VIBRATION EFFECTS IN SHORT RAYLEIGH LENGTH FELS


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Abstract

The short-Rayleigh length FEL configuration leaves the optical resonator near the cold-cavity stability limit. Studies show that the electron beam interaction stabilizes the optical modes and establishes limits to the vibrations of mirrors and the electron beam. Several types of vibrations are considered.

INTRODUCTION

Two of the central issues in designing a compact high-power free electron laser are overall system size and the ability of the cavity mirrors to withstand intense optical fields without damage. One design solution is to use a short cavity to reduce the system length, along with a short Rayleigh range \( Z_0 \) in order to spread the optical mode over the mirrors and thereby reduce the optical intensity for a given power. Such an optical mode has narrow radius \( W_0 \) at the waist and wide radius \( W \) at the mirror – the magnification \( W/W_0 \) is then \( S/2Z_0 \) for a cavity of length \( S \) [1]. To achieve this small \( Z_0 \) ratio, the mirror radii of curvature \( R \) must be only slightly greater than the cavity half-length \( S/2 \); for such a configuration the cavity stability factor \( g = 1 - S/R \) is only slightly above the cold-cavity stability limit \( g = -1 \). Above this limit, if \( g \) is only slightly greater than \(-1\), small fluctuations in the mirror position and orientation will cause large geometric tilts of the cavity axis and may have a significant effect on the FEL performance.

The effect of mirror tilt has been studied previously for short Rayleigh length 100 kW and 1 MW FEL designs [2]. For the case of no gain, the effects on the optical mode of mirror tilt, transverse and longitudinal mirror shift, and mirror focal length change have been reported [3, 4]. Here we report simulations of our latest compact design, and include, along with mirror tilt \( \theta_m \), the effects of mirror shift \( y_m \) and of transverse electron beam shift \( y_e \).

Since mirror motions are relatively slow (~ 100 ms) compared with the optical round trip time (~ ns) the motions are assumed to be fixed over many passes of the optical mode through the cavity. Simulation methods use the self-consistent Maxwell-Lorentz equations in the presence of the electron beam. In order to accommodate the large scale difference between the optical mode at its waist and at the mirror, an expanding coordinate system is utilized [5] which allows the simulation to follow the optical mode all the way to the mirror, even when the cavity length is very long compared with the cavity length, \( S \approx 300Z_0 \).

The electron beam is characterized by dimensionless current density, radius, angular spread and initial phase velocity; the undulator by its period, length, and undulator parameter \( K \); and the cavity by its length, mirror shape and size, and energy loss per pass. The cavity mirrors may be adjusted for tilt and transverse shift, and the electron beam may be shifted, as shown in Fig. 1. Both the weak-field gain and energy extraction (defined as the electron beam energy loss per pass divided by the electron beam energy) are reported as a function of the pass number.

The FEL explored has parameters: cavity length \( S = 18 \) m, Rayleigh range \( Z_0 = 6 \) cm, optical wavelength \( \lambda = 1 \mu \text{m} \), and optical power outcoupling/pass 25%. The electron beam energy is 80 MeV, peak beam current \( I_{\text{peak}} = 400 \) A, with bunch charge 0.2 nC. Since \( Z_0/S = 0.0033 \), this design will be subject to the short Rayleigh length effects described above.

MIRROR TILT

If one mirror of a laser cavity is tilted by angle \( \theta_m \), geometric considerations show that a new cavity axis will be defined which tilts with respect to the old axis by amount \( \phi \), where \( \phi = (S^2 + 4Z_0^2)/(8Z_0^2)\theta_m \) [4]. For \( Z_0 < S \), as is the case here, \( \phi \) can become quite large. Figure 2 shows the result of mirror tilt on the weak-field gain, showing that the laser will continue to operate up to \( \theta_m \approx 2.6 \mu \text{rad} \) at the laser threshold of 33% gain (corresponding to 25% cavity loss). Figure 2 also shows the extraction as a function of \( \theta_m \). If we take half of the peak power as a measure of the useful range of mirror tilt, we see that a tilt \( \theta_m \approx 1.4 \mu \text{rad} \) is acceptable. In fact, for both graphs, very little decrease in gain or extraction is evident out to \( \theta_m \approx 0.5 \mu \text{rad} \). With active alignment mirrors can currently be held to a design tolerance \( \approx 0.1 \mu \text{rad} \), therefore mirror tilt does not appear...
Figure 2: Dependence of weak-field gain and extraction on mirror tilt angle. Lasing continues out to 2.5 μrad; no appreciable change occurs over the current design tolerance of ≈ 0.1 μrad.

Figure 3: Simulation output showing the optical mode \(|a(0, y, z)|\) along the undulator for mirror tilt 2.2 μrad. The white lines represent 5% or the optical mode peak, the red curves show the untilted initial optical mode profile, the yellow lines are the final optical mode profile, and the vertical red lines show the electron distribution. The optical mode is tilted by 7 mrad, which is much less than the predicted cavity tilt.

**ELECTRON BEAM SHIFT**

A transverse shift of the electron beam will leave the cavity axis untilted, but the electron beam and optical mode will no longer be aligned. Figure 5 shows the gain and extraction for beam shifts up to 0.9 mm where lasing no longer occurs. Taking the half-power extraction as an acceptable beam shift range, the simulation tolerance is ≈ 0.4 μm.

Figure 4: Dependence of weak field gain and extraction on transverse mirror shift. Lasing continues to 22 μm; no appreciable change occurs out to a few μm.

**MIRROR SHIFT**

A transverse shift of one cavity mirror by amount \(y_m\) will also cause a geometric cavity axis tilt \(\phi\), where here \(\phi = (2S^2/8Z_0^2)y_m\). For our case of \(Z_0 \ll S\), a small mirror shift may cause a large axis tilt. Figure 4 shows the gain and extraction for mirror shifts up to the laser threshold at \(y_m = 22 \mu m\). Taking the half-power extraction for an acceptable mirror shift range, the simulation tolerance is \(y_m \approx 13 \mu m\). We have also examined the optical mode for a shift \(y_m = 21 \mu m\) and found a tilt of 0.5 mrad. The appearance of this optical mode is very similar to Fig. 3. Again, the simulation tilt is much less than the predicted cavity axis tilt \(\phi\), showing that the electron beam is stabilizing the optical mode.

Figure 3: Simulation output showing the optical mode \(|a(0, y, z)|\) along the undulator for mirror tilt 2.2 μrad. The white lines represent 5% or the optical mode peak, the red curves show the untilted initial optical mode profile, the yellow lines are the final optical mode profile, and the vertical red lines show the electron distribution. The optical mode is tilted by 7 mrad, which is much less than the predicted cavity tilt.
mm, a figure well beyond the current design tolerance of $\approx 10 \, \mu m$.

Although the cavity axis remains untilted, examination of optical mode diagrams similar to Fig. 3 shows that the optical mode does tilt, with the downstream side of the optical mode displaced in the same direction as the electron beam offset. [6] We attribute this tilt to the increase in electron bunching and hence the gain near the downstream end of the undulator. For an electron mode shift $y_e = 0.44$ mm, we observe an optical mode tilt of 1.3 mrad. Nevertheless, the optical mode overlaps the electron beam through the undulator.

**CONCLUSIONS**

The susceptibility of a free electron laser to vibrations – motions of the mirrors and of the electron beam – may be pronounced if the Rayleigh length of the cavity is small. We have investigated the dependence of weak-field gain and extraction on mirror tilt, mirror shift, and electron beam shift for a laser with $Z_0 / S = 0.0033$. From our simulations the laser maintains greater than half its power for mirror tilt $< 1.4 \, \mu rad$, transverse mirror shift $< 13 \, \mu m$, and electron beam shift $< 0.4$ mm. These tolerances are beyond known design tolerances and should be acceptable. We also observe that mirror tilt and shift tends to rotate the optical mode, but the rotation is considerably less than the cavity axis tilt predicted from geometric considerations, thereby showing that the electron beam strongly stabilizes the optical mode. In the case of electron beam shift, the optical mode also rotates, but the rotation is caused by asymmetric gain in the undulator rather than by cavity geometry.

**REFERENCES**