Effect of Transverse Wakefield in Low Energy Operation (Giga-Z)

200508xx K.Kubo

Analytical Calculation of wakefield effect

Emiitance increase due to cavity misalignment

(A) Constant betafunction

$$\langle \Delta\gamma\varepsilon \rangle \propto \beta \frac{a^2}{g} \log\left(\frac{E_f}{E_i} \right)$$

a : rms misalignment

g : gradient

E_i, E_f : initial and final energy

(B) Betafunction ~ sqrt(E)

$$\langle \Delta\gamma\varepsilon \rangle \propto \frac{\beta}{\sqrt{E}} \frac{a^2}{g} 2\left(\sqrt{E_f} - \sqrt{E_i} \right)$$

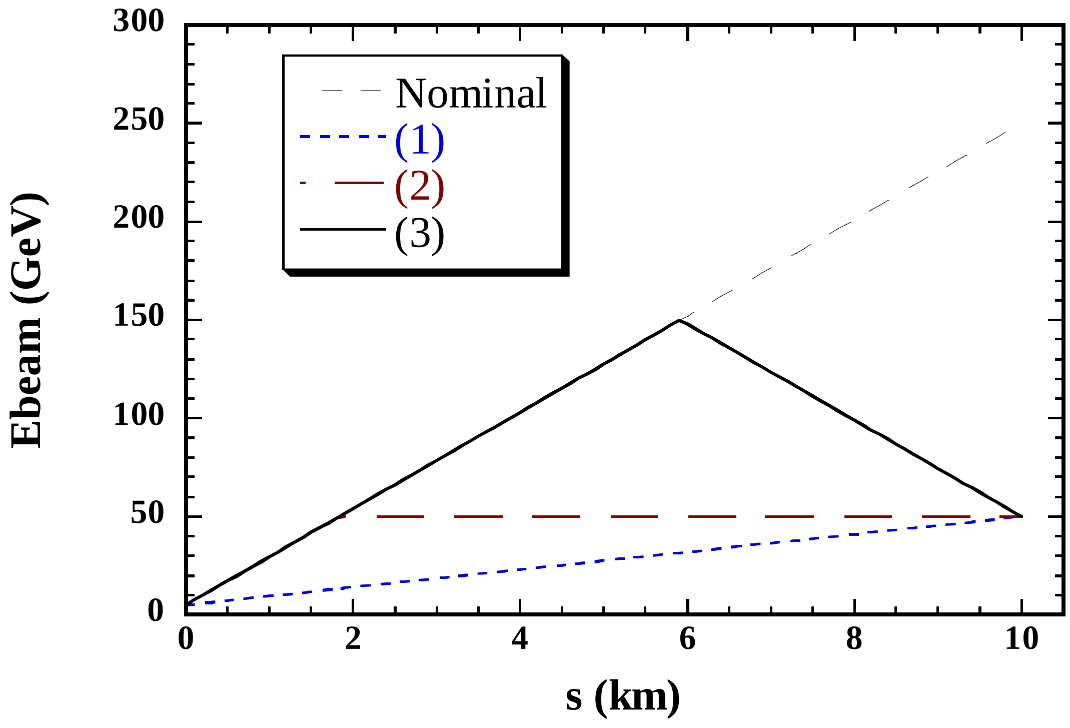
[1] K. Kubo, Methods to Estimate Emittance Dilutions Due to Misalignment of Accelerating Structures with Tracking, NLCNote-13 (1995), http://www-project.slac.stanford.edu/lc/local/NLCNotes/NLCnote 13.pdf .

[2] K.L.F. Bane, et.al, ISSUES IN MULTIBUNCH EMITTANCE PRESERVATION IN THE NLC, Proceedings, EPAC 94, vol. 2 1114-1116,

Three cases

(Final Beam energy 50 GeV where Nominal is 250 GeV)

1. Constant gradient
2. Accelerate first, then, no acceleration
3. Accelerate to 150 GeV, then, deccelerate to 50 GeV



Emittance increase ratio compared with nominal operation

		Constant beta	Beta ~ sqrt(E)
(1)	Constant gradient from 5 to 50 GeV	3.20	1.94
(2)	Accelerate with nominal gradient, then, no acceleration	1.61	1.40
(3)	Accelerate to 150 GeV, then, decelerate to 50 GeV	1.15	1.12

Wakefield effect in low gradient operation

revised 20050812 Kiyoshi Kubo

Analytical estimation of emittance increase from wakefield due to cavity misalignment for GigaZ operation, where the final beam energy is about 50 GeV, is compared with nominal operation, where the final beam energy is 250 GeV.

From analytical considerations as in references [1][2], emittance increase from wakefield due to cavity misalignment is expected to be

(A) Constant Betafunction

$$\langle \Delta\gamma\varepsilon \rangle \propto \frac{a^2}{g} \log\left(\frac{E_f}{E_i}\right),$$

(B) $\beta \propto \sqrt{E}$

$$\langle \Delta\gamma\varepsilon \rangle \propto \frac{a^2}{g} 2\left(\sqrt{E_f} - \sqrt{E_i}\right)$$

where a is rms misalignment of cavities, g the accelerating gradient, E_i and E_f the initial and final beam energy.

If there is no acceleration ($g = 0,\ E_i = E_f$),

(A) $$\langle \Delta\gamma\varepsilon \rangle \propto \frac{a^2 l}{E_i},$$ (B) $$\langle \Delta\gamma\varepsilon \rangle \propto \frac{a^2 l}{\sqrt{E_i}}$$

where l is the total length of the cavities.

For operation of final beam energy 50GeV, where nominal beam energy is 250 GeV, we consider three cases as follow. Initial energy is assumed to be 5 GeV.

(1) Operate all cavities with lower gradient, (50-5)/(250-5) of nominal.
For the same misalignment,

(A) $$\frac{\langle \Delta\gamma\varepsilon \rangle_{\text{case(1)}}}{\langle \Delta\gamma\varepsilon \rangle_{\text{nominal}}} = \frac{\dfrac{1}{50-5}\log\left(\dfrac{50}{5}\right)}{\dfrac{1}{250-5}\log\left(\dfrac{250}{5}\right)} \approx 3.20 \quad .$$

(B) $$\frac{\langle \Delta\gamma\varepsilon \rangle_{\text{case(1)}}}{\langle \Delta\gamma\varepsilon \rangle_{\text{nominal}}} = \frac{\dfrac{2}{50-5}\left(\sqrt{50}-\sqrt{5}\right)}{\dfrac{2}{250-5}\left(\sqrt{250}-\sqrt{5}\right)} \approx 1.94$$

It means emittance increase will be larger than nominal operation by factor of 3.20 or 1.94.

(2) Operate the first (50-5)/(250-5) part of cavities with nominal gradient, and no acceleration by others (probably de-tuned).

For the same misalignment,

(A) $$\frac{\langle \Delta\gamma\varepsilon \rangle_{\text{case(2)}}}{\langle \Delta\gamma\varepsilon \rangle_{\text{nominal}}} = \frac{\dfrac{l}{250-5}\log\left(\dfrac{50}{5}\right) + \dfrac{l_2}{50}}{\dfrac{l}{250-5}\log\left(\dfrac{250}{5}\right)} \approx 1.61,$$

(B) $$\frac{\langle \Delta\gamma\varepsilon \rangle_{\text{case(2)}}}{\langle \Delta\gamma\varepsilon \rangle_{\text{nominal}}} = \frac{\dfrac{2l}{250-5}\left(\sqrt{50}-\sqrt{5}\right) + \dfrac{l_2}{\sqrt{50}}}{\dfrac{2l}{250-5}\left(\sqrt{250}-\sqrt{5}\right)} \approx 1.40$$

where l is the total active length and l_2 is the length of the second part cavities ($l_2/l=200/245$).
It means emittance increase will be larger than nominal operation by factor 1.61 or 1.40.

(3) Accelerate up to 150 GeV, then, decelerate to 50 GeV. Gradient is as same as the nominal operation (opposite sign in the second part).

For the same misalignment,

(A) $$\frac{\langle \Delta\gamma\varepsilon \rangle_{\text{case(3)}}}{\langle \Delta\gamma\varepsilon \rangle_{\text{nominal}}} = \frac{\log\left(\frac{150}{5}\right) - \log\left(\frac{50}{150}\right)}{\log\left(\frac{250}{5}\right)} \approx 1.15 \quad ,$$

(B) $$\frac{\langle \Delta\gamma\varepsilon \rangle_{\text{case(3)}}}{\langle \Delta\gamma\varepsilon \rangle_{\text{nominal}}} = \frac{\left(\sqrt{150} - \sqrt{5}\right) - \left(\sqrt{50} - \sqrt{150}\right)}{\sqrt{250} - \sqrt{5}} \approx 1.12$$

It means emittance increase will be larger by factor 1.15 or 1.12.

Summary

		Emittance increase factor compared with nominal operation	
		Constant beta	Beta ~ sqrt(E)
(1)	Constant gradient from 5 to 50 GeV	3.20	1.94
(2)	Accelerate with nominal gradient, then, no acceleration	1.61	1.40
(3)	Accelerate to 150 GeV, then, decelerate to 50 GeV	1.15	1.12

References:

[1] K. Kubo, Methods to Estimate Emittance Dilutions Due to Misalignment of Accelerating Structures with Tracking, NLCNote-13 (1995), http://www-project.slac.stanford.edu/lc/local/NLCNotes/NLCnote 13.pdf .
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