Inclusive semileptonic decays of the B meson

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This talk is a short review on the theoretical issues and uncertainties in the calculation of partial decay rates in inclusive B decays. The main emphasis is on charmless semileptonic decays, the $\bar{B} \to X_s\gamma$ photon spectrum, and the extraction of $|V_{ub}|$ using model-independent methods.

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1. Introduction

Over-constraining the unitarity triangle using many different physical processes is a crucial test of the CKM picture of $\mathcal{CP}$ violation and the validity of the Standard Model. The topic of this talk is the theoretical framework and uncertainty estimation for partial rates in $\bar{B} \to X_u\, l^-\bar{\nu}$ decays and the extraction of the element V_{ub}. Its magnitude is proportional to the length of the side opposite the well-measured angle β of the unitarity triangle.

Because of an overwhelming background from $\bar{B} \to X_c\, l^-\bar{\nu}$ decays in a large portion of the phase space, experimental cuts are employed which suppress such background events. The bulk of this talk deals with the theoretical techniques used for calculating the surviving partial decay rates. We may identify two regions of phase-space for partial rates according to the integration domains in the hadronic variables

$$P_\pm = E_X \mp |\vec{P}_X|\,, \qquad (1)$$

where E_X and $\vec{P}_X$ are the energy and 3-momentum of the hadronic final state in the B meson rest frame. The available phase space in these variables is $M_\pi^2/P_- \le P_+ \le M_B - 2E_l \le P_- \le M_B$, where M_π^2 is the invariant mass of the lightest possible hadronic final state, i.e. the pion mass squared, and E_l is the energy of the charged lepton l^-. For the rest of this talk we will neglect the finite pion mass for simplicity. Note that the product $P_+P_- = M_X^2$ gives the invariant mass of the hadronic state. Hence the charmed background is eliminated as long as the phase space $P_+P_- \ge M_D^2$ is cut out. This is achieved in various ways, e.g. by imposing restrictions on the leptonic invariant mass $q^2 = (M_B - P_+)(M_B - P_-)$, or E_l, or P_+, or M_X, or combinations of them. The theoretical tools used for the calculation of such partial rates depend on whether the range of integrations are such that both P_+ and P_- are much larger than $\Lambda_{\rm QCD}$ (operator product expansion, OPE), or P_+ of order $\Lambda_{\rm QCD}$ while P_- much larger (QCD factorization theorems in this "shape-function region"). If both P_+ and P_- are small then resonances are not sufficiently smeared and the calculations are unreliable due to a breakdown of quark-hadron duality.

It is convenient to use a set of dimensionless variables which we define as $y = (P_- - P_+)/(M_B - P_+)$ and $\varepsilon = 1 - 2E_l/(M_B - P_+)$, for which the phase-space is simply $0 \le \varepsilon \le y \le 1$. The dependence of the fully differential decay rate on the charged lepton energy – or on ε in the dimensionless variables – can be separated from the dependence on hadronic variables, and the differential decay rate reads [1]

$$\begin{aligned}\frac{d^3\Gamma_u}{dP_+\,dy\,d\varepsilon} \;=\; & \frac{G_F^2|V_{ub}|^2}{16\pi^3} U_y(\mu_h,\mu_i)\,(M_B - P_+)^5 \\ & \Big[(y-\varepsilon)(1-y+\varepsilon)\,\mathcal{F}_1 \\ & + y(1-y)\,\mathcal{F}_2 + \varepsilon(y-\varepsilon)\,\mathcal{F}_3\Big]\,,\end{aligned}$$

where the structure functions $\mathcal{F}_i$ depend on both the hadronic variables P_+ and y and on the factorization scales μ_h and μ_i, but not on ε. It is worth noting that the above formula is written entirely in terms of hadronic quantities, so that at this step no uncertainty associated with our inability to determine the b-quark mass m_b is introduced.

The differential decay rate is formally independent of the factorization scales μ_h and μ_i. The scale dependence of the structure functions $\mathcal{F}_i$ cancels against the evolution factor $U_y(\mu_h,\mu_i)$, which sums (double) logarithms of the ratio of both scales and carries also dependence on y. The "natural" scaling for the scales, that avoids large logarithms, is such that $\mu_i^2 \sim \langle P_+P_-\rangle$ and $\mu_h \sim \langle P_-\rangle$, where the brackets indicate an average over the considered region in phase space when integrating the differential decay rate. In most applications the phase space surviving the experimental cut is such that the shape-function region, where QCD factorization theorems hold, is included and dominates the event rate. However, it is desirable to have a description of the decay rate in the entire phase space for e.g. studies of detector resolution effects. In the next part of this talk we will first address the theoretical issues of the differential decay rate in the shape-function region, and discuss the non-trivial transition into the OPE region. The second part deals with relations between semileptonic decays and the radiative $\bar{B} \to X_s\gamma$ photon spectrum, which are constructed in such a way to be model-independent at leading power.

2. Factorization

In the shape-function region there are three relevant scales in the problem: the hard scale $\mu_h \sim \langle P_- \rangle \sim m_b$, the intermediate scale $\mu_i^2 \sim \langle P_+ P_- \rangle \sim m_b \Lambda_{\rm QCD}$, and the non-perturbative scale $\Lambda_{\rm QCD}$. When systematically separating the physics effects at these scales, the structures $\mathcal{F}_i$ factorize into three parts correspondingly, the hard function, the jet function, and the shape function [2]. In modern language this factorization is achieved via a series of effective-field theory calculations QCD→SCET→HQET. The matching coefficients are by construction free of infra-red physics and capture the physics at the hard and jet scale [3, 4]. Symbolically we can write $\mathcal{F}_i = H_i(y,\mu_h) J(y,P_+,\mu_i) \otimes \hat{S}(\mu_i) + \ldots$, where $\otimes$ denotes a convolution integral, and the ellipsis denote terms that are suppressed by $\Lambda_{\rm QCD}/m_b$ as compared with the leading term. It was shown explicitly in [5, 6] that these power corrections factorize themselves in a similar fashion. The major difference is, however, that the leading-power shape function is unique, while at subleading order more than one soft function appear.

Let us return to the leading-power formula. Not only is the shape function universal, but in fact also the jet function, and we may combine the hard functions with the kinematic prefactors to write

$$\frac{d^3\Gamma_u^{(0)}}{dP_+\,dy\,d\varepsilon} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3} U_y \,(M_B - P_+)^5 H_u(y,\varepsilon) \times \int_0^{P_+} d\hat{\omega}\, y m_b J(y m_b (P_+ - \hat{\omega})) \hat{S}(\hat{\omega}) \,. \quad (2)$$

In the above expressions we have neglected the scale dependence, which is as discussed above. The superscript $^{(0)}$ on the left-hand side indicates that this formula is the leading power contribution only. The hard and jet functions have been calculated at one-loop order in [3, 4], and recently the two-loop result for the jet function has become available [7]. The shape function $\hat{S}(\hat{\omega})$, on the other hand, is not calculable in perturbation theory and must be determined by other means. This is where the universality of the shape function comes into play. Considering the well-measured $\bar{B} \to X_s \gamma$ photon spectrum near the endpoint of maximal photon energy E_γ, one can state a formula similar to equation (2), namely

$$\frac{1}{\Gamma_s} \frac{d\Gamma_s^{(0)}}{dP_+} = \frac{U}{H_\Gamma} \frac{(M_B - P_+)^3}{m_b^3} \times \int_0^{P_+} d\hat{\omega}\, m_b J(m_b(P_+ - \hat{\omega})) \hat{S}(\hat{\omega}) \,. \quad (3)$$

For later convenience we have normalized the photon spectrum to the total decay rate.

Many of the ingredients of this equation have already been discussed. The main difference to the semileptonic decay is that a) the short-distance physics is different leading to a different hard function, and b) the phase space is given by $P_- = M_B$ (hence $y = 1$) and $P_+ = M_B - 2E_\gamma$. Taking this into account, U still resums logarithms of the ratio μ_h/μ_i, but no longer carries a dependence[1] on y. We encounter the same shape function and jet function, with the latter evaluated with $y = 1$.

The general idea for the determination of $|V_{ub}|$ from inclusive B decays is to use the $\bar{B} \to X_s\gamma$ photon spectrum to extract the shape function, and subsequently use it for predictions of arbitrary decay rates by integrating the differential decay rate (2). This is typically achieved by assuming a reasonable functional form for the shape function in the domain where $\hat{\omega} \sim \Lambda_{\rm QCD}$ with adjustable parameters, and fitting the parameters to the data. The current state-of-the-art parameterizations involve exponential-type, gaussian-type, or hyperbolic-type functions with two free parameters, which can be linked to the calculation of the first few moments of the shape function. Therefore the extraction of the leading shape function is mainly an experimental issue, while theory contributes via the calculation of moment constraints (where a link to other decay processes is established, e.g. to $\bar{B} \to X_c\, l^- \bar{\nu}$, see for example [8–10] and scheme translations in [11]), and via the explicit calculation of the radiative tail of the shape function.

Beyond the leading-power approximation there are several corrections that one should take into account. Non-perturbative ("hadronic") corrections are encoded in subleading shape functions [5, 6], of which there are three new, independent ones at tree level called $\hat{t}(\hat{\omega}), \hat{u}(\hat{\omega})$ and $\hat{v}(\hat{\omega})$. Their functional form is unknown and the only information available at present are their first few moments at tree level. Furthermore there are corrections proportional to arbitrary powers of the ratio of P_+/m_b, called "kinematical" corrections, which start at order $\mathcal{O}(\alpha_s)$ and come with the leading shape function.

To make the extraction of the leading shape function from the $\bar{B} \to X_s\gamma$ photon spectrum feasible, a certain combination of $\hat{t}(\hat{\omega}), \hat{u}(\hat{\omega})$ and $\hat{v}(\hat{\omega})$ is absorbed into the redefined leading shape function. The only structure surviving at subleading power is then proportional to $(\bar{\Lambda} - \hat{\omega})\hat{S}(\hat{\omega})$, where $\bar{\Lambda} = M_B - m_b$ is a heavy-quark parameter. It has recently been proposed to cross-check the findings by comparing with $\bar{B} \to X_c\, l^- \bar{\nu}$ decay spectra, where a shape-function region also exists [12]. However, so far many important corrections – for example subleading shape-function contributions – have not been included.

[1] The relation is $U_y(\mu_h,\mu_i) = U(\mu_h,\mu_i)\cdot y^{-2a_\Gamma(\mu_h,\mu_i)}$, where $a_\Gamma(\mu_h,\mu_i)$ is the integrated cusp anomalous dimension. See [1] for details.

3. Transition into OPE region

The region where an operator product expansion applies is reached when the size of the integration domain over shape functions, i.e. the maximal P_+, is much larger than $\Lambda_{\rm QCD}$. The key observation is that the factorization theorems for the shape-function region connect with the traditional OPE calculation via *moments* of the shape functions. More generally, if the integration domain Δ in an integral of the form (with some function $f(\hat\omega)$)

$$\int_0^\Delta d\hat\omega\; f(\hat\omega)\,\hat S(\hat\omega) \tag{4}$$

is much larger than $\Lambda_{\rm QCD}$, then one can perform an operator product expansion in the quantity $\Lambda_{\rm QCD}/\bar\Delta$ [13] with $\bar\Delta = \Delta - \bar\Lambda$. An interpolation between the shape-function region and the OPE region, and therefore a description for the entire phase space, can now be achieved by essentially brute force. Let us demonstrate the procedure for the partial decay rate with a cut on P_+, i.e.

$$\Gamma_u(\Delta) = \int_0^\Delta dP_+ \frac{d\Gamma_u}{dP_+}\,. \tag{5}$$

We will first consider the case of small Δ, where the factorization theorems are valid, and identify a certain contribution (with label "SF"). Then we will ask what happens to this contribution when increasing Δ so that an OPE calculation is valid (labeled "OPE").

The leading-power "SF" contribution (2) feeds into all powers in the OPE via the moments of the shape function, which involve the heavy-quark parameters $m_b, \mu_\pi^2, \ldots$, but also misses some contributions at every level in the power counting. We will return to this last point shortly. The first subleading hadronic "SF" corrections (from subleading shape functions), which are known at tree level, do not contribute to the leading-power "OPE" piece, but again feed into all subleading powers below. Similarly the second subleading hadronic "SF" corrections give no contribution to the leading or the first subleading "OPE" piece. (Of course, when expanding in $1/m_b$ there is no first-order power correction in the OPE calculation.) Currently there is no complete categorization of subsubleading shape functions at order $1/m_b^2$. However, it is via the above observation that at least those which contribute to the $1/m_b^2$ piece in the OPE calculation can be simulated [1].

As mentioned earlier, there are also kinematical radiative corrections that are power suppressed in the shape-function region simply because they are proportional to $\alpha_s(\bar\mu)\cdot(P_+/m_b)^k$, $k \geq 1$. Such "SF" contributions come with the leading shape function and are promoted to leading power "OPE" terms when P_+ becomes of order m_b. The exact $\mathcal{O}(\alpha_s)$ kinematic corrections are known by comparing the factorized expressions with the fixed-order partonic calculation in full QCD [14].

By including all of the above contribution in the shape-function region we correctly reproduce the OPE result when allowing Δ to become large, up to order $1/m_b^3$ corrections; thereby achieving a reliable interpolation between the two phase-space regions.

4. Theoretical uncertainties

Predictions for partial semileptonic decay rates can be broken up into the individual contributions from different powers according to

$$\begin{aligned}\Gamma_u \;&=\; \Gamma_u^{(0)} + (\Gamma_u^{\rm kin(1)} + \Gamma_u^{\rm hadr(1)}) \\ &\quad + (\Gamma_u^{\rm kin(2)} + \Gamma_u^{\rm hadr(2)}) + \dots\,,\end{aligned}$$

where the superscript indicates both the nature of the correction and their order in power counting in the shape-function region. For the kinematical corrections $\Gamma_u^{\rm kin(n)}$ the sum of all terms is known and can be used without truncating the series. We discuss the uncertainty estimates term by term in the above formula.

The leading-power contribution has been factorized, as stated in (2), into hard, jet, and shape function. As mentioned earlier, the leading shape-function uncertainty is coming from experimental limitations, and is not accounted for in the list of theoretical uncertainties. The hard and jet function have perturbative expansions in the strong coupling constant α_s, evaluated at the scale μ_h and μ_i, respectively. While $\Gamma_u^{(0)}$ is formally independent of the choice of these scales, a residual scale dependence is introduced by truncating the perturbative series. A variation of the hard scale μ_h is used to estimate the unknown 2-loop contribution to the hard function. The uncertainty in the jet function has been estimated by assigning $\pm[\alpha_s(\mu_i)/\pi]^2$ as a relative error. Here no scale variation has been used since a change in μ_i also changes the shape function $\hat S(\hat\omega, \mu_i)$, which is assumed to be extracted at a fixed scale, which is chosen to be $\mu_i = 1.5$ GeV. However, the uncertainty from the intermediate scale is somewhat outdated today, ever since the appearance of the recent 2-loop calculation of the jet function [7].

The kinematical corrections $\Gamma_u^{\rm kin}$ start at order α_s and are convoluted with the leading shape function. They can thus be viewed as the product of subleading hard and jet functions. However, for them the factorized expressions have not been worked out, and the effects from the scales μ_h and μ_i have not been disentangled. Instead, the expressions are evaluated at one common scale $\bar\mu$ which is independent of, but around the intermediate scale. Higher-order effects are again estimated by a variation of $\bar\mu$.

So far we have accounted for three different perturbative uncertainties resulting from the scales $\mu_h, \mu_i, \bar\mu$, which can be added in quadrature. Next, we turn the discussion toward the hadronic corrections. It is necessary to obtain an idea about the severeness of our ignorance of the functional form of subleading shape functions. To make even a central-value prediction we need to model $\hat{t}(\hat{\omega}), \hat{u}(\hat{\omega}), \hat{v}(\hat{\omega})$ while respecting the moment constraints from OPE calculations. The idea for an estimator for the associated uncertainty is to find many different realistic models and study the resulting deviations from the default value. This can be achieved by altering the functional forms of each of the subleading shape functions via additional functions that have vanishing first few moments. Using four such functions and either adding or subtracting them, or leaving the subleading shape function unchanged, already gives a set of $(2\cdot 4+1)^3 = 729$ different models for the set of subleading shape functions. To be on the conservative side, we use the maximal deviation from the central value as an estimator for the subleading shape-function uncertainty.

At order $(1/m_b)^2$ we expect to find many new shape functions. As described earlier, we only include those contributions that feed into the $(1/m_b)^2$ terms of the OPE result and neglect even smaller contributions. At this level we need not worry about their precise form and may model them using $(\mu_\pi^2/m_b^2)\,\hat{S}(\hat{\omega})$ and $(\lambda_2/m_b^2)\,\hat{S}(\hat{\omega})$. The differences between these expressions and their true functional forms can be absorbed into the subleading shape-function uncertainty.

Lastly there is one non-negligible error estimate at third and higher order in power counting, which is the weak annihilation effect. This error must be included whenever the experimental cut includes the phase-space region near the origin, where $P_+ \sim P_- \sim \Lambda_{\rm QCD}$. A recent study has put a limit of $\pm 1.8\%$ on the total rate by analyzing CLEO data [15]. A second possibility is to cut out this region in phase space, at the moderate cost (few events are located in that region) of a smaller efficiency. We have found that this might improve the overall error estimate slightly.

In summary, we may split the theoretical uncertainty into three categories: perturbative, hadronic, and weak annihilation. The sizes of the individual errors in each of the categories depend on the specific cut employed in the measurement. For example, cutting on large charged-lepton energy is typically low in efficiency and is significantly sensitive to subleading shape functions. In this example the weak annihilation and hadronic errors dominate over the perturbative one unless the cut is relaxed below 2.1 GeV. (The charm background starts at 2.3 GeV.) On the other hand, for more efficient cuts, like a cut on the hadronic invariant mass or on P_+ for example, the perturbative uncertainty is typically dominant. At present, the combined theoretical error on $|V_{ub}|$ – excluding the uncertainty from the leading shape function – is in the neighborhood of 5%, but may be larger for certain cuts.

5. Comment on “Dressed Gluon Exponentiation”

A different approach in calculating inclusive B-decay spectra recently surfaced, named Dressed Gluon Exponentiation (DGE) by Andersen and Gardi [16, 17]. This computation is based on perturbation theory using an on-shell b-quark state “dressed” with gluons instead of the hadronic B-meson state. An interesting consequence of this state is that the kinematic range extends beyond the phase space of a single on-shell b-quark state. This observation motivates to use the calculation in comparing it with real data, so as to judge how realistic a dressed heavy quark can mimic a heavy meson.

The procedure of dressing the b-quark with gluons requires a complete knowledge of the anomalous dimensions of the jet and soft functions (i.e. to all orders in α_s), essentially because of renormalization-group running deep into the non-perturbative regime. Some limited information is available via the perturbative calculation to first few orders, but the remaining aspects must be modeled. Such models are guided by the large-β_0 approximation and certain renormalon cancellations.

For these reasons the DGE calculation should be viewed as a model, and not as a rigorous QCD prediction, the latter being a systematic expansion and model-independent. While it might be interesting to test this model against experimental data, it would be dangerous to use it for the extraction of $|V_{ub}|$, since the uncertainty introduced by the underlying assumptions are not under control. The statement that no non-perturbative function is needed and that the “prediction for the spectrum depends *only* on α_s and on the quark short-distance mass” [16] (emphasis as in that reference) is not quite accurate; the same could be said about a simple one-parameter model for the shape function, where the model dependence is uncontrolled.

Lastly it should be noted that the DGE calculation has not been performed to a comprehensive level comparable to the one described in the previous sections, i.e. many power corrections that we addressed above have yet to find their way into the DGE framework.

6. Model-independent relations

The universality of the leading shape function allows for infra-red safe relations between different inclusive B-decay spectra. In fact, the program outlined so far – using the experimental data on the $\bar{B} \to X_s\gamma$ photon spectrum to extract the leading shape function, and subsequently plugging this function into the formula for the semileptonic differential decay rate – can be viewed as a “manually shape-function free relation”. However, a more direct relation is desirable because the extraction of the shape function is somewhat cumbersome. Such relations would eliminate the issues arising from the parameterizing the shape function, and different fitting procedures that avoid sensitivity to resonances. The idea is to "invert" the jet function acting on the shape function in (3) so that instead of convolving with the shape function (which is not calculable) one convolves with the photon spectrum (which is measured). The desired relation reads

$$\Gamma_u\Big|_{\rm cut} = |V_{ub}|^2 \int_0^\Delta dP_+\, W(\Delta,P_+)\, \frac{1}{\Gamma_s}\frac{d\Gamma_s}{dP_+} + |V_{ub}|^2\, \Gamma_{\rm rhc}\Big|_{\rm cut}\,, \qquad (6)$$

where the second term $\Gamma_{\rm rhc}$ collects residual power corrections, e.g. terms that arise from the fact that different combinations of subleading shape functions appear in the semileptonic and radiative decay rates. The left-hand side of the equation denotes the partial semileptonic decay rate as obtained after imposing a cut. For the determination of $|V_{ub}|$ the partial $\bar B \to X_u\, l^-\bar\nu$ decay rate, as well as the photon energy spectrum, enter as experimental inputs. The weight function $W(\Delta,P_+)$ and the residual terms $\Gamma_{\rm rhc}$, on the other hand, are theoretical quantities.

The main obstacle in finding an infra-red safe weight function is that the jet function $J(p^2)$ in (3) is a distribution, which does not "invert" easily. Although the jet function is universal, it cannot be eliminated entirely either, because it is called with different arguments in (2) and in (3). The difference is that in radiative decays $y=1$, while this is generally not true for semileptonic decays. This problem is solved by defining a "jet kernel" $Y(k,\ln y)$ which is used to extract the y-dependence from the jet function by means of the following equation [18]

$$\int_0^{y m_b \Omega} dp^2\, J(p^2) = \int_0^{\Omega} dk\, Y(k,\ln y) \int_0^{m_b(\Omega-k)} dp^2\, J(p^2)\,. \qquad (7)$$

This equation defines the jet kernel to all orders in perturbation theory. The explicit expressions for $Y(k,\ln y)$ have been calculated to complete 2-loop order. We now proceed in constructing the weight function.

The experimental cut on the $\bar B \to X_u\, l^-\bar\nu$ phase space can be encoded as

$$\begin{aligned} 0 &\le P_+ \le \Delta\,, \\ 0 &\le y \le y_{\rm max}(P_+)\,, \\ 0 &\le \varepsilon \le \varepsilon_{\rm max}(P_+,y)\,. \end{aligned} \qquad (8)$$

As written in (6) the weight function depends on Δ. More generally, the weight function (and also the power correction $\Gamma_{\rm rhc}$) changes as the specifics of the cut change, that is, it depends also on the functions $y_{\rm max}(P_+)$ and $\varepsilon_{\rm max}(P_+,y)$ that encode the cut. When integrating the leading-power differential decay rate formula (2) over this phase space one can show via simple interchanges of integrations that the weight function $W^{(0)}(\Delta,P_+)$ itself factorizes symbolically as

$$W^{(0)}(\Delta,P_+) \sim H_\Gamma\, H_u(y,\varepsilon) \otimes Y(k,\ln y)\,, \qquad (9)$$

where this time $\otimes$ is a three-folded convolution in the variables ε, y and k. Because the specifics of the cut, $\Delta, y_{\rm max}(P_+)$, and $\varepsilon_{\rm max}(P_+,y)$ enter only as integration limits in this structure, weight functions can be computed in an automated fashion.

Before we turn the discussion to the error analysis when using relation (6) for the extraction of $|V_{ub}|$, we have to note a feature that contrasts direct partial decay-rate prediction described in Section 2. There, the b-quark mass enters only indirectly into the factorization formula (2) through the shape function. When integrating the differential decay rate to obtain a partial rate, the m_b dependence is generated dynamically. For example, the total decay rate is proportional to $\int dP_+\,(M_B-P_+)^5\hat S(P_+) \approx m_b^5$ at tree level. The analogous statement is true for the $\bar B \to X_s\gamma$ photon spectrum. This is the reason for the m_b^3 factor in the denominator in (3) for the *normalized* photon spectrum. Therefore the weight function $W(\Delta,P_+)$ will pick up this explicit m_b dependence. Previous works [19, 20] used the *absolute* photon spectrum, which introduces other, more severe uncertainties. Firstly, without the normalization one can only extract the ratio $|V_{ub}|^2/|V_{ts}^*V_{tb}|^2$. Secondly, the radiative corrections are then unacceptably large due to large operator mixing effects [21]. Thirdly, event fractions in $\bar B \to X_s\gamma$ can be calculated with higher precision than the absolute branching ratio [13].

As an example let us consider the theoretical uncertainties on the partial decay rate when using relation (6) for a pure cut on P_+. To this end we may pretend for the moment that the experimental data on the shape of the photon spectrum had no uncertainty. The analysis was carried out in [21], and reads

$$\begin{aligned} &\Gamma_u(P_+ \le 0.65~{\rm GeV}) \\ &= (46.5\ \pm 1.4\,{\scriptstyle [\rm pert]}\ \pm 1.8\,{\scriptstyle [\rm hadr]}\ \pm 1.8\,{\scriptstyle [m_b]} \\ &\quad \pm 0.8\,{\scriptstyle [\rm pars]}\ \pm 2.8\,{\scriptstyle [\rm norm]})\ |V_{ub}|^2{\rm ps}^{-1} \end{aligned} \qquad (10)$$

Again, the perturbative uncertainty stems from a variation of the renormalization scales, and the hadronic uncertainty from scanning over many models of subleading shape functions. While the latter is comparable to the error analysis of the direct next-to-leading order partial rate prediction, the perturbative error is significantly reduced due to the fact that the jet kernel has been evaluated to complete 2-loop order. (In the direct prediction [1] we find $\pm 2.5\,{\scriptstyle [\rm pert]}\ |V_{ub}|^2{\rm ps}^{-1}$ with the 1-loop jet function.) Varying the input parameters needed for the weight function, i.e. quark masses and HQET parameters, leads to the next two stated errors. Finally we have to consider the norm of the relative photon spectrum, which impacts $|V_{ub}|$ directly. Because the shape of the photon spectrum is not measured over the full kinematic region but only above some photon energy E_0, we will need the theoretical prediction for the event fraction that falls into that window [13].

7. Conclusions

The CKM matrix element $|V_{ub}|$ is a fundamental parameter of the Standard Model, which is already the first motivation in pursuing its precise determination. It also allows for testing the CKM picture of $\mathcal{CP}$ violation and for indirect New Physics searches. A determination with overall uncertainty around and below the 10% level is both experimentally and theoretically challenging, but feasible. In this talk we have touched upon many of the theoretical issues and possibilities, from direct predictions of partial rates using QCD-factorization theorems to relations between different inclusive B-decay spectra, which are model-independent at leading power.

The theoretical error can be reduced in several ways. Higher-order computations allow for a reduction of perturbative uncertainties. The recently published 2-loop result for the jet function [7] will have an impact on both the direct theoretical prediction of partial rates – via reduced scale dependence – and the weight-function relations. While the weight function itself was already known to 2-loop order at the intermediate scale, the improved jet function will impact the theoretical error on the event fraction in $\bar{B} \to X_s\gamma$, i.e. the *norm* of the photon spectrum. Other possibilities for improving the theoretical errors include the computation of higher-order hard functions; however, a complete next-to-next-to leading order description in renormalization-group improved perturbation theory would require 3-loop anomalous dimensions and the 4-loop cusp anomalous dimension, which seems currently out of reach. On the non-perturbative side, further studies of subleading shape function contributions, and improved limits on weak annihilation can have a similar overall impact.

A precise determination of $|V_{ub}|$ requires a variety of different methods and measurements. The study of inclusive B decays remains a very active field of research with further improvements in both theory and experiment.

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