

Merging Parton Showers with NLO QCD

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INTRODUCTION

Why do we need NLO or higher order?

- We need accuracy.
- The strong coupling is big and the leading order predictions are very poor.
- The leading order results has very strong dependence on the arbitrary non-physical scales (renormalization and factorization)

Why do we need parton shower?

- We want more realistic final state.
- In the fix order calculations we are able to calculate processes only with few partons in the final state.
- But in the detector we see lots of **hadrons**.
- If we want more realistic picture we have to deal with the hadronization. The hadronization is long distance physics. We cannot calculate but we can “measure” it. It is **universal**.
- We need “bridge” between the short distance and long distance part. $\implies$
Parton shower

BORN LEVEL CALCULATION

The Born level cross section is an m -parton phase space integral:

$$\sigma^{LO} = \int d\Gamma(\{p\}_m, Q) |M(\{p, f\}_m)|^2 F^{(m)}(p_1, \dots, p_m) + \mathcal{O}(\alpha_s^{m-1})$$

- Trivially no UV singularities. (No integral over infinite phase space.)
- No IR singularities from the phase space integral ensured by the $F^{(m)}$ measurement function.
- At this level the main task (challenge) is to calculate the **matrix element squares**.
 - the matrix element automatically generated up to $2 \rightarrow 6$ or even $2 \rightarrow 8$ (**MADGRAPH, ALPGEN, HELAC, AMEGIC++,...**)
 - plus automatic integration over the phase space (**PHEGAS, MADEVENT, SHERPA,...**)

PARTON SHOWER

Take the primary hard process (in e^+e^- annihilation it is the $e^+e^- \rightarrow q\bar{q}$) and calculate the rest of the event by parton shower algorithm

$$\sigma^{LO,S} = \int d\Gamma(\{p\}_2, Q) |M(\{p, f\}_2)|^2 (S(\{p, f\}_2)|F) (1 + \mathcal{O}(\alpha_s L))$$

- Based on that physical picture that every parton produce a jet (jet: A “spray” of collinear hadrons).
- In the collinear limit the QCD matrix element has factorization properties. This factorization property allows us to calculate the $(S(\{p, f\}_2)|F)$ recursively.
- There are several program available: APACIC++, ARIADNE, COJET, HERWIG(++), PYTHIA(++),...
- This is an all order expression but gives good approximation only at LL level. Out of this region the performance is very poor.

MATRIX ELEMENT + PARTON SHOWER

(CKKW: Catani-Krauss-Kuhn-Webber Method)

Defining the jet clustering sequence for a tree level m -parton process using the $k_\perp$ jet algorithm, that is $d_2 > d_3 > \dots > d_n > d_{\text{ini}}$, the cross section is given by

$$\sigma^{LO+S}[F] = \sum_{m=2}^{m_{\text{max}}} \tilde{\sigma}_m^{B+S}[F] \ ,$$

where

$$\begin{aligned} \tilde{\sigma}_m^{B+S}[F] = & \int_m d\Gamma(\{p\}_m, Q) |M(\{p, f\}_m)|^2 \theta(d_m > d_{\text{ini}}) \\ & \times W_m(\{p, f\}_m) (S(\{p, f\}_m; d < d_{\text{ini}}) | F) \end{aligned}$$

- The scale $d_{\text{ini}} > 0$ helps to keep away from the singular region in the hard matrix element. It is arbitrary but not zero.
- The $\theta(d_m > d_{\text{ini}})$ introduces large logarithms in the variable d_{ini} but they are cancelled at NLL level.
- This cancellation is ensured by the Sudakov reweighting and the vetoed shower.

NLO CROSS SECTIONS

To eliminate the IR singularities from the real part the best way is the **dipole method**:

$$\sigma^{NLO} = \int_m d\sigma^B + \int_{m+1} [d\sigma^R|_{\epsilon=0} - d\sigma^A|_{\epsilon=0}] + \int_m [d\sigma^V + \int_1 d\sigma^A]_{\epsilon=0}$$

massless case :

S. Catani and M.H. Seymour

massive case : S. Catani, S. Dittmaier, M.H. Seymour, Z. Trócsányi

The $d\sigma^A$ is a local counterterm for $d\sigma^R$ with same pointwise behaviour as $d\sigma^R$. Furthermore it is integrable in $d = 4 - 2\epsilon$ dimension over the single parton subspaces.

$$\begin{aligned} \sigma^{NLO} = & \int_m d\sigma^B + \int_{m+1} [d\sigma^R|_{\epsilon=0} - \sum_{\text{dipoles}} d\sigma^B \otimes dV|_{\epsilon=0}] \\ & + \int_m [d\sigma^V + d\sigma^B \otimes I^R(\epsilon)]_{\epsilon=0} \end{aligned}$$

where the $I(\epsilon)$ singular factor in the massless case is

$$I^R(\epsilon) = -\frac{\alpha_s}{2\pi} \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \sum_{j \neq i} \frac{\mathbf{T}_i \cdot \mathbf{T}_j}{\mathbf{T}_i^2} \left(\frac{\mu^2}{s_{ij}} \right)^\epsilon \left[\mathbf{T}_i^2 \left(\frac{1}{\epsilon^2} - \frac{\pi^2}{3} \right) + \gamma_i \frac{1}{\epsilon} + \gamma_i + K_i + \mathcal{O}(\epsilon) \right]$$

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- several program: EKS, JETRAD, EVENT(2), EERAD, DISENT, DISASTER++,MEPJET, AYLEN/EMILIA, PHOX, ...
- in some case we can calculate up to $2 \rightarrow 3$ (NLOJET++, MCFM)
- some interesting process is computed but many of them still missing
- There is **no automated** program.

NLO + SHOWER

There are two other approaches on the market:

- **MC@NLO** approach by Frixione, Webber and Nason
 - the method is **not general, not Lorentz covariant**
 - the matching is **not exact**
 - it is worked out for the processes with no final state colored particle or only with massive quarks.
 - The method is **specific** to a **particular Monte Carlo** implementation (HERWIG).
- By M. Krämer and D.E. Soper
 - It is worked out only for $e^+e^- \rightarrow 3 \text{ jets}$.
 - It is based on a fully numeric NLO method which is also specific to the $e^+e^- \rightarrow 3 \text{ jets}$ process.
 - It is not Lorentz covariant.
 - But the basic idea is very general and it gives **exact matching**.
 - It can work together with any shower program.
- There is **no algorithm** defined to achieve the “**NLO matrix element + Parton Shower**” project (**CKKW@NLO**).

HIGHLIGHTS OF OUR METHOD

In our approach we want the most of all, to have an algorithm that can be used in a reasonably straightforward manner:

- It is based on the Catani-Seymour subtraction method $\Rightarrow$ Lorentz covariance, *easy to implement*
- The first hardest step of the shower is included in the NLO program and the subsequent shower is calculated by the user's favorite shower algorithm $\Rightarrow$ *not specific to a particular MC implementation*
- We have full control on the first step in every singular regions $\Rightarrow$ *exact matching*
- The algorithm can deal with *any number of the colored particles* in the final and initial states.
- We implement the *CKKW* matching scheme at *NLO level*.

NLO + PARTON SHOWER

In the CKKW matching scheme the cross section is sum of the partial cross sections:

$$\sigma^S[F] = \sum_{m=2}^{m_{\text{NLO}}} \sigma_m^{NLO+S}[F] + \sum_{m=m_{\text{NLO}}+1}^{m_{\text{max}}} \tilde{\sigma}_m^{B+S}[F]$$

At NLO level one term in the CKKW cross section should be

$$\sigma_m^{NLO+S}[F] = \sigma_m^{B+S}[F] + \sigma_m^{R+S}[F] + \sigma_m^{V+S}[F]$$

The simplified Born term should be

$$\begin{aligned} \tilde{\sigma}_m^{B+S}[F] = & \sum_{\{f\}_m} \frac{1}{m!} \int d\Gamma(\{p\}_m; Q) \theta(d_m > d_{\text{ini}}) W_m(\{p, f\}_m) \\ & \times |\mathcal{M}(\{p, f\}_m)|^2 (S(\{\hat{p}, \hat{f}\}_m; d < d_{\text{ini}})|F) \end{aligned}$$

NLO + PARTON SHOWER

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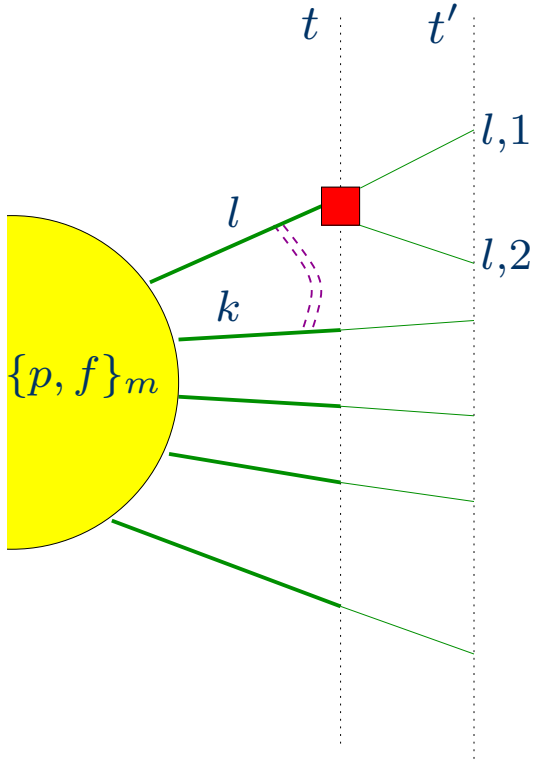
At NLO level one term in the CKKW cross section should be

$$\sigma_m^{NLO+S}[F] = \sigma_m^{B+S}[F] + \sigma_m^{R+S}[F] + \sigma_m^{V+S}[F]$$

But the Born term in the NLO part must be matched to the NLO calculation

$$\begin{aligned} \sigma_m^{B+S}[F] &= \sum_{\{f\}_m} \frac{1}{m!} \int d\Gamma(\{p\}_m; Q) \theta(d_m > d_{\text{ini}}) W_m(\{p, f\}_m) \\ &\times \sum_{l=1}^m \sum_{k \neq l} \langle \mathcal{M}(\{p, f\}_m) | \int dY_l \mathbf{E}_{l,k}(Y_l) | \mathcal{M}(\{p, f\}_m) \rangle \\ &\times (S(\{\hat{p}, \hat{f}\}_{m+1}; d < d_{\text{ini}}) | F) \end{aligned}$$

EMISSION OPERATOR



$$\begin{aligned}
 \mathbf{E}_{l,k}(Y_l) = & \frac{\mathbf{T}_l \cdot \mathbf{T}_k}{-\mathbf{T}_l^2} \int_0^\infty dt \delta(t - T_l(z_l, y_l)) \\
 & \times \frac{\alpha_s(t)}{2\pi} \mathbf{S}_l(Y_l) \theta(d_l(z_l, y_l) < d_{\text{ini}}) \\
 & \times \prod_{l'} \exp \left(- \int_0^1 \frac{dy}{y} \int_0^1 dz \theta(T_{l'}(z, y) > t) \right. \\
 & \quad \left. \times \theta(d_{l'}(z, y) < d_{\text{ini}}) \frac{\alpha_s}{2\pi} S(y, z, f_{l'}) \right)
 \end{aligned}$$

The $Y_l = \{z, y, \phi, \hat{f}_{l,1}, \hat{f}_{l,2}\}$ represent the splitting variables

$$\int dY_l \equiv \int_0^1 \frac{dy}{y} \int_0^1 dz \int_0^{2\pi} \frac{d\phi}{2\pi} \frac{1}{2} \sum_{\hat{f}_{l,1}, \hat{f}_{l,2}} \delta_{\hat{f}_{l,1} + \hat{f}_{l,2}}^{f_l}$$

It is not a simple object but its integral is very simple

$$\sum_{k \neq l} \frac{\mathbf{T}_l \cdot \mathbf{T}_k}{-\mathbf{T}_l^2} = 1 \quad \Rightarrow \quad \sum_l \sum_{k \neq l} \int dY_l \mathbf{E}_{l,k}(Y_l) = 1 .$$

SPLITTING KINEMATICS

The daughter partons those are emitted from the line l are labeled by $l, 1$ and $l, 2$. Their flavors must correspond to a QCD vertex that $f_l \rightarrow \hat{f}_{l,1} + \hat{f}_{l,2}$.

The momenta are defined according to the Sudakov parametrization

$$\hat{p}_{l,1} = z p_l + y(1-z)p_k + k_\perp, \quad \hat{p}_{l,2} = (1-z)p_l + yz p_k - k_\perp,$$

and the spectator (recoiled) momentum is $\hat{p}_k = (1-y)p_k$.

$$p_l + p_k = \hat{p}_{l,1} + \hat{p}_{l,2} + \hat{p}_k, \quad \hat{p}_{l,1}^2 = \hat{p}_{l,2}^2 = 0.$$

The transverse momentum is perpendicular both to the emitter and spectator

$$k_\perp \cdot p_l = k_\perp \cdot p_k = 0, \quad k_\perp^2 = -2p_l \cdot p_k yz(1-z).$$

The phase space can be written in factorized form

$$d\Gamma^{(m+1)}(\{\hat{p}\}_{m+1}; Q) \frac{1}{2\hat{p}_{l,1} \cdot \hat{p}_{l,2}} = d\Gamma^{(m)}(\{p\}_m; Q) \frac{dy}{y} dz \frac{d\phi}{2\pi} \frac{1-y}{16\pi^2}$$

$\Rightarrow$ There is no approximation in the phase space.

SPLITTING KERNELS

The **S splitting kernels** are based on the Catani-Seymour dipole factorization formulas. For example for the $q \rightarrow q + g$ splitting

$$\langle s | \mathbf{S}_{qg}(z, y) | s' \rangle = C_F (1 - y) \left[\frac{2}{1 - z(1 - y)} - (1 + z) \right] \delta_{ss'} ,$$

and similarly for the $g \rightarrow g + g$ splitting

$$\begin{aligned} \langle s | \mathbf{S}_{gg}(p_l, z, y, k_\perp) | s' \rangle = \\ 2C_A (1 - y) \epsilon_\mu^*(p, s) \left[-g^{\mu\nu} \left(\frac{1}{1 - z(1 - y)} + \frac{1}{1 - (1 - z)(1 - y)} - 2 \right) \right. \\ \left. + 2z(1 - z) \frac{k_\perp^\mu k_\perp^\nu}{-k_\perp^2} \right] \epsilon_\nu(p, s') . \end{aligned}$$

The evolution variable is chosen to be proportional to the transverse momentum

$$T_l(z, y) = s_l yz(1 - z) ,$$

where s_l is a hard scale but it is independent off the spectator label k .

NLO CORRECTION

At NLO level one term in the CKKW cross section should be

$$\sigma_m^{NLO+S}[F] = \sigma_m^{B+S}[F] + \sigma_m^{R+S}[F] + \sigma_m^{V+S}[F]$$

The contributions of the $m + 1$ parton matrix element should be

$$\begin{aligned} \sigma_m^{R+S}[F] = & \sum_{\{f\}_{m+1}} \frac{1}{(m+1)!} \int_{m+1} d\Gamma^{(m+1)}(\{p\}_{m+1}; Q) \\ & \times \left\{ |\mathcal{M}(\{p\}_{m+1})|^2 \theta(d_m > d_{\text{ini}}) \theta(d_{m+1} < d_{\text{ini}}) \right. \\ & \quad \left. - \sum_{\substack{i,j \\ \text{pairs}}} \sum_{k \neq i,j} \mathcal{D}_{ij,k}(\{p\}_{m+1}) \theta(d_m > d_{\text{ini}}) \theta(d_{ij} < d_{\text{ini}}) \right\} \\ & \times (S(\{p, f\}_{m+1})|F) \end{aligned}$$

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At NLO level one term in the CKKW cross section should be

$$\sigma_m^{NLO+S}[F] = \sigma_m^{B+S}[F] + \sigma_m^{R+S}[F] + \sigma_m^{V+S}[F]$$

The contributions of the m parton one-loop matrix element is

$$\begin{aligned} \sigma_m^{V+S}[F] = & \sum_{\{f\}_m} \frac{1}{m!} \int_m d\Gamma^{(m)}(\{p\}_m; Q) \\ & \times \left\{ \text{Re} \langle \mathcal{M}(\{p, f\}_m) | \left[\mathbf{I}(\epsilon) | \mathcal{M}(\{p, f\}_m) \rangle + 2 | \mathcal{M}^{(1)}(\{p, f\}_m; \epsilon) \rangle \right]_{\epsilon=0} \right. \\ & - \frac{\alpha_s}{2\pi} W_m^{(1)}(\{p, f\}_m) \langle \mathcal{M}(\{p, f\}_m) | \mathcal{M}(\{p, f\}_m) \rangle \\ & \left. - \sum_l \sum_{k \neq l} C_{l,k}(\{p\}_m, d_{\text{ini}}) \right\} \\ & \times (S(\{p, f\}_m) | F) \end{aligned}$$

NLO EXPANSION

The secondary shower has the property that

$$(S(\{p, f\}_n)|F) = F(\{p\}_n) + \mathcal{O}(\alpha_s) + \mathcal{O}(1 \text{ GeV}/\sqrt{s})$$

If the measurement function F sensitive for the N -jet region then the main contribution comes from the partial cross section $\sigma_{m=N}^{NLO+S}$.

The expansion of the Born term is

$$\begin{aligned} \sigma_N^{B+S}[F] = & \sum_{\{f\}_N} \frac{1}{N!} \int d\Gamma(\{p\}_N; Q) \langle \mathcal{M}(\{p, f\}_N) | \mathcal{M}(\{p, f\}_N) \rangle \\ & \times F(\{p\}_N) \theta(d_N > d_{\text{ini}}) \left(1 + \frac{\alpha_s}{2\pi} W_N^{(1)} \right) \\ & + \sum_{\{f\}_{m+1}} \frac{1}{(N+1)!} \int_{N+1} d\Gamma^{(N+1)}(\{p\}_{N+1}; Q) \\ & \times \sum_{\substack{i,j \\ \text{pairs}}} \sum_{k \neq i,j} \mathcal{D}_{ij,k}(\{p\}_{N+1}) \theta(d_{ij} < d_{\text{ini}} < d_N^{ij,k}) \\ & \times \left\{ F(\{p\}_{N+1}) - F(\{\tilde{p}\}_N^{ij,k}) \right\} \\ & + \mathcal{O}(\alpha_s^2) + \mathcal{O}(1 \text{ GeV}/\sqrt{s}) \end{aligned}$$

NLO EXPANSION

Finally, the perturbative expansion of the N-jet cross section is

$$\begin{aligned}
 \sigma^S[F] = & \sigma_m^{NLO}[F] + \\
 & + \sum_{\{f\}_N} \frac{1}{N!} \int d\Gamma(\{p\}_N; Q) \theta(d_N > d_{\text{ini}}) F(\{p\}_N) \\
 & \times \left[\langle \mathcal{M}(\{p, f\}_N) | \mathcal{M}(\{p, f\}_N) \rangle (W_N^{(1)} - W_N^{(1)}) \right. \\
 & \quad \left. - \sum_l \sum_{k \neq l} C_{l,k}(\{p\}_N, d_{\text{ini}}) \right] \\
 & + \sum_{\{f\}_{N+1}} \frac{1}{(N+1)!} \int_{m+1} d\Gamma^{(N+1)}(\{p\}_{N+1}; Q) \\
 & \times \sum_{\substack{i,j \\ \text{pairs}}} \sum_{k \neq i,j} \mathcal{D}_{ij,k}(\{p\}_{N+1}) \theta(d_N^{ij,k} > d_{\text{ini}}) \\
 & \times \left\{ F(\{p\}_{N+1}) - F(\{p\}_{N+1}) + F(\{p\}_N^{ij,k}) \theta(d_{ij} \geq d_{\text{ini}}) \right\} \\
 & + \mathcal{O}(\alpha_s^2) + \mathcal{O}(1 \text{ GeV}/\sqrt{s})
 \end{aligned}$$

CONCLUSIONS AND OUTLOOKS

- With some modifications in the first step of the shower we are able to merge the parton shower with the NLO cross section avoiding the double counting.
- The method is **Lorentz covariant**.
- It is based on the Catani-Seymour dipole subtraction method.
- The method is **not specific** to a **particular Monte Carlo** implementation.
- The algorithm is **fully accurate** in the **soft region** (in every singular regions). There is no left over singularities that needs special treatment.
- This method works for the processes with **incoming hadrons**,
- and with **massive particles**
- With this method we can also add “**NLO matrix element**” corrections **to the parton shower** (**CKKW@NLO**).
- The coding is a big challenge. We need a general NLO program that can work together with the automated matrix element generators.

CONCLUSIONS AND OUTLOOKS

- Based on the C-S dipole factorization one can define a new transverse ordered shower.
 - The shower is **Lorentz invariant/covariant**.
 - The phase space is the **exact m -body phase space** with the **exact phase space weight**.
 - The angular ordering is provided by the kinematics. $\Rightarrow$ There is **no awkward cut parameters** in the algorithm.
 - Actually there is only one external parameter, the infrared cutoff parameter.
- The $k_{\perp}$ ordered shower helps to have better understanding on the “**Matrix element + Shower**” matching
 - Two ways: **Slicing** (CKKW) and **Subtraction** ($\Rightarrow$ Peter Skands talk)
 - It is not obvious but they are completely equivalent.
 - The slicing method is **artificially complicated**.
 - The subtraction method is **more suitable** for **NLO+Shower** matching.
- The shower with exact phase space is the best way to include **higher order effects**. $\Rightarrow$ complete **NNLO subtraction** method with **exact phase space factorization** at least at leading color level