

# Standard Model expectations on $\sin 2\beta(\phi_1)$ from $b \rightarrow s$ penguins

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Recent results of the standard model expectations on  $\sin 2\beta_{\text{eff}}$  from penguin-dominated  $b \rightarrow s$  decays are briefly reviewed.

## 1. Introduction

Although the Standard Model is very successful, New Physics is called for in various places, such as neutrino-oscillation, dark matter (energy) and baryon-asymmetry. Possible New Physics beyond the Standard Model is being intensively searched via the measurements of time-dependent CP asymmetries in neutral  $B$  meson decays into final CP eigenstates defined by

$$\frac{\Gamma(\overline{B}(t) \rightarrow f) - \Gamma(B(t) \rightarrow f)}{\Gamma(\overline{B}(t) \rightarrow f) + \Gamma(B(t) \rightarrow f)} = \mathcal{S}_f \sin(\Delta mt) + \mathcal{A}_f \cos(\Delta mt), \quad (1)$$

where  $\Delta m$  is the mass difference of the two neutral  $B$  eigenstates,  $\mathcal{S}_f$  monitors mixing-induced CP asymmetry and  $\mathcal{A}_f$  measures direct CP violation. The CP-violating parameters  $\mathcal{A}_f$  and  $\mathcal{S}_f$  can be expressed as

$$\mathcal{A}_f = -\frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \quad \mathcal{S}_f = \frac{2 \text{Im}\lambda_f}{1 + |\lambda_f|^2}, \quad (2)$$

where

$$\lambda_f = \frac{q_B}{p_B} \frac{A(\overline{B}^0 \rightarrow f)}{A(B^0 \rightarrow f)}. \quad (3)$$

In the standard model  $\lambda_f \approx \eta_f e^{-2i\beta}$  for  $b \rightarrow s$  penguin-dominated or pure penguin modes with  $\eta_f = 1$  ( $-1$ ) for final CP-even (odd) states. Therefore, it is expected in the Standard Model that  $-\eta_f \mathcal{S}_f \approx \sin 2\beta$  and  $\mathcal{A}_f \approx 0$  with  $\beta$  being one of the angles of the unitarity triangle.

The mixing-induced CP violation in  $B$  decays has been already observed in the golden mode  $B^0 \rightarrow J/\psi K_S$  for several years. The current world average the mixing-induced asymmetry from tree  $b \rightarrow c\bar{c}s$  transition is [1]

$$\sin 2\beta = 0.687 \pm 0.032. \quad (4)$$

However, the time-dependent CP-asymmetries in the  $b \rightarrow sq\bar{q}$  induced two-body decays such as  $B^0 \rightarrow (\phi, \omega, \pi^0, \eta', f_0)K_S$  are found to show some indications of deviations from the expectation of the Standard Model (SM) [1] (see Fig. 1). In the SM, CP asymmetry in all above-mentioned modes should be equal to  $\mathcal{S}_{J/\psi K}$  with a small deviation *at most*

$\mathcal{O}(0.1)$  [2]. As discussed in [2], this may originate from the  $\mathcal{O}(\lambda^2)$  truncation and from the subdominant (color-suppressed) tree contribution to these processes. Since the penguin loop contributions are sensitive to high virtuality, New Physics beyond the SM may contribute to  $\mathcal{S}_f$  through the heavy particles in the loops. In order to detect the signal of New Physics unambiguously in the penguin  $b \rightarrow s$  modes, it is of great importance to examine how much of the deviation of  $\mathcal{S}_f$  from  $\mathcal{S}_{J/\psi K}$ ,

$$\Delta \mathcal{S}_f \equiv -\eta_f \mathcal{S}_f - \mathcal{S}_{J/\psi K_S}, \quad (5)$$

is allowed in the SM [2–15].

The decay amplitude for the pure penguin or penguin-dominated charmless  $B$  decay in general has the form

$$M(\overline{B}^0 \rightarrow f) = V_{ub}V_{us}^*F^u + V_{cb}V_{cs}^*F^c + V_{tb}V_{ts}^*F^t. \quad (6)$$

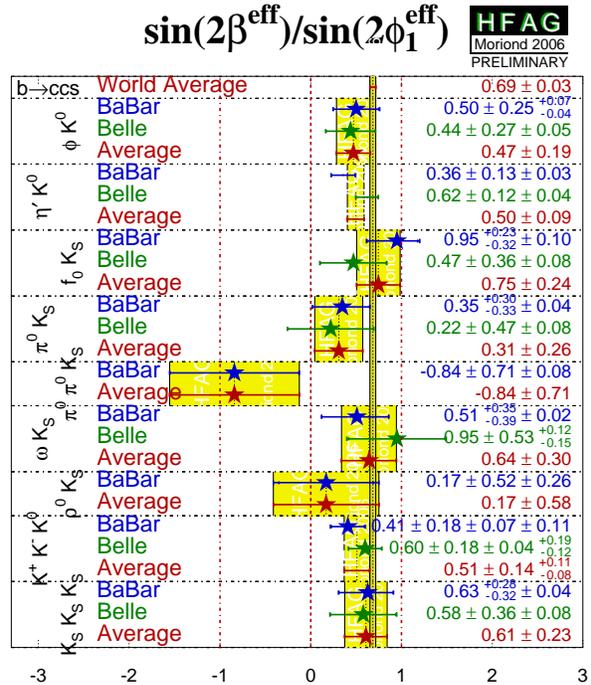


Figure 1: Experimental results for  $\sin 2\beta_{\text{eff}}$  from  $b \rightarrow s$  penguin decays [1].

Unitarity of the CKM matrix elements leads to

$$M(\bar{B}^0 \rightarrow f) = V_{ub}V_{us}^*A_f^u + V_{cb}V_{cs}^*A_f^c \approx \lambda^4 R_b e^{-i\gamma} A_f^u + \lambda^2 A_f^c, \quad (7)$$

where  $A_f^u = F^u - F^t$ ,  $A_f^c = F^c - F^t$ ,  $R_b \equiv |V_{ud}V_{ub}|/|V_{cd}V_{cb}| = \sqrt{\bar{\rho}^2 + \bar{\eta}^2}$ . The first term is suppressed by a factor of  $\lambda^2$  relative to the second term. For a pure penguin decay such as  $B^0 \rightarrow \phi K^0$ , it is naively expected that  $A_f^u$  is in general comparable to  $A_f^c$  in magnitude. Therefore, to a good approximation  $-\eta_f S_f \approx \sin 2\beta \approx S_{J/\psi K}$ . For penguin-dominated modes such as  $\omega K_S, \rho^0 K_S, \pi^0 K_S$ ,  $A_f^u$  also receives tree contributions from the  $b \rightarrow u\bar{u}s$  tree operators. Since the Wilson coefficient for the penguin operator is smaller than the one for the tree operator,  $A_f^u$  could be significantly larger than  $A_f^c$ . As the first term carries a weak phase  $\gamma$ , it is possible that  $S_f$  is subject to a significant ‘‘tree pollution’’. To quantify the deviation, it is known that to the first order in  $r_f \equiv (\lambda_u A_f^u)/(\lambda_c A_f^c)$  [5, 16]

$$\Delta\mathcal{S}_f = 2|r_f| \cos 2\beta \sin \gamma \cos \delta_f, \quad \mathcal{A}_f = 2|r_f| \sin \gamma \sin \delta_f,$$

with  $\delta_f = \arg(A_f^u/A_f^c)$ . Hence, the magnitude of the CP asymmetry difference  $\Delta\mathcal{S}_f$  and direct CP violation are both governed by the size of  $A_f^u/A_f^c$ . However, for the aforementioned penguin-dominated modes, the tree contribution is color suppressed and hence in practice the deviation of  $\mathcal{S}_f$  is expected to be small [2]. It is useful to note that  $\Delta\mathcal{S}_f$  is proportional to the real part of  $A_f^u/A_f^c$  as shown in the above equation.

Below I will review the results of the SM expectations on  $\Delta\mathcal{S}_f$  from short-distance and long-distance calculations. Recent reviews of results obtained from the SU(3) approach can be found in [17].

## 2. $\Delta\mathcal{S}_f$ from short-distance calculations

There are several QCD-based approaches in calculating hadronic  $B$  decays [18–20].  $\Delta\mathcal{S}_f$  from calculations of QCDF [9, 10], pQCD [11], SCET [12] are shown in Table 1. The QCDF calculations on  $PP$ ,  $VP$  modes are from [9]<sup>1</sup>, while those in  $SP$  modes are from [10]. It is interesting to note that (i)  $\Delta\mathcal{S}_f$  are small and positive in most cases, while experimental central values for  $\Delta\mathcal{S}_f$  are all negative, except the one from  $f_0 K_S$ ; (ii) QCDF and pQCD results agree with each other, since the main difference of these two approaches is the (penguin) annihilation contribution, which hardly affects  $S_f$ ; (iii) The SCET results involve some non-perturbative contributions fitted from

Table I  $\Delta\mathcal{S}_f$  from various short-distance calculations.

$\Delta\mathcal{S}_f$	QCDF	pQCD	SCET	Expt
$\phi K_S$	$0.02 \pm 0.01$	$0.020_{-0.008}^{+0.005}$		$-0.22 \pm 0.19$
$\omega K_S$	$0.13 \pm 0.08$			$-0.06 \pm 0.30$
$\rho^0 K_S$	$-0.08_{-0.12}^{+0.08}$			$-0.52 \pm 0.58$
$\eta' K_S$	$0.01 \pm 0.01$		$-0.02 \pm 0.01$ $-0.01 \pm 0.01$	$-0.19 \pm 0.09$
$\eta K_S$	$0.10_{-0.07}^{+0.11}$		$-0.03 \pm 0.17$ $+0.07 \pm 0.14$	
$\pi^0 K_S$	$0.07_{-0.04}^{+0.05}$	$0.06_{-0.03}^{+0.02}$	$0.08 \pm 0.03$	$-0.38 \pm 0.26$
$f_0 K_S$	$0.02 \pm 0.00$			$+0.06 \pm 0.24$
$a_0 K_S$	$0.02 \pm 0.01$			
$\bar{K}_0^{*0} \pi^0$	$0.00_{-0.05}^{+0.03}$ $0.02_{-0.02}^{+0.00}$			

data. These contributions affect  $\Delta\mathcal{S}_f$  and give results in the  $\eta' K_S$  mode different from the QCDF ones.

It is instructive to understand the size and sign of  $\Delta\mathcal{S}_f$  in the QCDF approach [9], for example. Recall that  $\Delta\mathcal{S}_f$  is proportional to the real part of  $A_f^u/A_f^c$ . We follow [9] to denote a complex number  $x$  by  $[x]$  if  $\text{Re}(x) > 0$ . In QCDF the dominant contributions to  $A_f^u/A_f^c$  are basically given by [9, 21]

$$\begin{aligned} \frac{A_{\phi K_S}^u}{A_{\phi K_S}^c} &\sim \frac{[-(a_4^u + r_\chi^\phi a_6^u)]}{[-(a_4^c + r_\chi^\phi a_6^c)]} \sim \frac{[-P^u]}{[-P^c]}, \\ \frac{A_{\omega K_S}^u}{A_{\omega K_S}^c} &\sim \frac{+[a_4^u - r_\chi^\phi a_6^u] + [a_2^u R]}{+[a_4^c - r_\chi^\phi a_6^c]} \sim \frac{+[P^u] + [C]}{+[P^c]}, \\ \frac{A_{\rho K_S}^u}{A_{\rho K_S}^c} &\sim \frac{-[a_4^u - r_\chi^\phi a_6^u] + [a_2^u R]}{-[a_4^c - r_\chi^\phi a_6^c]} \sim \frac{-[P^u] + [C]}{-[P^c]}, \quad (8) \\ \frac{A_{\pi^0 K_S}^u}{A_{\pi^0 K_S}^c} &\sim \frac{[-(a_4^u + r_K^\phi a_6^u)] + [a_2^u R']}{[-(a_4^c + r_K^\phi a_6^c)]} \sim \frac{[-P^u] + [C]}{[-P^c]}, \\ \frac{A_{\eta' K_S}^u}{A_{\eta' K_S}^c} &\sim \frac{-[-(a_4^u + r_K^\phi a_6^u)] + [a_2^u R'']}{-[-(a_4^c + r_K^\phi a_6^c)]} \sim \frac{[-P^u] - [C]}{[-P^c]}, \end{aligned}$$

where  $a_i^p$  are effective Wilson coefficients<sup>2</sup>,  $r_\chi = O(1)$  are the chiral factors and  $R^{(t, \prime)}$  are (real and positive) ratios of form factors and decay constants.

From Eq.(8), it is clear that  $\Delta\mathcal{S}_f > 0$  for  $\phi K_S, \omega K_S, \pi^0 K_S$ , since their  $\text{Re}(A_f^u/A_f^c)$  can only be positive. Furthermore, due to the cancellation between  $a_4$  and  $r_\chi a_6$  in the  $\omega K_S$  amplitude, the corresponding penguin contribution is suppressed. This leads to a large and positive  $\Delta\mathcal{S}_{\omega K_S}$  as shown in Table I. For the cases of  $\rho^0 K_S$  and  $\eta' K_S$ , there are chances for  $\Delta\mathcal{S}_f$  to be positive or negative. The different signs

<sup>1</sup>Results obtained agree with those in [13].

<sup>2</sup>In general, we have  $\text{Re}(a_2) > 0$ ,  $\text{Re}(a_6) < \text{Re}(a_4) < 0$ .

Table II Direct CP asymmetry parameter  $\mathcal{A}_f$  and the mixing-induced CP parameter  $\Delta\mathcal{S}_f^{SD+LD}$  for various modes. The first and second theoretical errors correspond to the SD and LD ones, respectively [13].

Final State	$\Delta\mathcal{S}_f$			$\mathcal{A}_f$ (%)		
	SD	SD+LD	Expt	SD	SD+LD	Expt
$\phi K_S$	$0.02^{+0.01}_{-0.02}$	$0.04^{+0.01+0.01}_{-0.02-0.02}$	$-0.22 \pm 0.19$	$0.8^{+0.5}_{-0.2}$	$-2.3^{+0.9+2.2}_{-1.0-5.1}$	$9 \pm 14$
$\omega K_S$	$0.12^{+0.06}_{-0.05}$	$0.02^{+0.03+0.03}_{-0.04-0.02}$	$-0.06 \pm 0.30$	$-6.8^{+2.4}_{-4.0}$	$-13.5^{+3.5+2.4}_{-5.7-1.5}$	$44 \pm 23$
$\rho^0 K_S$	$-0.08^{+0.03}_{-0.10}$	$-0.04^{+0.07+0.10}_{-0.10-0.12}$	$-0.52 \pm 0.58$	$7.8^{+4.5}_{-2.0}$	$48.9^{+15.8+5.8}_{-13.7-12.5}$	$-64 \pm 48$
$\eta' K_S$	$0.01^{+0.01}_{-0.02}$	$0.00^{+0.01+0.00}_{-0.02-0.00}$	$-0.19 \pm 0.09$	$1.7^{+0.4}_{-0.3}$	$2.1^{+0.2+0.1}_{-0.5-0.4}$	$7 \pm 7$
$\eta K_S$	$0.07^{+0.03}_{-0.03}$	$0.07^{+0.03+0.00}_{-0.03-0.01}$	—	$-5.7^{+2.0}_{-5.5}$	$-3.9^{+1.8+2.5}_{-5.0-1.6}$	—
$\pi^0 K_S$	$0.06^{+0.03}_{-0.03}$	$0.04^{+0.01+0.02}_{-0.02-0.02}$	$-0.38 \pm 0.26$	$-3.2^{+1.1}_{-2.3}$	$3.7^{+1.9+1.7}_{-1.6-1.7}$	$2 \pm 13$

in front of  $[P]$  in  $\rho^0 K_S$  and  $\omega K_S$  are originated from the second term of the wave functions  $(u\bar{u} \pm d\bar{d})/\sqrt{2}$  of  $\omega$  and  $\rho^0$  in the  $\bar{B}^0 \rightarrow \omega$  and  $\bar{B}^0 \rightarrow \rho^0$  transitions, respectively. The  $[P]$  in  $\rho^0 K_S$  is also suppressed as the one in  $\omega K_S$ , resulting a negative  $\Delta\mathcal{S}_{\rho^0 K_S}$ . On the other hand,  $[-P]$  in  $\eta' K_S$  is not only unsuppressed (no cancellation in the  $a_4$  and  $a_6$  terms), but, in fact, is further enhanced due to the constructive interference of various penguin amplitudes [22]. This enhancement is responsible for the large  $\eta' K_S$  rate [22] and also for the small  $\Delta\mathcal{S}_{\eta' K_S}$  [9, 13].

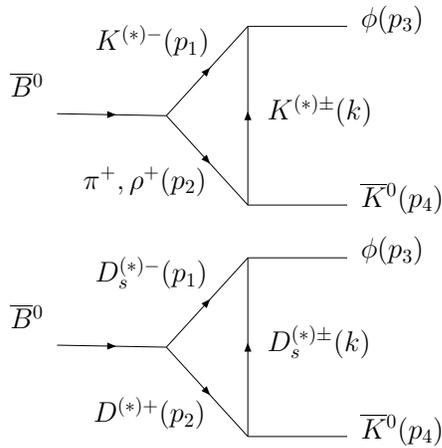
### 3. FSI contributions to $\Delta\mathcal{S}_f$

Evidence of direct CP violation in the decay  $\bar{B}^0 \rightarrow K^-\pi^+$  is now established, while the combined BaBar and Belle measurements of  $\bar{B}^0 \rightarrow \rho^\pm\pi^\mp$  imply a sizable direct CP asymmetry in the  $\rho^+\pi^-$  mode [1]. In fact, direct CP asymmetries in these channels are much bigger than expectations (of many people) and may be indicative of appreciable LD rescattering ef-

fects, in general, in  $B$  decays [23]. The possibility of final-state interactions in bringing in the possible tree pollution sources to  $\mathcal{S}_f$  are considered in [13]. Both  $A_f^u$  and  $A_f^c$  will receive long-distance tree and penguin contributions from rescattering of some intermediate states. In particular there may be some dynamical enhancement of light  $u$ -quark loop. If tree contributions to  $A_f^u$  are sizable, then final-state rescattering will have the potential of pushing  $\mathcal{S}_f$  away from the naive expectation. Take the penguin-dominated decay  $\bar{B}^0 \rightarrow \omega\bar{K}^0$  as an illustration. It can proceed through the weak decay  $\bar{B}^0 \rightarrow K^{*-}\pi^+$  followed by the rescattering  $K^{*-}\pi^+ \rightarrow \omega\bar{K}^0$ . The tree contribution to  $\bar{B}^0 \rightarrow K^{*-}\pi^+$ , which is color allowed, turns out to be comparable to the penguin one because of the absence of the chiral enhancement characterized by the  $a_6$  penguin term. Consequently, even within the framework of the SM, final-state rescattering may provide a mechanism of tree pollution to  $\mathcal{S}_f$ . By the same token, we note that although  $\bar{B}^0 \rightarrow \phi\bar{K}^0$  is a pure penguin process at short distances, it does receive tree contributions via long-distance rescattering. Note that in addition to these charmless final states contributions, there are also contributions from charmed  $D_s^{(*)}D^{(*)}$  final states, see Fig. 2. These final-state rescatterings provide the long-distance  $u$ - and  $c$ -penguin contributions.

An updated version of results in [13] are shown in Table II. Several comments are in order. (i)  $\phi K_S$  and  $\eta' K_S$  are the theoretical and experimental cleanest modes for measuring  $\sin 2\beta_{\text{eff}}$  in these penguin modes. The constructive interference behavior of penguins in the  $\eta' K_S$  mode is still hold in the LD case, resulting a tiny  $\Delta\mathcal{S}_{\eta' K_S}$ . (ii) Tree pollutions in  $\omega K_S$  and  $\rho^0 K_S$  are diluted due to the LD  $c$ -penguin contributions.

It is found that LD tree contributions are in general not large enough in producing sizable  $\Delta\mathcal{S}_f$ , since their contributions are overwhelmed by LD  $c$ -penguin contributions from  $D_s^{(*)}D^{(*)}$  rescatterings. On the other hand, while it may be possible to have a large  $\Delta\mathcal{S}_f$  from rescattering models that enhance the contributions from charmless states, a sizable direct CP vi-


 Figure 2: Final-state rescattering contributions to the  $\bar{B}^0 \rightarrow \phi\bar{K}^0$  decay.

olation will also be generated. Since direct CP violations are sensitive to strong phases generated from FSI, these approaches will also give a sizable direct CP violation at the same time when a large  $\Delta\mathcal{S}_f$  is produced. The present data on the  $\phi K_S$  and  $\eta' K_S$  modes do not support large direct CP violations in these modes. Consequently, it is unlikely that FSI will enlarge their  $\Delta\mathcal{S}_f$ . In order to constrain or to refine these calculations, it will be very useful to have more and better data on direct CP violations.

#### 4. $\Delta\mathcal{S}_f$ in $KKK$ modes

$\overline{B}^0 \rightarrow K^+K^-K_{S,L}$  and  $\overline{B}^0 \rightarrow K_S K_S K_S$  are penguin-dominated and pure penguin decays, respectively. They are also used to extract  $\sin 2\beta_{\text{eff}}$  with results shown in Fig. 1. Three-body modes are in general more complicated than two-body modes. For example, while the  $K_S K_S K_S$  mode remains as a CP-even mode, the  $K^+K^-K_{S(L)}$  mode is not a CP-eigen state<sup>3</sup>. Furthermore, the mass spectra of these modes are in general complicated and non-trivial.

A factorization approach is used to study these  $KKK$  modes [14]. In the factorization approach, the  $\overline{B}^0 \rightarrow K^+K^-K_S$  amplitude, for example, basically consists of two factorized terms:  $\langle \overline{B}^0 \rightarrow K_S \rangle \times \langle 0 \rightarrow K^+K^- \rangle$  and  $\langle \overline{B}^0 \rightarrow K^+K_S \rangle \times \langle 0 \rightarrow K^- \rangle$ , where  $\langle A \rightarrow B \rangle$  denotes a  $A \rightarrow B$  transition matrix element. The dominant contribution is from the  $\langle \overline{B}^0 \rightarrow K_S \rangle \times \langle 0 \rightarrow K^+K^- \rangle$  term, which is a penguin induced term, while

Table III Mixing-induced and direct CP asymmetries  $\Delta\mathcal{S}_f$  (top) and  $\mathcal{A}_f$  (in %, bottom), respectively, in  $B^0 \rightarrow K^+K^-K_S$  and  $K_S K_S K_S$  decays. Results for  $(K^+K^-K_L)_{CP\pm}$  are identical to those for  $(K^+K^-K_S)_{CP\mp}$ .

Final State	$\Delta\mathcal{S}_f$	Expt.
$(K^+K^-K_S)_{\phi K_S}$ excluded	$0.03^{+0.08+0.02+0.00}_{-0.01-0.01-0.02}$	$-0.12^{+0.18}_{-0.17}$
$(K^+K^-K_S)_{CP+}$	$0.05^{+0.11+0.04+0.00}_{-0.03-0.02-0.01}$	
$(K^+K^-K_L)_{\phi K_L}$ excluded	$0.03^{+0.08+0.02+0.00}_{-0.01-0.01-0.02}$	$-0.60 \pm 0.34$
$K_S K_S K_S$	$0.02^{+0.00+0.00+0.01}_{-0.00-0.00-0.02}$	$0.19 \pm 0.23$
$K_S K_S K_L$	$0.02^{+0.00+0.00+0.01}_{-0.00-0.00-0.02}$	
	$\mathcal{A}_f$ (%)	Expt.
$(K^+K^-K_S)_{\phi K_S}$ excluded	$0.2^{+1.0+0.3+0.0}_{-0.1-0.3-0.0}$	$-8 \pm 10$
$(K^+K^-K_S)_{CP+}$	$-0.1^{+0.7+0.2+0.0}_{-0.0-0.3-0.0}$	
$(K^+K^-K_L)_{\phi K_L}$ excluded	$0.2^{+1.0+0.3+0.0}_{-0.1-0.3-0.0}$	$-54 \pm 24$
$K_S K_S K_S$	$0.7^{+0.0+0.0+0.1}_{-0.1-0.0-0.1}$	$31 \pm 17$
$K_S K_S K_L$	$0.8^{+0.1+0.1+0.1}_{-0.3-0.1-0.1}$	

<sup>3</sup>However, it is found that  $K^+K^-K_S$  is dominated by the CP-even part and hence it is still useful in extracting  $\sin 2\beta_{\text{eff}}$ .

the sub-leading  $\langle \overline{B}^0 \rightarrow K^+K_S \rangle \times \langle 0 \rightarrow K^- \rangle$  term contains both tree and penguin contributions. In fact,  $\overline{B}^0 \rightarrow K^+K_S$  transition is a  $b \rightarrow u$  transition, which has a color allowed tree contribution.

Results of CP asymmetries for these modes are given in Table III. The first uncertainty is from hadronic parameter in  $\overline{B}^0 \rightarrow K^+K_{S,L}$  transition in  $K^+K^-K_{S,L}$  mode (and a similar term in  $K_S K_S K_S$  mode), the second uncertainty is from other hadronic parameters, while the last uncertainty is from the uncertainty in  $\gamma$ .

To study  $\Delta\mathcal{S}_f$  and  $\mathcal{A}_f$ , it is crucial to know the size of the  $b \rightarrow u$  transition term ( $A_f^u$ ). For the pure-penguin  $K_S K_S K_S$  mode, the smallness of  $\Delta\mathcal{S}_{K_S K_S K_S}$  and  $\mathcal{A}_{K_S K_S K_S}$  can be easily understood. For the  $K^+K^-K_S$  mode, there is a  $b \rightarrow u$  transition in the  $\langle \overline{B}^0 \rightarrow K^+K_S \rangle \times \langle 0 \rightarrow K^- \rangle$  term. It has the potential of giving large tree pollution in  $\Delta\mathcal{S}_{K^+K^-K_S}$ . It requires more efforts to study the size and the impact of this term.

It is important to note that the  $b \rightarrow u$  transition term in the  $K^+K^-K_S$  mode is not a CP self-conjugated term, since under a CP conjugation, this term will be turned into a  $\langle \overline{B}^0 \rightarrow K^-K_S \rangle \times \langle 0 \rightarrow K^+ \rangle$  term, which is, however, missing in the original amplitude. Hence, this term contributes to both CP-even and CP-odd configurations with similar strength. Therefore, information in the CP-odd part can be used to constrain its size and its impact on  $\Delta\mathcal{S}_f$  and  $\mathcal{A}_f$ . Indeed, it is found recently [24] that the CP-odd part is highly dominated by  $\phi K_S$ , where other contributions (at  $m_{K^+K^-} \neq m_\phi$ ) are highly suppressed. Since the  $\langle \overline{B}^0 \rightarrow K^+K_S \rangle \times \langle 0 \rightarrow K^- \rangle$  term favors a large  $m_{K^+K^-}$  region, which is clearly separated from the  $\phi$ -resonance region, the result of the CP-odd configuration strongly constrains the contribution from this  $b \rightarrow u$  transition term. Consequently, the tree pollution is constrained and the  $\Delta\mathcal{S}_{K^+K^-K_S}$  should not be large. Note that results shown in Table III were obtained without fully incorporating these information. The first uncertainty in Table III will be reduced, if the CP-odd result is taken into account. To further refine the results it will be very useful to perform a detail Dalitz-plot analysis.

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