August 18 - 19, 2005

LoopFest IV

Radiative Corrections for the International Linear Collider

Snowmass, Colorado

Subtraction Terms for NNLO Calculations

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Objective:

We have heard several talks about attempts to construct general purpose algorithms for NNLO. I am less ambitious but more impatient. My goal is:

A fully inclusive Parton-Level Monte Carlo calculation of Drell-Yan at NNLO.

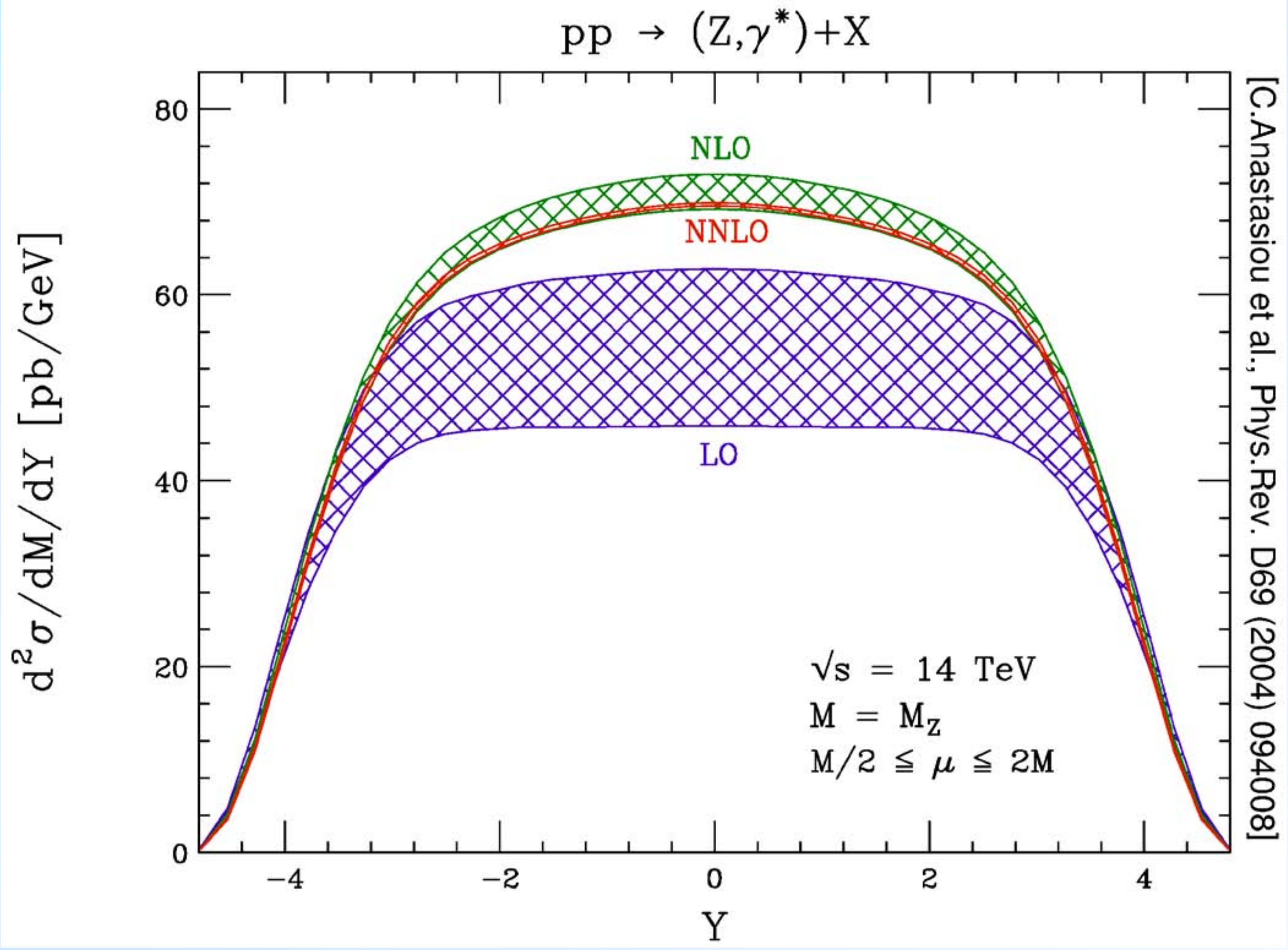
I am not trying to construct a general algorithm. Instead, the idea is to leverage the techniques of inclusive calculations to obtain exclusive information.

Why keep flogging Drell-Yan?

- Inclusive NNLO known for more than 10 years.

[Hamberg et al. ;Harlander & W.K.]

- Rapidity Distributions at NNLO are now known.



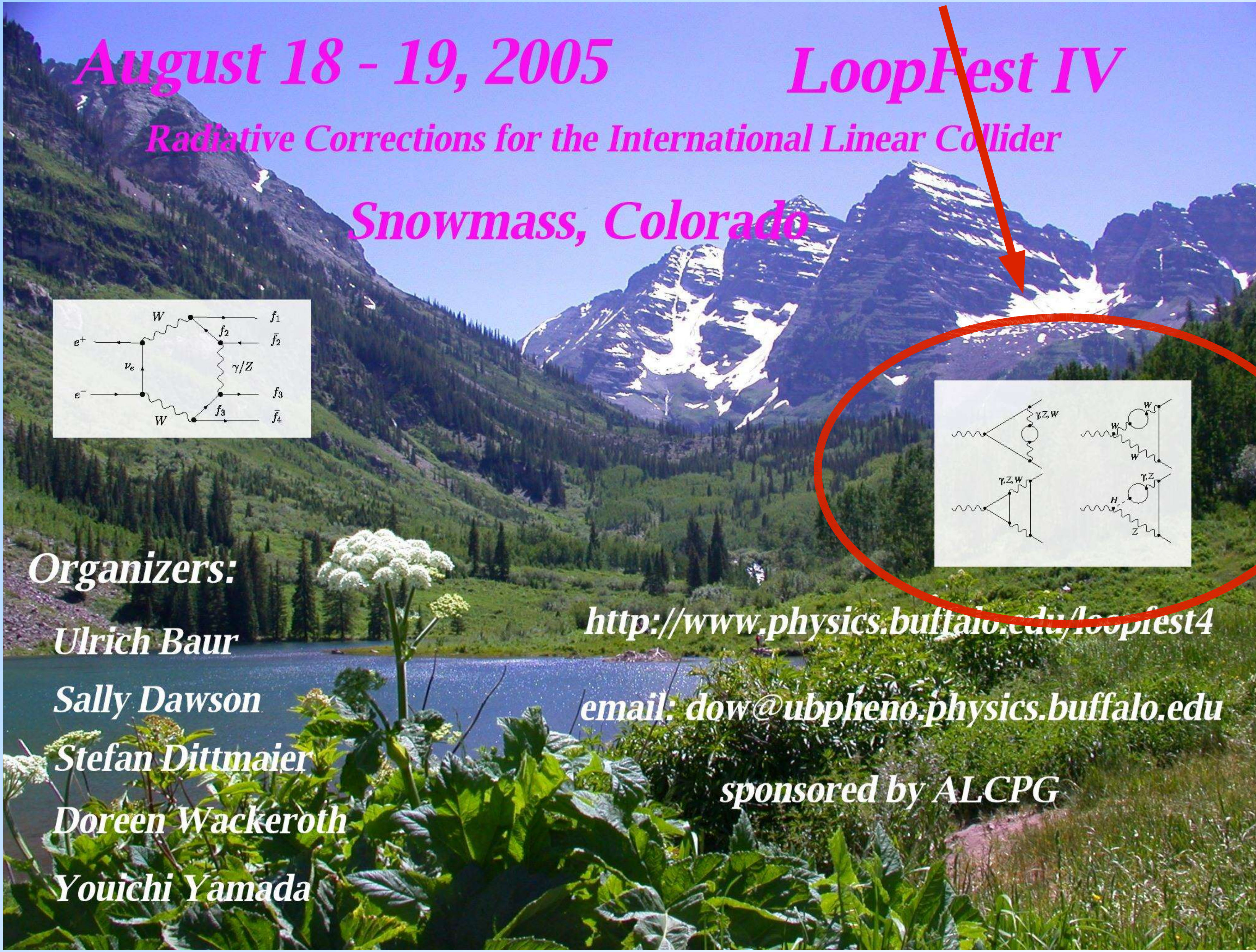
Why keep flogging Drell-Yan?

- We do not yet have a fully exclusive calculation of Drell-Yan.

  Existing NNLO calculations compute massive vector boson production, not di-lepton production. Particle detectors detect leptons. Also, there are measurements to exploit (forward-backward asymmetry) with a fully exclusive calculation of Drell-Yan.

- A Parton-level Monte Carlo will permit a fully exclusive calculation of Drell-Yan production.

Why keep flogging Drell-Yan?



How can I do this?

Petriello has told us about one method for doing this calculation: Sector Decomposition.

I want to try a different approach, more in line with methods developed for Next-to-Leading order calculations over the last ~15 years.

I want to implement a subtraction scheme, using local counter-terms, in a more-or-less standard Monte Carlo framework.

Subtraction at NLO

Next-to-Leading Order calculations consist of two contributions:

Virtual Corrections to one loop.

Single Real Emission Corrections at tree-level.

$$\sigma^{NLO} = \int_{n+1} d\sigma_{n+1}^{(0)} + \int_{n} d\sigma_{n}^{(1)}$$

Both terms are infrared singular.

Subtraction at NLO

A subtraction scheme adds a local counter-term to both Virtual and Real Correction terms, canceling the infrared singularities.

$$\sigma^{NLO} = \int_{n+1} \left(d\sigma^{(0)}_{n+1} - d\alpha^{(0)}_{n+1} \right) + \int_{n} d\sigma^{(1)}_{n} + \int_{n+1} d\alpha^{(0)}_{n+1}$$

Both terms are now infrared finite.

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Both terms are now infrared finite.

Subtraction at NNLO

At Next-to-Next-to-Leading Order, there are three contributions:

- Virtual Corrections to two loops.
- Single Real Emission Corrections to one loop.
- Double Real Emission Corrections at tree-level.

$$\sigma^{NNLO} = \int_{n+2} d\sigma_{n+2}^{(0)} + \int_{n+1} d\sigma_{n+1}^{(1)} + \int_{n} d\sigma_{n}^{(2)}$$

All three terms are infrared singular.

Subtraction at NNLO

A normal NLO subtraction scheme will take care of the singly-infrared singular regions.

$$\sigma^{NNLO} = \int_{n+2} \left(d\sigma^{(0)}_{n+2} - d\alpha^{(0)}_{n+2} \right) + \int_{n+1} d\sigma^{(1)}_{n+1} + \int_{n+2} d\alpha^{(0)}_{n+2} + \int_{n} d\sigma^{(2)}_{n}$$

We still must deal with doubly-infrared regions of both $d\sigma$ and $d\alpha$.

Subtraction at NNLO

We need counter-terms to tree-level double real emission, one-loop single real emission and a counter-term to single real emission from the counter-term!

$$\sigma^{NNLO} = \int_{n+2} \left(d\sigma^{(0)}_{n+2} - d\alpha^{(0)}_{n+2} + d\beta^{(0)}_{n+2} - d\gamma^{(0)}_{n+2} \right) + \int_{n+1} \left(d\sigma^{(1)}_{n+1} - d\alpha^{(1)}_{n+1} \right) + \int_{n+2} \left(d\alpha^{(0)}_{n+2} - d\beta^{(0)}_{n+2} \right) + \int_{n} d\sigma^{(2)}_{n} + \int_{n+1} d\alpha^{(1)}_{n+1} + \int_{n} d\gamma^{(0)}_{n+2}$$

Subtraction at NNLO

We need counter-terms to tree-level double real emission, one-loop single real emission and a counter-term to single real emission from the counter-term!

$$
\begin{aligned}
\sigma^{NNLO} = & \int_{n+2} \left(d\sigma_{n+2}^{(0)} - d\alpha_{n+2}^{(0)} + d\beta_{n+2}^{(0)} - d\gamma_{n+2}^{(0)} \right) \\
& + \int_{n+1} \left(d\sigma_{n+1}^{(1)} - d\alpha_{n+1}^{(1)} \right) + \int_{n+2} \left(d\alpha_{n+2}^{(0)} - d\beta_{n+2}^{(0)} \right) \\
& + \int_{n} d\sigma_{n}^{(2)} + \int_{n+1} d\alpha_{n+1}^{(1)} + \int_{n} d\gamma_{n+2}^{(0)}
\end{aligned}
$$

NNLO Counter-terms

We have already seen a few talks about efforts to construct a general formula for the counter-terms we need. As I am not interested in generality, a very simple choice is available to me:

I can use the matrix elements themselves as the counter-terms and map them to the appropriate doubly singular point in phase space.

$$d\alpha_{n+1}^{(1)} = d\sigma_{n+1}^{(1)} \ ,$$

$$d\gamma_{n+2}^{(0)} = d\sigma_{n+2}^{(0)} \ ,$$

$$d\beta_{n+2}^{(0)} = d\alpha_{n+2}^{(0)} \ .$$

Production Observables

In a process like Drell-Yan, production variables like the gauge boson rapidity are readily obtained in the radiative (2→3) phase space, but must also be made available to the integrated counter-term in the virtual phase space.

$$\sigma^{NNLO} = \int_{n+2} \left(d\sigma^{(0)}_{n+2} - d\alpha^{(0)}_{n+2} + d\beta^{(0)}_{n+2} - d\gamma^{(0)}_{n+2}\right)$$
$$+ \int_{n+1} \left(d\sigma^{(1)}_{n+1} - d\alpha^{(1)}_{n+1}\right) + \int_{n+2} \left(d\alpha^{(0)}_{n+2} - d\beta^{(0)}_{n+2}\right)$$
$$\boxed{+ \int_{n} d\sigma^{(2)}_{n} + \int_{n+1} d\alpha^{(1)}_{n+1} + \int_{n} d\gamma^{(0)}_{n+2}}$$

The integral in the virtual phase space cannot be just the inclusive cross section!

A First Attempt

My initial proposal for capturing the production information envisioned computing each component, $\sigma(q\bar{q}\to V\,g\,g),\sigma(g\,q\to V\,q),\ldots$, and then decomposing the result into a convolution reminiscent of mass factorization.

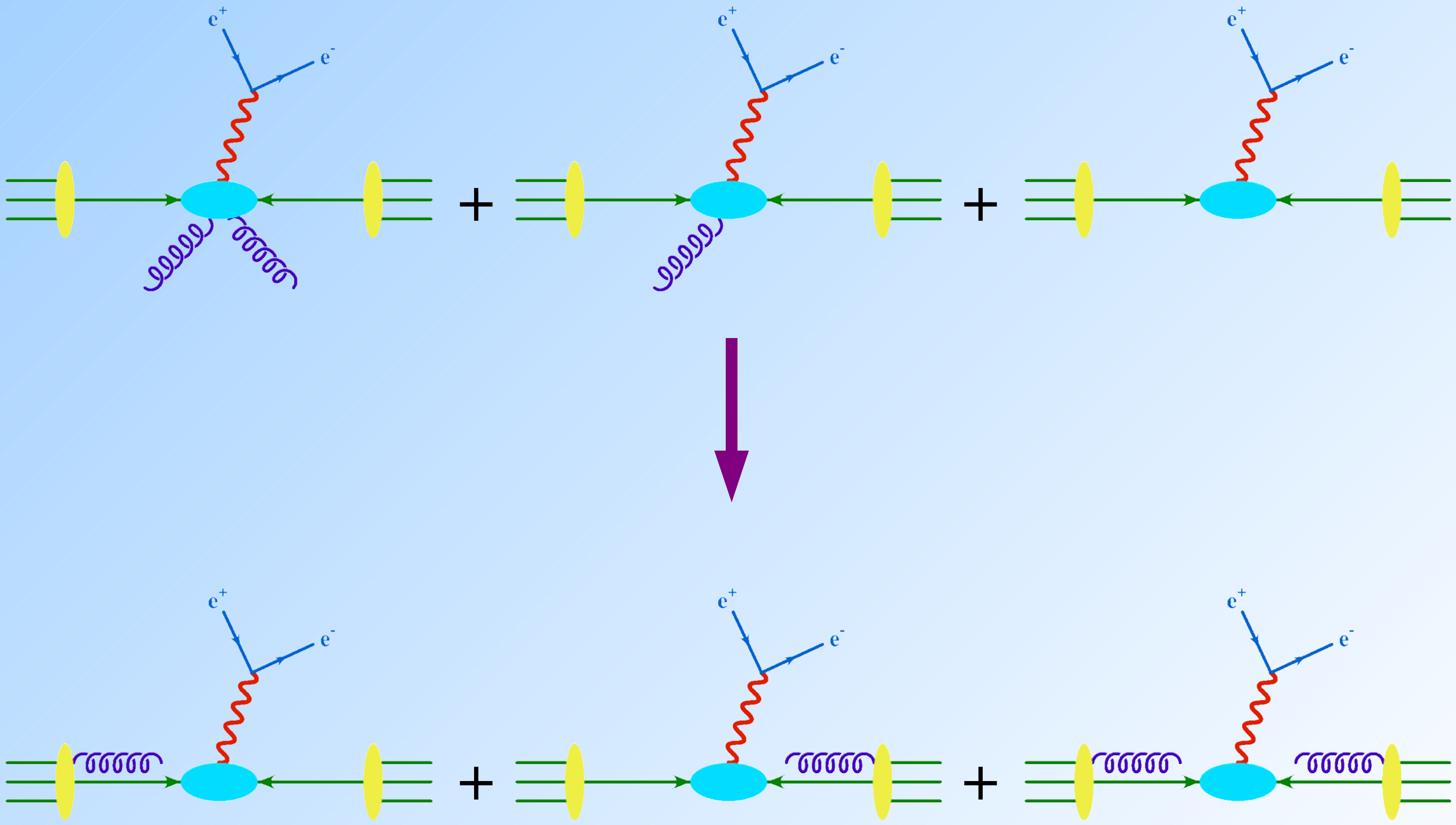
$$d\sigma^{(m)}_{n+2-m}=\sum_{k=0}^{m}\sum_{j=0}^{2-k-m} d\hat{\sigma}^{(k)}_{n}\otimes\tilde{\Gamma}^{(j)}_{1}\otimes\tilde{\Gamma}^{(2-j-k-m)}_{2}\quad ,$$ where,

$$d\hat{\sigma}_{n}(z) \;=\; \delta(1-z)\left[a_0+\frac{\alpha_s}{\pi}\,a_1+\frac{\alpha_s^2}{\pi^2}\,a_2+\ldots\right] \;,$$

$$\tilde{\Gamma}_{i}(w_i) \;=\; \delta(1-w_i)+\frac{\alpha_s}{\pi}\,\tilde{\Gamma}^{(1)}_{i}(w_i)+\frac{\alpha_s^2}{\pi^2}\,\tilde{\Gamma}^{(2)}_{i}(w_i)+\ldots \quad .$$

First Attempt

Schematically, this solution is:



Rapidities

The rapidity of the gauge boson would be determined by the momentum fractions of the incoming partons, $\hat{s} = x_1 x_2 s$, and the fractions of those momentum fractions that go into gauge boson production, $z = w_1 w_2 = M_V^2 / \hat{s}$.

$$y_n = \frac{1}{2} \ln \frac{x_1}{x_2}$$

$$y_{n+1} = \frac{1}{2} \ln \frac{x_1}{x_2} \pm \frac{1}{2} \ln z$$

$$\frac{1}{2} \ln \frac{x_1}{x_2} + \frac{1}{2} \ln z \le y_{n+2} \le \frac{1}{2} \ln \frac{x_1}{x_2} - \frac{1}{2} \ln z$$

Convolution Terms

The convolution terms could be solved for iteratively. For instance, for $\sigma(q\bar{q}\to V\,g\,g)$:

$$\widetilde{\Gamma}^{(2)}=\widetilde{\Gamma}_1^{(2)}+\widetilde{\Gamma}_2^{(2)}+\widetilde{\Gamma}_1^{(1)}\otimes\widetilde{\Gamma}_2^{(1)}\quad,$$

$$\widetilde{\Gamma}_1^{(1)}\otimes\widetilde{\Gamma}_2^{(1)}=\frac{g_{(-4)}}{\epsilon^4}+\frac{g_{(-3)}}{\epsilon^3}+\frac{g_{(-2)}}{\epsilon^2}+\frac{g_{(-1)}}{\epsilon}+g_{(\ 0)}+\ldots,$$

$$\widetilde{\Gamma}_i^{(1)}=\frac{a_{i,(-2)}}{\epsilon^2}+\frac{a_{i,(-1)}}{\epsilon}+a_{i,(\ 0)}+a_{i,(\ 1)}\epsilon+a_{i,(\ 2)}\epsilon^2+\ldots,$$

$$g_{(-4)}=a_{1,(-2)}\otimes a_{2,(-2)},$$

$$g_{(-3)}=a_{1,(-1)}\otimes a_{2,(-2)}+a_{1,(-2)}\otimes a_{2,(-1)},$$

$$g_{(-2)}=a_{1,(\ 0)}\otimes a_{2,(-2)}+a_{1,(-1)}\otimes a_{2,(-1)}+a_{1,(-2)}\otimes a_{2,(\ 0)},$$

$$g_{(-1)}=a_{1,(\ 1)}\otimes a_{2,(-2)}+a_{1,(\ 0)}\otimes a_{2,(-1)}+a_{1,(-1)}\otimes a_{2,(\ 0)}+a_{1,(-2)}\otimes a_{2,(\ 1)},$$

$$g_{(\ 0)}=a_{1,(\ 2)}\otimes a_{2,(-2)}+a_{1,(\ 1)}\otimes a_{2,(-1)}+a_{1,(\ 0)}\otimes a_{2,(\ 0)}+a_{1,(-1)}\otimes a_{2,(\ 1)}$$
$$+a_{1,(-2)}\otimes a_{2,(\ 2)}.$$

Flaws with the First Attempt

This first attempt is not entirely satisfactory.

- It seems to be more of a parameterization than a derivation.
- It is not clear that the solutions for the convolution terms are unique. There may be cancellations in the integrated cross section from terms that give differences in the differential cross section.

Second Attempt: Turn things around

Another approach is to turn things around. Instead of mapping integrated matrix elements onto convolutions, map convolutions into matrix element integrals.

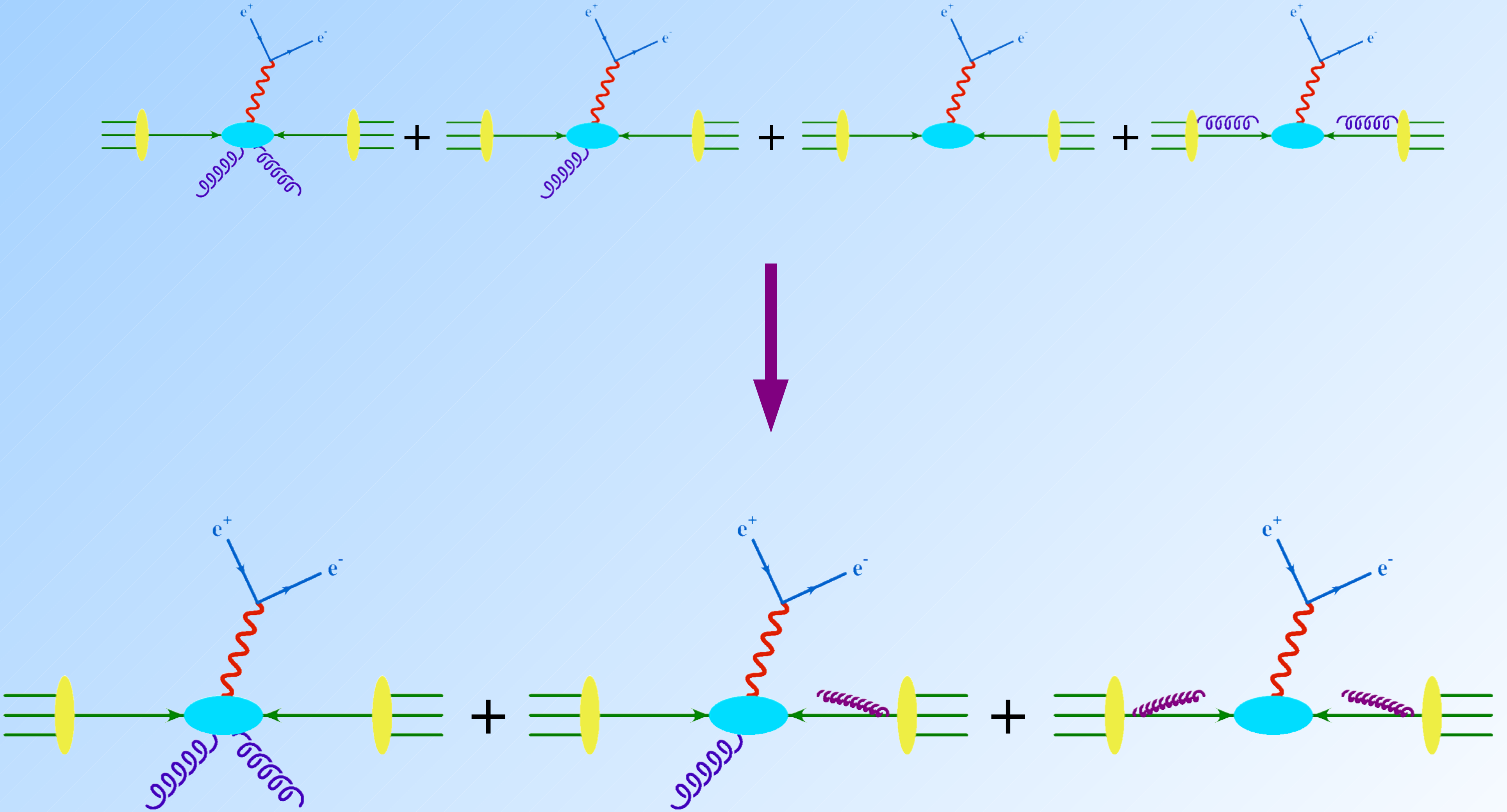
- All contributions are mapped into 2→3 body phase space integrals using the factorization properties of phase space and QCD amplitudes.

$$\lim_{u\|B} \frac{1}{s_{aB}} d^n\Phi(a+B\to u+Q) \to d^n\Phi(B\to u+b)\times\frac{1}{s_{ab}} d^n\Phi(a+b\to Q)$$

$$\lim_{u\|B} |\mathcal{M}(a+B\to u+Q)|^2 \to \hat{c}_{split}^{B\to ub}\times|\mathcal{M}(a+b\to Q)|^2$$

Turning things around

Schematically,



Parameterizing Phase Space

In calculating the inclusive cross section, van Neerven *et al.* chose different parameterizations of phase space to simplify the calculation of each term. If we instead choose a particular parameterization, we can focus on a single kinematic invariant to determine the production characteristics.

- Specifically, for a given $\hat{s}$, M_V, a single invariant, say s_{1V}, will determine the rapidity of the gauge boson at the doubly-singular point.

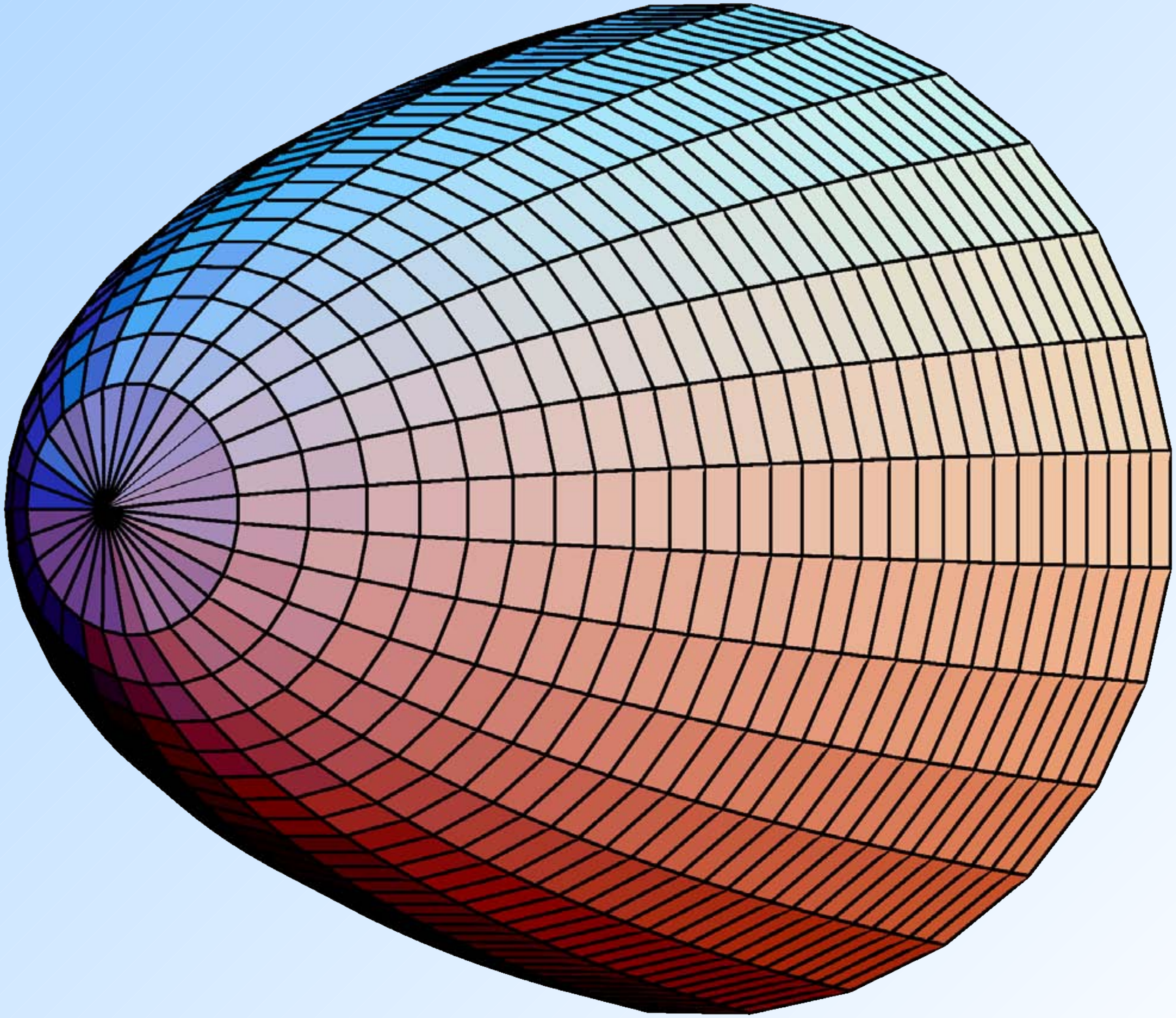
Double Singularities

In general, one needs to know both s_{1V} and s_{2V} to determine the rapidity of the gauge boson. In doubly-singular configurations however, all particles are confined to the beam axis.

In this constrained kinematics, it is sufficient to know only one of the two invariants. So, if we map all configurations with a particular value of s_{1V} to the corresponding doubly-singular configuration, we have a suitable subtraction counter-term.

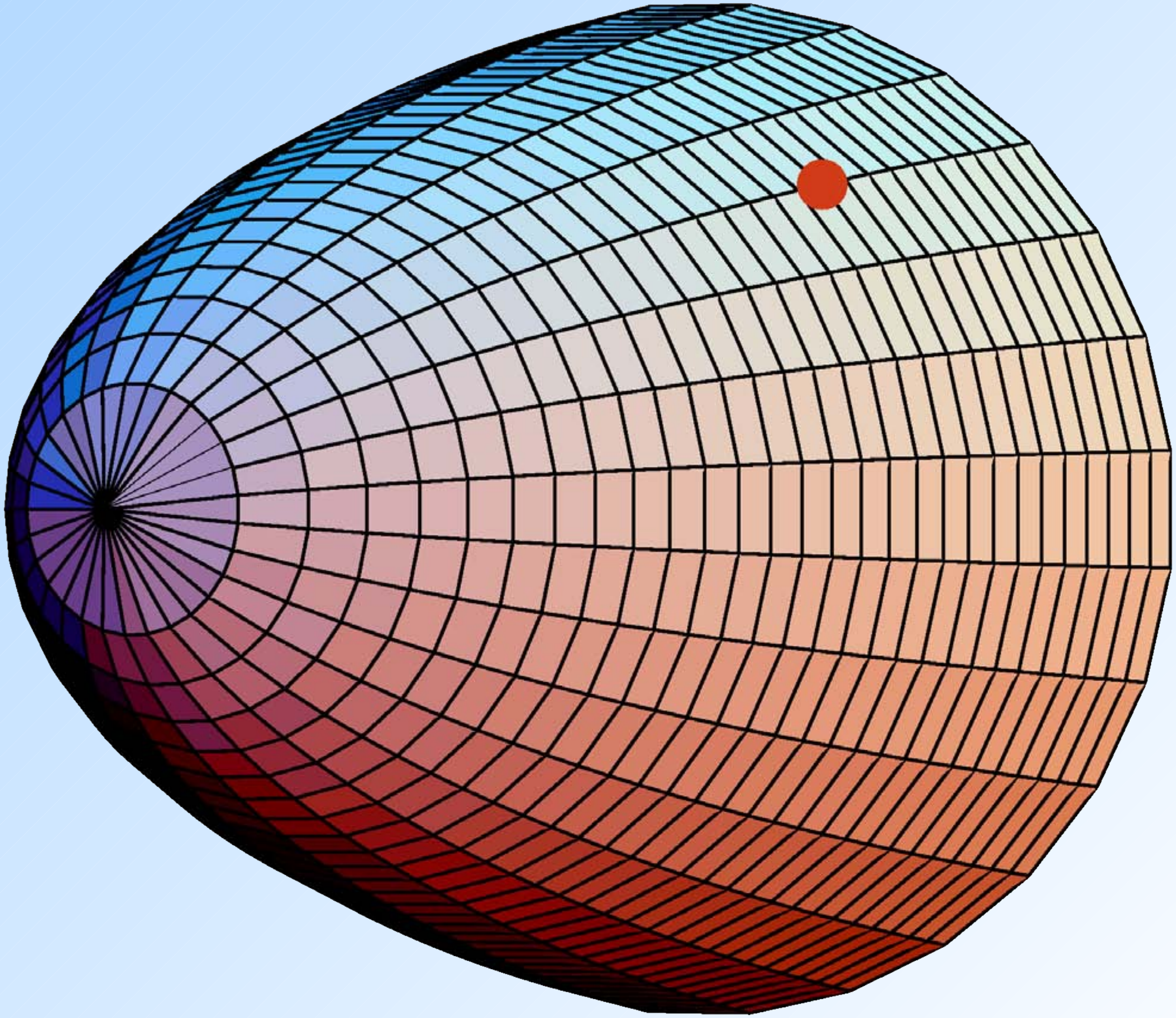
Mapping to the Doubly-Singular Point

The surface describes the possible values of p_2 for given values of $\hat{s}$, M_V, s_{1V} .



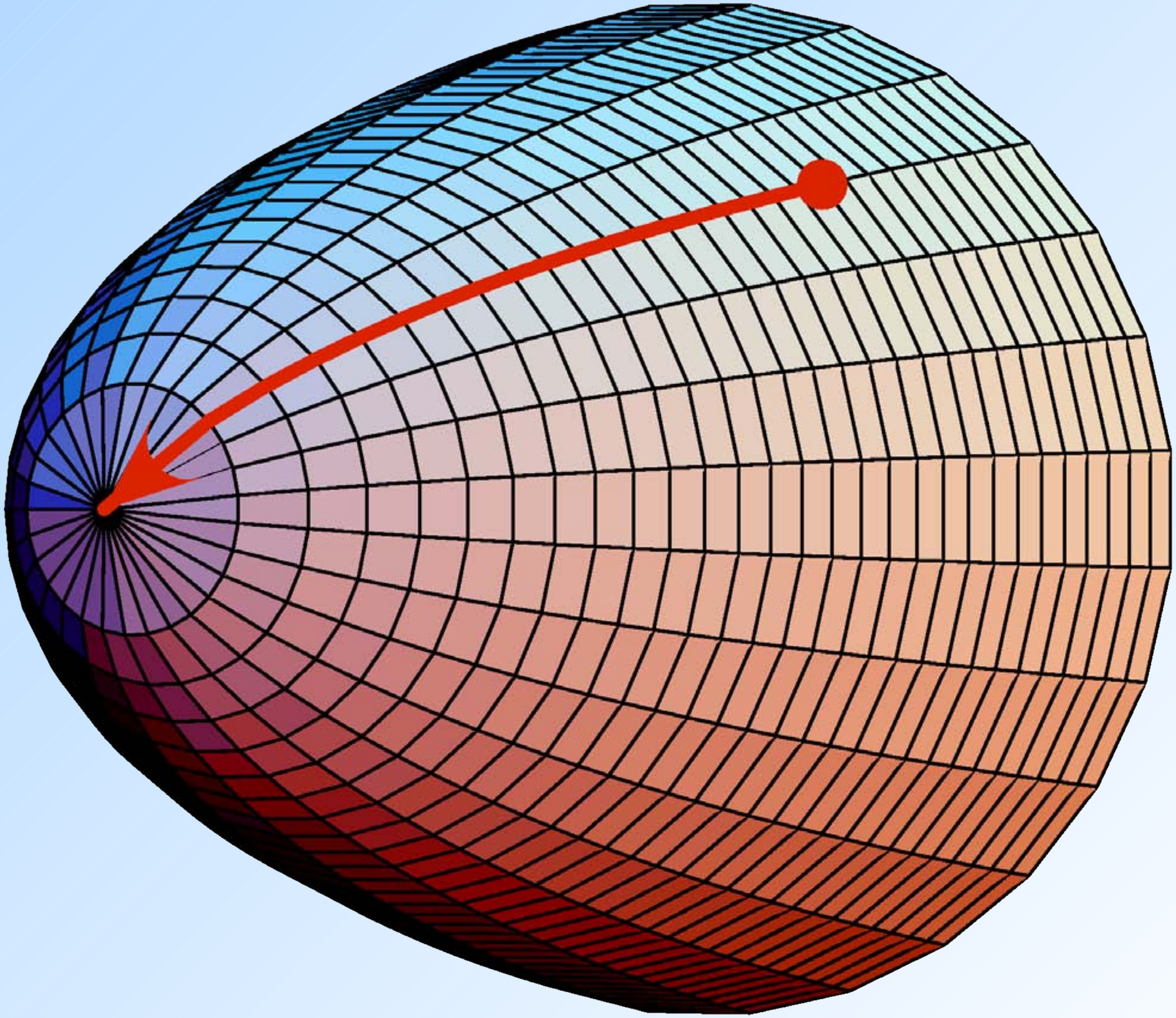
Mapping to the Doubly-Singular Point

All points on the surface map to the doubly-singular point.



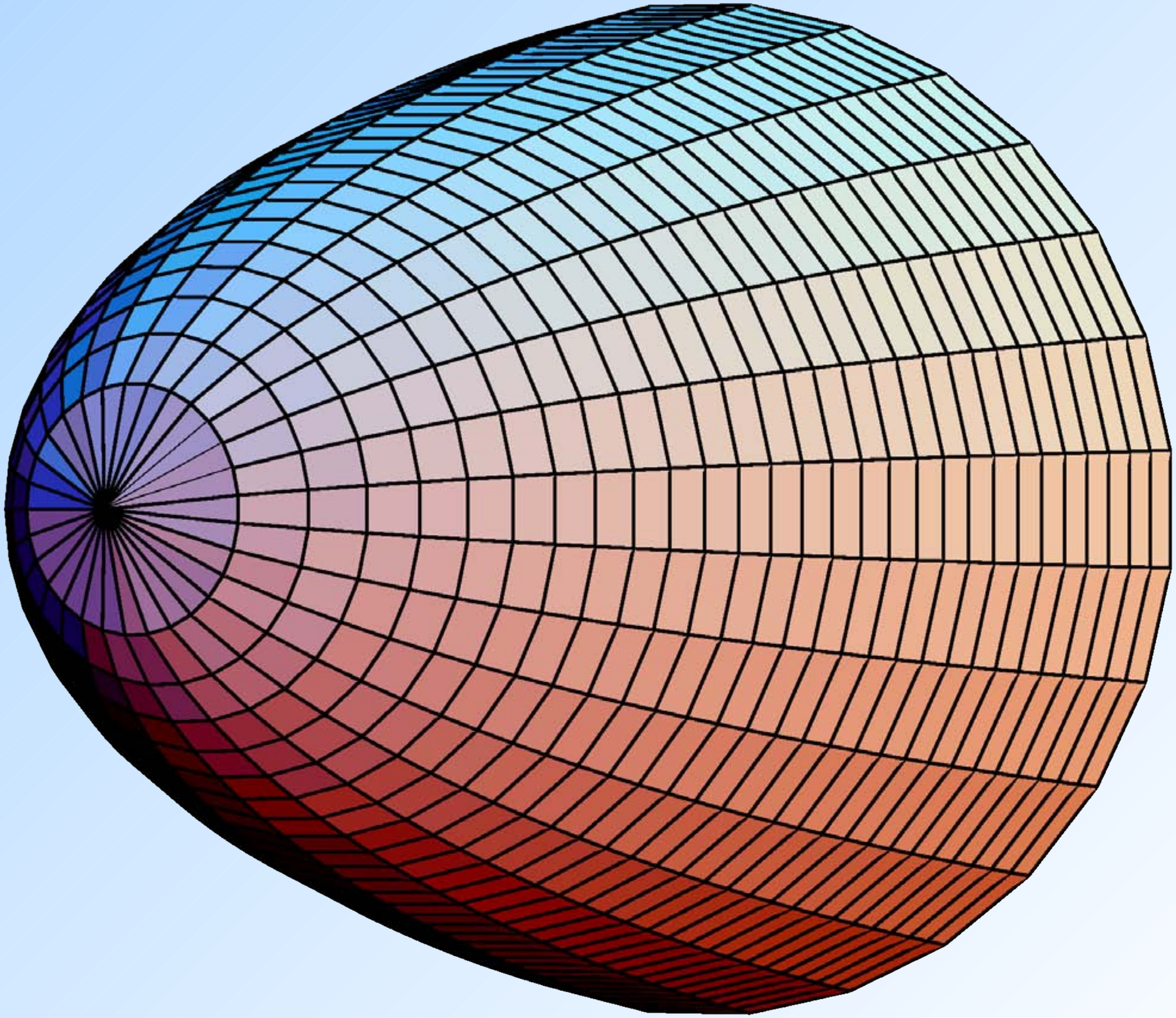
Mapping to the Doubly-Singular Point

All points on the surface map to the doubly-singular point.



Mapping to the Doubly-Singular Point

In the 2→3 phase space, all points are mapped and then integrated.
In the 2→1 phase space all points are integrated and then mapped.



Fully Exclusive Drell-Yan

Existing calculations of Drell-Yan production at NNLO have been fully inclusive of leptonic observables. With a parton-level Monte Carlo, it is possible to be fully exclusive of these observables.

This means capturing the spin correlations. Writing the cross section as the scalar product of hadronic and leptonic tensors, $\sigma = H^{\mu\nu} L_{\mu\nu}$. After averaging over the lepton observables, $\int L_{\mu\nu}$ is proportional to the metric tensor. So, fully inclusive calculations retain only the trace of the hadronic tensor, $H^{\mu}{}_{\mu}$.

Fully Exclusive Drell-Yan

Being exclusive in the lepton observables means that we need the full structure of the hadronic tensor.

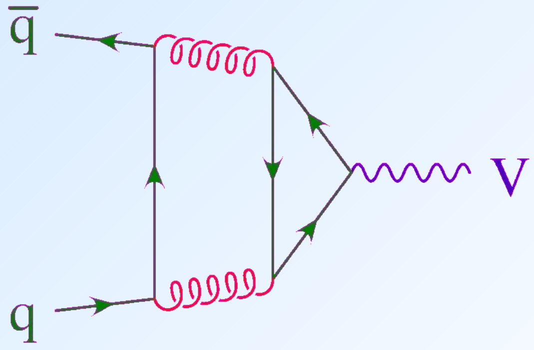
This means that the calculation must be performed in an oriented phase space. The frame I use for the calculation is the gauge boson rest frame, with incoming parton p_1 defining the z axis and the lepton momenta defining the x-z plane. s_{1V} defines the rapidity of the subtraction point.

Axial Terms

Capturing the full structure of the hadronic tensor means computing axial-vector terms too, which were annihilated by taking the trace in inclusive results.

No new two-loop calculation is needed (almost). Consistency (modulo anomaly terms) between σ^{AA} and σ^{VV} and the universal structure of infrared singularities fully determines σ^{AV}.

In the soft limit, the only anomaly contribution comes from:



Consistency conditions

The soft factorization properties of QCD and Catani's formula state that

$$\lim_{g\,g\to soft} \sigma_{n+2}^{(0)} \to \sigma_n^{(0)} \frac{\alpha_s^2}{\pi^2}\left[\frac{d_4}{\epsilon^4}+\frac{d_3}{\epsilon^3}+\frac{d_2}{\epsilon^2}+\frac{d_1}{\epsilon}+d_0+\ldots\right] ,$$

$$\lim_{g\to soft} \sigma_{n+1}^{(1)} \to \sigma_n^{(1)} \frac{\alpha_s}{\pi}\left[\frac{s_2^{(0)}}{\epsilon^2}+\frac{s_1^{(0)}}{\epsilon}+s_0^{(0)}+s_{-1}^{(0)}\epsilon+s_{-2}^{(0)}\epsilon^2+\ldots\right]$$

$$+\sigma_n^{(0)} \frac{\alpha_s^2}{\pi^2}\left[\frac{s_4^{(1)}}{\epsilon^4}+\frac{s_3^{(1)}}{\epsilon^3}+\frac{s_2^{(1)}}{\epsilon^2}+\frac{s_1^{(1)}}{\epsilon}+s_0^{(1)}+\ldots\right] ,$$

$$\sigma_n^{(2)}=\sigma_n^{(1)} \frac{\alpha_s}{\pi}\left[\frac{c_2^{(0)}}{\epsilon^2}+\frac{c_1^{(0)}}{\epsilon}\right]+\sigma_n^{(0)} \frac{\alpha_s^2}{\pi^2}\left[\frac{c_4^{(1)}}{\epsilon^4}+\frac{c_3^{(1)}}{\epsilon^3}+\frac{c_2^{(1)}}{\epsilon^2}+\frac{c_1^{(1)}}{\epsilon}+c_0^{(1)}+\ldots\right] .$$

Gauge invariance demands (modulo anomaly terms) $\sigma_{soft}^{VV}=\sigma_{soft}^{AA}$.
Once all coefficients are fixed, σ_{soft}^{AV} is determined and is exactly proportional to σ_{soft}^{VV} .

Status of the Calculation

I am collaborating with John Campbell to construct a full NNLO Monte Carlo program.

There are two main projects going on:

- Analytic integration of the subtraction term into the $2 \to 1$ phase space.
- Numerical integration of subtracted cross sections in the $2 \to 3$ and $2 \to 2$ phase spaces.

We expect results by early Fall.

Conclusions

- I have outlined a program for constructing a fully exclusive Monte Carlo calculation of Drell-Yan production at NNLO.
- This program leverages the techniques developed for single-inclusive production processes, but cannot be readily applied to a more general framework.
- Most of the technical issues have been worked out and we expect a working implementation by early Fall.