Automating radiative corrections in Bhabha scattering

Alejandro Lorca

DESY Zeuthen

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I. Motivation

Motivation: ILC and Bhabha physics

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($A^b_{\rm FB}$, $\sin\theta_{\rm eff}$, N_ν,. . .) require better control of theoretical uncertainties.

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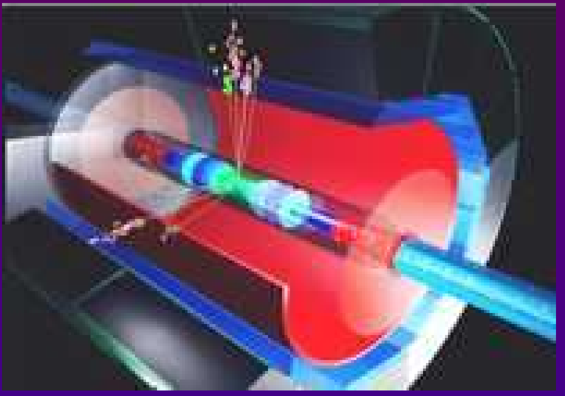
$$\boxed{e^+e^- \text{ beam}, \quad E_{\mathrm{CM}} \approx 1\mathrm{TeV}, \quad ‰ \text{ precision}}$$



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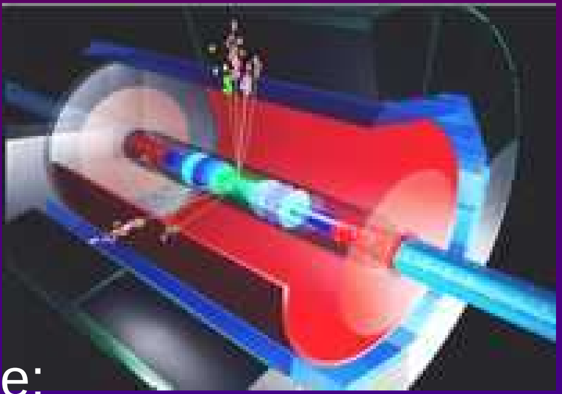

Bhabha scattering ($e^+e^- \to e^+e^-$) plays a dominant role:

- Luminosity monitoring and precise parameter determination
- Disentangle limits on *New Physics* from SM predictions and backgrounds

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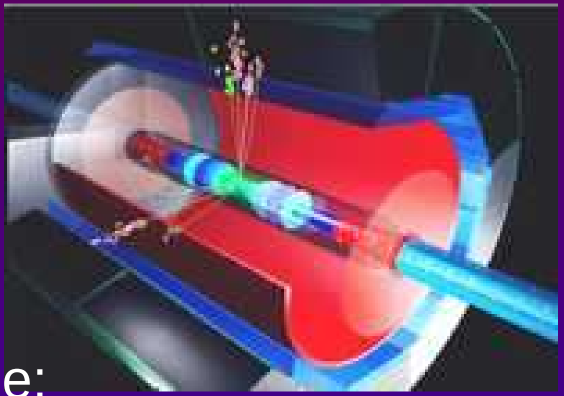

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Massive effects important for heavy (t, b) fermions $\longrightarrow$ test of Higgs

Motivation: computing

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- Documented and easy to install
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☞ Do other tools in the market satisfy these 3-4 points? ☹

Packages in the market to be mentioned:

FEYNARTS, GRACE, SANC, COMPHEP, MADGRAPH ...

Motivation: Bhabha calculations

An incomplete historical summary . . .

Who?	When?	What?
Bhabha	1935	Tree-level QED
Readhead	1953	Leading 1-loop QED
Berends, Gaemers and Gastmans	1974	Complete 1-loop QED with hard γ
Consoli	1979	1-loop Electro-weak (approx. weak boxes)
Böhm, Denner, Hollik and Sommer	1984	Complete 1-loop Electro-weak
Bardin, Hollik and Riemann	1991	Leading weak 2-loop at Z-resonance
Jadach and others	90's	Precise Monte Carlo
Bern, Dixon and Ghinculov	2000	2-loops QED
Fleischer et al	2002	$(1\text{-loop})^2$ massive QED
Penin, others (Bonciani *et al*, Gluza *et al*)	2005	leading 2-loops massive QED (ILC precision)

Introduction: electroweak model

Electroweak interactions (Glashow-Salam-Weinberg 60's): local gauge group $G^{\mathrm{EW}}_{\mathrm{local}} = SU(2)_L \otimes U(1)_Y$ broken to $U(1)_{\mathrm{em}}$ invariant $\longrightarrow$ QED

$$\mathcal{L} = \mathcal{L}_{\mathrm{YM}} + \mathcal{L}_{\mathrm{H}} + \mathcal{L}_F + \mathcal{L}_{\mathrm{fix}} + \mathcal{L}_{\mathrm{FP}}$$

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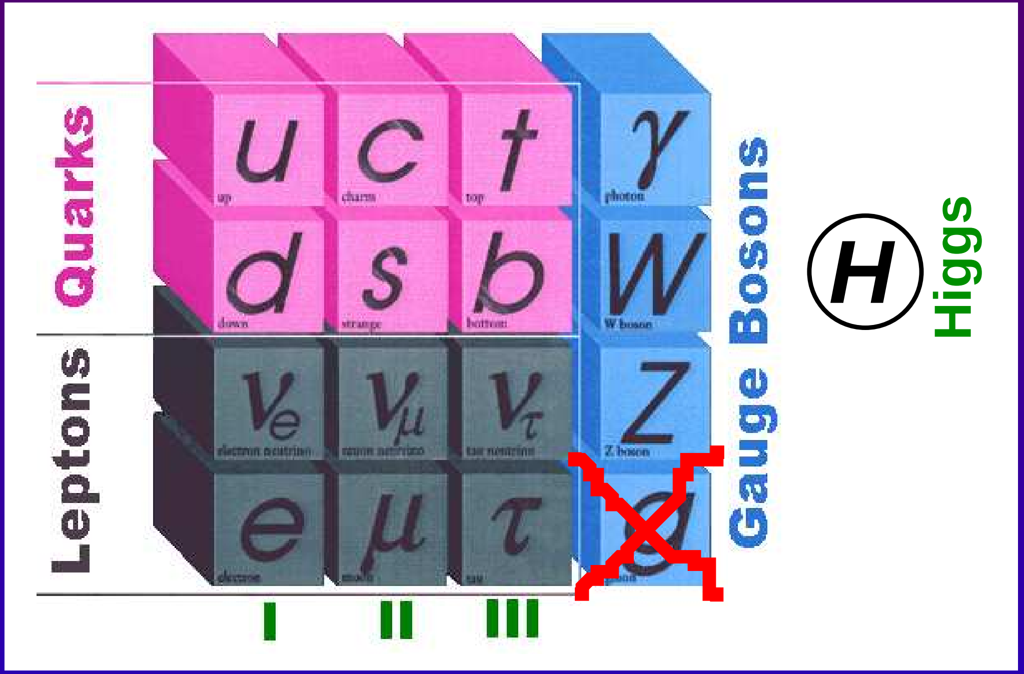
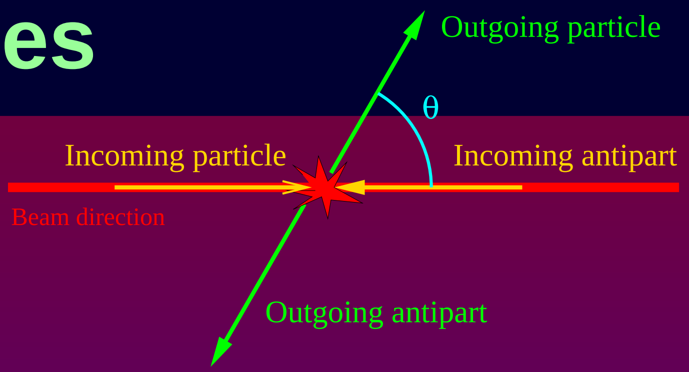
Particle content and couplings $\longrightarrow$ Feynman Rules ('t Hooft '71)



Introduction: observables



In $2 \longrightarrow 2$ fermion processes we calculate

- Differential Cross Section (pb) $[l^2]$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} = \frac{1}{32\pi} \frac{\beta_{\mathrm{out}}\beta_{\mathrm{in}}^{-1}}{s} \sum_{\mathrm{conf}} |\mathcal{M}(\cos\theta)|^2$$

- Total Cross Section:

$$\sigma_{\mathrm{tot}} = \int_{-1}^{1} \mathrm{d}\cos\theta \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}$$

- Forward–Backward Asymmetry:

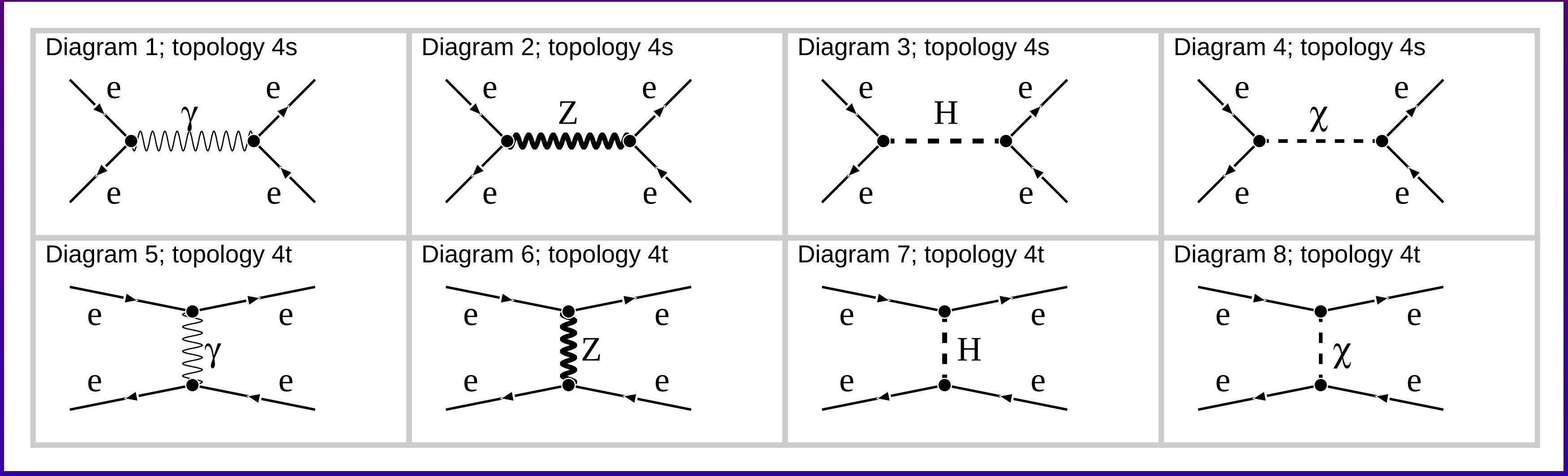
$$A_{FB} = \frac{\sigma_{\mathrm{F}} - \sigma_{\mathrm{B}}}{\sigma_{\mathrm{tot}}} = \frac{\left[\int_0^1 - \int_{-1}^0\right] \mathrm{d}\cos\theta \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}}{\sigma_{\mathrm{tot}}}$$

Introduction: perturbation theory

Thanks to the perturbative approach (Dyson, Feynman 50's), we represent the different contributions through Feynman diagrams.

Example: massive Bhabha scattering: $\boxed{e^- e^+ \longrightarrow e^- e^+}$

Vector Scalar



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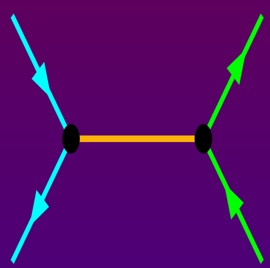

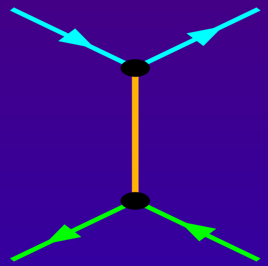

$$
\begin{aligned}
\mathcal{M}_S^{(0)} = {} & \bar{v}_e \; ie\gamma^\mu Q_e \; u_e \; \frac{-ig_{\mu\nu}}{s} \; \bar{u}_e \; ie\gamma^\nu Q_e \; v_e \\
& + \bar{v}_e ie\gamma^\mu \left(V_e - A_e\gamma_5\right) u_e \; \frac{-ig_{\mu\nu}}{s-m_Z^2} \; \bar{u}_e i\varepsilon\gamma^\nu \left(V_e - A_e\gamma_5\right) v_e \\
& + \bar{v}_e \; -i\frac{1}{2s_W}\frac{m_e}{m_W} \; u_e \; \frac{i}{s-m_H^2} \; \bar{u}_e \; -i\frac{1}{2s_W}\frac{m_e}{m_W} \; v_e \\
& + \bar{v}_e \; -i\frac{1}{2s_W}\frac{m_e}{m_W}\gamma_5 \; u_e \; \frac{i}{s-m_Z^2} \; \bar{u}_e \; -i\frac{1}{2s_W}\frac{m_e}{m_W}\gamma_5 \; v_e \\
\mathcal{M}_T^{(0)} = {} & \bar{u}_e \; ie\gamma^\mu Q_e \; u_e \; \frac{-ig_{\mu\nu}}{t} \; \bar{v}_e \; ie\gamma^\nu Q_e \; v_e \\
& + \bar{u}_e ie\gamma^\mu \left(V_e - A_e\gamma_5\right) u_e \; \frac{-ig_{\mu\nu}}{t-m_Z^2} \; \bar{v}_e ie\gamma^\nu \left(V_e - A_e\gamma_5\right) v_e \\
& + \bar{u}_e \; -i\frac{1}{2s_W}\frac{m_e}{m_W} \; u_e \; \frac{i}{t-m_H^2} \; \bar{v}_e \; -i\frac{1}{2s_W}\frac{m_e}{m_W} \; v_e \\
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\end{aligned}
$$

Coupling   Propag.   Coupling

Introduction: one-loop decomposition

Going to next perturbation order . . .

$$
\begin{aligned}
|\mathcal{M}|^2 &= |\mathcal{M}^{(0)} + \mathcal{M}^{(1)} + \ldots|^2 \\
&= \underbrace{\mathcal{M}^{(0)}\mathcal{M}^{(0)^*}}_{\mathrm{LO}\,\mathcal{O}(\alpha^2)} + \underbrace{2\Re e\left(\mathcal{M}^{(1)}\mathcal{M}^{(0)^*}\right)}_{\mathrm{NLO}\,\mathcal{O}(\alpha^3)} + \ldots
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The invariant amplitude $\mathcal{M}$ is decomposed into a sum:

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\mathcal{M} = \sum_{i}^{\substack{\text{complete}\\ \text{set}}} \mathbf{M}_i \;\left(\mathbf{F}_i^{(0)} + \mathbf{F}_i^{(1)} + \ldots \right)
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- Arbitrary basis of Matrix Elements (kinematics)
- Polarization independent, scalar Form Factors (dynamics)

Introduction: matrix elements

We use 9×4 elements for each channel (S, T or U) = 36

Example for the fermionic S-channel:

$$
\begin{array}{lcl}
\mathbf{M}_{\mathbf{S}\ 1,j} &=& \bar{v}_e \{\mathbb{1},\gamma_5\}_j u_e \otimes \bar{u}_f \{\mathbb{1},\gamma_5\}_j v_f \\
\mathbf{M}_{\mathbf{S}\ 2,j} &=& \bar{v}_e \not{p}_a \{\mathbb{1},\gamma_5\}_j u_e \otimes \bar{u}_f \{\mathbb{1},\gamma_5\}_j v_f \\
\mathbf{M}_{\mathbf{S}\ 3,j} &=& \bar{v}_e \{\mathbb{1},\gamma_5\}_j u_e \otimes \bar{u}_f \not{p}_b \{\mathbb{1},\gamma_5\}_j v_f \\
\mathbf{M}_{\mathbf{S}\ 4,j} &=& \bar{v}_e \not{p}_a \{\mathbb{1},\gamma_5\}_j u_e \otimes \bar{u}_f \not{p}_b \{\mathbb{1},\gamma_5\}_j v_f \\
\hline
\mathbf{M}_{\mathbf{S}\ 5,j} &=& \bar{v}_e \gamma^\mu \{\mathbb{1},\gamma_5\}_j u_e \otimes \bar{u}_f \gamma_\mu \{\mathbb{1},\gamma_5\}_j v_f \\
\mathbf{M}_{\mathbf{S}\ 6,j} &=& \bar{v}_e \gamma^\mu \not{p}_a \{\mathbb{1},\gamma_5\}_j u_e \otimes \bar{u}_f \gamma_\mu \{\mathbb{1},\gamma_5\}_j v_f \\
\mathbf{M}_{\mathbf{S}\ 7,j} &=& \bar{v}_e \gamma^\mu \{\mathbb{1},\gamma_5\}_j u_e \otimes \bar{u}_f \gamma_\mu \not{p}_b \{\mathbb{1},\gamma_5\}_j v_f \\
\mathbf{M}_{\mathbf{S}\ 8,j} &=& \bar{v}_e \gamma^\mu \not{p}_a \{\mathbb{1},\gamma_5\}_j u_e \otimes \bar{u}_f \gamma_\mu \not{p}_b \{\mathbb{1},\gamma_5\}_j v_f \\
\hline
\mathbf{M}_{\mathbf{S}\ 9,j} &=& \bar{v}_e \gamma^\mu \gamma^\nu \{\mathbb{1},\gamma_5\}_j u_e \otimes \bar{u}_f \gamma_\mu \gamma_\nu \{\mathbb{1},\gamma_5\}_j v_f
\end{array}
$$

Introduction: renormalization

It turns out that some loop integrals are divergent!

- Ultra-Violet: High energy limit in virtual momenta
- Infra-Red: Massless photon between external legs

How can we still make predictions?

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Redefinition of bare parameters and fields, absorbing ϵ

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☞ Only physical parameters and fields plus counter-terms appear

Introduction: photon emission

✗ Real detectors cannot observe soft photons

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$$\text{SOFT Contribution:}\quad \frac{d\sigma}{d\cos\theta}\Big|_{\mathrm{Soft}} = \mathrm{Factor}(\gamma_{\mathrm{soft}})\,\frac{d\sigma}{d\cos\theta}\Big|_{\mathrm{Born}}$$

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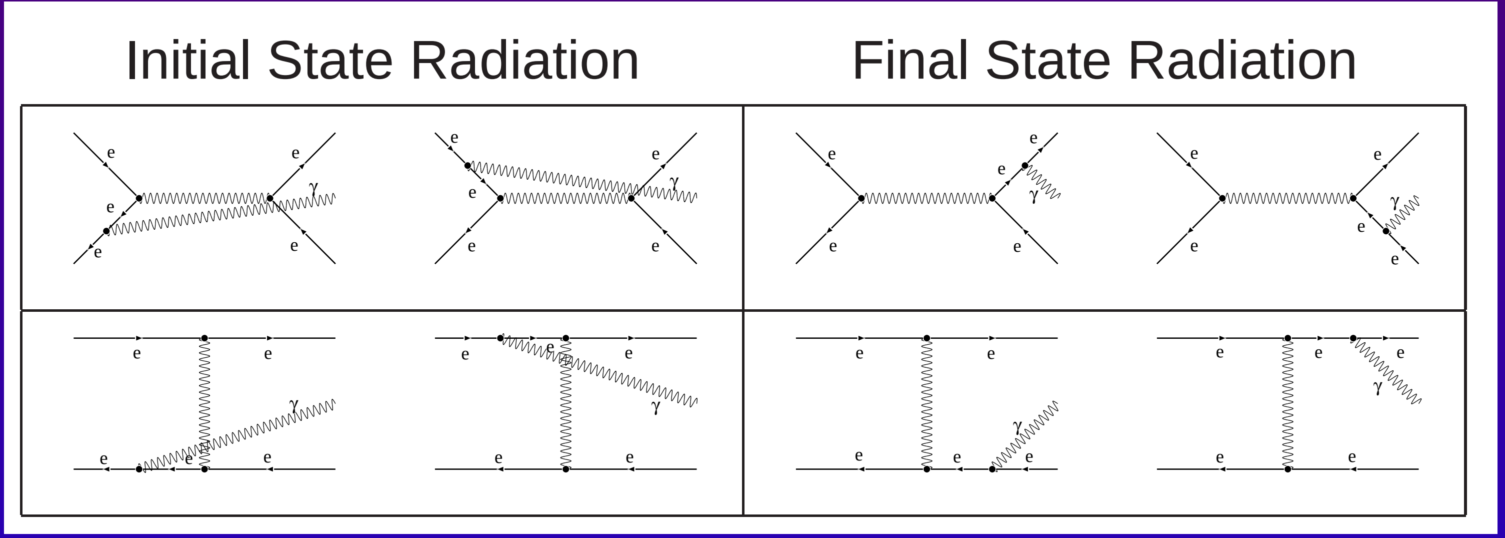

Introduction: on the Z-peak

Strict one-loop corrections in $2 \longrightarrow 2$ fermions lead to

$\boxed{\sigma(s = m_Z^2) \longrightarrow \infty}$, coming from lowest order Z-boson propagator

$$\frac{1}{s - m_Z^2}$$

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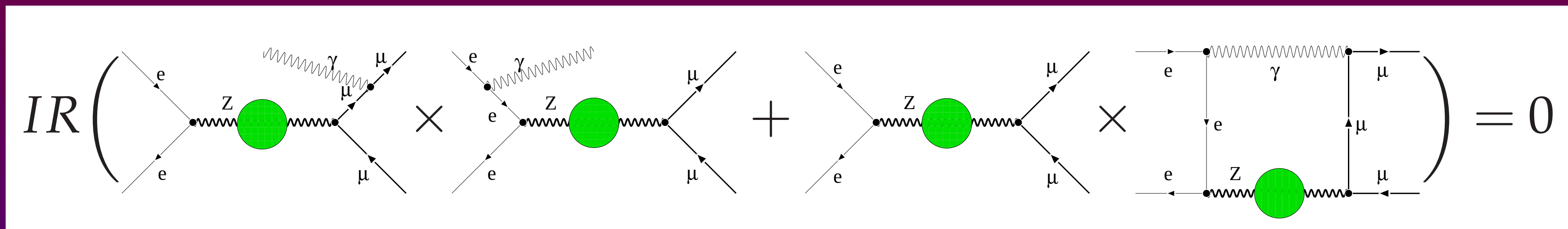
In order to have $\mathcal{O}(\alpha)$ accuracy beyond IBA we implemented *fixed width* scheme:

- Dyson summation $\frac{1}{s-m_Z^2} \longrightarrow \frac{1}{s-m_Z^2+i\Gamma_Z m_Z}$
- Discard self-energies topologies from 1-loop amplitude
- Ensure Infrared finiteness when adding soft-photon emission

Introduction: IR-finiteness on the Z-peak

The infrared cancellation reads



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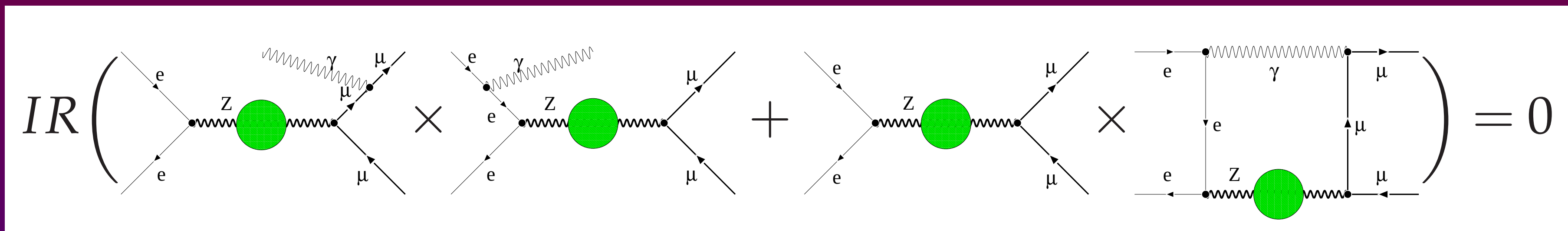

Minimally we need

- D_0 with complex mass argument
  Beenakker and Denner. Nucl. Phys. B338 (1990)
- C_0, B_0, A_0 with arbitrary complex arguments (due to the reduction to masters of projected $D_{i,ij}$)
  't Hooft and Veltman. Nucl. Phys. B153 (1979)

II. Automated tool: *a*ITALC

Automation: aITALC overview

an Integrated Tool for Automated Loop Calculations

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- Restricted to automated 2 $\longrightarrow$ 2 fermions (EWSM and QED)
- GNU/LINUX tool, GPL licensed, free available since Oct'04
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Three structural blocks:

Diagram generation	Algebra simplification	Numerical evaluation
DIANA 2.35 (QGRAF)	FORM 3.1	FORTRAN 77 LOOPTOOLS (FF)

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Three structural blocks: all running under MAKE environment

Diagram generation	$\longrightarrow$	Algebra simplification	$\longrightarrow$	Numerical evaluation
DIANA 2.35 (QGRAF)		FORM 3.1		FORTRAN 77 LOOPTOOLS (FF)

Automation: Feynman Diagram Analizer DIANA

Developed at U.Bielefeld 1997-2004 (Fleischer and Tentyukov)

Automation: Feynman Diagram Analizer DIAN

Developed at U.Bielefeld 1997-2004 (Fleischer and Tentyukov)

- C program, based on Nogueira's FORTRAN generator QGRAF2
- Command line: requires a driver file and model file
- High portability, running in many UNIX systems
- Front-end topology editor (tedi) included for GNU/LINUX

http://www.physik.uni-bielefeld.de/~tentukov/diana.html

Automation: Feynman Diagram Analizer DIAN

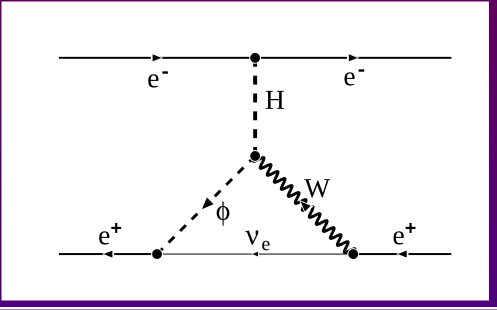
Developed at U.Bielefeld 1997-2004 (Fleischer and Tentyukov)

What do we ask?

```
SET _processname = Bhabha
\Begin(model,EWSM.model)
\Begin(process)
ingoing le(;p1),Le(;p4);
outgoing le(;-p2),Le(;-p3);
loops = 1;
options = onshell,notadp;
*\excludevertex(Le,le,H)
SET MakeEps = "!"
...
```

What does Diana answer?

Bhabha626.eps




```
G Amplitude =
(-1)*F(1,1,1,0,0)*(-i_)*e/2/sw*Mle/MW*F(2,2,1,-1,0)*
(-i_)*e/2/sqrt2/sw*Mle/MW*FF(3,2,+q,Mne)*i_*
F(3,2,mu1,1,-1,1)*(+i_)*e/2/sqrt2/sw*SS(4,0)*i_*
SS(1,2)*i_*VV(2,mu2,mu1,-q-k2,2)*i_*
V(4,mu2,+p1+p2-(+q+k1),1)*(-i_)*e/2/sw;
#define COUNTER "626" #define LINE "4"
#define LOOPTYPE "c" ...
```

Automation: aITALC algebra

DIANA
(symbolic level)

¿¿¿ 1−Loop Library ???

FORTRAN
(numeric level)

Written in FORM

```
#call feynmanrules()
...
#call tracefermiloops()
#call integration()
#call chisholm()
#call dimensionfour()
#call gammaalgebra()
#call onshell()
#call diracequation()
#call massiveformfactors()
.end
```

These general procedures perform all algebra simplifications

✓ Write automatically FORTRAN subroutines from DIANA output

Automation: numerics with aITALC

For numerical evaluation language FORTRAN 77 is used

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The code is decomposed into different routines

- **Local**: Process-dependent automatically generated (`me, ff`)
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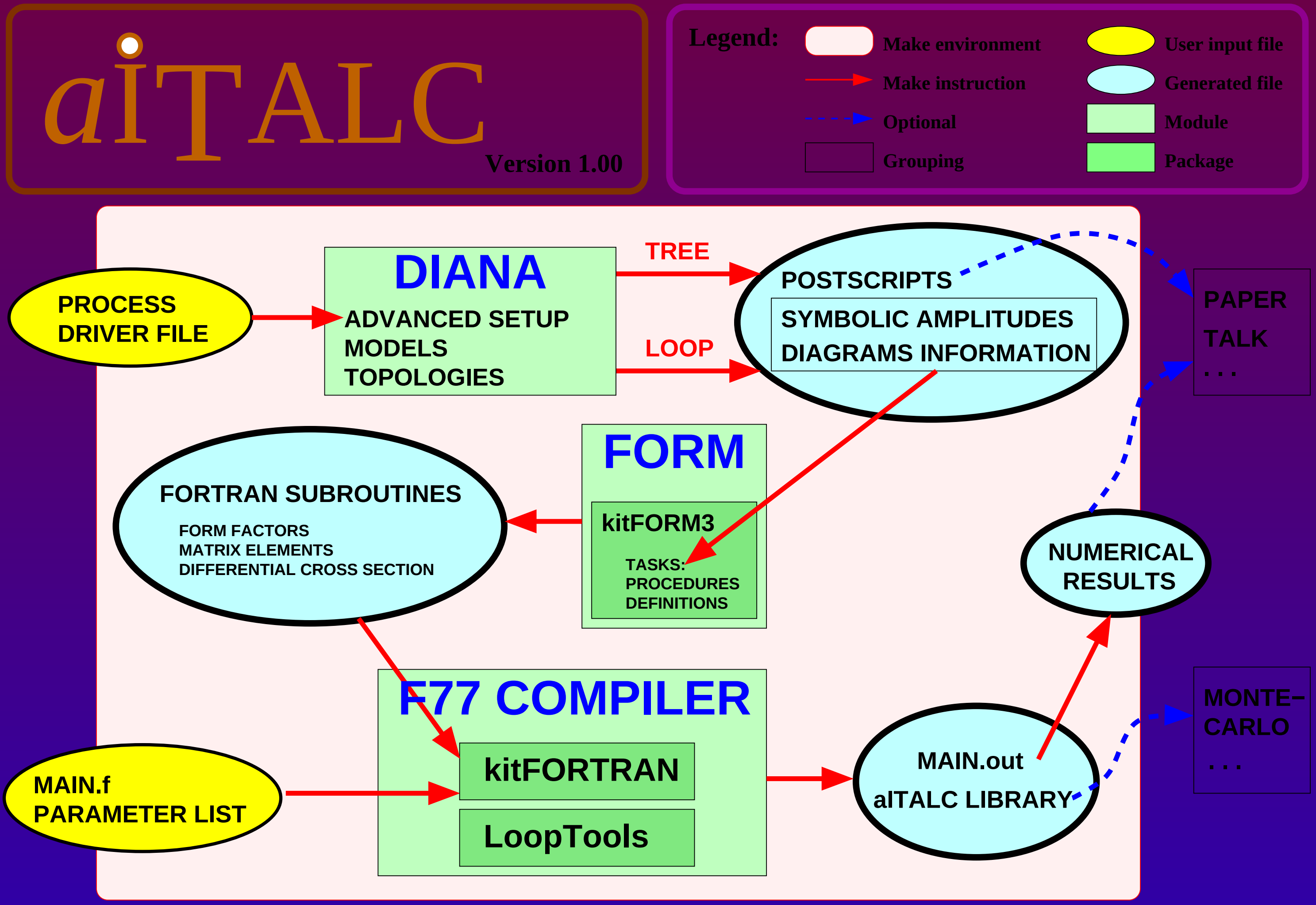
Executable file $\boxed{\texttt{main.out}}$

Input $\longrightarrow$ parameter list, control flags.

Output $\longleftarrow$ tables for differential and integrated cross sections and forward-backward asymmetries

Tests ✔ ultraviolet and infrared finiteness against parameter variation. Quadruple precision

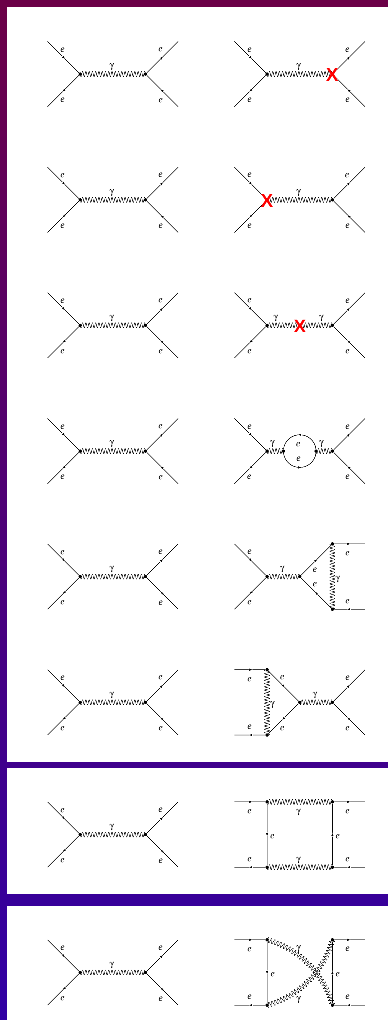
Automation: aITALC Flowchart



III. Results

Results: Bhabha analytical

aITALC helped in getting analytical results: massless QED



$$e^4\,\frac{t^2+u^2}{s^2}\left(2\delta Z_e+\delta Z_{AA}+2\delta Z_e^{f,V}\right),$$

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$$e^4\,\frac{t^2+u^2}{s^2}\left(-2\delta Z_{AA}\right),$$

$$\frac{e^6}{\pi^2}\,\frac{t^2+u^2}{s^2}\left(+\frac{1}{18}-\frac{1}{6}B_s^e\right),$$

$$\frac{e^6}{\pi^2}\,\frac{t^2+u^2}{s^2}\left(-\frac{1}{4}+\frac{1}{2}B_e-\frac{3}{8}B_s^e-\frac{1}{4}\tilde{C}_s^e\right),$$

$$\frac{e^6}{\pi^2}\,\frac{t^2+u^2}{s^2}\left(-\frac{1}{4}+\frac{1}{2}B_e-\frac{3}{8}B_s^e-\frac{1}{4}\tilde{C}_s^e\right),$$

$$\frac{e^6}{\pi^2}\left(-\frac{1}{4}\,\frac{u}{s}\left(B_s^\gamma-B_t^e\right)-\frac{1}{4}\,\frac{t-u}{s}\tilde{C}_t^e+\frac{1}{4}\,\frac{3t^2+u^2}{s^2}\tilde{C}_s^\gamma-\frac{1}{8}\,\frac{s^2+u^2}{st}\tilde{D}_{ts}\right),$$

$$\frac{e^6}{\pi^2}\left(\frac{1}{4}\,\frac{t}{s}\left(B_s^\gamma-B_u^e\right)-\frac{1}{4}\,\frac{t-u}{s}\tilde{C}_u^\gamma-\frac{1}{8}\,\frac{t^2+3u^2}{s^2}\left(2\tilde{C}_s^\gamma-\tilde{D}_{su}\right)\right),$$

Results: Bhabha comparisons

$e^-e^+ \to e^-e^+\ (\gamma)$ at ILC: $\sqrt{s} = 500\ \mathrm{GeV}, \quad E_{\max}(\gamma_{\mathrm{soft}}) = \frac{\sqrt{s}}{10}$

$\cos\theta$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\mathrm{Born}}$ (pb)	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\mathcal{O}(\alpha^3)=\mathrm{Born+QED+weak+soft}}$ (pb)	Tool
-0.9	0.21699 88288 10920	0.19344 50785 26862 70315 89 …	aITALC
-0.9	0.21699 88288 10920	0.19344 50785 26862	FeynArts
-0.9	0.21699 88288 41513	0.19344 50785 62638	$m_e = 0$
$+0.0$	0.59814 23072 50331	0.54667 71794 69423 03528 77 …	aITALC
$+0.0$	0.59814 23072 50329	0.54667 71794 69422	FeynArts
$+0.0$	0.59814 23072 88584	0.54667 71794 99961	$m_e = 0$
$+0.9$	$0.18916\ 03223\ 32271 \cdot 10^3$	$0.17292\ 83490\ 66508\ 29307\ 47 \ldots \cdot 10^3$	aITALC
$+0.9$	$0.18916\ 03223\ 32271 \cdot 10^3$	$0.17292\ 83490\ 66508 \cdot 10^3$	FeynArts
$+0.9$	$0.18916\ 03223\ 31849 \cdot 10^3$	$0.17292\ 83490\ 61347 \cdot 10^3$	$m_e = 0$
$+0.9999$	$0.20842\ 90676\ 46391 \cdot 10^9$	$0.19140\ 17861\ 11883\ 04292\ 09 \ldots \cdot 10^9$	aITALC
$+0.9999$	$0.20842\ 90676\ 46436 \cdot 10^9$	$0.19140\ 17861\ 11979 \cdot 10^9$	FeynArts

Great independent agreement saturating limit in double precision

Thanks to T. Hahn for supplying FeynArts' numbers

Results: Bhabha within different models

Forward region, SABH (Small Angle BHabha scattering)

$e^- e^+ \longrightarrow e^- e^+ \ (\gamma)$ at ILC: $\sqrt{s} = 500$ GeV, $\quad E_{\max}(\gamma_{\text{soft}}) = \frac{\sqrt{s}}{10}$

rad	$\cos\theta$	Born EWSM	Born QED	$\mathcal{O}(\alpha)$ EWSM	$\mathcal{O}(\alpha)$ QED $N_f{=}9$	$\mathcal{O}(\alpha)$ QED $N_f{=}1$
0.451	$+0.9000$	$1.89160 \cdot 10^2$	-0.0222%	-8.58%	-6.71%	-15.60%
0.142	$+0.9900$	$2.06556 \cdot 10^4$	-0.0840%	-7.72%	-7.16%	-14.06%
0.045	$+0.9990$	$2.08236 \cdot 10^6$	0.0031%	-7.98%	-7.53%	-12.50%
0.014	$+0.9999$	$2.08429 \cdot 10^8$	0.0005%	-8.17%	-7.75%	-11.02%

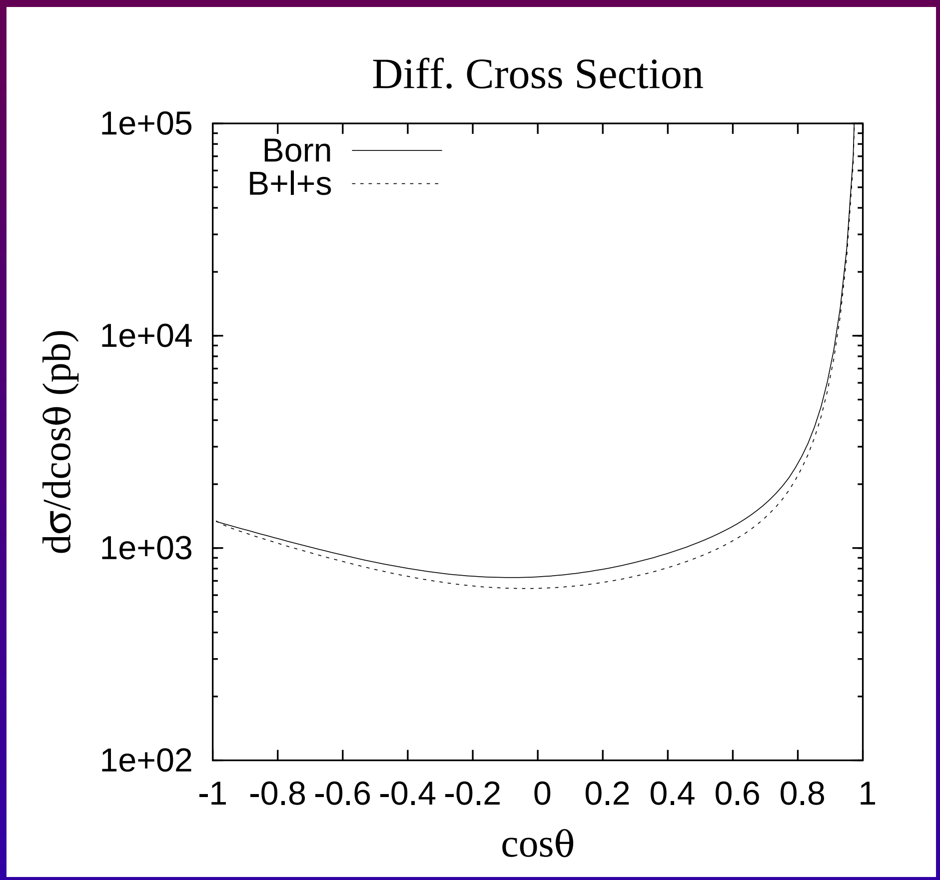
Precision required at ILC for SABH is achieved incorporating

fully flavour QED two-loops corrections to the one-loop EWSM ones.

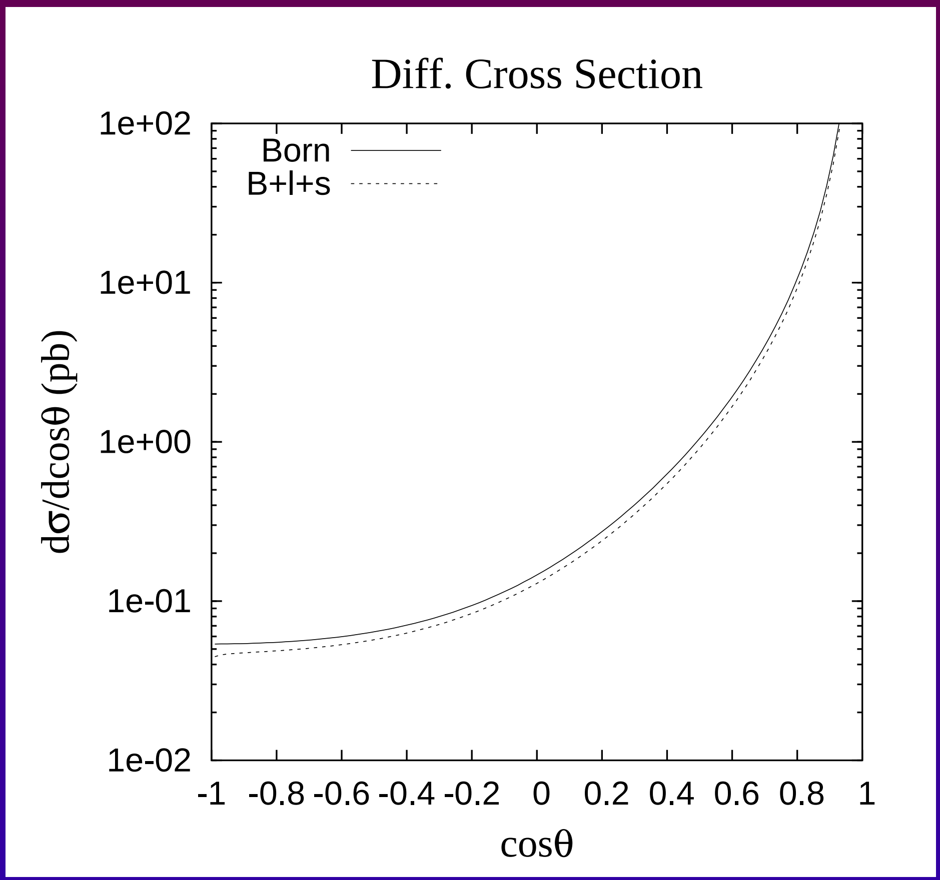
Results: some plots

Differential cross sections

(a) $\sqrt{s} = m_Z$



(b) $\sqrt{s} = 1000$ GeV



Conclusions & Outlook

- Complete $\mathcal{O}(\alpha)$ electroweak corrections to Bhabha scattering (also other different $2 \to 2$ fermion processes not shown)
  - Resonances and masses included
  - Other contributions still required (hard γ, QCD, kin. cuts...)

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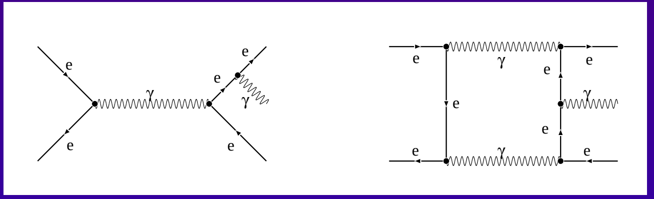
Radiative Bhabha scattering:



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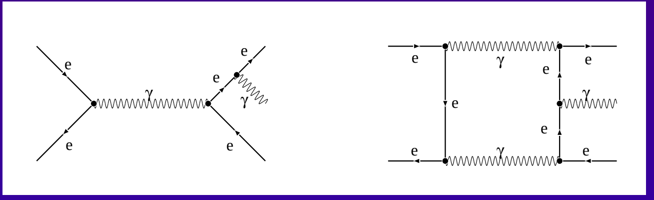
  Radiative Bhabha scattering: 

? Should we join into a project in loop calculations for colliders?