Why multi-jet studies?

A. Banfi$^1$

$^1$ Università degli Studi di Milano-Bicocca
Dipartimento di Fisica “G. Occhialini”

13 January 2006 / FRIF Workshop
Outline

1. Introduction
   - Why event-shape variables?
   - Power corrections and Feynman tube model

2. Three-jet event shapes
   - PQCD and NP corrections
   - $K_{\text{out}}$ and $D$-parameter in $e^+e^-$ annihilation
   - $K_{\text{out}}$ in DY and underlying event
Outline

1 Introduction
   - Why event-shape variables?
   - Power corrections and Feynman tube model

2 Three-jet event shapes
   - PQCD and NP corrections
   - $K_{out}$ and $D$-parameter in $e^+e^-$ annihilation
   - $K_{out}$ in DY and underlying event
Event-shape variables

- **Event-shape variables** $V(p_1, \ldots, p_n)$ are continuous measures of the geometrical properties of hadron energy-momentum flow.

- **Thrust**: longitudinal particle alignment
  
  \[ T = \frac{1}{Q} \max_{\vec{n}_T} \sum_i |\vec{p}_i \cdot \vec{n}_T| \quad t = 1 - T \]

  Pencil-like event: $t \geq 0$

  Planar event: $t \simeq 1/3$
Event-shape variables $V(p_1, \ldots, p_n)$ are continuous measures of the geometrical properties of hadron energy-momentum flow.

- **Thrust**: longitudinal particle alignment

\[
T \equiv \frac{1}{Q} \max_{\vec{n}_T} \sum_i |\vec{p}_i \cdot \vec{n}_T| \quad t \equiv 1 - T
\]

Pencil-like event: $t \geq 0$  
Planar event: $t \approx 1/3$
Event-shape variables

- Event-shape variables $V(p_1, \ldots, p_n)$ are continuous measures of the geometrical properties of hadron energy-momentum flow.
- Thrust: longitudinal particle alignment

$$T \equiv \frac{1}{Q} \max_{\vec{n}_T} \sum_i |\vec{p}_i \cdot \vec{n}_T| \quad t \equiv 1 - T$$

Pencil-like event: $t \geq 0$

Planar event: $t \approx 1/3$
Event-shape variables

- Event-shape variables $V(p_1, \ldots, p_n)$ are continuous measures of the geometrical properties of hadron energy-momentum flow.
- **Thrust:** longitudinal particle alignment

\[
T \equiv \frac{1}{Q} \max_{\vec{n}_T} \sum_i |\vec{p}_i \cdot \vec{n}_T| \quad t \equiv 1 - T
\]

Pencil-like event: $t \geq 0$

Planar event: $t \simeq 1/3$
From short to long distances

Decay scheme (udscb)

\[ Q = M_z \]

- ALEPH
- DELPHI
- L3
- OPAL
- SLD

NP hadronisation  \[ \text{PT resummation} \]  \[ \text{PT fixed order} \]
Example: mean value of the thrust

\[ \langle t \rangle_{\text{PT}} = \langle t \rangle_1 \alpha_s(Q^2) + \langle t \rangle_2 \alpha_s^2(Q^2) + \ldots \]

PT theory cannot be the whole story…

\[ \langle t \rangle = \langle t \rangle_{\text{PT}} + \langle t \rangle_{\text{NP}} \quad \langle t \rangle_{\text{NP}} \sim \frac{1 \text{ GeV}}{Q} \]

Sterman’s lemma \( \Rightarrow 18\alpha_s^3 \simeq 1/Q \)

\[ \langle t \rangle_{\text{PT}} \simeq \frac{4C_F\alpha_s}{\pi} \sum_{n=0}^{\infty} n! \left( \frac{2\beta_0\alpha_s}{4\pi} \right)^n n^{\beta_1} \]
NP corrections to mean values

Example: mean value of the thrust

\[
\langle t \rangle_{\text{PT}} = \langle t \rangle_1 \alpha_s(Q^2) + \langle t \rangle_2 \alpha_s^2(Q^2) + \ldots
\]

PT theory cannot be the whole story…

\[
\langle t \rangle = \langle t \rangle_{\text{PT}} + \langle t \rangle_{\text{NP}} \quad \langle t \rangle_{\text{NP}} \sim \frac{1 \text{ GeV}}{Q}
\]

Sterman’s lemma \( \Rightarrow \quad 18\alpha_s^3 \sim \frac{1}{Q} \)

\[
\langle t \rangle_{\text{PT}} \sim \frac{4C_F\alpha_s}{\pi} \sum_{n=0}^{\infty} n! \left( \frac{2\beta_0\alpha_s}{4\pi} \right)^n n^\beta_1
\]
NP corrections to mean values

Example: mean value of the thrust

\[ \langle t \rangle_{\text{PT}} = \langle t \rangle_1 \alpha_s(Q^2) + \langle t \rangle_2 \alpha_s^2(Q^2) + \ldots \]

PT theory cannot be the whole story...

\[ \langle t \rangle = \langle t \rangle_{\text{PT}} + \langle t \rangle_{\text{NP}} \quad \langle t \rangle_{\text{NP}} \sim \frac{1 \text{ GeV}}{Q} \]

Sterman’s lemma ⇒ \( 18\alpha_s^3 \approx \frac{1}{Q} \)

\[ \langle t \rangle_{\text{PT}} \approx \frac{4C_F\alpha_s}{\pi} \sum_{n=0}^{\infty} n! \left( \frac{2\beta_0\alpha_s}{4\pi} \right)^n n^{\beta_1} \]
Example: mean value of the thrust

\[
\langle t \rangle_{PT} = \langle t \rangle_1 \alpha_s(Q^2) + \langle t \rangle_2 \alpha_s^2(Q^2) + \ldots
\]

PT theory cannot be the whole story...

\[
\langle t \rangle = \langle t \rangle_{PT} + \langle t \rangle_{NP} \quad \langle t \rangle_{NP} \sim \frac{1 \text{ GeV}}{Q}
\]

Sterman’s lemma \( \Rightarrow \) \( 18 \alpha_s^3 \sim 1/Q \)

\[
\langle t \rangle_{PT} \approx \frac{4C_F \alpha_s}{\pi} \sum_{n=0}^{\infty} n! \left( \frac{2\beta_0 \alpha_s}{4\pi} \right)^n n^{\beta_1}
\]
NP emissions give rise to a shift of the PT distribution!

\[ t = t_{PT} + \delta t \simeq t_{PT} + \frac{\epsilon}{Q} \]
\[ \epsilon \sim \Lambda_{QCD} \]

\[ \frac{1}{\sigma} \frac{d\sigma}{dt} = \int_0^{tQ} d\epsilon f(\epsilon) \frac{d\sigma_{PT}(t - \frac{\epsilon}{Q})}{\sigma} \sim \frac{d\sigma_{PT}(t - \frac{\langle \epsilon \rangle}{Q})}{\sigma} \]

1/Q power correction to \( \langle V \rangle \Rightarrow \) full (PT+NP) event shape distributions for \( V \gg \Lambda_{QCD}/Q \)

\[ \langle t \rangle_{NP} \simeq \int d\epsilon \frac{\epsilon}{Q} f(\epsilon) = \frac{\langle \epsilon \rangle}{Q} \]
NP emissions give rise to a shift of the PT distribution!

\[ t = t_{PT} + \delta t \simeq t_{PT} + \frac{\epsilon}{Q} \]

\[ \epsilon \sim \Lambda_{QCD} \]

\[
\frac{1}{\sigma} \frac{d\sigma}{dt} = \int_0^{tQ} d\epsilon \ f(\epsilon) \frac{d\sigma_{PT}(t - \frac{\epsilon}{Q})}{\sigma \ dt} \sim \frac{d\sigma_{PT}(t - \frac{\langle \epsilon \rangle}{Q})}{\sigma \ dt}
\]

1/Q power correction to \( \langle V \rangle \Rightarrow \) full (PT+NP) event shape distributions for \( V \gg \Lambda_{QCD}/Q \)

\[
\langle t \rangle_{NP} \simeq \int d\epsilon \ \frac{\epsilon}{Q} f(\epsilon) = \frac{\langle \epsilon \rangle}{Q}
\]
NP emissions give rise to a shift of the PT distribution!

\[ t = t_{PT} + \delta t \approx t_{PT} + \frac{\varepsilon}{\mathcal{Q}} \]

\[ \varepsilon \sim \Lambda_{QCD} \]

\[
\frac{1}{\sigma} \frac{d\sigma}{dt} = \int_0^{tQ} d\varepsilon \frac{f(\varepsilon)}{\sigma} \frac{d\sigma_{PT}(t - \frac{\varepsilon}{\mathcal{Q}})}{dt} \approx \frac{d\sigma_{PT}(t - \frac{\langle \varepsilon \rangle}{\mathcal{Q}})}{dt}
\]

1/\mathcal{Q} power correction to \( \langle V \rangle \Rightarrow \) full (PT+NP) event shape distributions for \( V \gg \Lambda_{QCD}/\mathcal{Q} \)

\[ \langle t \rangle_{NP} \approx \int d\varepsilon \frac{\varepsilon}{\mathcal{Q}} f(\varepsilon) \equiv \frac{\langle \varepsilon \rangle}{\mathcal{Q}} \]
Outline

1. Introduction
   - Why event-shape variables?
   - Power corrections and Feynman tube model

2. Three-jet event shapes
   - PQCD and NP corrections
   - $K_{out}$ and $D$-parameter in $e^+e^-$ annihilation
   - $K_{out}$ in DY and underlying event
Feynman tube model and leading power corrections

- Hadron multiplicity is **uniform** in rapidity

\[
\frac{dn_h}{d\eta \, d \ln k_t} = \Phi_h(k_t)
\]

- Power correction to linear event shapes:

\[
\delta V(\{k_i\}) \simeq \sum_i \frac{k_{ti}}{Q} f_V(\eta_i)
\]

\[
\langle \delta V \rangle = \int \frac{dk_t}{k_t} \frac{k_t}{Q} \sum_h \Phi_h(k_t) \int d\eta f_V(\eta) = \frac{\langle k_t \rangle}{Q} c_V
\]

- Leading power corrections are **universal**, i.e. after factorisation of rapidity dependence, they depend only on \(\langle k_t \rangle\)

\[
\frac{\langle C' \rangle_{NP}}{\langle t \rangle_{NP}} = \frac{4 \text{ GeV}}{1 \text{ GeV}} \simeq \frac{3\pi}{2} = \frac{c_C}{c_t}
\]
Feynman tube model and leading power corrections

- Hadron multiplicity is **uniform** in rapidity

\[
\frac{d n_h}{d \eta \ d \ln k_t} = \Phi_h(k_t)
\]

- Power correction to **linear** event shapes:

\[
\delta V(\{k_i\}) \simeq \sum_i \frac{k_{ti}}{Q} f_V(\eta_i)
\]

\[
\langle \delta V \rangle = \int \frac{d k_t}{k_t} \frac{k_t}{Q} \sum_h \Phi_h(k_t) \int d \eta f_V(\eta) = \frac{\langle k_t \rangle}{Q} c_V
\]

- Leading power corrections are **universal**, i.e. after factorisation of rapidity dependence, they depend only on \(\langle k_t \rangle\)

\[
\frac{\langle C' \rangle_{NP}}{\langle t \rangle_{NP}} = \frac{4 \ \text{GeV}}{1 \ \text{GeV}} \simeq \frac{3\pi}{2} = \frac{c_C}{c_t}
\]
Feynman tube model and leading power corrections

- Hadron multiplicity is **uniform** in rapidity

\[
\frac{d n_h}{d \eta \, d \ln k_t} = \Phi_h(k_t)
\]

- Power correction to **linear** event shapes:

\[
\delta V(\{k_i\}) \simeq \sum_i \frac{k_{ti}}{Q} f_V(\eta_i)
\]

\[
\langle \delta V \rangle = \int \frac{d k_t}{k_t} \frac{k_t}{Q} \sum_h \Phi_h(k_t) \int d \eta f_V(\eta) = \frac{\langle k_t \rangle}{Q} c_V
\]

- Leading power corrections are **universal**, i.e. after factorisation of rapidity dependence, they depend only on \( \langle k_t \rangle \)

\[
\frac{\langle C \rangle_{NP}}{\langle t \rangle_{NP}} = \frac{4 \text{ GeV}}{1 \text{ GeV}} \sim \frac{3\pi}{2} = \frac{c_C}{c_t}
\]
Introduction

- Why event-shape variables?
- Power corrections and Feynman tube model

Three-jet event shapes

- PQCD and NP corrections
- $K_{out}$ and $D$-parameter in $e^+e^-$ annihilation
- $K_{out}$ in DY and underlying event
From two- to multi-jet events

- **Intra-jet** hadrons (collinear to jet direction $\vec{n}_i$)
  \[
  \frac{d n_h}{d \ln k_t^{(i)}} = \Phi_h^{(i)}(k_t) d\eta^{(i)}
  \]

- **Inter-jet** hadrons (large angles)
  \[
  \frac{d n_h}{d \ln k_t} = \Phi_h(k_t) d\eta \frac{d\phi}{2\pi} \Psi_h(\eta, \phi, \{\vec{n}_i \cdot \vec{n}_j\})
  \]
From two- to multi-jet events

- **Intra-jet** hadrons (collinear to jet direction $\vec{n}_i$)

  $$\frac{d n_h}{d \ln k_t} = \Phi_h^{(i)}(k_t) d\eta^{(i)}$$

- **Inter-jet** hadrons (large angles)

  $$\frac{d n_h}{d \ln k_t} = \Phi_h(k_t) d\eta \frac{d\phi}{2\pi} \Psi_h(\eta, \phi, \{\vec{n}_i \cdot \vec{n}_j\})$$
Soft radiation and confinement

• Soft dressed gluon emission from a $q\bar{q}$ dipole

\[ \frac{dw}{d \ln k_t d\eta} = 2 A_q [\alpha_s^{\overline{MS}}(k_t)] = 2 C_F \frac{\alpha_s^{\overline{CMW}}(k_t)}{\pi} \sum_h \Phi_h(k_t) \]

\[ k_t^2 = \frac{(2pk)(2k\bar{p})}{2p\bar{p}} \quad \eta = \frac{1}{2} \ln \frac{p\bar{k}}{pk} \]

• Soft dressed gluon emission from more dipoles

\[ dw = \sum_{i<j} (-2\vec{T}_i \cdot \vec{T}_j) \frac{d\kappa_{ij}}{\kappa_{ij}} d\eta_{ij} \frac{\alpha_s^{\overline{CMW}}(\kappa_{ij})}{\pi} \sum_h \frac{d\kappa_{ij}}{\kappa_{ij}} d\eta_{ij} \sum_h \Phi_h^{(ij)}(\kappa_{ij})? \]

\[ \kappa_{ij}^2 = \frac{(2p_i k)(2k p_j)}{2p_i p_j} \quad \eta_{ij} = \frac{1}{2} \ln \frac{p_j k}{p_i k} \]
Soft radiation and confinement

- Soft dressed gluon emission from a $q\bar{q}$ dipole

$$\frac{dw}{d\ln k_t d\eta} = 2A_q [\alpha_s^{\overline{\text{MS}}} (k_t)] = 2C_F \frac{\alpha_s^{\text{CMW}} (k_t)}{\pi} \rightarrow \sum_h \Phi_h (k_t)$$

$$k_t^2 = \frac{(2p_k)(2k_p)}{2p_k p_p} \quad \eta = \frac{1}{2} \ln \frac{p_k}{p_p}$$

- Soft dressed gluon emission from more dipoles

$$dw = \sum_{i<j} (-2\vec{T}_i \cdot \vec{T}_j) \frac{d\kappa_{ij}}{\kappa_{ij}} d\eta_{ij} \frac{\alpha_s^{\text{CMW}} (\kappa_{ij})}{\pi} \rightarrow \sum_{i<j} \frac{d\kappa_{ij}}{\kappa_{ij}} d\eta_{ij} \sum_h \Phi_h^{(ij)} (\kappa_{ij})?$$

$$\kappa_{ij}^2 = \frac{(2p_i k)(2k_p j)}{2p_i p_j} \quad \eta_{ij} = \frac{1}{2} \ln \frac{p_j k}{p_i k}$$
Validity of the soft approximation

Power corrections for $V \gg \Lambda_{QCD}$ arise from $\langle k_t \rangle \ll VQ$.

What about hard collinear emissions?

- $V$ damped in rapidity (example: thrust)

$$\delta V(k) \sim k_t e^{-|\eta|} \quad |\eta| \to \infty$$

$\langle k_t \rangle$ receives contributions only from $\eta \sim 0$ (soft large angle)

- $V$ uniform in rapidity (example: broadening)

$$\delta V(k) \sim k_t \quad |\eta| \to \infty$$

$k_t$ is measured with respect to thrust axis. Hard partons recoil with $p_t \lesssim VQ \Rightarrow \langle k_t \rangle \ll VQ \Rightarrow k_t \ll p_t$ (soft large angle)
Validity of the soft approximation

Power corrections for $V \gg \Lambda_{QCD}$ arise from $\langle k_t \rangle \ll V Q$.

What about hard collinear emissions?

- $V$ damped in rapidity (example: thrust)
  
  \[ \delta V(k) \sim k_t e^{-|\eta|} \quad |\eta| \to \infty \]
  
  $\langle k_t \rangle$ receives contributions only from $\eta \sim 0$ (soft large angle)

- $V$ uniform in rapidity (example: broadening)
  
  \[ \delta V(k) \sim k_t \quad |\eta| \to \infty \]

  $k_t$ is measured with respect to thrust axis. Hard partons recoil with $p_t \lesssim V Q \Rightarrow \langle k_t \rangle \ll V Q \Rightarrow k_t \ll p_t$ (soft large angle)
Validity of the soft approximation

Power corrections for $V \gg \Lambda_{QCD}$ arise from $\langle k_t \rangle \ll VQ$.

What about hard collinear emissions?

- $V$ damped in rapidity (example: thrust)
  \[ \delta V(k) \sim k_t e^{-|\eta|} \quad |\eta| \to \infty \]
  $\langle k_t \rangle$ receives contributions only from $\eta \sim 0$ (soft large angle)

- $V$ uniform in rapidity (example: broadening)
  \[ \delta V(k) \sim k_t \quad |\eta| \to \infty \]
  $k_t$ is measured with respect to thrust axis. Hard partons recoil with $p_t \lesssim VQ \Rightarrow \langle k_t \rangle \ll VQ \Rightarrow k_t \ll p_t$ (soft large angle)
The CMW coupling can be extended at low scales via the dispersion relation

\[ \alpha_s(\kappa^2) = \kappa^2 \int_0^\infty \frac{dm^2}{(m^2 + \kappa^2)^2} \alpha_{\text{eff}}(m^2) \]

- LPHD philosophy \(\Rightarrow\) hadron flow \(\sim\) parton flow
- Hadronisation corrections are due to extra-soft gluons (gluers) with transverse momenta \(\kappa \sim \Lambda_{QCD}\), whose emission probability is ruled by the NP part of the dispersive coupling.
The CMW coupling can be extended at low scales via the dispersion relation

\[ \alpha_s(\kappa^2) = \kappa^2 \int_0^\infty \frac{d\sigma^2}{(m^2 + \kappa^2)^2} \alpha_{eff}(m^2) \]

LPHD philosophy \(\Rightarrow\) hadron flow \(\sim\) parton flow

Hadronisation corrections are due to extra-soft gluons (gluers) with transverse momenta \(\kappa \sim \Lambda_{QCD}\), whose emission probability is ruled by the NP part of the dispersive coupling.
The CMW coupling can be extended at low scales via the dispersion relation

\[ \alpha_s(\kappa^2) = \kappa^2 \int_0^\infty \frac{dm^2}{(m^2 + \kappa^2)^2} \alpha_{eff}(m^2) \]

LPHD philosophy \( \Rightarrow \) hadron flow \( \sim \) parton flow

Hadronisation corrections are due to extra-soft gluons (gluers) with transverse momenta \( \kappa \sim \Lambda_{QCD} \), whose emission probability is ruled by the NP part of the dispersive coupling.
**Power corrections in the DMW approach**

- Combine **real and virtual** corrections and obtain

\[
\frac{\alpha_s(\kappa^2)}{\kappa^2} \to \int_0^\infty \frac{d\kappa^2}{(m^2 + \kappa^2)^2} \alpha_{\text{eff}}(m^2)
\]

- Allow gluon to decay inclusively \( \kappa^2 \to \kappa^2 + m^2 \)

\[
\langle \delta V \rangle_{\text{naive}} = \frac{\langle \kappa \rangle_{\text{naive}}}{Q} \sum_{i<j} (-2 \vec{T}_i \cdot \vec{T}_j) c_{ij}^{(V)}
\]

\[
\langle \kappa \rangle_{\text{naive}} = \int_0^\infty \frac{d\kappa^2 dm^2}{(m^2 + \kappa^2)^2} \sqrt{\kappa^2 + m^2} \alpha_{\text{eff}}(m^2) \approx 2 \int_0^\infty \frac{dm^2}{m^2} m \delta \alpha_{\text{eff}}(m^2)
\]

- Take into account **non-inclusiveness** via the **Milan factor**

\[
\langle \kappa \rangle \to M \langle \kappa \rangle_{\text{naive}}
\]
Power corrections in the DMW approach

- Combine real and virtual corrections and obtain

\[
\frac{\alpha_s(\kappa^2)}{\kappa^2} \rightarrow \int_0^\infty \frac{d\mathcal{m}^2}{(m^2 + \kappa^2)^2} \alpha_{\text{eff}}(m^2)
\]

- Allow gluon to decay inclusively \( \kappa^2 \rightarrow \kappa^2 + m^2 \)

\[
\langle \delta V \rangle_{\text{naive}} = \frac{\langle \kappa \rangle_{\text{naive}}}{Q} \sum_{i<j} (-2\vec{T}_i \cdot \vec{T}_j) c_{V}^{(ij)}
\]

\[
\langle \kappa \rangle_{\text{naive}} = \int_0^\infty \frac{d\kappa^2 d\mathcal{m}^2}{(m^2 + \kappa^2)^2} \sqrt{\kappa^2 + m^2} \alpha_{\text{eff}}(m^2) \approx 2 \int_0^\infty \frac{d\mathcal{m}^2}{m^2} m \delta \alpha_{\text{eff}}(m^2)
\]

- Take into account non-inclusiveness via the Milan factor

\[
\langle \kappa \rangle \rightarrow \mathcal{M} \langle \kappa \rangle_{\text{naive}}
\]
Power corrections in the DMW approach

- Combine real and virtual corrections and obtain

\[
\frac{\alpha_s(\kappa^2)}{\kappa^2} \rightarrow \int_0^\infty \frac{dm^2}{(m^2 + \kappa^2)^2} \alpha_{\text{eff}}(m^2)
\]

- Allow gluon to decay inclusively \( \kappa^2 \rightarrow \kappa^2 + m^2 \)

\[
\langle \delta V \rangle_{\text{naive}} = \frac{\langle \kappa \rangle_{\text{naive}}}{Q} \sum_{i<j} (-2 \vec{T}_i \cdot \vec{T}_j) c^{(ij)}_V
\]

\[
\langle \kappa \rangle_{\text{naive}} = \int_0^\infty \frac{d\kappa^2 dm^2}{(m^2 + \kappa^2)^2} \sqrt{\kappa^2 + m^2} \alpha_{\text{eff}}(m^2) \simeq 2 \int_0^\infty \frac{dm^2}{m^2} m \delta\alpha_{\text{eff}}(m^2)
\]

- Take into account non-inclusiveness via the Milan factor

\[
\langle \kappa \rangle \rightarrow M \langle \kappa \rangle_{\text{naive}}
\]
Power corrections to multi-jet shapes

- **Two-jet event shapes** ⇒ \( \langle \delta V \rangle \) same as in Feynman tube model

\[
\langle \delta V \rangle = 2C_F \left( \frac{\langle \kappa \rangle}{Q} c_V \right) = \frac{\langle k_t \rangle}{Q} c_V
\]

Universality of \( 1/Q \) corrections ⇒ \( dn_h/d \ln k_t d\eta \) is uniform in \( \eta \)!

- **Multi-jet event shapes** ⇒ test PQCD-inspired hadronisation

\[
\langle \delta V \rangle = \frac{\langle \kappa \rangle}{Q} \sum_{i<j} (-2\vec{T}_i \cdot \vec{T}_j) c_{ij}^{(ij)}
\]

1. \( \langle \kappa \rangle \) same as in two-jet event shapes ⇒ Universality
2. dependence on hard parton colour through \( \vec{T}_i \cdot \vec{T}_j \)
3. dependence on event geometry through \( c_{ij}^{(ij)} \)
Power corrections to multi-jet shapes

- **Two-jet event shapes** ⇒ ⟨δV⟩ same as in Feynman tube model

  \[ \langle \delta V \rangle = 2C_F \frac{\bar{\kappa}}{Q} c_V = \frac{\bar{k}_t}{Q} c_V \]

  **Universality of 1/Q corrections** ⇒ \( dn_h/d \ln k_t d\eta \) is uniform in \( \eta \)!

- **Multi-jet event shapes** ⇒ test PQCD-inspired hadronisation

  \[ \langle \delta V \rangle = \frac{\bar{\kappa}}{Q} \sum_{i<j} (-2 \vec{T}_i \cdot \vec{T}_j) c_V^{(ij)} \]

  1. ⟨κ⟩ same as in two-jet event shapes ⇒ Universality
  2. dependence on hard parton colour through \( \vec{T}_i \cdot \vec{T}_j \)
  3. dependence on event geometry through \( c_V^{(ij)} \)
Power corrections to multi-jet shapes

- **Two-jet event shapes** ⇒ \( \langle \delta V \rangle \) same as in Feynman tube model

\[
\langle \delta V \rangle = 2C_F \frac{\langle \kappa \rangle}{Q} c_V = \frac{\langle k_t \rangle}{Q} c_V
\]

**Universality of** \( 1/Q \) **corrections** ⇒ \( dn_h/d \ln k_t d\eta \) **is uniform in** \( \eta \)!

- **Multi-jet event shapes** ⇒ test PQCD-inspired hadronisation

\[
\langle \delta V \rangle = \frac{\langle \kappa \rangle}{Q} \sum_{i<j} (-2 \vec{T}_i \cdot \vec{T}_j) c_V^{(ij)}
\]

\( \langle \kappa \rangle \) **same as in two-jet event shapes** ⇒ **Universality**

2. Dependence on hard parton colour through \( -\vec{T}_i \cdot \vec{T}_j \)

3. Dependence on event geometry through \( c_V^{(ij)} \)
Power corrections to multi-jet shapes

- **Two-jet event shapes** ⇒ $\langle \delta V \rangle$ same as in Feynman tube model

  \[
  \langle \delta V \rangle = 2C_F \frac{\langle \kappa \rangle}{Q} c_V = \frac{\langle k_t \rangle}{Q} c_V
  \]

  Universality of $1/Q$ corrections ⇒ $dn_h / d \ln k_t d\eta$ is uniform in $\eta$!

- **Multi-jet event shapes** ⇒ test PQCD-inspired hadronisation

  \[
  \langle \delta V \rangle = \frac{\langle \kappa \rangle}{Q} \sum_{i<j} (-2\vec{T}_i \cdot \vec{T}_j) c_{V}^{(ij)}
  \]

  1. $\langle \kappa \rangle$ same as in two-jet event shapes ⇒ **Universality**
  2. dependence on hard parton colour through $\vec{T}_i \cdot \vec{T}_j$
  3. dependence on event geometry through $c_{V}^{(ij)}$
Outline

1. Introduction
   - Why event-shape variables?
   - Power corrections and Feynman tube model

2. Three-jet event shapes
   - PQCD and NP corrections
   - $K_{out}$ and $D$-parameter in $e^+e^-$ annihilation
   - $K_{out}$ in DY and underlying event
Three-jet events

- **Thrust-major:**
  \[ T_M Q \equiv \max_{\vec{n}_M \cdot \vec{n}_T = 0} \sum_i |\vec{p}_i \cdot \vec{n}_M| \]

- **Event plane:** \( \langle \vec{n}_T; \vec{n}_M \rangle \)
  \[ \vec{p}_{ti} = p_i^{\text{in}} \vec{n}_{\text{in}} + p_i^{\text{out}} \vec{n}_{\text{out}} \]
  \[ \vec{n}_{\text{out}} \equiv \vec{n}_T \times \vec{n}_M \]
$D$-parameter and Thrust-minor

- **$D$-parameter**: determinant of the momentum tensor $\theta_{\alpha\beta}$

$$
\theta_{\alpha\beta} \, Q \equiv \sum_h \frac{p_h^\alpha p_h^\beta}{|\vec{p}_h|} \quad D \equiv 27 \, \text{det} \, \theta
$$

$$
\delta D \simeq 27 \lambda_1 \lambda_2 \sum_i \frac{\kappa_i^2 \sin^2 \phi_i}{\omega_i Q} \quad \text{damped in rapidity}
$$

$$
c_D = C_F \, g_q(T, T_M) + C_F \, g_{\bar{q}}(T, T_M) + C_A \, g_g(T, T_M)
$$

- **Thrust-minor**: sum of the momenta out of the event plane

$$
K_{\text{out}} \equiv \sum_h |p_h^{\text{out}}| \quad \delta K_{\text{out}} \simeq \sum_i \kappa_i |\sin \phi_i| \quad \text{uniform in rapidity}
$$

$$
c_{K_{\text{out}}} = \langle |\sin \phi| \rangle \left( C_F \ln \frac{Q_{q\bar{q}}}{|p_q^{\text{out}}|} + C_F \ln \frac{Q_{q\bar{q}}}{|p_{\bar{q}}^{\text{out}}|} + C_A \ln \frac{Q_{gg} Q_{g\bar{q}}}{|p_g^{\text{out}}| Q_{q\bar{q}}} \right)
$$
**D-parameter and Thrust-minor**

- **D-parameter:** determinant of the momentum tensor $\theta_{\alpha\beta}$

\[
\theta_{\alpha\beta} Q \equiv \sum_h \frac{p_h^\alpha p_h^\beta}{|\vec{p}_h|} \quad D \equiv 27 \det \theta
\]

\[
\delta D \simeq 27 \lambda_1 \lambda_2 \sum_i \frac{\kappa_i^2 \sin^2 \phi_i}{\omega_i Q}
\]

damped in rapidity

\[
c_D = C_F g_q(T, T_M) + C_F g_{\bar{q}}(T, T_M) + C_A g_g(T, T_M)
\]

- **Thrust-minor:** sum of the momenta out of the event plane

\[
K_{\text{out}} \equiv \sum_h |p_h^{\text{out}}| \quad \delta K_{\text{out}} \simeq \sum_i \kappa_i |\sin \phi_i|
\]

uniform in rapidity

\[
c_{K_{\text{out}}} = \langle |\sin \phi| \rangle \left( C_F \ln \frac{Q_{q\bar{q}}}{|p_{q}^{\text{out}}|} + C_F \ln \frac{Q_{q\bar{q}}}{|p_{\bar{q}}^{\text{out}}|} + C_A \ln \frac{Q_{gg} Q_{g\bar{g}}}{|p_g^{\text{out}}| Q_{q\bar{q}}} \right)
\]
$D$-parameter: theory vs data

- Select 3-jet events with $y_3 > y_{\text{cut}}$
- $D$-parameter distribution with LO matching
- $D$-parameter means for different values of $y_{\text{cut}}$
**D-parameter: theory vs data**

- **Select 3-jet events with** $y_3 > y_{cut}$
- **$D$-parameter distribution** with LO matching
- **$D$-parameter means** for different values of $y_{cut}$

\[ \sigma^{-1} d\sigma/dD \]

- $\alpha_s(M_Z)=0.118$
- $\alpha_s(2 \text{ GeV})=0.52$
- $y_{cut}=0.1$

\[ \text{NLL+LO} \quad \text{NLL+LO+NP} \quad \text{ALEPH} \]
$D$-parameter: theory vs data

- Select 3-jet events with $y_3 > y_{cut}$
- $D$-parameter distribution with LO matching
- $D$-parameter means for different values of $y_{cut}$
Outline

1. Introduction
   - Why event-shape variables?
   - Power corrections and Feynman tube model

2. Three-jet event shapes
   - PQCD and NP corrections
   - $K_{\text{out}}$ and $D$-parameter in $e^+e^-$ annihilation
   - $K_{\text{out}}$ in DY and underlying event
$K_{\text{out}}$ in hadronic $Z_0$ production

- Event plane: $\langle \vec{q}; \vec{p}_1 \rangle \Rightarrow \vec{n}_{\text{out}} = \frac{\vec{q} \times \vec{p}_1}{|\vec{q}| |\vec{p}_1|}$

$$K_{\text{out}} \equiv \sum_h |p_{h\text{out}}| \times \Theta(|\eta_h| - \eta_0) \quad \text{rapidity cut around beam pipe}$$

- Soft approximation

$$\delta K_{\text{out}} \approx \sum_i \kappa_i |\sin \phi_i| + \sum_i \kappa_{ti} |\sin \phi_i|$$

QCD emissions \hspace{1cm} \text{beam remnant}
$K_{\text{out}}$ in hadronic $Z_0$ production

Event plane: $\langle \vec{q} ; \vec{p}_1 \rangle \Rightarrow \vec{n}_{\text{out}} = \frac{\vec{q} \times \vec{p}_1}{|\vec{q}| |\vec{p}_1|}$

$K_{\text{out}} \equiv \sum_{h} |p_{h}^{\text{out}}| \times \Theta(|\eta_h| - \eta_0)$ \hspace{1cm} \text{rapidity cut around beam pipe}

Soft approximation

$\delta K_{\text{out}} \simeq \sum_i \kappa_i |\sin \phi_i| + \sum_i \kappa_{ti} |\sin \phi_i|$ \\
QCD emissions \hspace{1cm} \text{beam remnant}$
$K_{out}$ in hadronic $Z_0$ production

- Event plane: $\langle \vec{q} ; \vec{p}_1 \rangle \Rightarrow \vec{n}_{out} = \frac{\vec{q} \times \vec{p}_1}{|\vec{q}||\vec{p}_1|}$

$$K_{out} \equiv \sum_h |p_{h}^{out}| \times \Theta(|\eta_h| - \eta_0) \quad \text{rapidity cut around beam pipe}$$

- Soft approximation

$$\delta K_{out} \simeq \sum_i \kappa_i |\sin \phi_i| + \sum_i k_{ti} |\sin \phi_i|$$

QCD emissions \hspace{2cm} beam remnant
Power corrections for $K_{\text{out}}$ in DY

- QCD emissions with $\kappa \sim \Lambda_{QCD}$ (gluers)

\[ cK_{\text{out}} = \langle |\sin \phi| \rangle \left( C_1(\eta_0 - \eta_3) + C_2(\eta_0 + \eta_3) + C_3 \ln \frac{Q_t}{|p_3^{\text{out}}|} \right) \]

- Extra $\delta K_{\text{out}}$ produced in beam remnant interactions

\[ \delta K_{\text{out}}^{\text{remnant}} = \frac{\langle k_t \rangle}{Q_t} \times \langle |\sin \phi| \rangle \times 2\eta_0 \]
Power corrections for $K_{\text{out}}$ in DY

- **QCD emissions with $\kappa \sim \Lambda_{QCD}$ (gluers)**

\[ cK_{\text{out}} = \langle | \sin \phi | \rangle \left( C_1 (\eta_0 - \eta_3) + C_2 (\eta_0 + \eta_3) + C_3 \ln \frac{Q_t}{|p_{\text{out}}^3|} \right) \]

- **Extra $\delta K_{\text{out}}$ produced in beam remnant interactions**

\[ \delta K_{\text{out}}^{\text{remnant}} = \frac{\langle k_t \rangle}{Q_t} \times \langle | \sin \phi | \rangle \times 2\eta_0 \]
Conclusions

- Multi-jet event shapes benefits
  - More stringent tests of universality
  - $1/Q$ corrections depend on hard parton colour and geometry

- Three-jet event shapes are coming soon
  1. NLO+PC already there for $D$ parameter means
  2. Work in progress for NLL+NLO+PC

- Four jets and more
  1. Quantum evolution of colour?
  2. Soft underlying event (already there in DY)