup> ]	3.72 <sup>+0.15</sup> <sub>-0.14</sub>	+0.30 -0.29	+0.48 -0.43
arg $[V_{ud}V_{ub}^*]$ (deg)	59.8 <sup>+4.9</sup> <sub>-4.0</sub>	+13.9 -7.8	+20.9 -12.1
arg $[-V_{ts}V_{tb}^*]$ (deg)	1.043 <sup>+0.061</sup> <sub>-0.057</sub>	+0.151 -0.114	+0.238 -0.176
$ V_{cd}V_{cb}^* $ [10 <sup>-3</sup> ]	9.49 <sup>+0.15</sup> <sub>-0.15</sub>	+0.30 -0.30	+0.45 -0.45
arg $[-V_{cd}V_{cb}^*]$ (deg)	0.0339 <sup>+0.0021</sup> <sub>-0.0020</sub>	+0.0050 -0.0040	+0.0077 -0.0060
$ V_{td}V_{tb}^* $ [10 <sup>-3</sup> ]	8.27 <sup>+0.33</sup> <sub>-0.29</sub>	+0.93 -0.57	+1.38 -0.85
arg $[V_{td}V_{tb}^*]$ (deg)	-22.84 <sup>+1.00</sup> <sub>-0.99</sub>	+1.98 -2.02	+2.93 -3.21
Re $\lambda_c$	-0.22098 <sup>+0.00095</sup> <sub>-0.00091</sub>	+0.00188 -0.00184	+0.00282 -0.00275
Im $\lambda_c$ [10 <sup>-4</sup> ]	-1.377 <sup>+0.080</sup> <sub>-0.084</sub>	+0.161 -0.203	+0.244 -0.310
Re $\lambda_t$ [10 <sup>-4</sup> ]	-3.11 <sup>+0.13</sup> <sub>-0.14</sub>	+0.26 -0.36	+0.39 -0.57
Im $\lambda_t$ [10 <sup>-4</sup> ]	1.377 <sup>+0.084</sup> <sub>-0.080</sub>	+0.203 -0.161	+0.310 -0.244

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