Inclusive semileptonic B decays

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The present status of the measurement of the semileptonic B decays is reviewed. Emphasis is given on the factors affecting experimental errors of the Cabibbo-Kobayashi-Masawa matrix element $|V_{cb}|$ and $|V_{ub}|$.

1. Introduction

In the framework of the Standard Model, the quark sector is characterised by a rich pattern of flavour-changing transitions, described by the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The quark transitions $B \to X_c \ell\nu$ and $B \to X_u \ell\nu$ provide a way for determining these CKM matrix elements. This report reviews the experimental measurements used to extract the quark mixing parameters $|V_{cb}|$ and $|V_{ub}|$ from the inclusive branching fractions of semileptonic b-hadron decays $Br(B \to X\ell\nu)$. At present, the values of $|V_{qb}|$ obtained from inclusive semileptonics decays are more precise than the corresponding exclusive determinations.

All theoretical approaches used to extract $|V_{qb}|$ exploit the fact that the mass of the b quark is large compared to the scale Λ_{QCD} that determines low-energy hadronic physics. The basis for precise calculations is the expansion in powers of Λ_{QCD}/m_b , where effective-field-theory methods are used to separate non-perturbative from perturbative contributions.

Several studies have shown that the spectator model decay rate is the leading term in a well-defined expansion controlled by the parameter Λ_{QCD}/m_b. Non-perturbative corrections to this leading approximation arise only to order $1/m_b^2$. The key issue in this approach is the ability to separate non-perturbative corrections, that can be expressed as a series in powers of $1/m_b$, and perturbative corrections, expressed in powers of α_s. There are different methods [1] [2] [3] [4] [5] [6] to handle the energy scale μ used to separate long-distance from short-distance physics and they are discussed in the theory part of these proceedings. The coefficients of the $1/m_b$ power terms are expectation values of operators that include non-perturbative physics.

In this framework, non-perturbative corrections are parameterized by quark masses and matrix elements of higher dimensional operators that at present are poorly known. The experimental accuracy already achieved, and the one expected from the large data sets recorded by the B-factories, make the ensuing theory uncertainty a major limiting factor. The extraction of the non-perturbative parameters, describing the heavy quark masses, kinetic energy of the b quark and the $1/m_b^3$ corrections, directly from the data has therefore become a key issue. Recently, a lot of progress has been made on the use of the moments of energy and mass spectra in $B \to X_c \ell\bar{\nu}$ and $B \to X_s\gamma$ for performing these determinations.

2. $B \to X_c \ell\nu$

The determination of $|V_{cb}|$ from inclusive decays currently has an error of 2%, dominated by the knowledge of higher order perturbative and non-perturbative corrections. This method is based on the measurement of the total semileptonic decay rate, together with the leptonic energy and the hadronic invariant mass spectra of inclusive semileptonic decays and $B \to s\gamma$ transitions.

The non-calculable non-perturbative quantities are parametrised in terms of expectation values of hadronic matrix elements, which can be related to the shape (moments) of inclusive decay spectra. The shape of the lepton energy spectrum and of the hadronic mass spectrum provides constraints on the heavy quark expansion based on local Operator Product Expansion (OPE) that describes the properties of the $Br(B \to X_c \ell\nu)$ transitions.

So far, measurements of the hadronic mass distribution and the leptonic spectrum have been made by Babar, Belle, Cleo and DELPHI, while CDF provide only the measurement of the hadronic mass spectum [8] [9] [10] [11] [12] [13].

The measured hadronic mass distribution and lepton energy spectrum are affected by detector resolution, accessible phase space, radiative corrections, etc. It is particularly important for reducing both theoretical and experimental uncertainties to have the largest accessible phase space in which to measure the decay spectra. Each of the respective experiments has focused on lowering the lepton energy cut-off values.

All experiments performed measurements of the hadronic invariant mass spectrum M_X^2. Babar has measured up to the second moment of this distribution, DELPHI up to the third. In order to compare with theoretical predictions, the moments are measured with a well defined cut on the lepton momentum in the B rest frame.

Belle[8], Babar[9] and CLEO[14] explored the moments of the hadronic mass spectrum, as a function of the lepton momentum cuts. CLEO performs a fit for the contributions of signal and backgrounds to the full three-dimensional differential decay rate distribution as a function of the reconstructed quantities q^2, M_X^2, $\cos\theta_\ell$. Belle[8] and Babar[9] use a sample where one of the the B-mesons is fully reconstructed and the signal side is tagged by a well identified lepton. In this case the main sources of systematic errors are the uncertainties related the detector and signal modelling, and to reconstruction.

The hadronic mass spectrum in $B \to X_c \ell\nu$ decays can be split into three contributions corresponding to D, D* , and D**, where D** stands for any neutral charmed state, resonant or not, other than D and D*. Belle reconstructs the full hadronic mass spectrum and derives M_X^2 with an unfolding technique. Babar extract the moments from the measured distributions using a calibration curve derived from Monte Carlo data. Employing this method Babar reaches a minimum momentum for the electron in the B meson rest frame of 0.9 GeV, while Belle reaches 0.7 GeV, Figure 1.

DELPHI follow a different approach in extracting the moments. The D and D* component of the mass spectrum is well known, hence DELPHI measures the invariant mass distribution of the D** component only and fix the D and D* components. In fact, the D** component is not well known. For this reason DELPHI measures the first moment with respect to the spin averaged mass of D and D*. At LEP b-quarks were created with an energy of approximately 30 GeV. The large boost allowed DELPHI to measure the hadronic mass moments without a cut on the lepton energy [11].

The shape of the lepton spectrum provides further constraints on the OPE. Even if these measurements are less sensitive to OPE parameters they are considerably more precise experimentally. Moments of the lepton momentum with a cut $p_\ell \geq 1.0$ GeV/c have been measured by the CLEO collaboration [15]. Babar [10] extract up to the third moment of this distribution, using a low momentum cut of $p_\ell \geq 0.6$ GeV/c. Both Babar and CLEO use dilepton samples.

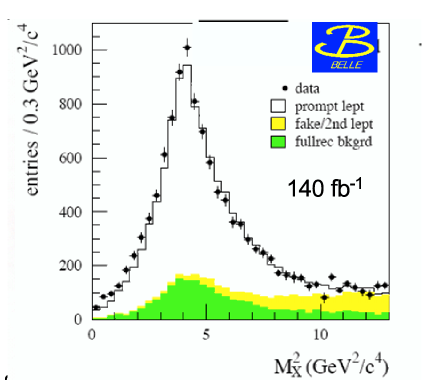

Figure 1: Belle hadronic invariant mass spectrum

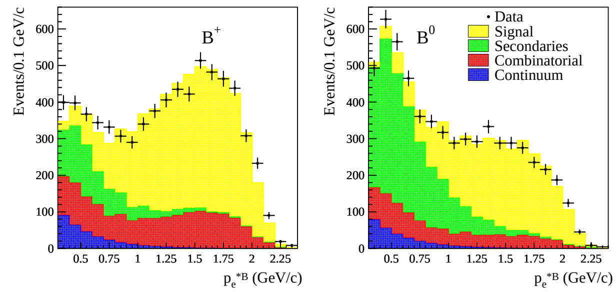

Figure 2: Belle:electron energy spectra for B^0 and B^+.

Belle [8] determines the first, second and third moments of the electron energy spectrum with a minimum lepton momentum cut ranging between 0.4 and 1.5 GeV in the B meson rest frame. Belle measures, independently, the electron energy moments for the semileptonic decays of the B^+ and the B^0 mesons [8], Figure 2. In Belle, events are selected by fully reconstructing one of the B mesons, produced in pairs from $\Upsilon(4S)$, in several hadronic decay modes. Prompt semileptonic decays ($b \to x\ell\nu$) of the non-tag side B mesons are separated from cascade charm decays ($B \to X_c \to X\ell\nu$), based on the correlation between the flavour of the tagged B and the lepton charge. DELPHI calculates up to the third moment for the lepton energy spectrum, without an explicit lepton momentum cut [11].

All the lepton moment measurements are consistent with theory and with the moment of the hadronic and $b \to s\gamma$ spectrum. Hadronic and lepton energy measurements are consistent within their errors. When compared with theory there is no sign of inconsistencies.

A global fit to all available hadron mass, electron energy and photon energy moments in the 1S scheme has been performed in [16], giving:

$$|V_{cb}| = (41.4 \pm 0.6 \pm 0.1) \times 10^{-3} \quad (1)$$

$$m_b^{1S} = 4.68 \pm 0.03 GeV \quad (2)$$

$$\lambda_1^{1S} = 0.27 \pm 0.04 GeV \quad (3)$$

where the first error includes experimental and theoretical uncertainties and the second error on $|V_{cb}|$ is due to the B lifetime.

A more updated set of data has been fitted in the kinetic scheme, resulting in [18]:

$$|V_{cb}| = (42.0 \pm 0.2 \pm 0.1) \times 10^{-3} \quad (4)$$

$$m_b^{kin} = 4.59 \pm 0.04 GeV \quad (5)$$

$$\mu_\pi^2 = 0.406 \pm 0.042 GeV \quad (6)$$

where the first error includes statistical and theoretical uncertainties and the second error on $|V_{cb}|$ is from the estimated accuracy of the HQE for the total semileptonic rate.

In both fits, the value of the b and c quark masses are in good agreement with independent determination of the same parameters.

3. $B \to X_u \ell \nu$

The decay rate for $B \to X_u \ell \nu$ is proportional to $|V_{ub}|^2$ and m_b^5. The theoretical description of inclusive $B \to X_u \ell \nu$ decays is based on the Heavy Quark Expansion, as for $B \to X_c \ell \nu$ decays [21, 22], which predicts the total decay rate with uncertainties of about 5%.

Experimentally, the principal challenge is to separate the signal $B \to X_u \ell \nu$ decays from the 50 times larger $B \to X_c \ell \nu$ background. This can be achieved by selecting regions of phase space in which this background is highly suppressed. In these regions the spectra are affected by the distribution of the b-quark momentum inside the B meson, which can be described by a structure or "shape function" (SF) [23, 24], in addition to weak annihilation and other non-perturbative effects.

Extrapolation from the limited momentum range near the endpoint to the full spectrum is a difficult task. The shape function is a universal property of B mesons at leading order [23, 24]. Sub-leading shape functions [25]-[31] arise at each order in $1/m_b$ and differ between semileptonic and radiative B decays. Several functional forms for the SF, which generally depend on two parameters related to the mass and kinetic energy of the b-quark, Λ or m_b , and λ_1 or π^2, have been proposed. The values and precise definitions of these parameters depend on the specific ansatz for the SF, the mass renormalization scheme, and the renormalization scale chosen.

In inclusive measurements, the most common kinematic variables discussed in the literature, each having their own advantages, are the lepton energy (E_e), the hadronic invariant mass (M_X), the leptonic invariant mass squared (q^2) and the light-cone momentum component $P_+ = E_X - |P_X|$. In all cases, the experiments need to model $B \to X_u \ell \nu$ decays in order to calculate acceptances and efficiencies.

3.1. Endpoint measurements

The first $|V_{ub}|$ measurement performed with this technique was by CLEO. In the rest frame of the B meson, the kinematic endpoint of the electron spectrum is about 2.3 GeV/c for the dominant $B \to X_c \ell \nu$ decays and about 2.6 GeV/c for $B \to X_u \ell \nu$ decays.

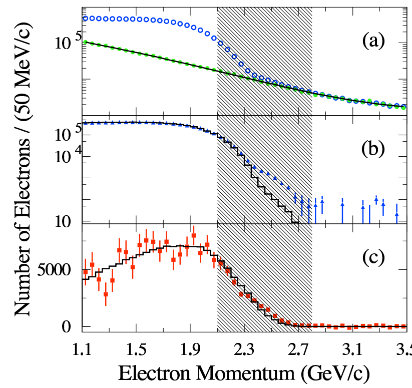

Figure 3: BaBar: Electron momentum spectra in the $\Upsilon(4S)$ rest frame: (a) on-resonance data (open circles) and scaled off-resonance data (solid circles); (b) on-resonance data after non-B$\bar{\text{B}}$ background subtraction (triangles) compared to simulated B$\bar{\text{B}}$ background (histogram); (c) on-resonance data after subtraction of all backgrounds (data points), compared to the simulated $B \to X_u \ell \nu$ signal spectrum (histogram).

The primary challenge in reducing the lepton momentum cut in the endpoint method is controlling the $B \to X_c \ell \nu$ background at the required precision. The spectrum above 2.3 GeV/c is dominated by electrons from $B \to X_u \ell \nu$ transitions, and this allows for a relatively precise measurement, largely free from B$\bar{\text{B}}$ background, in a 300 MeV/c interval that covers approximately 10% of the total electron spectrum for charmless semileptonic B decays. Figure 3 shows the Babar lepton energy spectrum distribution for the signal and background. Belle extracts the $B \to X_u \ell \nu$ signal in the momentum region $1.9 - 2.6$ GeV/c, Babar covers $2 - 2.6$ GeV/c and CLEO $2.3 - 2.6$ GeV/c [32–34].

3.2. M_X, q^2 and P_+

As described earlier, this method relies on a sample of events where one of the B mesons from the $\Upsilon(4S)$ decay is fully reconstructed, while the semileptonic decay of the signal side B meson, is identified by the presence of a high momentum electron or muon. With this sample we are able to construct all kinematic variables: the invariant meson mass M_X, the lepton neutrino mass squared q^2, and the hadronic light-cone momentum P_+, which are the best available discriminators of signal and background in inclusive $|V_{ub}|$ analyses.

BABAR selected semileptonic $B \to X_u \ell \nu$ decays using an approach based on simultaneous requirements for the electron energy, E_e, and the invariant mass squared of the $e\nu$ pair, q^2 [35]. The neutrino 4-momentum is reconstructed from the visible 4-momentum and knowledge of the e^+e^- initial state.

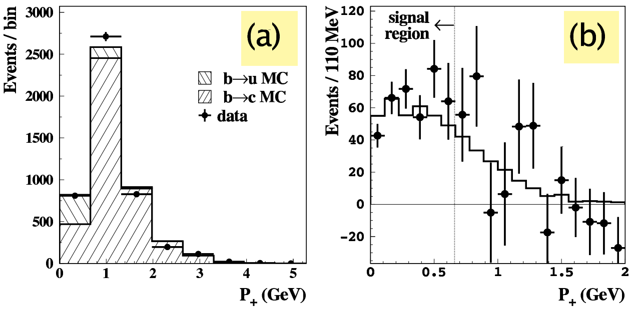

Figure 4: The electron momentum spectrum in the $\Upsilon(4S)$ restframe: (a) P_+ distribution for the selected events. (d) P_+ distribution after subtracting the $B \to X_c \ell \nu$ contribution.

The dominant charm background is suppressed by selecting a region of the q^2 -E_e phase space where correctly reconstructed $B \to X_c \ell \nu$ events are kinematically excluded.

Theoretical studies indicate that it is possible to reduce the theoretical error on the extrapolation by applying simultaneous cuts on $M_X$and q^2 in inclusive $B \to X_u \ell \nu$ decays [36]. In fact, while the M_X distribution has a large usable fraction of events, of the order of 70%, but depends on the shape function describing the Fermi motion of the b quark inside the B meson, the q^2 distribution is less sensitive to non-perturbative effects and less dependent on the calculation. Unfortunately, only a small fraction of events (about 20%) are usable with a pure q^2 selection. The study in [36] shows that combined cuts on M_X and q^2 mitigate the drawbacks of the two methods while retaining good statistical and systematic sensitivities. Babar performed a measurement of the $|V_{ub}|$ CKM matrix element, on the fully reconstructed B sample, by using the combined information of the M_X-q^2 distribution to discriminate signal and background and to minimize the theoretical uncertainties [37].

Belle has measured partial rates with cuts on M_X, q^2 , E_e, but the highlight is the first measurement of the light-cone momentum P_+ [38]. Belle measures the partial branching fraction in the kinematic region: P_+ <0.66 GeV/c, Figure 4. The direct measurement of P_+ is quite important as the shape function becomes influential when P_+ is not large compared to Λ_{QCD} .

3.3. Reduced Model Dependence

Babar used two new techniques to extract $|V_{ub}|$ from inclusive semileptonic B decays where the uncertainties due to m_b and the modeling of the Fermi motion of the b quark inside the B meson are significantly reduced [39].

Leibovich, Low, and Rothstein (LLR) presented a prescription to extract $|V_{ub}|$ with reduced model dependence from either the lepton energy or the hadronic mass M_X [40]. A technique to utilise weight functions had been proposed previously by Neubert [41] The calculations of LLR are accurate up to corrections of order α_s^2 and $(\Lambda m_B/(\xi m_b))^2$, where ξ is the experimental maximum hadronic mass up to which the $B \to X_u \ell \nu$ decay rate is determined and $\Lambda \sim \Lambda$QCD. This method combines the hadronic mass spectrum, integrated below ξ , with the high-energy end of the measured differential $B \to s\gamma$ photon energy spectrum via the calculations of LLR. An alternative method [42] to reduce the model dependence is to measure the $B \to X_u \ell \nu$ rate over the entire M_X spectrum. Since no extrapolation is necessary to obtain the full rate, systematic uncertainties from m_b and Fermi motion are considerably reduced. Perturbative corrections are known to order α_s^2. Babar extracts the $B \to X_u \ell \nu$ rate from the hadronic mass spectrum up to $\xi = 2.5$ GeV/c2 which corresponds to about 96% of the simulated hadronic mass spectrum, Figure 5.

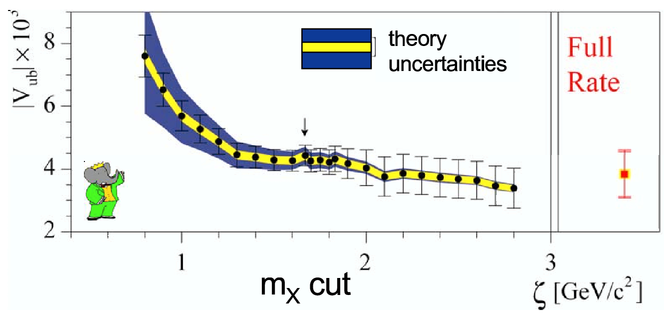

Figure 5: $|V_{ub}|$ as a function of ξ with LLR method (left) and for the determination with the full rate measurement (right). The errors are statistical only. The shaded area illustrates the theoretical uncertainty; the inner light shaded (yellow) area indicates the perturbative part of the uncertainty. The arrow indicates ξ =1.67GeV/c^2.

Using the weighting technique with the photon energy spectrum in $B \to s\gamma$ in Ref. [43] the hadronic mass spectrum up to a value of ξ= 1.67GeV/c2, Babar finds $|V_{ub}|/|V_{ts}| = 0.107 \pm 0.009_{stat} \pm 0.006_{syst} \pm 0.007_{theo}$. Assuming the CKM matrix is unitary gives $|V_{ts}| = |V_{cb}| \times (1 \pm O(1\%))$ and taking $|V_{cb}|$ from [18] one gets: $|V_{ub}| = (4.43 \pm 0.38 \pm 0.25 \pm 0.29) \times 10^{-3}$, where the first error is the statistical uncertainty, the second is systematic, the third theoretical. Babar also determines $|V_{ub}|$ from the full M_X spectrum, i.e., up to a value of ξ=2.5GeV/c^2, and find $|V_{ub}| = (3.84 \pm 0.70_{stat} \pm 0.30_{sys} \pm 0.10_{theo}) \times 10^{-3}$.

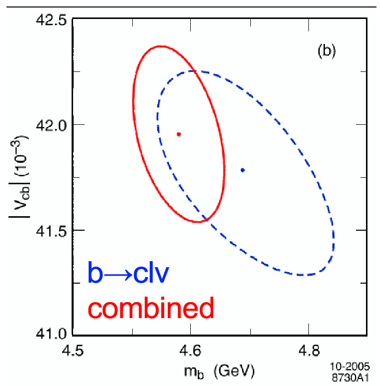

Figure 6: Kinetic mass fit from which the heavy quark parameters needed in calculating $B \to X_u \ell \nu$ partial rates are derived. The dotted curve shows the fit using information form $B \to X_c \ell \nu$ only, the red line includes information from $B \to s\gamma$.

3.4. HQE parameters and shape function input

The global fits to $B \to X_c \ell \nu$ and $B \to s\gamma$ moments in ref [18] discussed earlier provide input values for the heavy quark parameters needed in calculating $B \to X_u \ell \nu$ partial rates and to constrain the first and second moments of the shape function. Additional information on the leading shape function and HQE parameters is obtained from the photon energy spectrum in $B \to s\gamma$ decays. The results of the global fit are shown in Figure 6.

3.5. $|V_{ub}|$ extraction

The CKM parameter $|V_{ub}|$ is obtained directly from the partial branching fraction using

$$|V_{ub}|^2 = \Delta B(B \to X_u \ell \nu)/(R\tau_B),$$

where τ_B is the average B lifetime. R is the theoretical prediction of the partial rate and is calculated for a given signal region by using an inclusive $B \to X_u \ell \nu$ decay generator. The extracted values of $|V_{ub}|$ are given in Figure 7.

The world average is determined by HFAG. We have chosen the value extracted using the BLNP [44] theoretical framework as the world average of the CKM parameter $|V_{ub}|$. The value of m_b used in the world average is from the global fit in the Kinetic scheme to measurements of $B \to X_c \ell \nu$ and $B \to s\gamma$ decays.

HFAG also extracts $|V_{ub}|$ using the Dressed Gluon Exponentiation [45] framework, a recent new addition to the phenomenology landscape of inclusive B-meson decays. In this framework the on-shell b-quark calculation, converted into hadronic variables, can be directly used as an approximation to the meson decay spectrum, without need of a leading-power non-perturbative function i.e no shape function. The on-shell mass of the b-quark within the B-meson is required as an input (m_b). Theoretical uncertainties are assessed by varying m_b, the strong coupling constant (α_s), the number of light fermion flavours, and the method and scale of the matching scheme intrinsic to the approach. As the DGE error is not yet fully calculated (e.g. the weak anihilation contribution is missing), we prefer not to quote this method as the principal one. It must be noted that the value of $|V_{ub}|$ obtained within the DGE framework agrees very well with the BLNP determination.

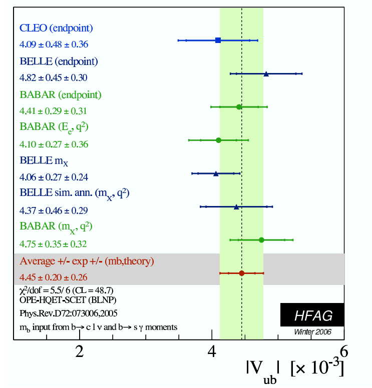

Figure 7: The world average for $|V_{ub}|$

At present, as indicated by the average given above, the uncertainty on $|V_{ub}|$ is at the 7% level. The uncertainty on m_b used here is 40 MeV, contributing an uncertainty of 4.5% on $|V_{ub}|$.

4. Conclusions

The global fits to the semileptonic and radiative decay spectral moments are used to extract the CKM matrix element $|V_{cb}|$ with 2% precision dominated by HQE measurements.

Various methods to extract $|V_{ub}|$ are used, giving a 7% precision in the BLNP scheme, dominated by HQE parameters.

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