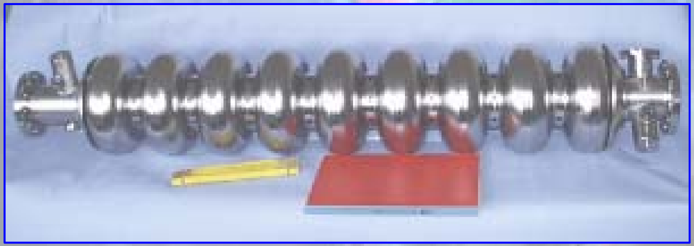
Emittance Dilution Due Transverse Mode Coupling

- **Injected emittance: $\varepsilon_{x,y}$ = 8e-6, 2e-8 m.rads**
- **Injection beam energy 5 GeV => O/P 500 GeV c.m. energy**
- **$\gamma_{i,f}$ ~ 9782, and 489,000**
- **All simulations are made with the code LIAR.**
- **500 bunches in all simulations.**
- **Initial vertical and horizontal offset ~10 μm and 400μm.**
- **c.f. initial beam $\sigma_{y,x}$ (= $[\beta\varepsilon_n/\gamma]^{1/2}$) ~ 10.1 μm, 270 μm.**
- **$\sigma_{x',y'}$ (= $[(1+\alpha^2)/\beta\ \varepsilon_n/\gamma]^{1/2}$) ~ 5.1, 0.26 μrads**
- **Initial $\beta_{x,y}$ ~ 89.3, 50.7 m**



TESLA 9-Cell Cavity

Thanks to *Chris Adolphsen*, Nicoleta Baboi,
Joe Frisch, *Roger Miller*, Tor Raubenheimer, Mark Ross, *Peter Tenenbaum.*

Overview

Make several injection offset simulations to investigate issue of emittance dilution.

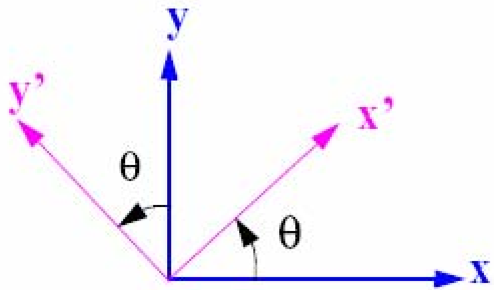
1. **Force the angle of all couplers relative to the dipole field to be static.**
2. **Allow complete azimuthal randomization of couplers (0 to 2π).**
3. **Limit the azimuthal spread of the couplers.**

- **For *detailed* simulations consider a single machine for the above cases.**
- **For general emittance dilution consider 200 separate machines.**

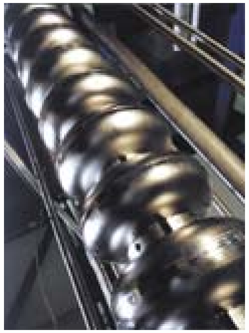
Two cases are investigated:

1. **'Properly' damped cavities (all modes damped according to the design)**
2. **'Rogue' mode miss-damped. What is the impact on the emittance dilution?**

Degenerate Dipole Mode Rotation and E.M. Field Coupling



$$V_{x'}(0) = V_x(0)\cos\theta$$
$$V_{y'}(0) = -V_x(0)\sin\theta$$



$$V_y(t) = V_{x'}(t)\sin\theta + V_{y'}(t)\cos\theta$$

$$\Rightarrow V_y(t) = V_x(0)\sin\theta\cos\theta\exp(i\,\omega_{x'}t) - V_x(0)\sin\theta\cos\theta\exp(i\,\omega_{y'}t)$$

$$\Rightarrow V_y(t) = i\,V_x(0)\sin 2\theta\exp(i\langle\omega\rangle t)\sin(\Delta\omega t/2)$$

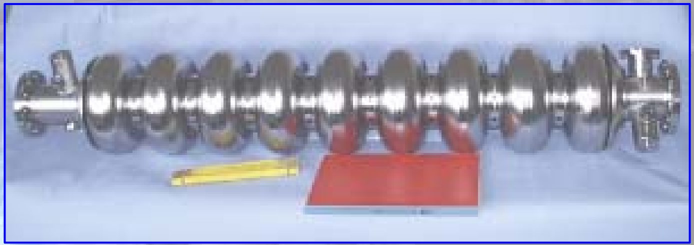
where the splitting of the degenerate frequency is given by:

$$\Delta\omega = \omega_{x'} - \omega_{y'}$$

and the mean frequency: $\langle\omega\rangle = (\omega_{x'} + \omega_{y'})/2$

$$V_y = iV_x(0)\sin 2\theta \exp(i\langle\omega\rangle t)\sin(\Delta\omega t/2)$$

Thus for $\theta = 0$ or $\pi/2$ $V_y(0) = 0$ as expected!



TESLA 9-Cell Cavity

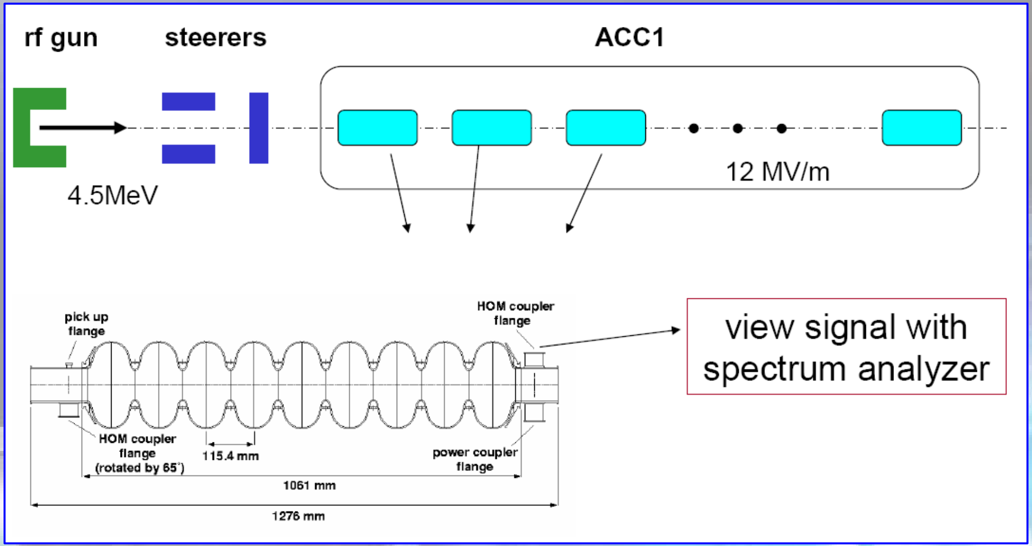
The additional kick to the beam is of the form:

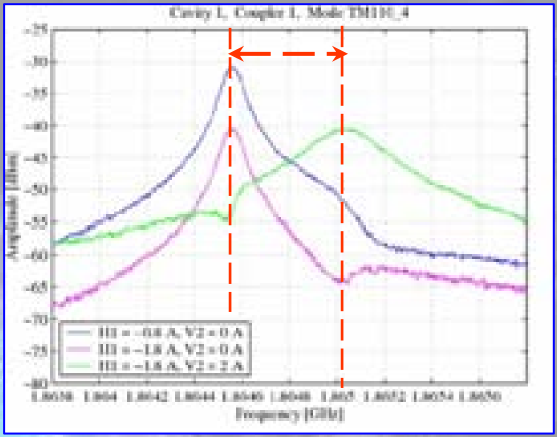
$$V(t) = \Sigma K_n \sin(\omega_p t)\exp(-\omega_{pt}/2Q_p)U(t)y +$$
$$\Sigma K_n \cos(\langle\omega_p\rangle t)\sin(\Delta\omega_p t/2)\exp(-\langle\omega_{p>} t/2Q_p)U(t)x$$

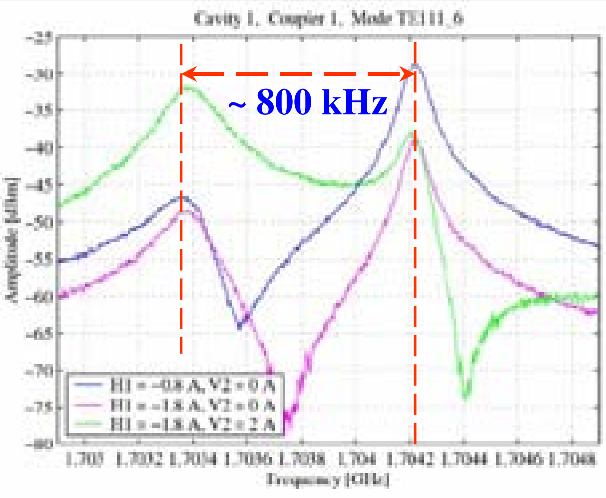
Where the sum is taken over all modes of interest and U(t) is the unit step function

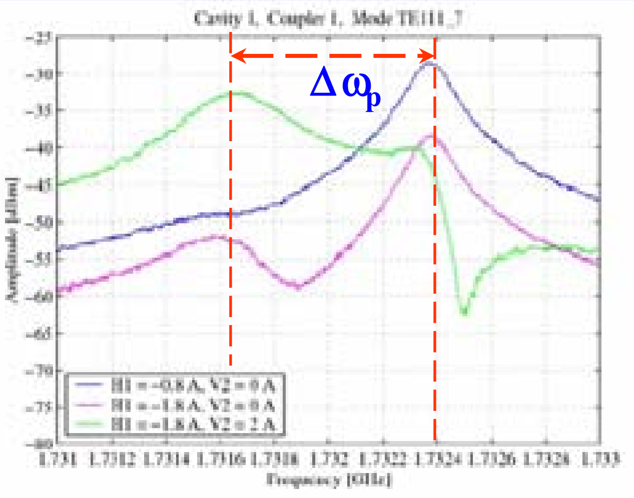
=> Recent TTF experiments (Ross, Napoly & Baboi) suggest that this frequency splitting varies by ~ 400 kHz to 800 kHz. In all simulations presented here we use 600 kHz frequency splitting for all modes.

Measurement of HOMs at TTF





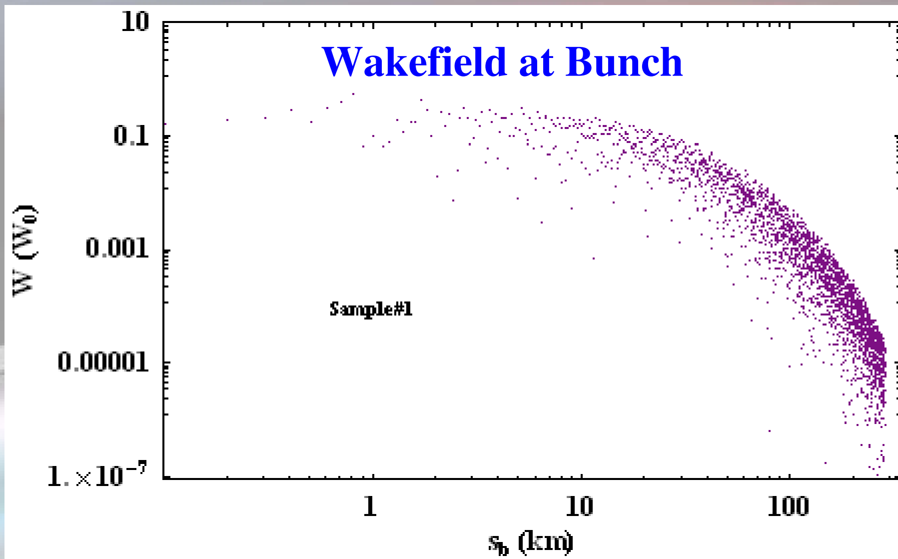


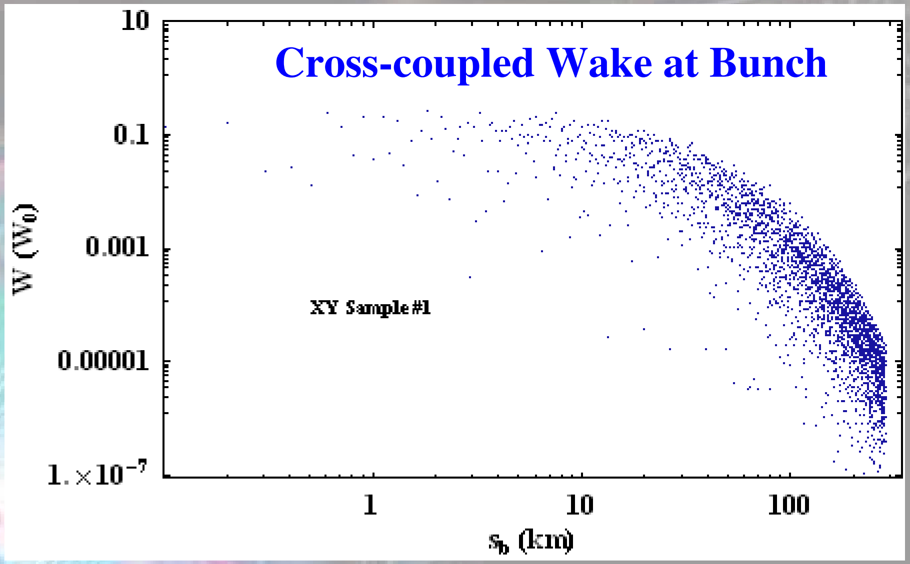


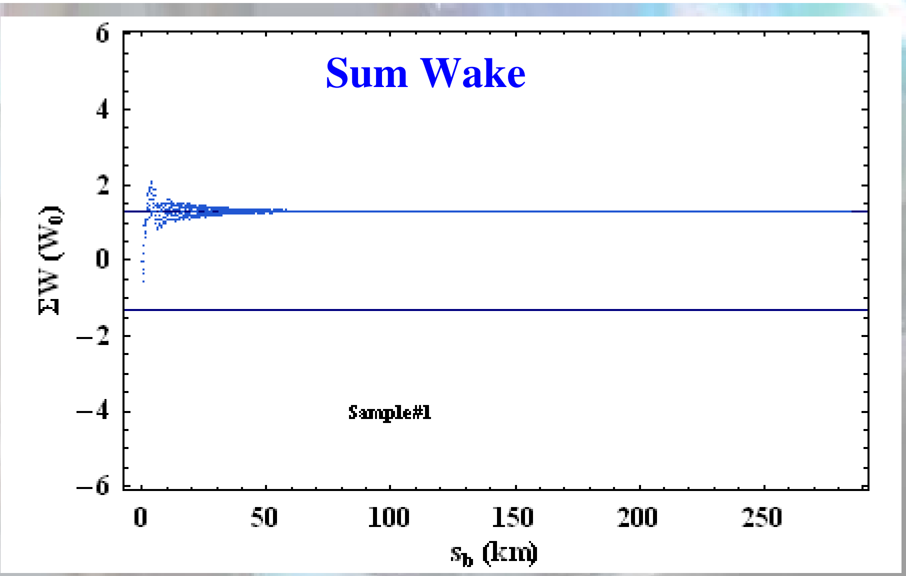
Taken from Ross, Baboi, Napoly et al. See Linac04 paper & SLAC-PUB 11190.

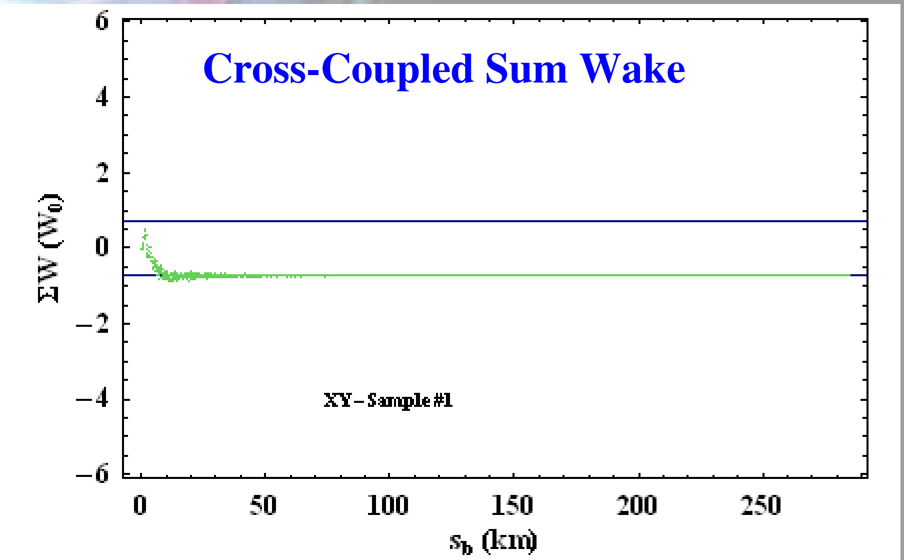
Transverse Wakefields in TESLA-like Cavities.

$W_y(t)= \Sigma K_n \sin(\omega_p t) \exp(-\omega_p t/2Q_p)U(t)$, $W_{xy}=\Sigma K_n \cos(\langle\omega_p\rangle t) \sin(\Delta\omega_p t/2) \exp(-\langle\omega_p\rangle t/2Q_p)U(t)$.

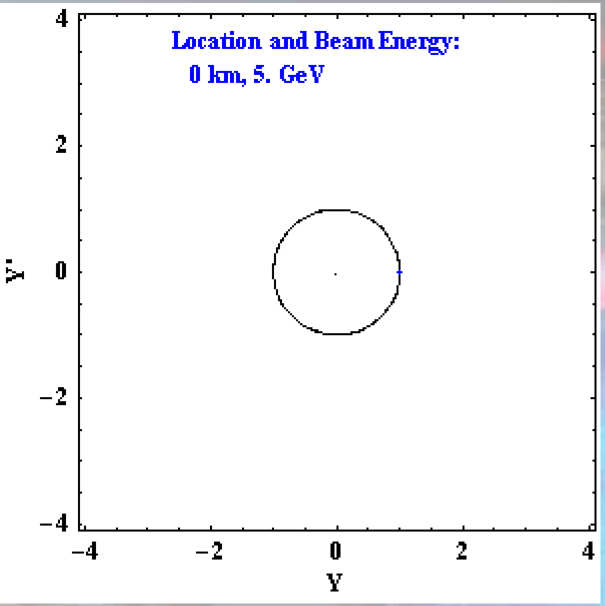


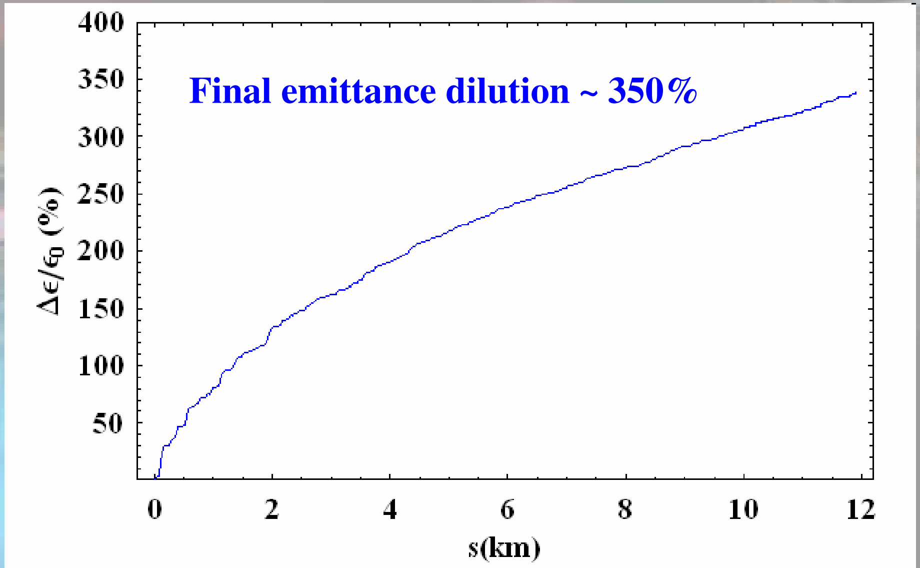


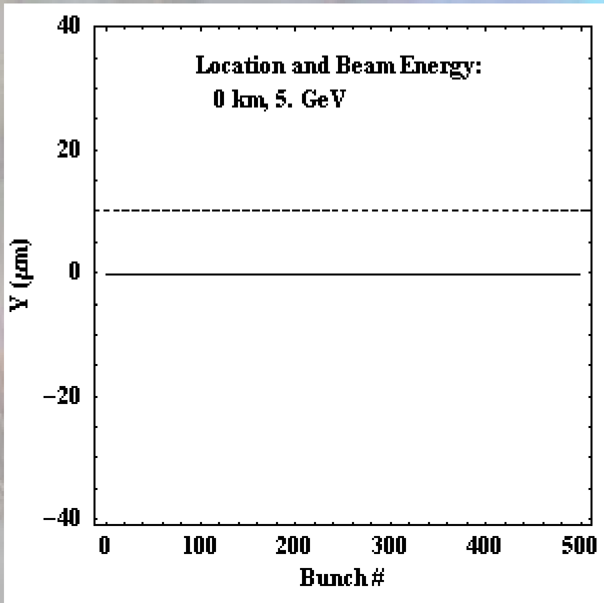




Cross-Coupled Motion *At* Fixed Azimuthal Phase Distribution (Yoff=0).

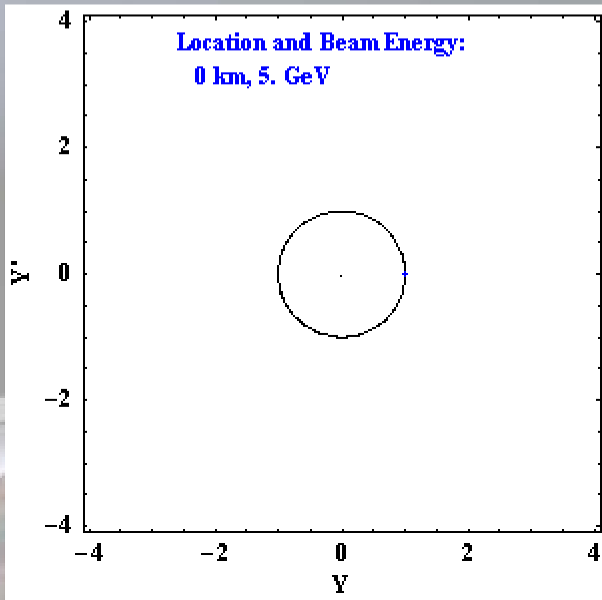


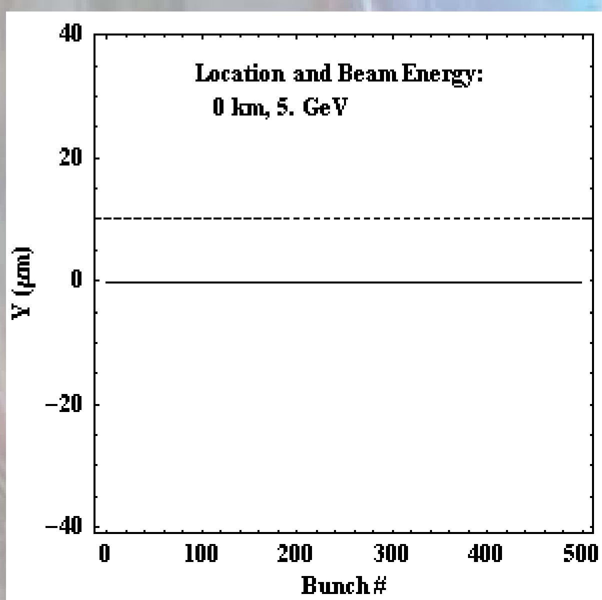


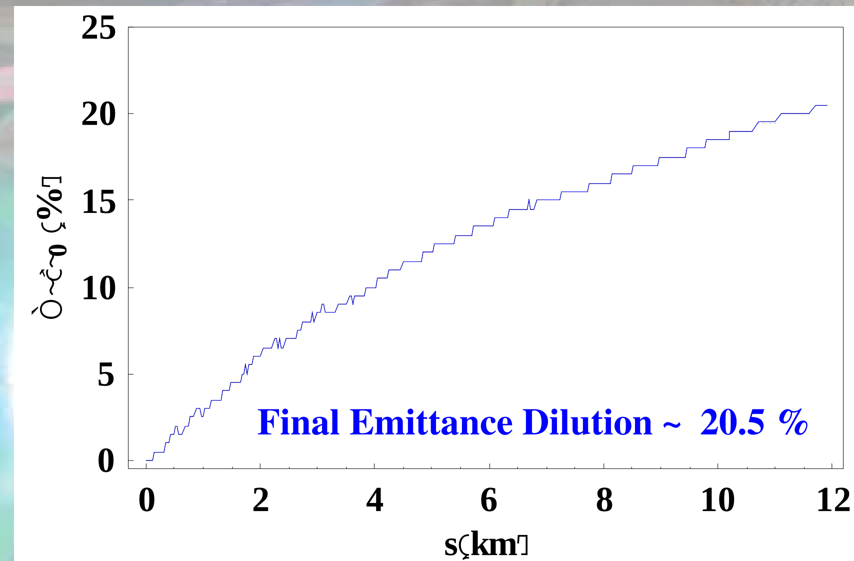


- 500 particles in all simulations
- Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x), respectively.
- (c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)
- *Fixed azimuthal orientation of couplers.*
- **Long range wakes included in simulations**

Emittance Dilution From Cross-Coupling Wakes => Yoffset =0

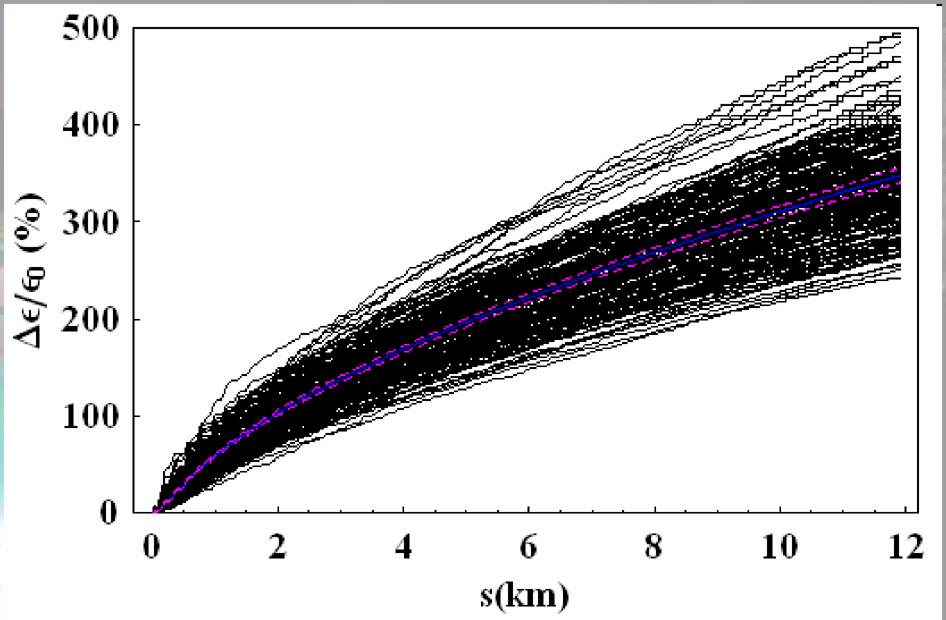


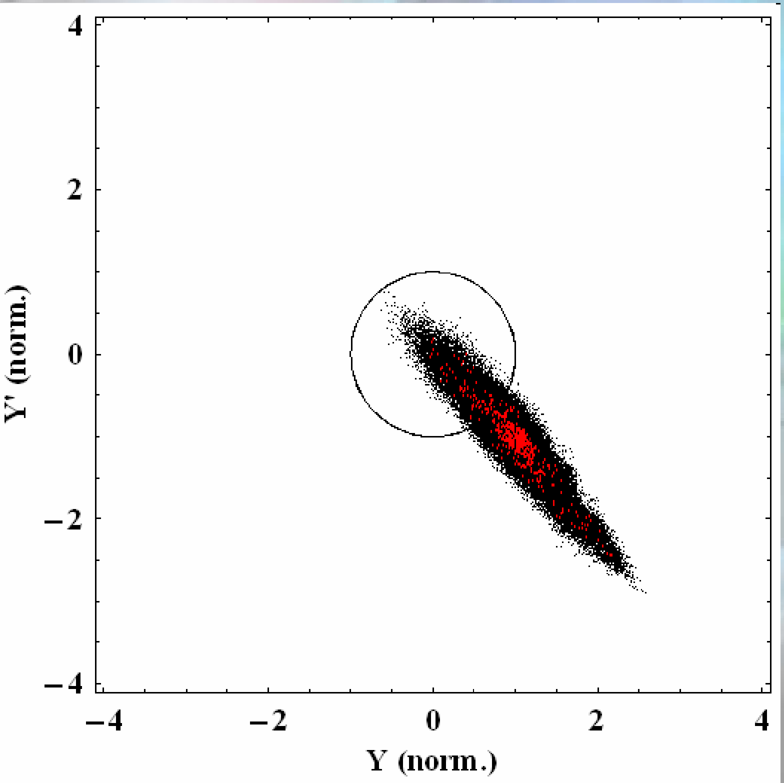




- 500 particles in all simulations.
- Initial horizonal offset ~400 μm
- No vertical offset!
- (c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)
- *Random azimuthal phase about mean of π/4*
- **Long + short range wakes included in simulations**

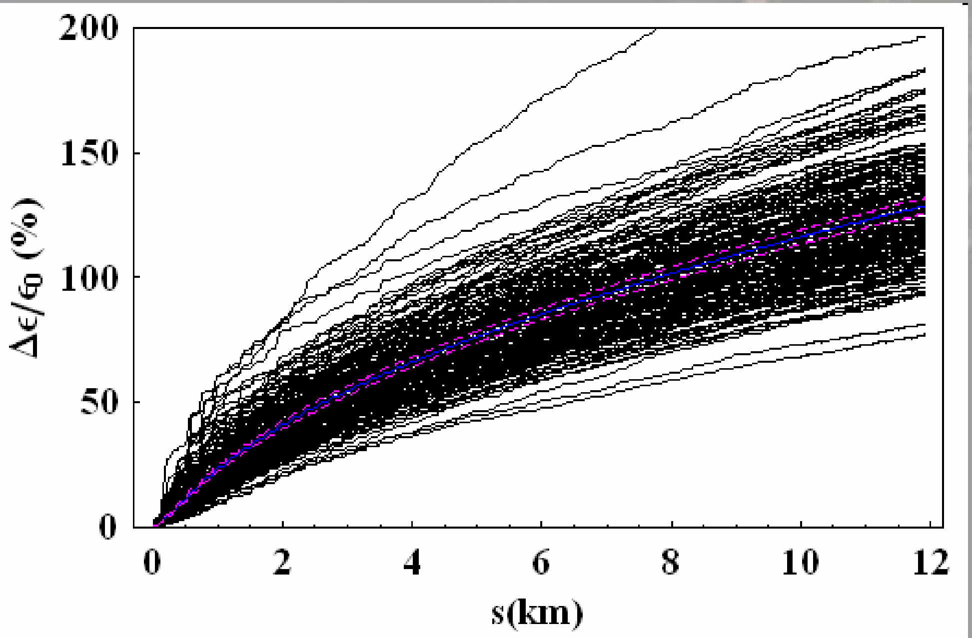
Fixed Azimuthal Phase Distribution (Yoff=0).

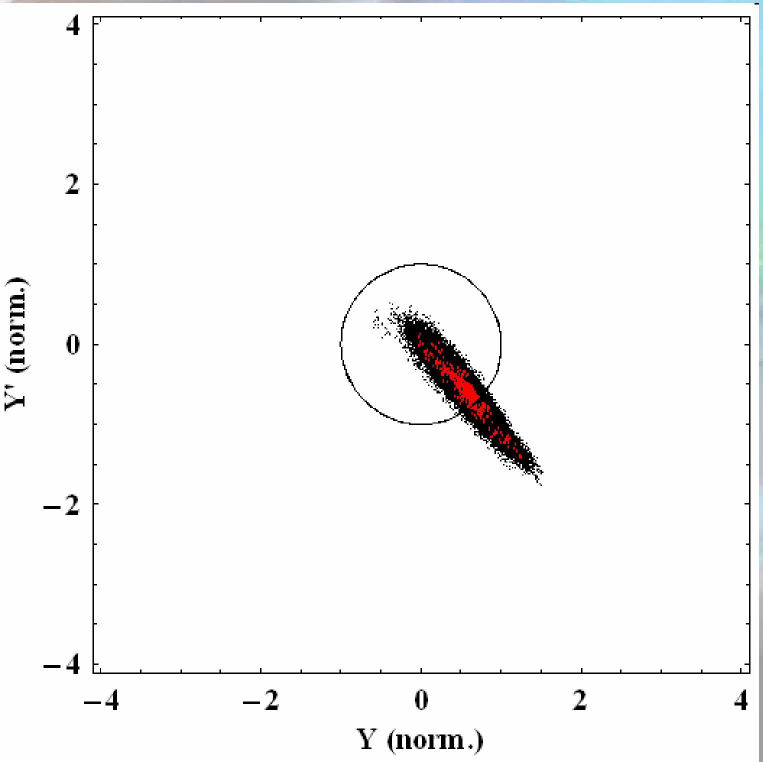




- 500 particles in all simulations and 200 machines shown
- Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x), respectively.
- (c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)
- *Fixed azimuthal orientation of couplers.*
- **Long range wakes included in simulations**

Fixed AzimuthalPhase Distribution (Yoff=0).





- 500 particles in all simulations and 200 machines shown
- Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x), respectively.
- (c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)
- *Random azimuthal orientation of couplers (0 to 2π about a mean value of $\pi/4$).*
- **Long range wakes included in simulations**

Amelioration of Large Emittance Dilution?

•Increase the damping of the HOM modes? At present the damping is ~10^4 - 10^5. It may be difficult to reduce the damping even further?

•The dipole frequency degeneracy's are already split –by ~ 400 kHz to 800 kHz. Increasing the splitting (which can be achieved by making markedly asymmetrical cavities) to say 10 MHz may allow the influence of mode coupling to be minimal over the length of the linac.

•Splitting the tune of the linac. Present design is 60/60 – horizontal/vertical. This may be the most straightforward to implement. => We investigate splitting the tune in this talk.

Re-entrant cavity shape is shown. Taken from Rong-Li Geng SRF 2005

Split Tune of Lattice

- Present lattice has phase advance per cell of 60 degrees per cell.

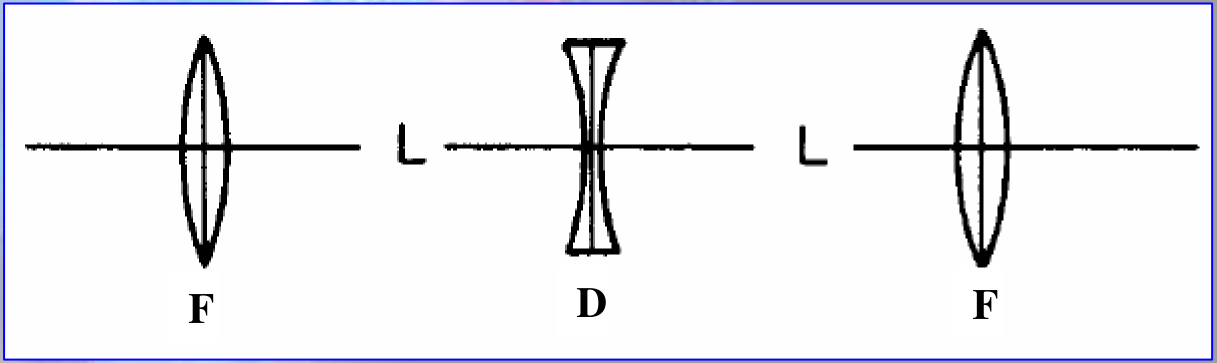
- Each lattice cell consists of two groups of:
quadupole + 2 cryo-module + quadrupole+ 2 cryo-module.
- There are 12 cavities per cryo-module.
- If we split the tune then the coupling should be out of resonance.

- We look at 61/60 through 90/60.

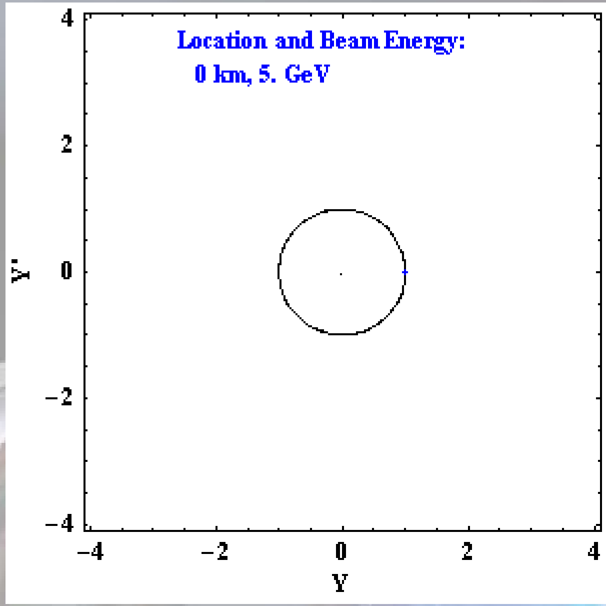
- Single FODO array shown.

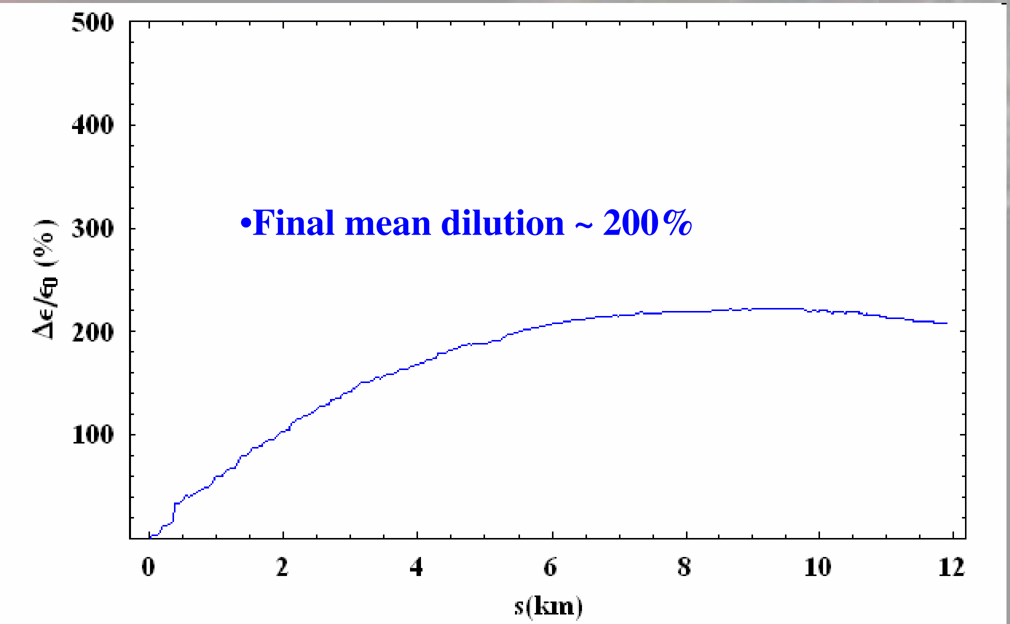
- We split the tune by raising the F functions for the focusing quadrupole with respect to the D quadrupole.

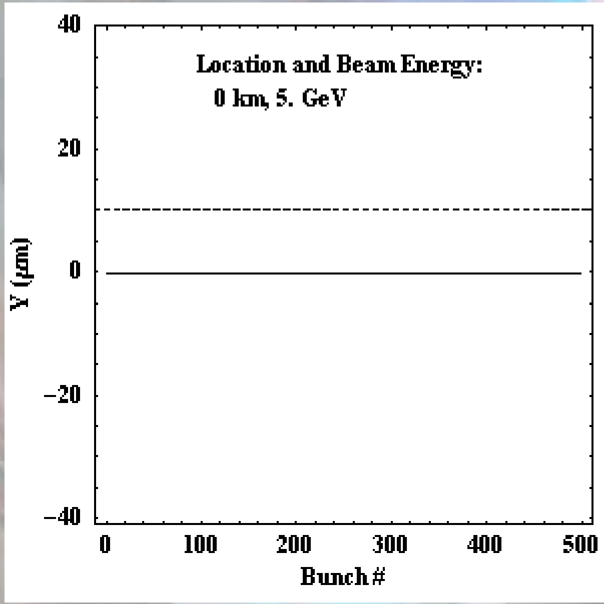
- The D quadrupole is then readjusted in order to achieve 60 in the vertical plane.



Cross-Coupled Motion *At* Fixed Azimuthal Phase Distribution (Yoff=0). *Split Tune 61/60*

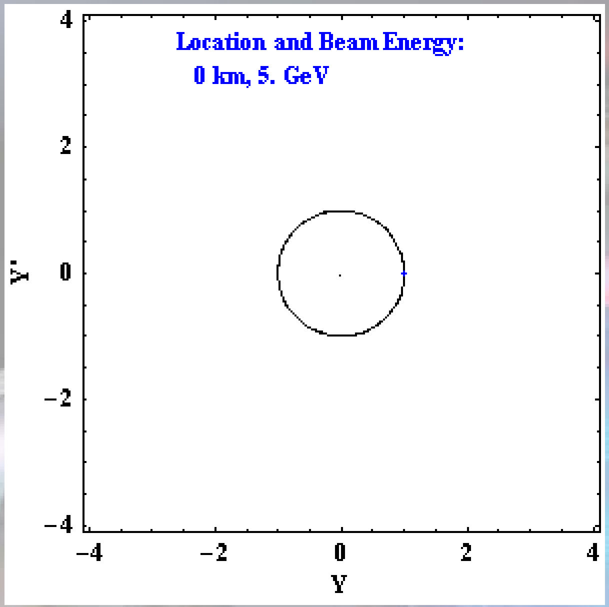


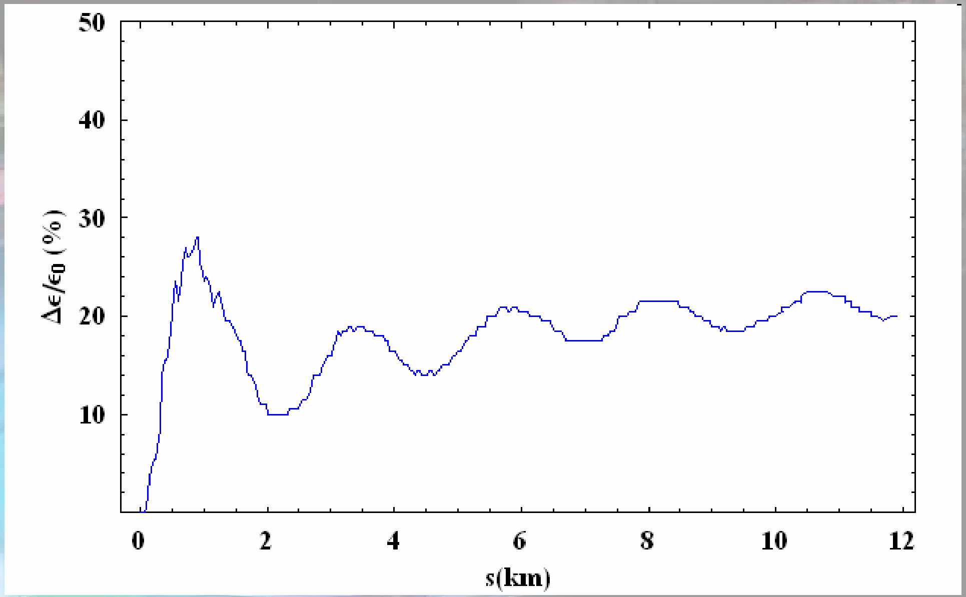


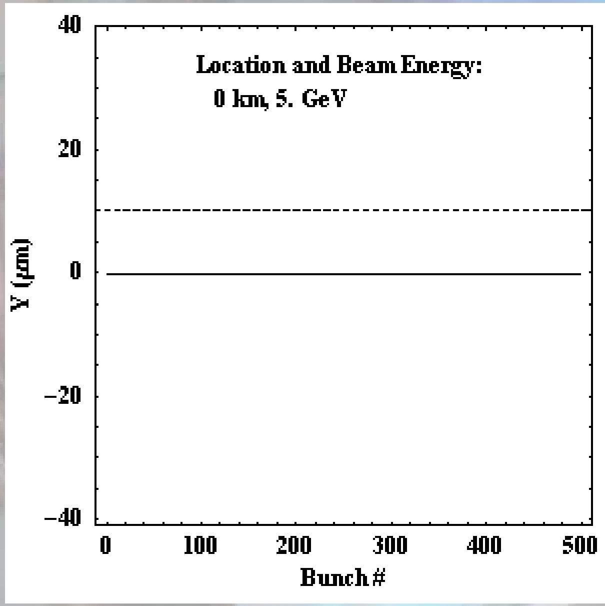


- •500 particles in all simulations
- •Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x), respectively.
- •(c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)
- •*Fixed azimuthal orientation of couplers.*
- •**Long range wakes included in simulations**

Cross-Coupled Motion *At* Fixed Azimuthal Phase Distribution (Yoff=0). *Split Tune 70/60*

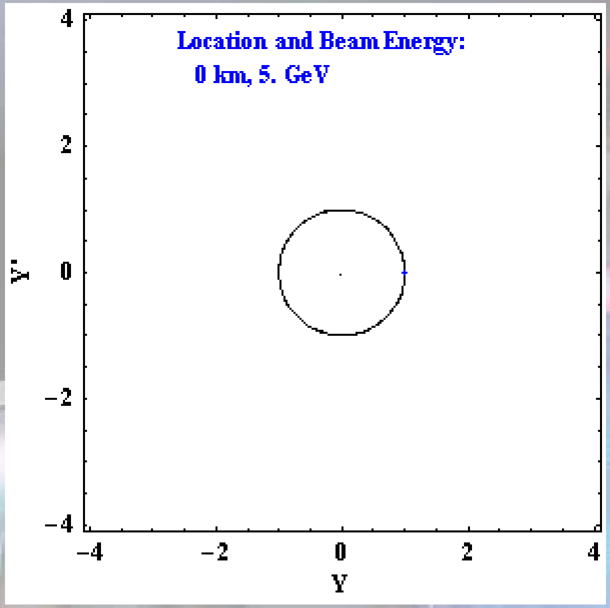


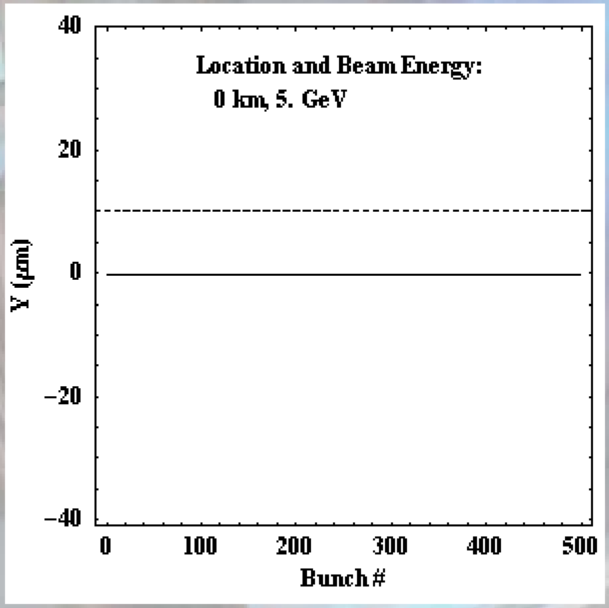


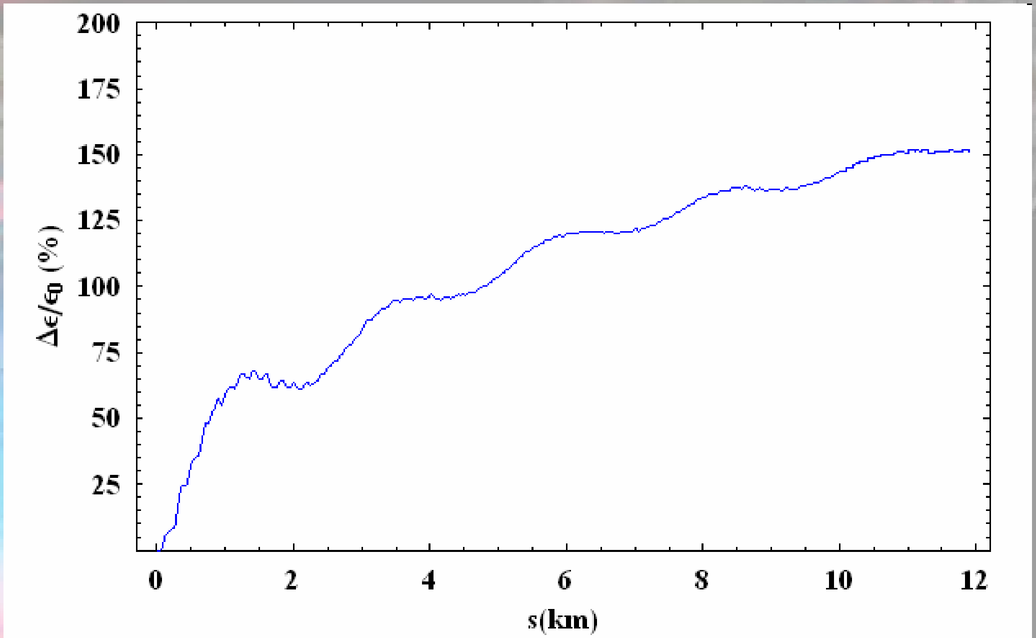


- **500 particles in all simulations**
- **Initial vertical and horizontal offset 0 µm and 270µm (~ σ_x), respectively.**
- **(c.f. initial beam $\sigma_{y,x}$ ~ 10.1 µm, 270 µm)**
- ***Fixed azimuthal orientation of couplers.***
- **Long range wakes included in simulations**

Cross-Coupled Motion *At* Fixed Azimuthal Phase Distribution (Yoff=0). *Split Tune 70/60*

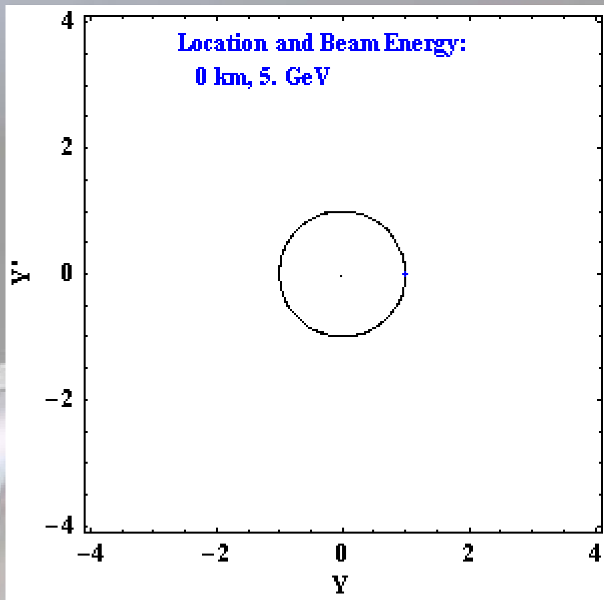


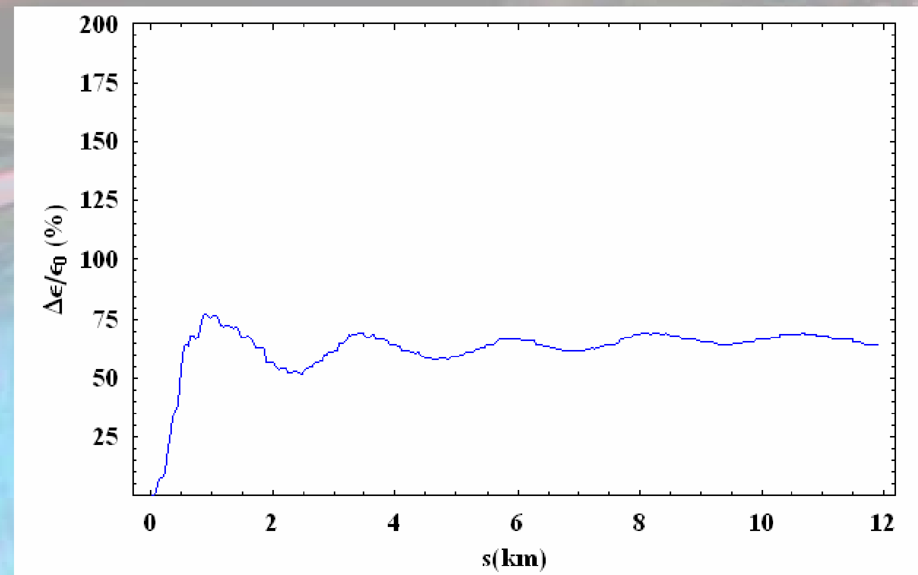


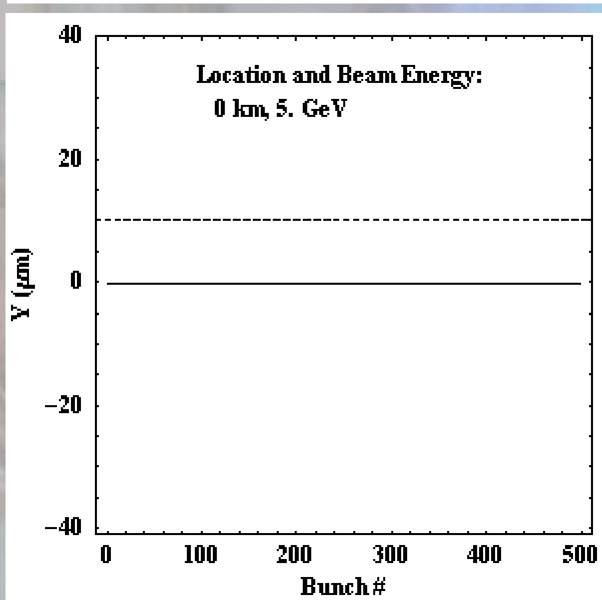


- 500 particles in all simulations
- Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x), respectively.
- (c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)
- *Fixed azimuthal orientation of couplers.*
- **Long + Short range wakes included in simulations**

Cross-Coupled Motion *At* Fixed Azimuthal Phase Distribution (Yoff=0). *Split Tune 70/60*

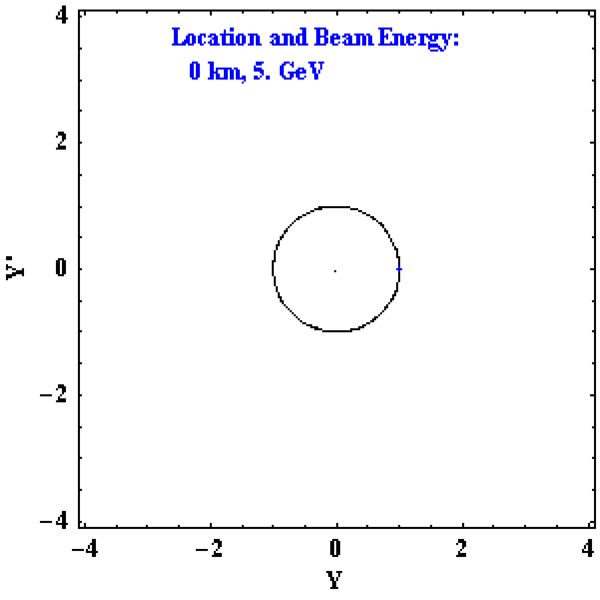


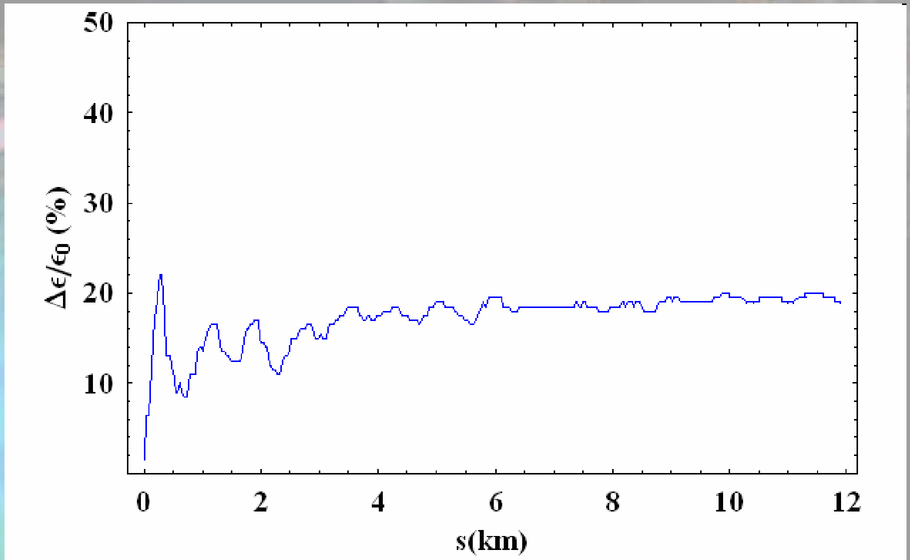


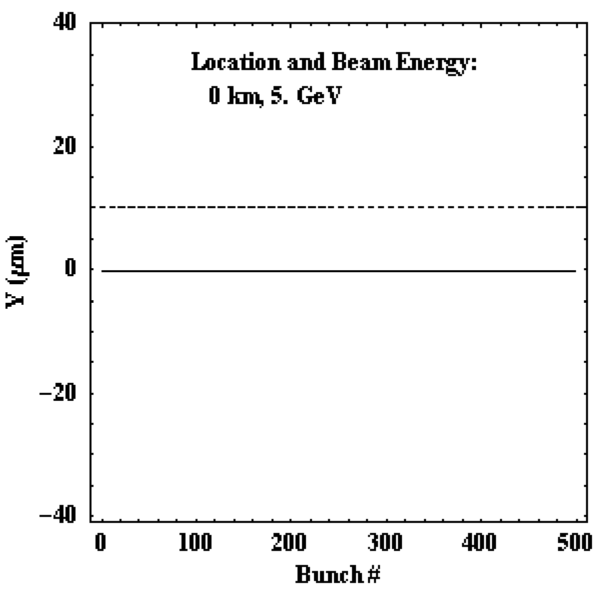


- •500 particles in all simulations
- •Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x),
- •(c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)
- •*Random azimuthal orientation of couplers (0 to 2π).*
- **•Long + Short range wakes included in simulations**

Cross-Coupled Motion *At* Fixed Azimuthal Phase Distribution (Yoff=0). *Split Tune 90/60*

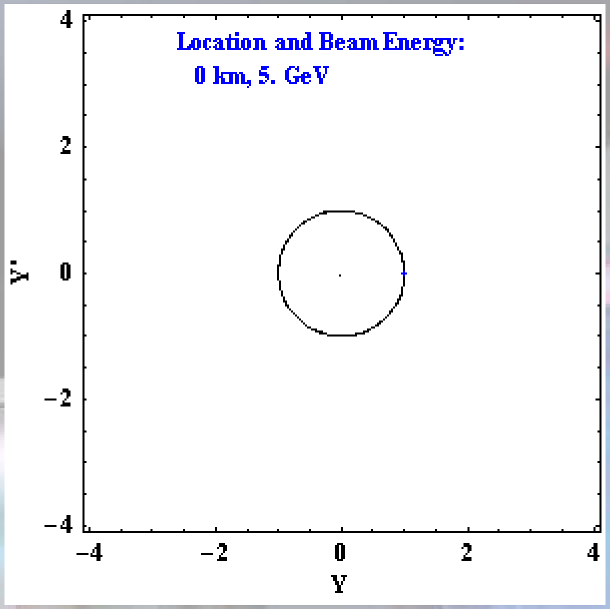


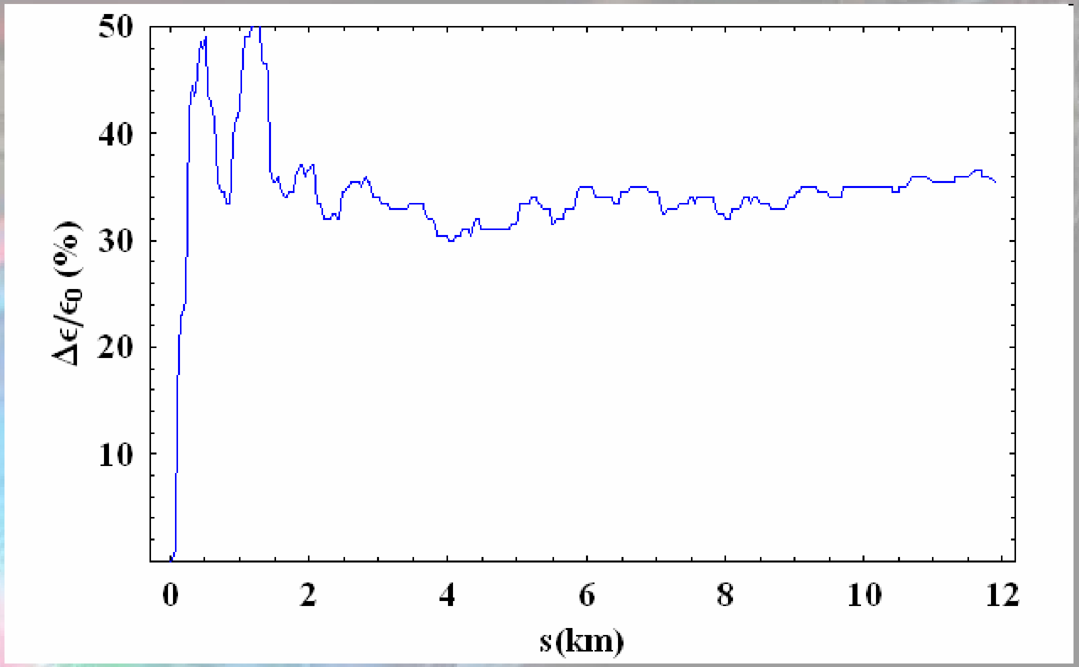


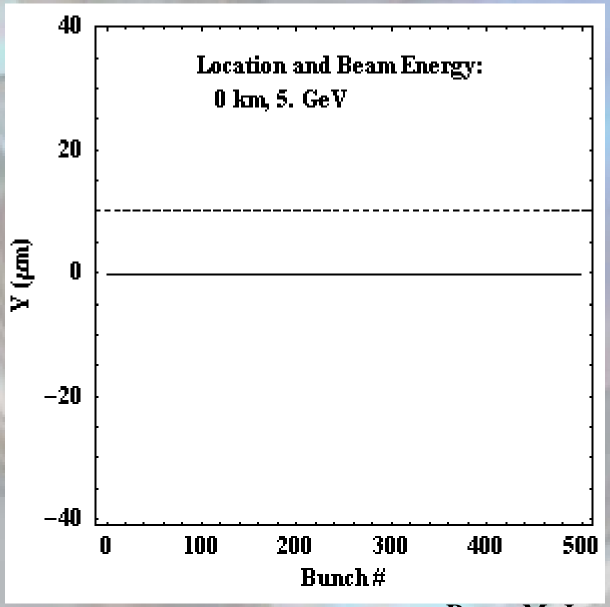


- 500 particles in all simulations
- Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x), respectively.
- (c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)
- *Fixed azimuthal orientation of couplers.*
- Long range wakes included in simulations

Cross-Coupled Motion *At* Fixed Azimuthal Phase Distribution (Yoff=0). *Split Tune 90/60*





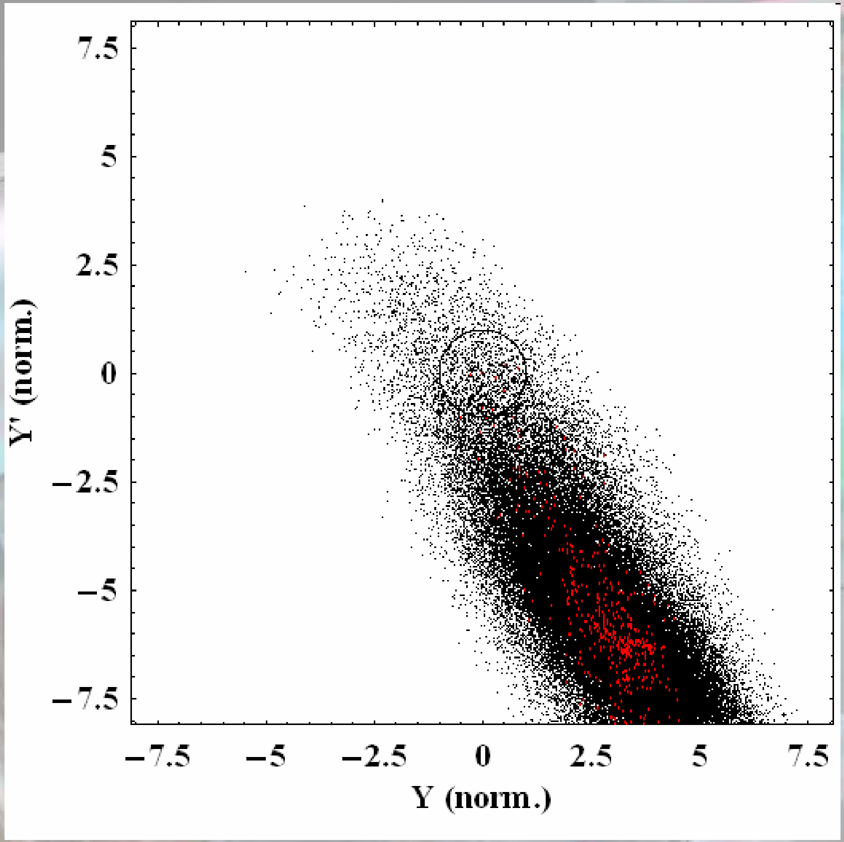


- 500 particles in all simulations
- Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x), respectively.
- (c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)
- *Limited randomization of azimuthal phase*

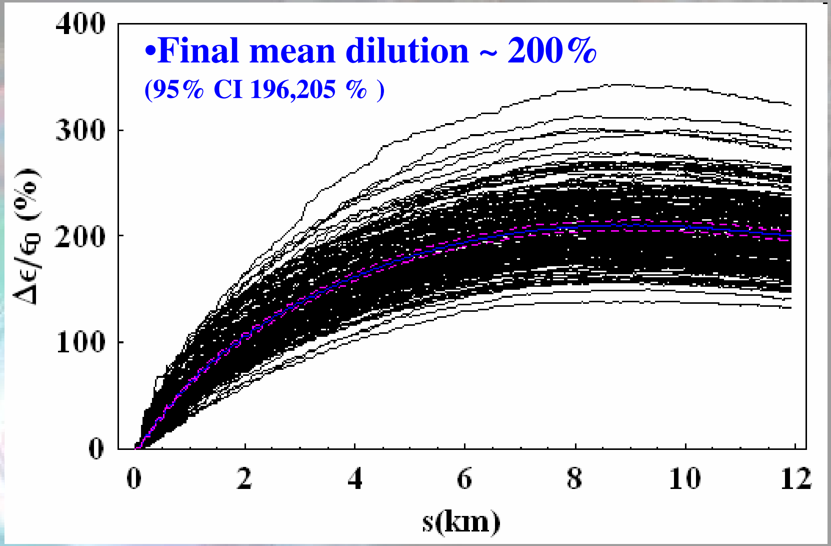
–Max 10% of mean.

- **Long range wakes included in simulations**

Cross-Coupled Motion *At* Fixed Azimuthal Phase Distribution (Yoff=0). *Split Tune 61/60*

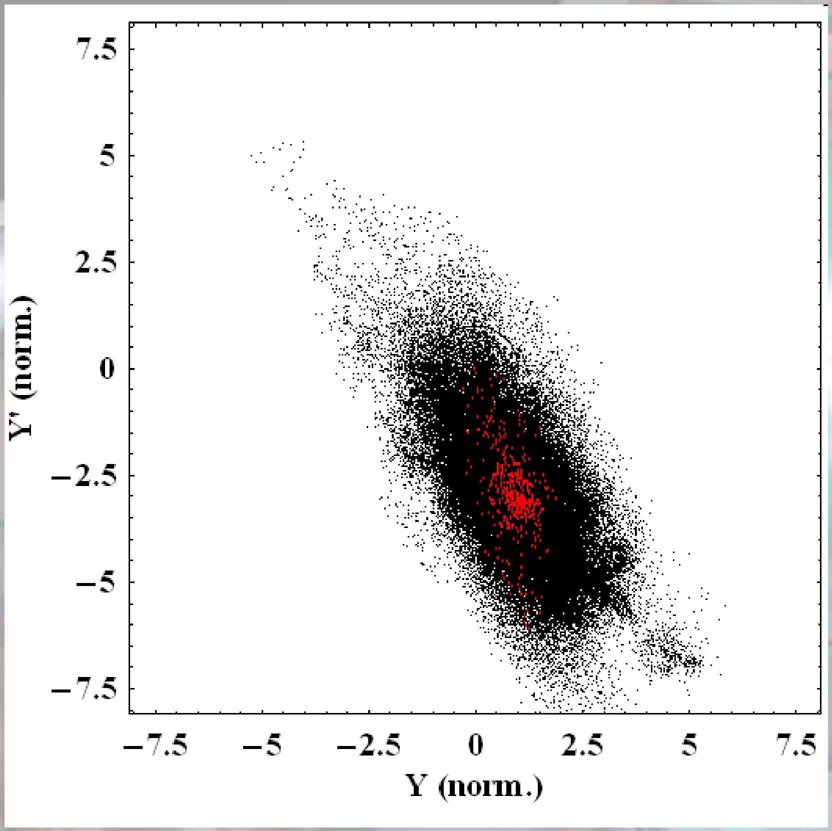


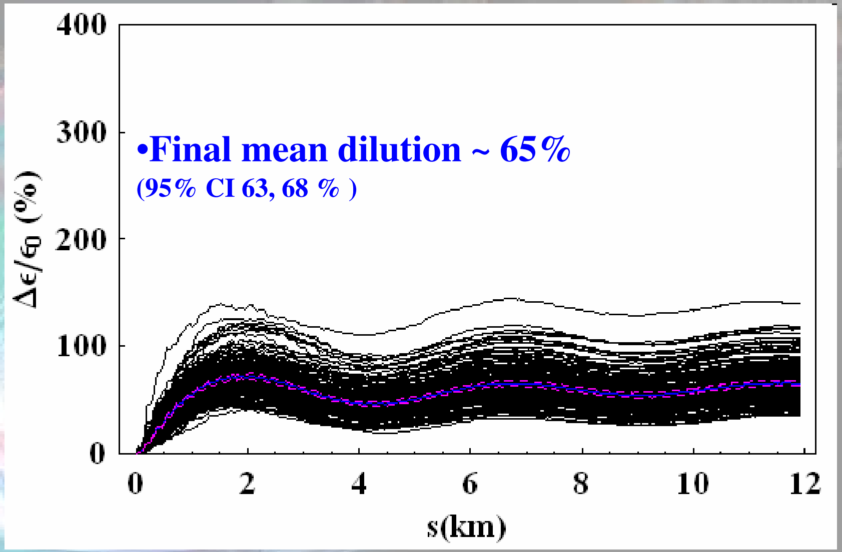
Small circle represents scaling w.r.t. energy at *end* of linac.



- •500 particles in all simulations and 200 machines are illustrated
- •The dashed line marks the injection offset of all bunches
- •Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x), respectively.
- •(c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)
- •*Fixed azimuthal phase.*
- **•Long range wakes included in simulations**

Cross-Coupled Motion *At* Fixed Azimuthal Phase Distribution (Yoff=0). *Split Tune 65/60*





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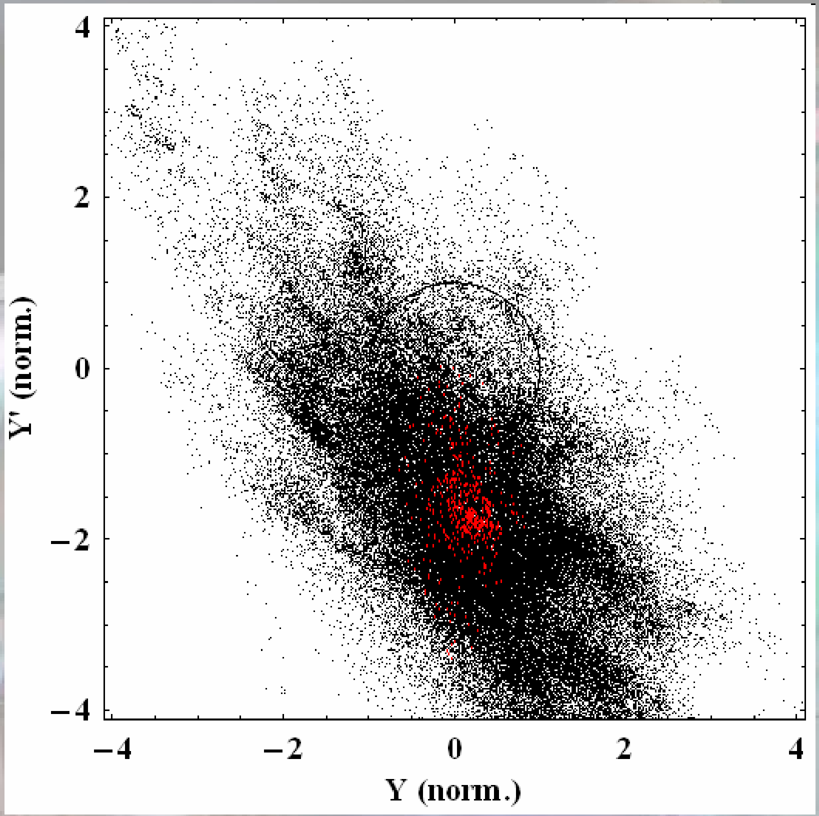
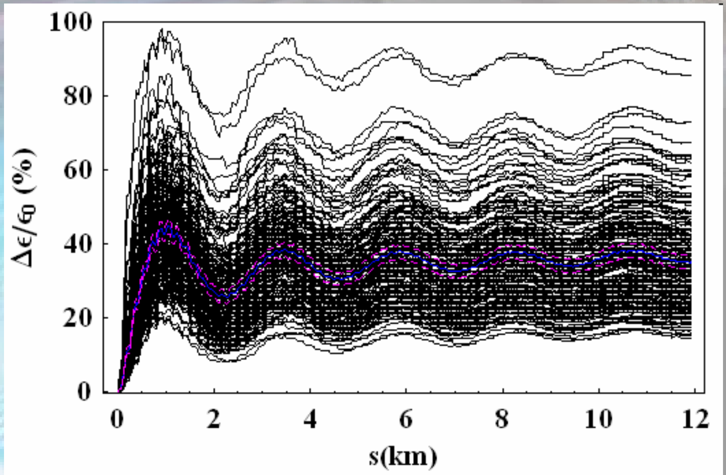
- Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x), respectively.
- (c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)
- *Fixed azimuthal phase.*
- **Long range wakes included in simulations**

Cross-Coupled Motion *At* Fixed Azimuthal Phase Distribution (Yoff=0). *Split Tune 70/60*



These have been rescaled relative to the previous ones

•Final mean dilution ~ 35% (95% CI 33, 37 %)



•500 particles in all simulations and 200 machines are illustrated

•The dashed line marks the injection offset of all bunches

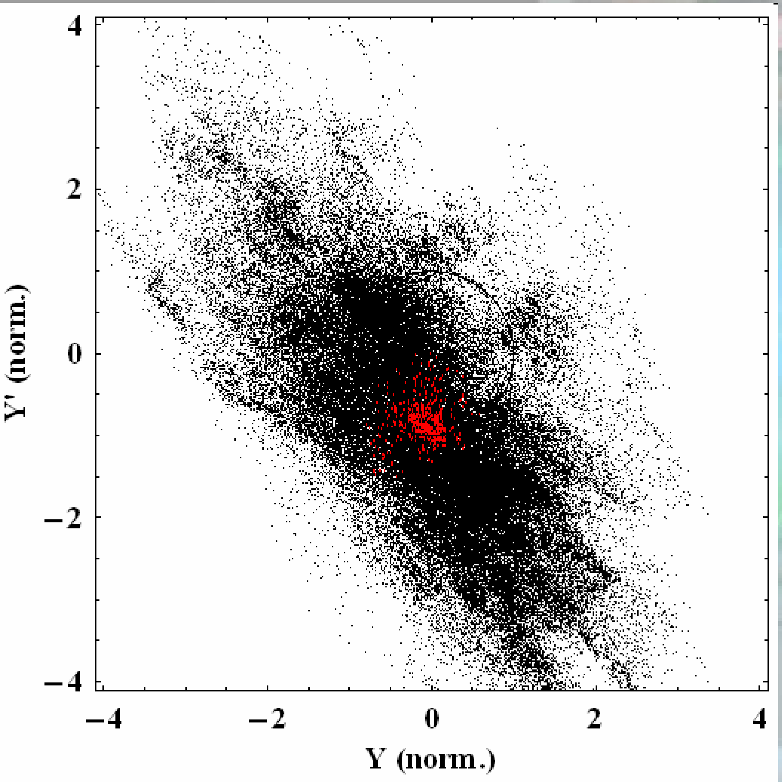
•Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x), respectively.

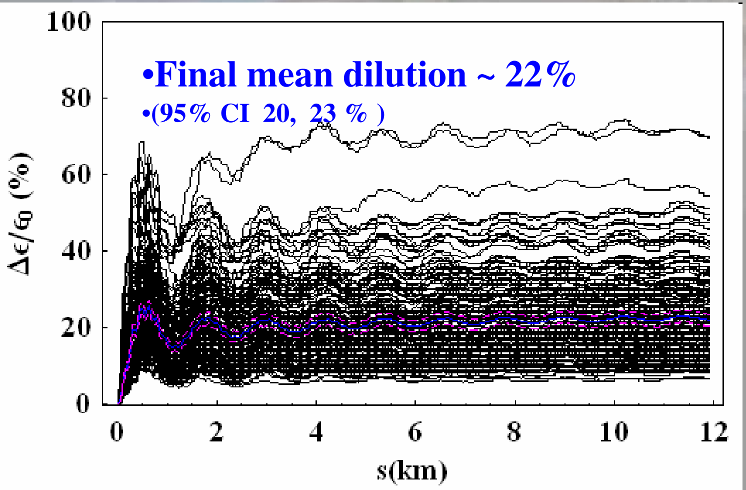
•(c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)

•*Fixed azimuthal phase.*

•**Long range wakes included in simulations**

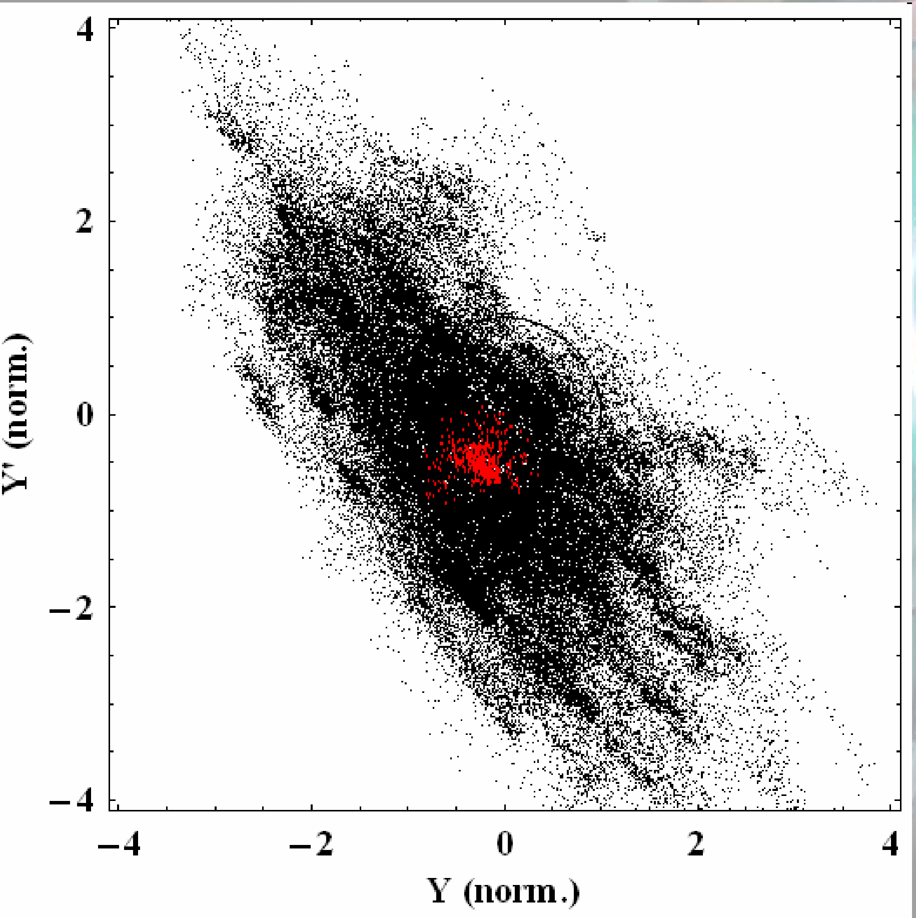
Cross-Coupled Motion *At* Fixed Azimuthal Phase Distribution (Yoff=0). *Split Tune 80/60*

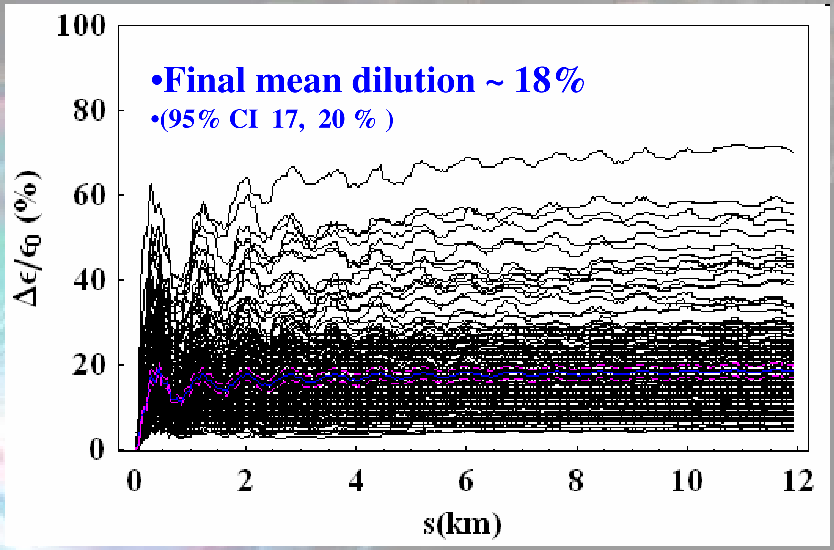




- 500 particles in all simulations and 200 machines are illustrated
- The dashed line marks the injection offset of all bunches
- Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x), respectively.
- (c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)
- *Fixed azimuthal phase.*
- **Long range wakes included in simulations**

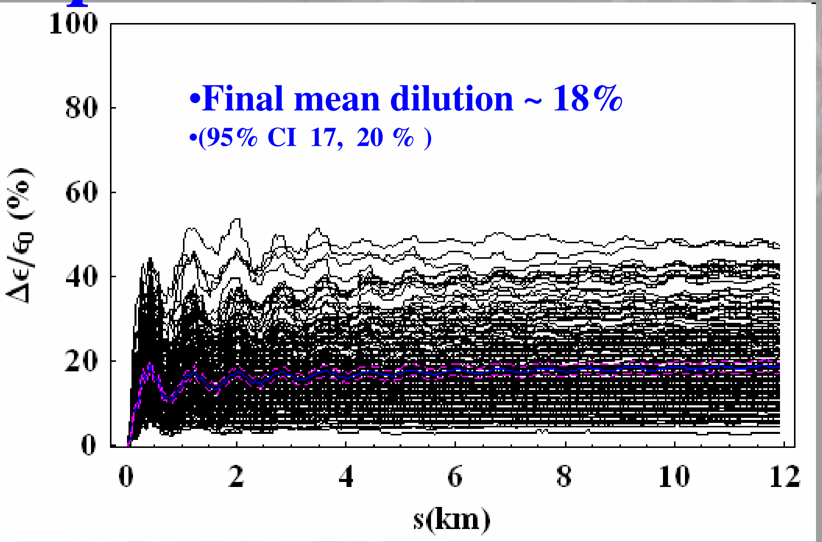
Cross-Coupled Motion *At* Fixed Azimuthal Phase Distribution (Yoff=0). *Split Tune 90/60*

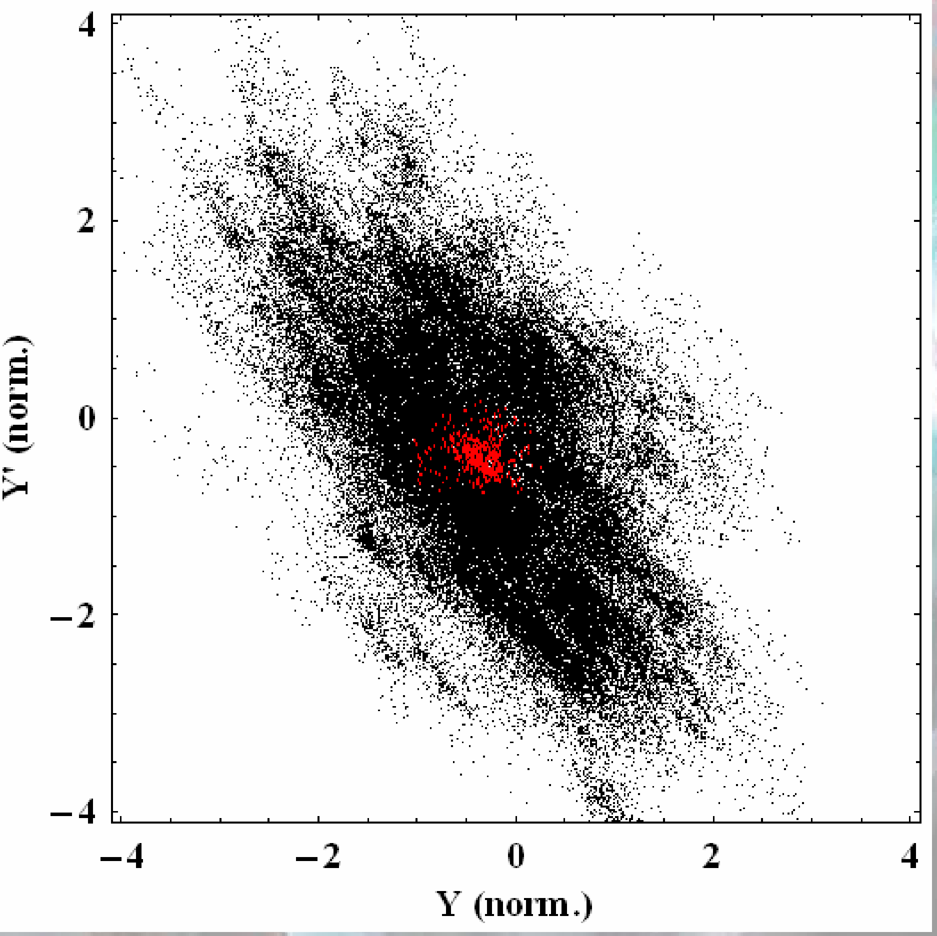




- 500 particles in all simulations and 200 machines are illustrated
- The dashed line marks the injection offset of all bunches
- Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x), respectively.
- (c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)
- *Fixed azimuthal phase.*
- **Long range wakes included in simulations**

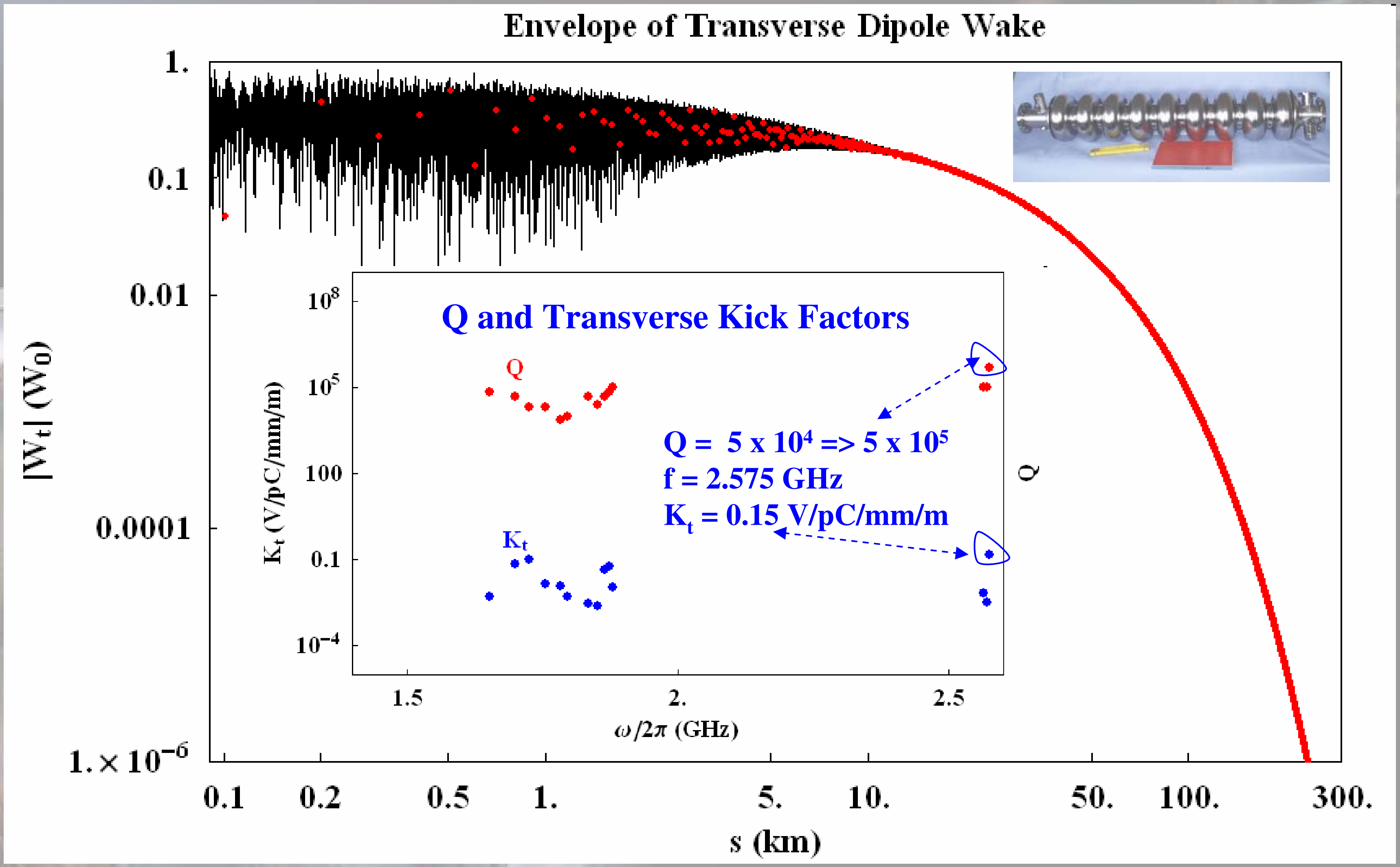
Cross-Coupled Motion, Limited Randomization of Azimuthal Phase Distribution of HOM Couplers (Yoff=0). *Split Tune 90/60*.





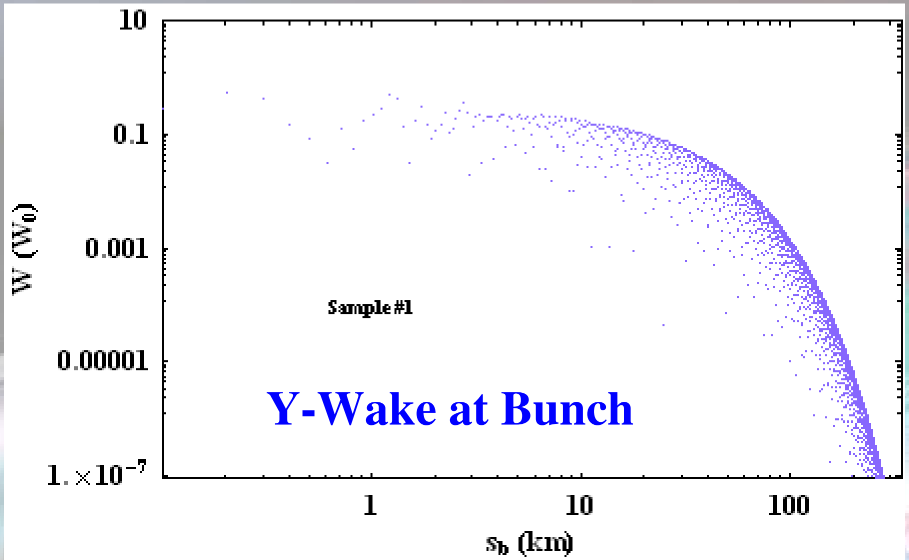
- 500 particles in all simulations and 200 machines are illustrated
- The dashed line marks the injection offset of all bunches
- Initial vertical and horizontal offset 0 µm and 270µm (~ σ_x), respectively.
- (c.f. initial beam $\sigma_{y,x}$ ~ 10.1 µm, 270 µm)
- *Limited randomization of azimuthal phase*
  - *Max 10% of mean.*
- **Long range wakes included in simulations**

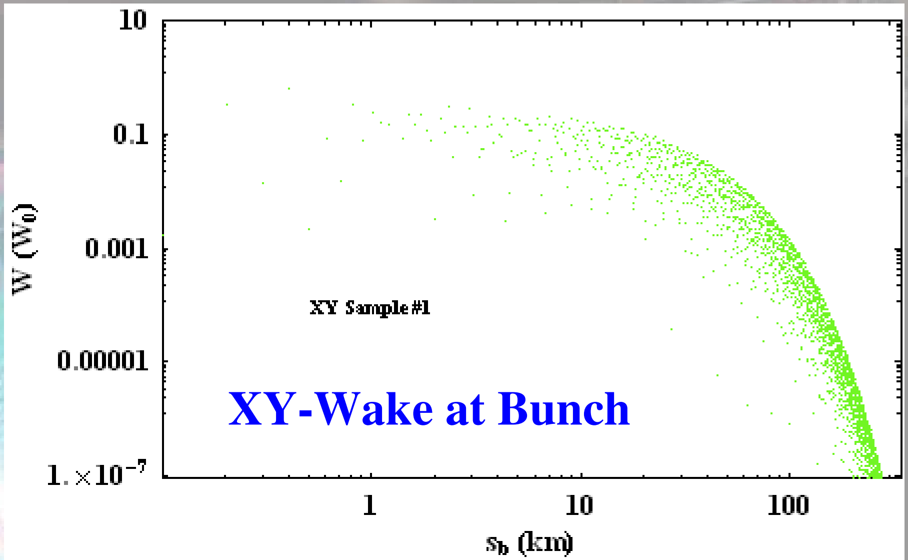
Wakes for Cross-Coupled Motion + Rogue Mode

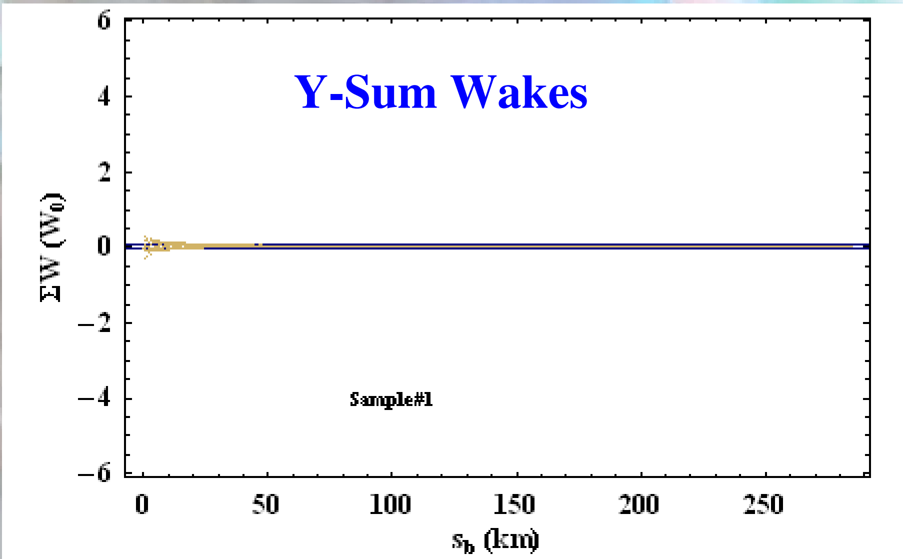


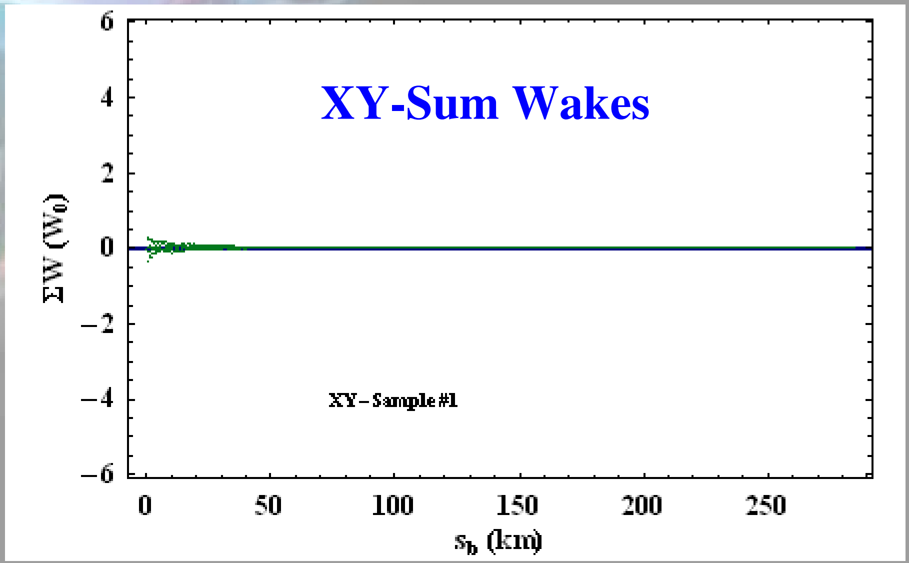
International Linear Collider at Stanford Linear Accelerator Center

Wakes Incl. Rogue Mode & Random Errors

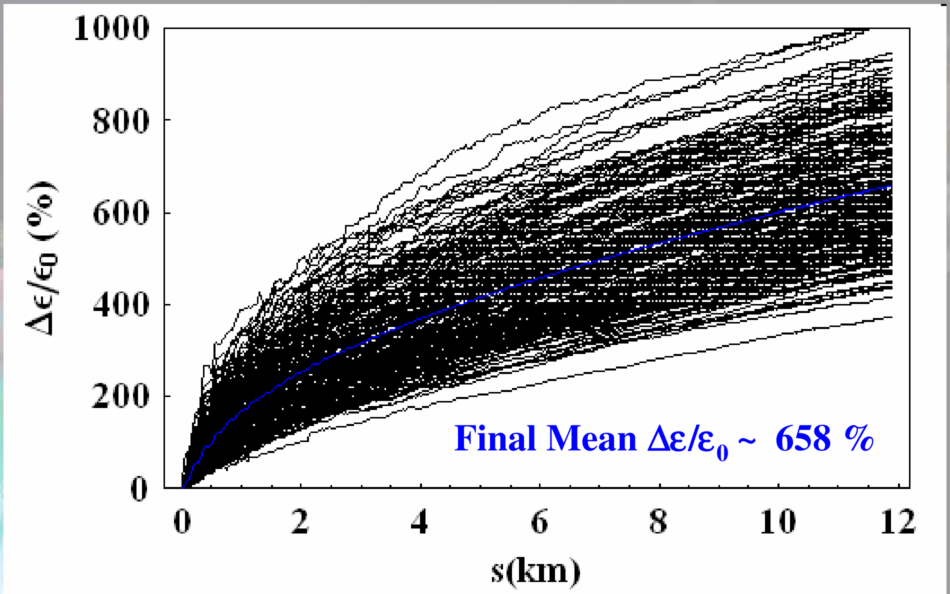


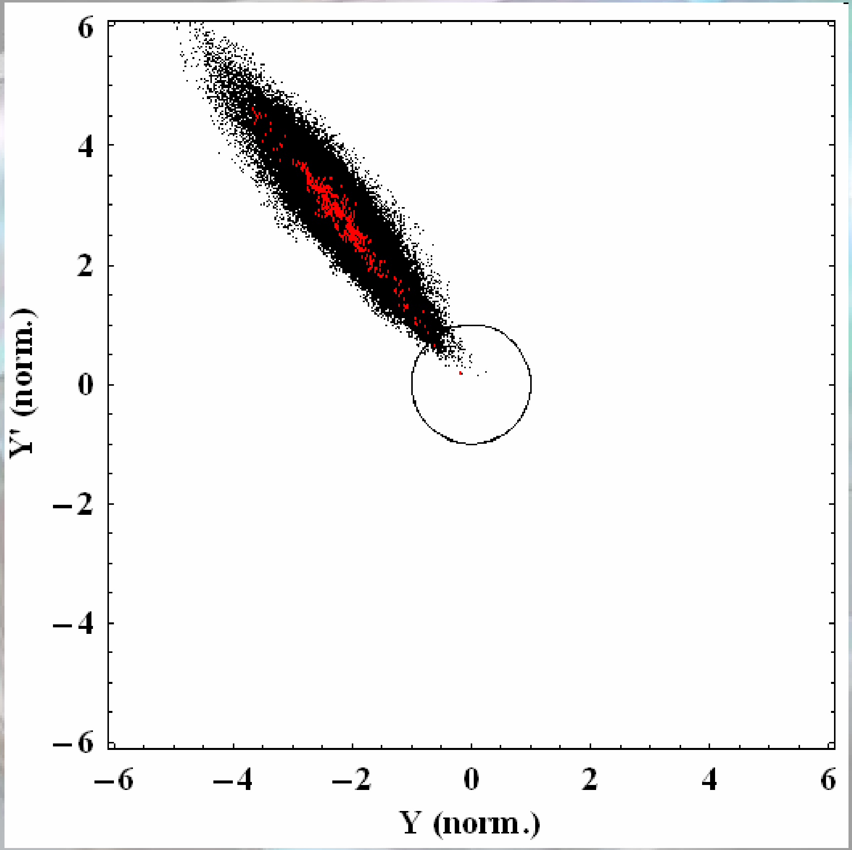






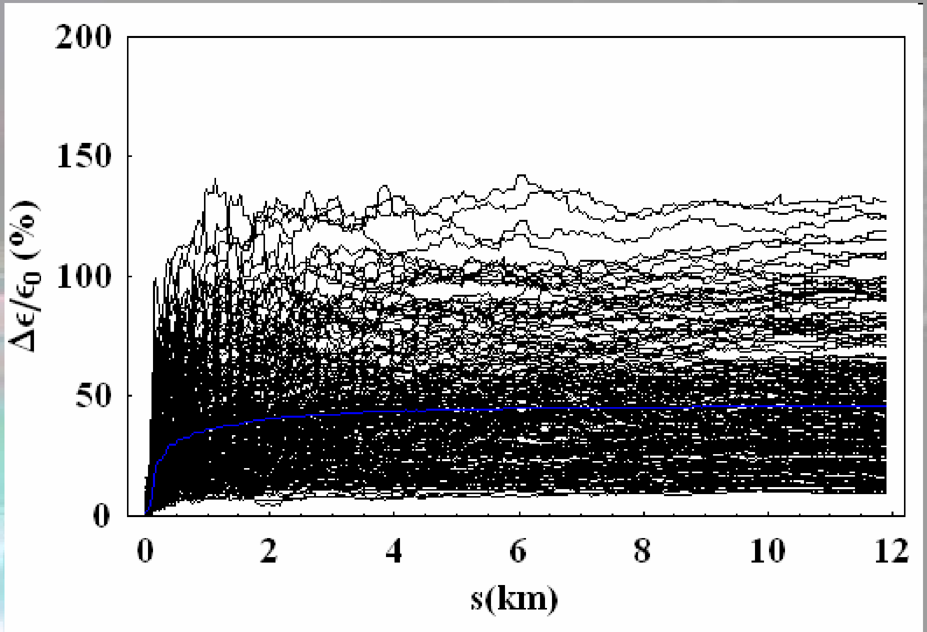
Cross-Coupled Motion At Fixed Azimuthal Phase Distribution + Rogue Mode

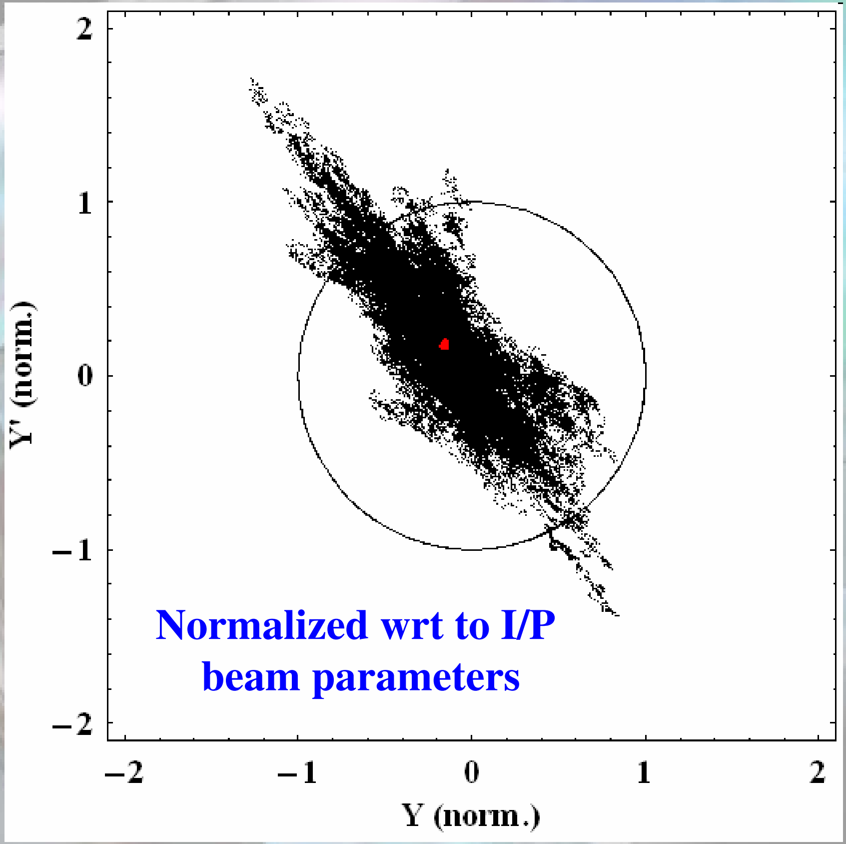




- 500 particles in all simulations.
- The initial position of all bunches in phase space is indicated by the blue dot.
- Initial vertical and horizontal offset ~10 μm and 400μm.
- (c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)
- *Azimuthal phase fixed at π/4*
- Final mean of the emittance dilution from 200 machines is indicated by the blue line
- **Long range wakes included in simulations**

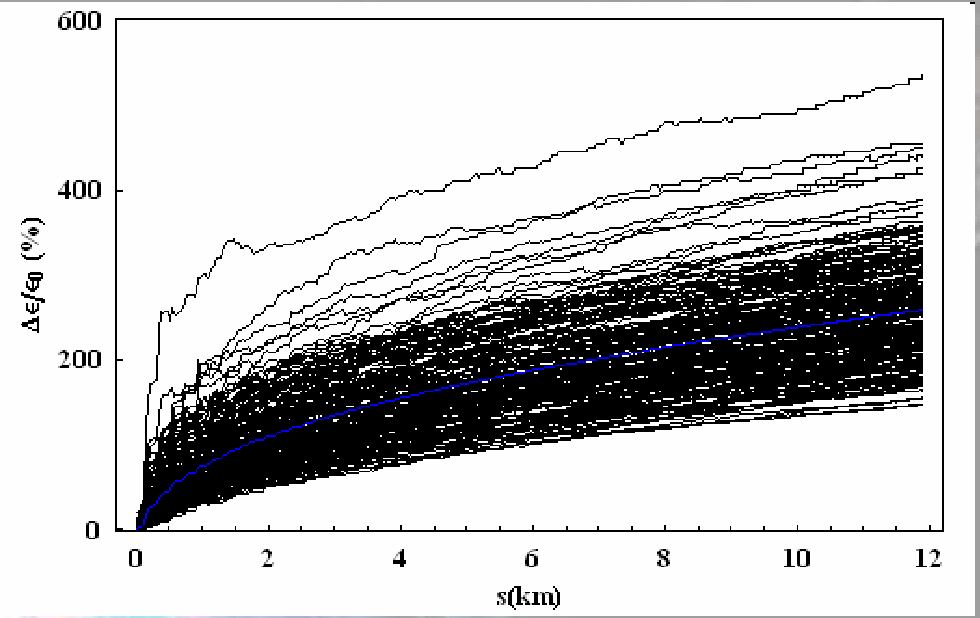
Cross-Coupled Motion With Random Azimuthal Phase Distribution + Rogue Mode

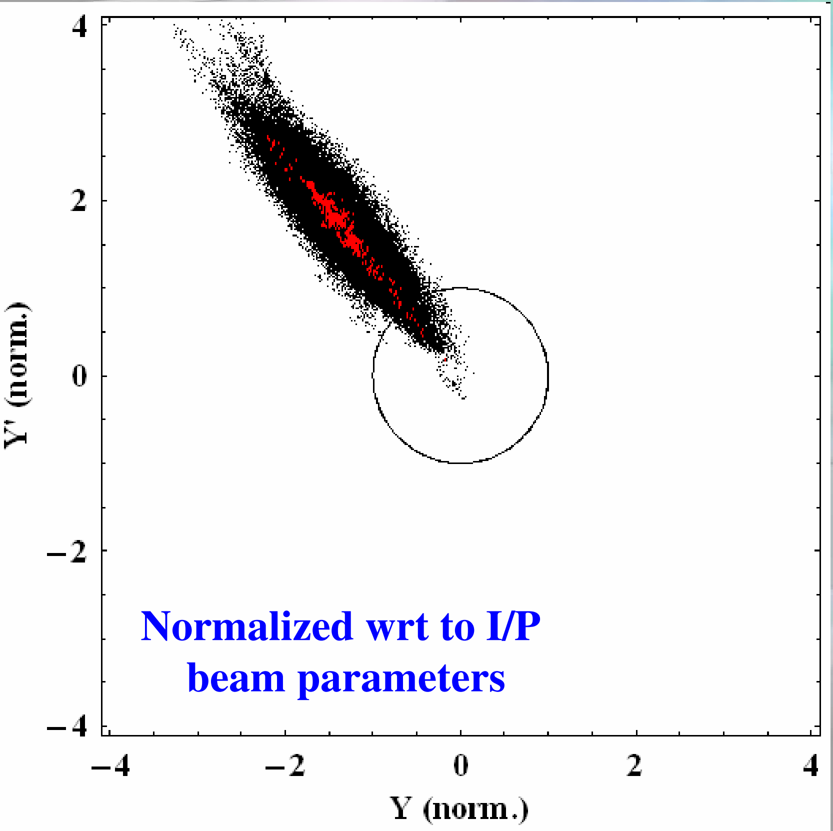




- 500 particles in all simulations.
- The initial position of all bunches in phase space is indicated by the blue dot.
- Initial vertical and horizontal offset ~10 µm and 400µm.
- (c.f. initial beam $\sigma_{y,x}$ ~ 10.1 µm, 270 µm)
- *Random Azimuthal phase (mean π/4)*
- Final mean of the emittance dilution from 200 machines is indicated by the blue line
- **Long range wakes included in simulations**

Cross-Coupled Motion With Limited Random Azimuthal Phase Distribution + Rogue Mode

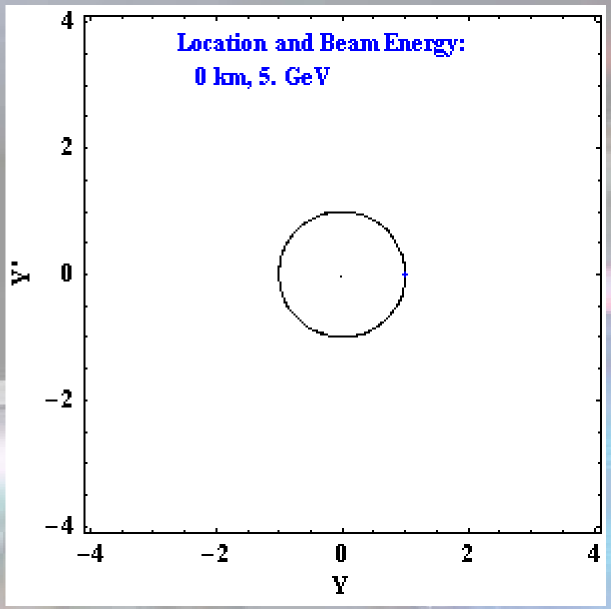


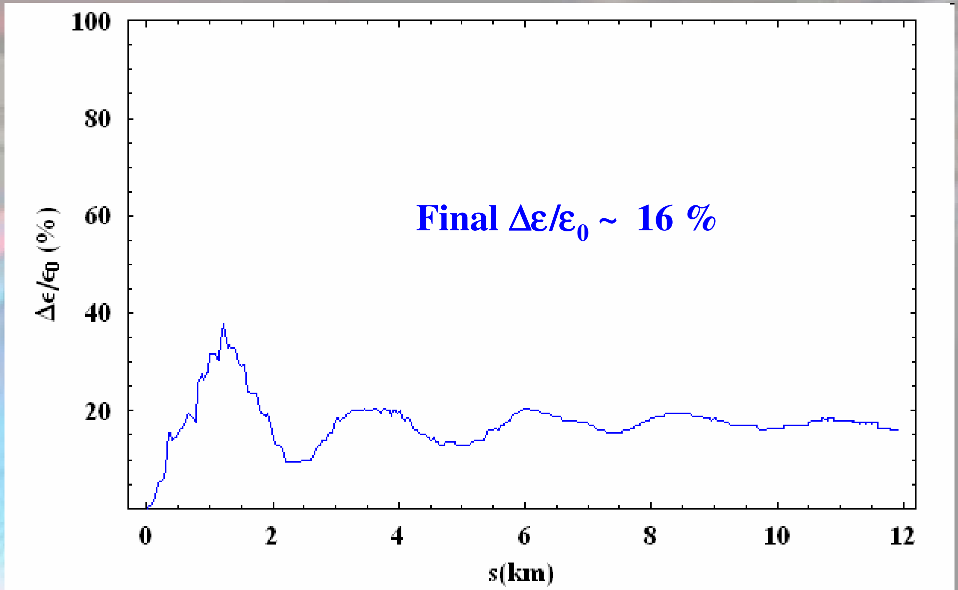


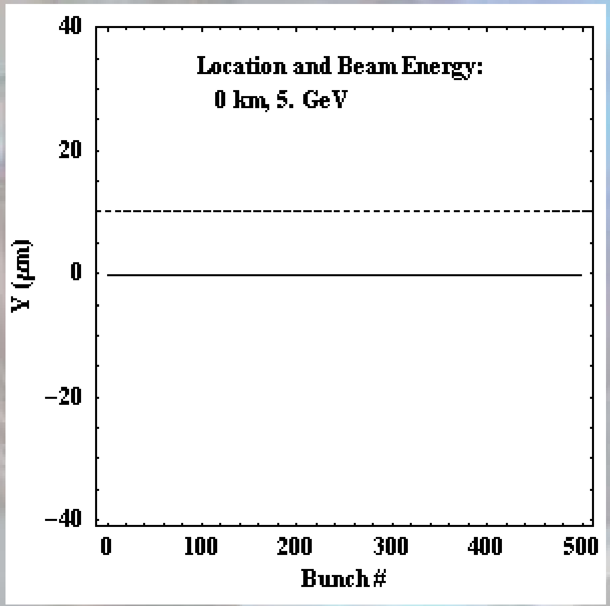
- 500 particles in all simulations.
- The initial position of all bunches in phase space is indicated by the blue dot.
- Initial vertical and horizontal offset ~10 μm and 400μm.
- (c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)
- *Azimuthal phase randomly distributed –Max 10% of mean.*
- Final mean of the emittance dilution from 200 machines is indicated by the blue line
- **Long range wakes included in simulations**

Mitigation of Emittance Growth Due to Rogue Mode.

Split Tune 70/60.



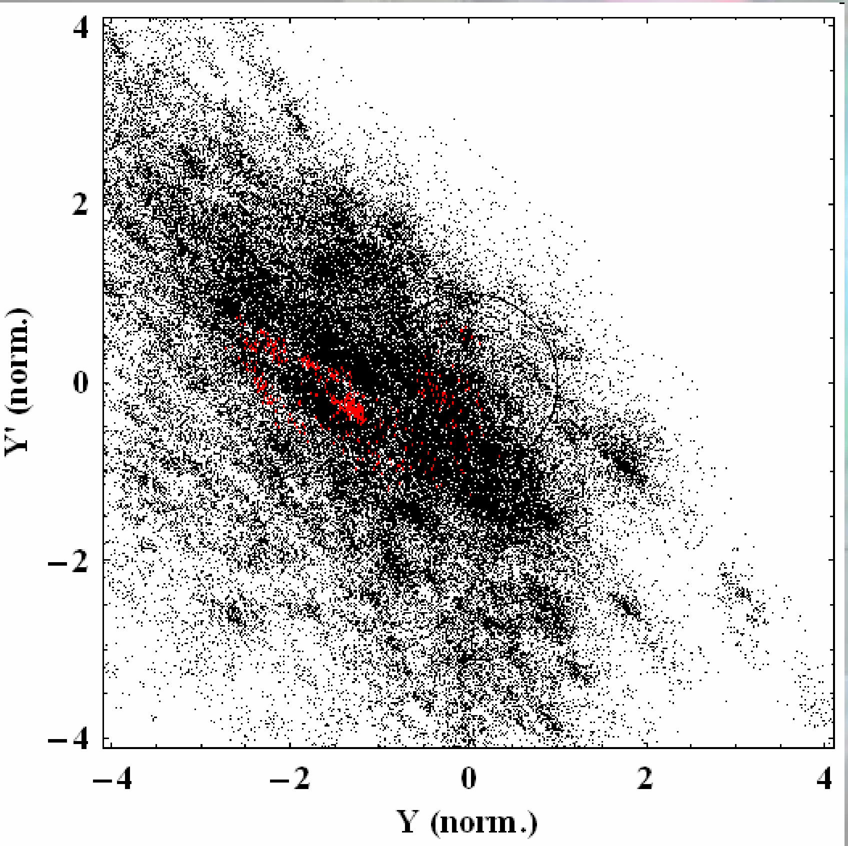


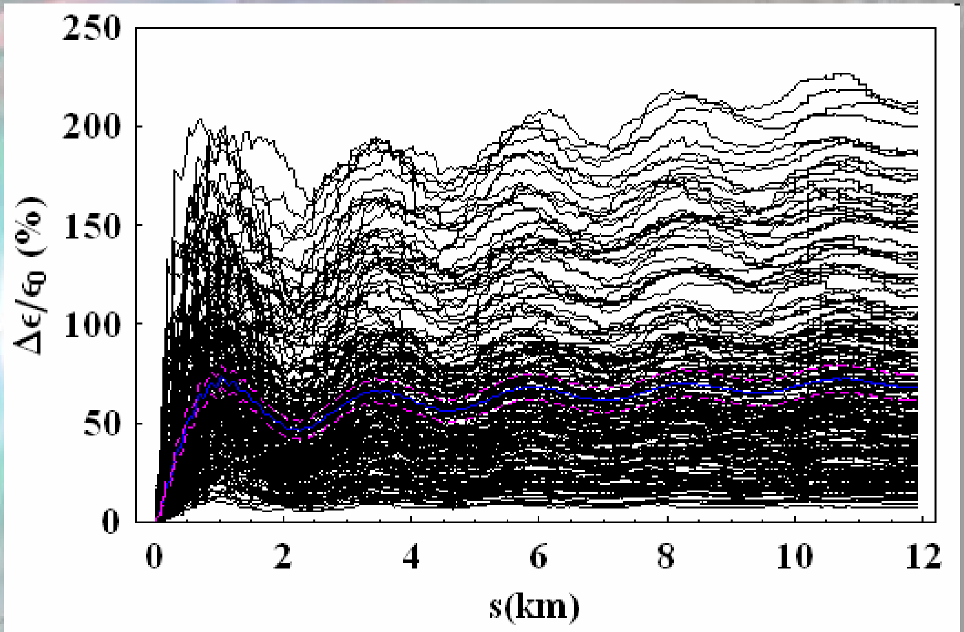


- **Only mode-cross coupling gives rise to emittance dilution**
- **500 particles in all simulations**
- **Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x), respectively.**
- **(c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)**
- ***Fixed azimuthal orientation of couplers.***
- **Long range wakes included in simulations**

Cross-Coupled Motion Zero Randomization of Azimuthal Orientation of HOM Couplers + Rogue Mode

$X_{off} \sim \sigma_x$, $Y_{off}=0$. *Split Tune 70/60.*

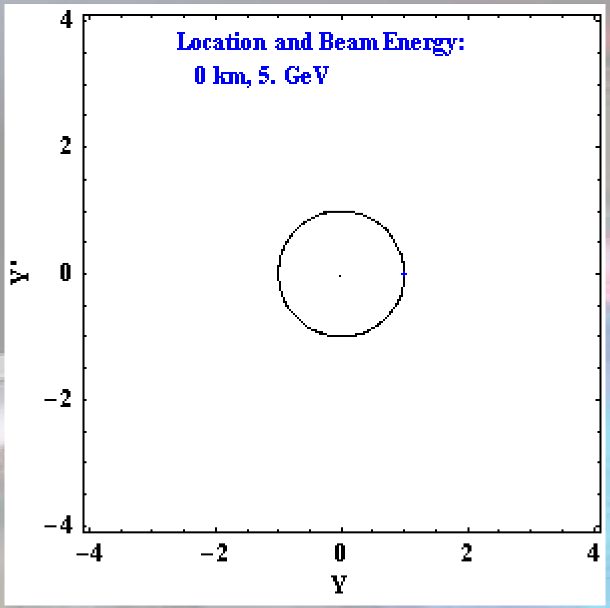


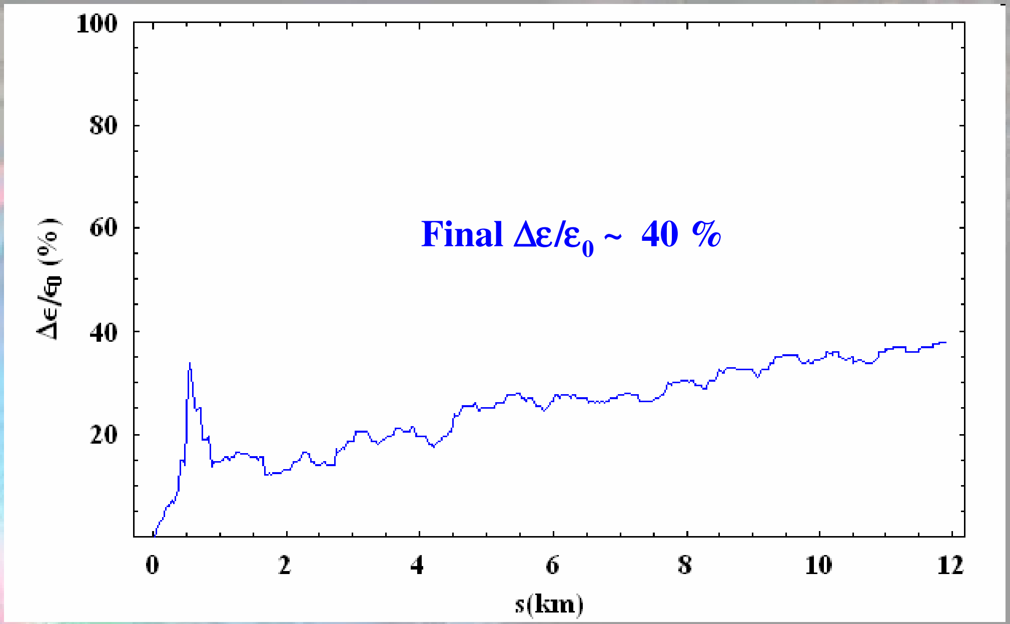


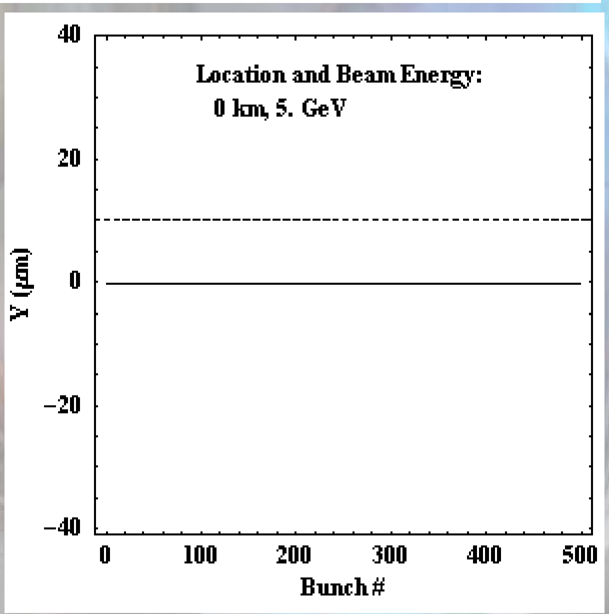
- 500 particles in all simulations.
- Initial vertical and horizontal offset =0 and 270 μm.
- c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm.
- Final mean of the emittance dilution from 200 machines is indicated by the blue line
- **Long range wakes included**

Mitigation of Emittance Growth Due to Rogue Mode.

Split Tune 90/60.



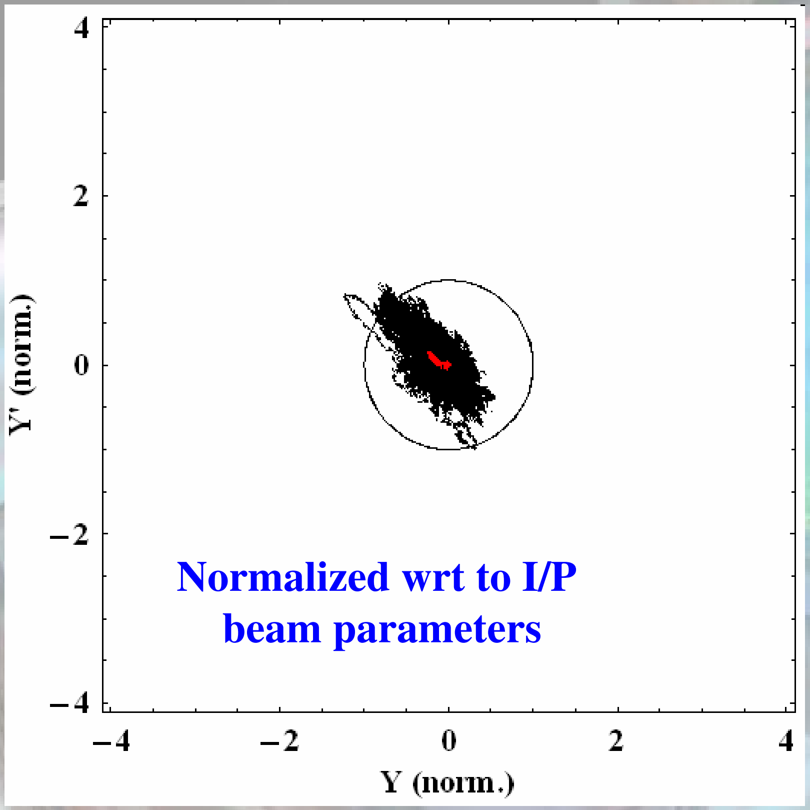


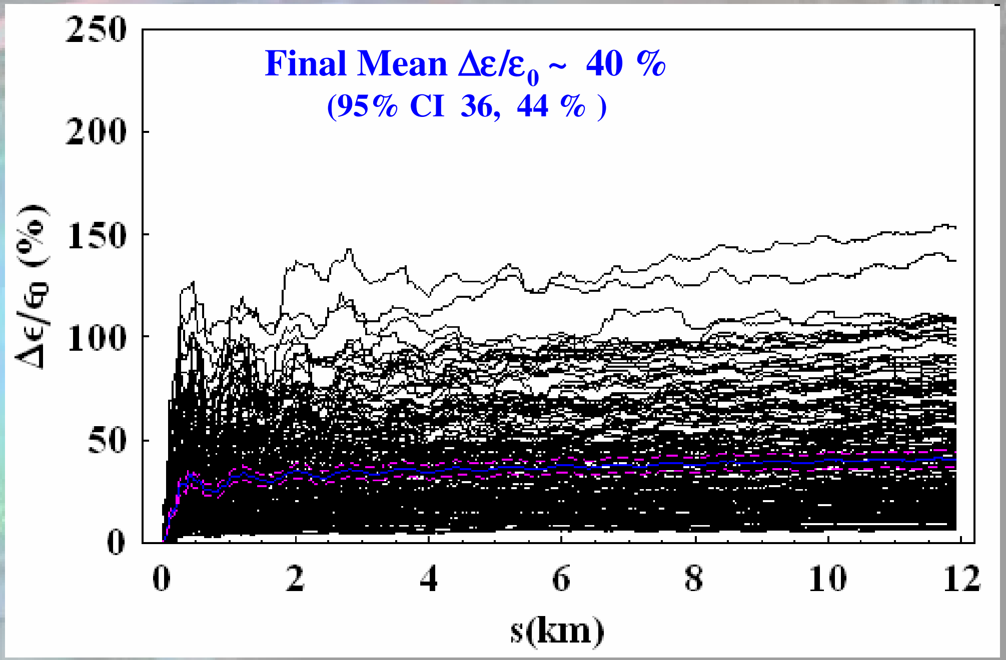


- **Only mode-cross coupling gives rise to emittance dilution**
- **500 particles in all simulations**
- **Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x), respectively.**
- **(c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)**
- ***Fixed azimuthal orientation of couplers.***
- **Long range wakes included in simulations**

Cross-Coupled Motion
Zero Randomization of Azimuthal Orientation of HOM Couplers + Rogue Mode

$X_{off} \sim \sigma_x$, $Y_{off}=0$. *Split Tune 90/60.*





- 500 particles in all simulations.
- Initial vertical and horizontal offset =0 and 270 μm.
- c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm.
- Final mean of the emittance dilution from 200 machines is indicated by the blue line
- **Long range wakes included**

Mode Coupling Summary

•Mode-coupling is a significant source of emittance dilution.

•Emittance dilution can be ameliorated by splitting the tune of the lattice.

•Further experimental data is needed on the actual splitting of the dipole degeneracies in frequencies that is likely to occur in the fabrication of several thousand ILC cavities (experiments at DESY, TTF already in progress will be helpful in this respect –Ross, Napoly, Baboi et al). In addition, simulations are in progress on the splitting of the tune encountered due to the HOM couplers at SLAC (Z. Li, K. Ko et al).

•Analysis of the influence of higher order bands on emittance dilution in the presence of mode coupling is required.

•The effect of purposely distorting the shapes of the cavities in order to increase the frequency degeneracy splitting requires further study.

EXTRA SLIDES

Cross-Coupled Motion
Zero Randomization of Azimuthal Orientation of HOM Couplers + Rogue Mode
$X_{off} \sim \sigma_x$, $Y_{off}=0$. *Split Tune 90/60.*