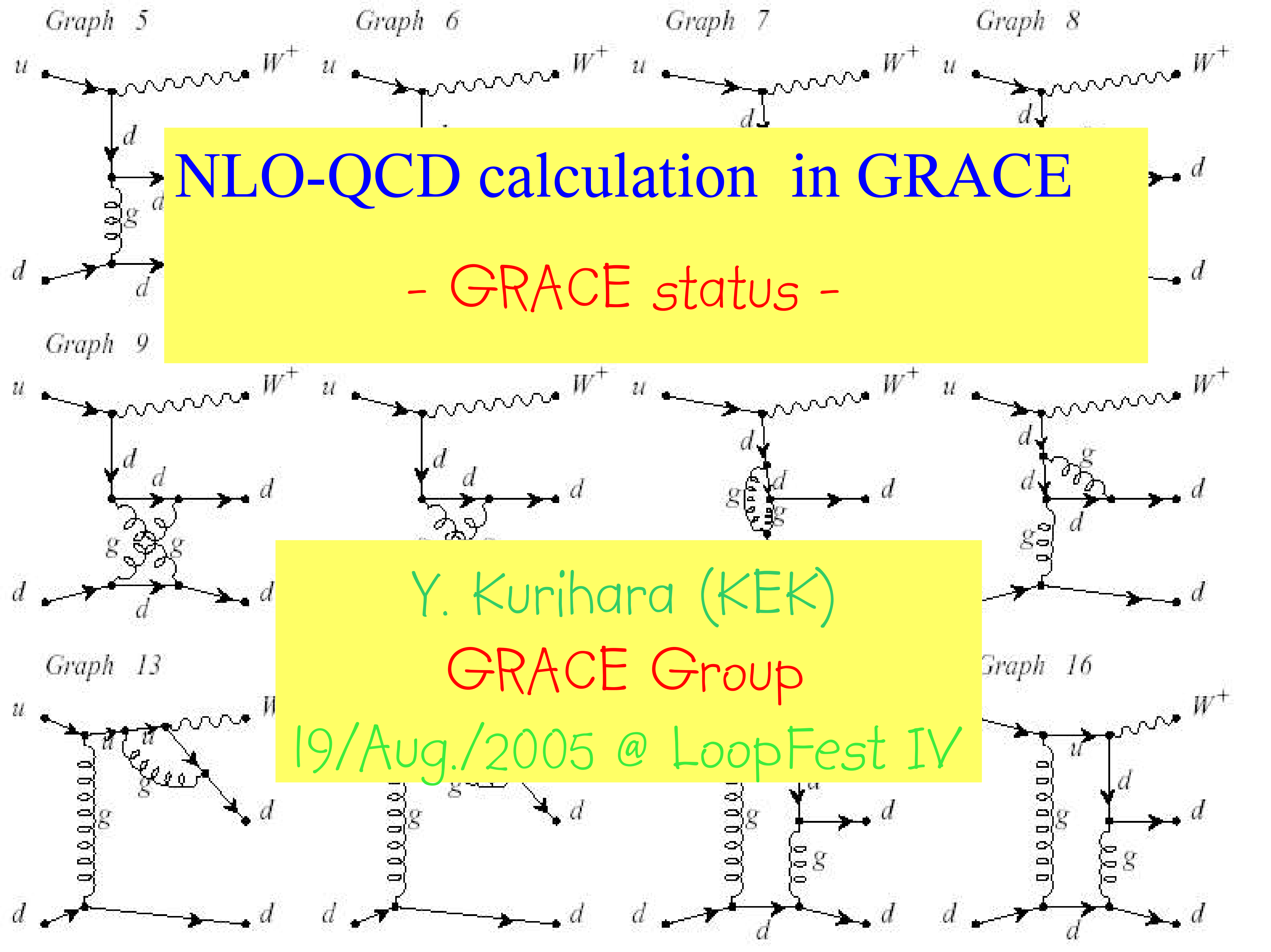


NLO-QCD calculation in GRACE

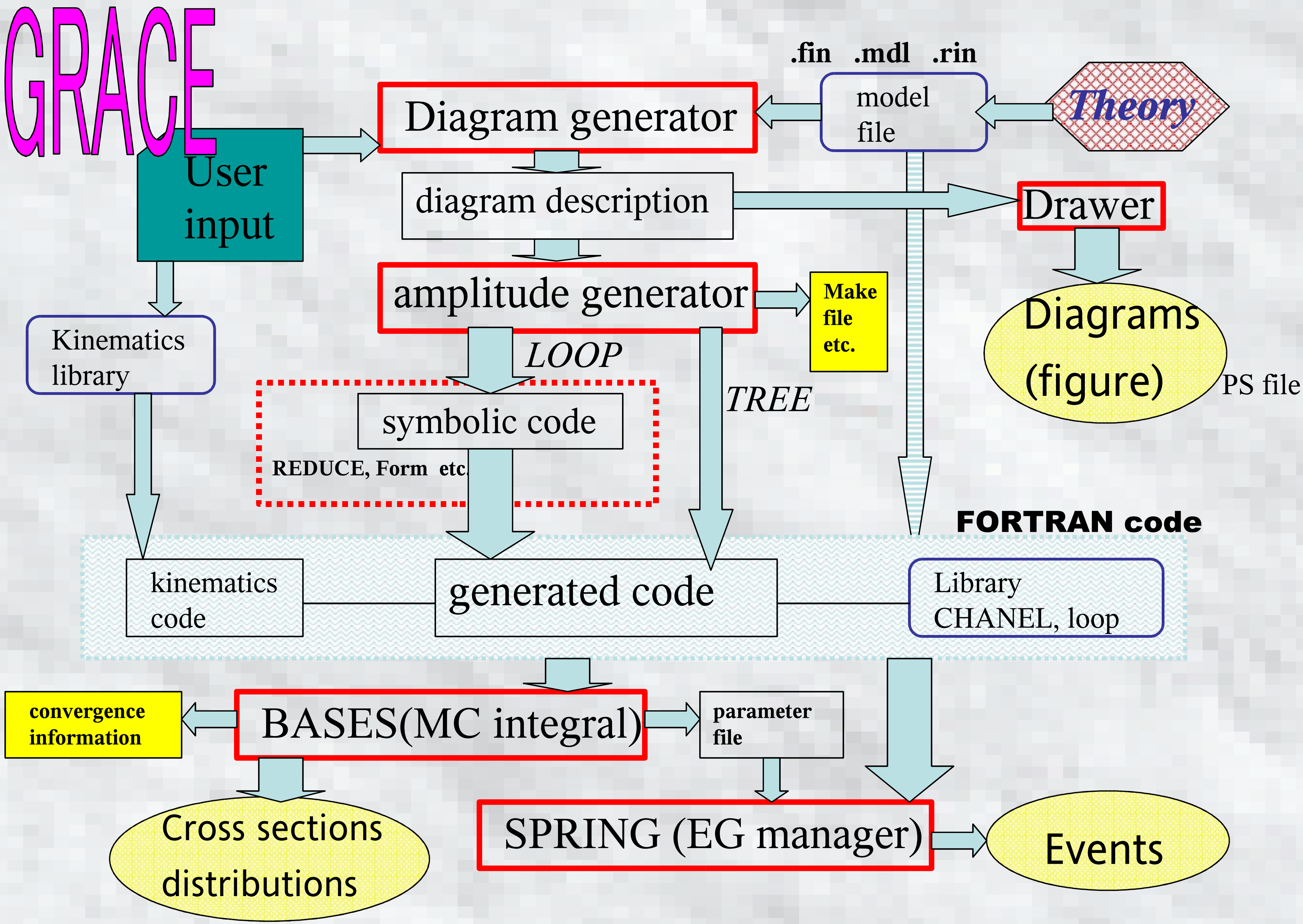
- GRACE status -

Y. Kurihara (KEK)
GRACE Group

19/Aug./2005 @ LoopFest IV

GRACE Author list

J. Fujimoto, T. Ishikawa,
M. Jimbo, T. Kaneko,
K. Kato, S. Kawabata,
T. Kon, Y. Kurihara,
M. Kuroda, N. Nakazawa,
Y. Shimizu, H. Tanaka,
Y. Yuasa


GRACE
.fin .mdl .rin
Theory
model file
Diagram generator
User input
diagram description
Drawer
amplitude generator
Make file etc.
Diagrams (figure)
PS file
Kinematics library
LOOP
TREE
symbolic code
REDUCE, Form etc.
FORTRAN code
kinematics code
generated code
Library CHANEL, loop
convergence information
BASES(MC integral)
parameter file
Cross sections distributions
SPRING (EG manager)
Events

Physics in LHC

- LHC Experimental requirement

New Particle Search/<u>Precision Measurements</u>

LO-QCD Event generator+K-factor

↓

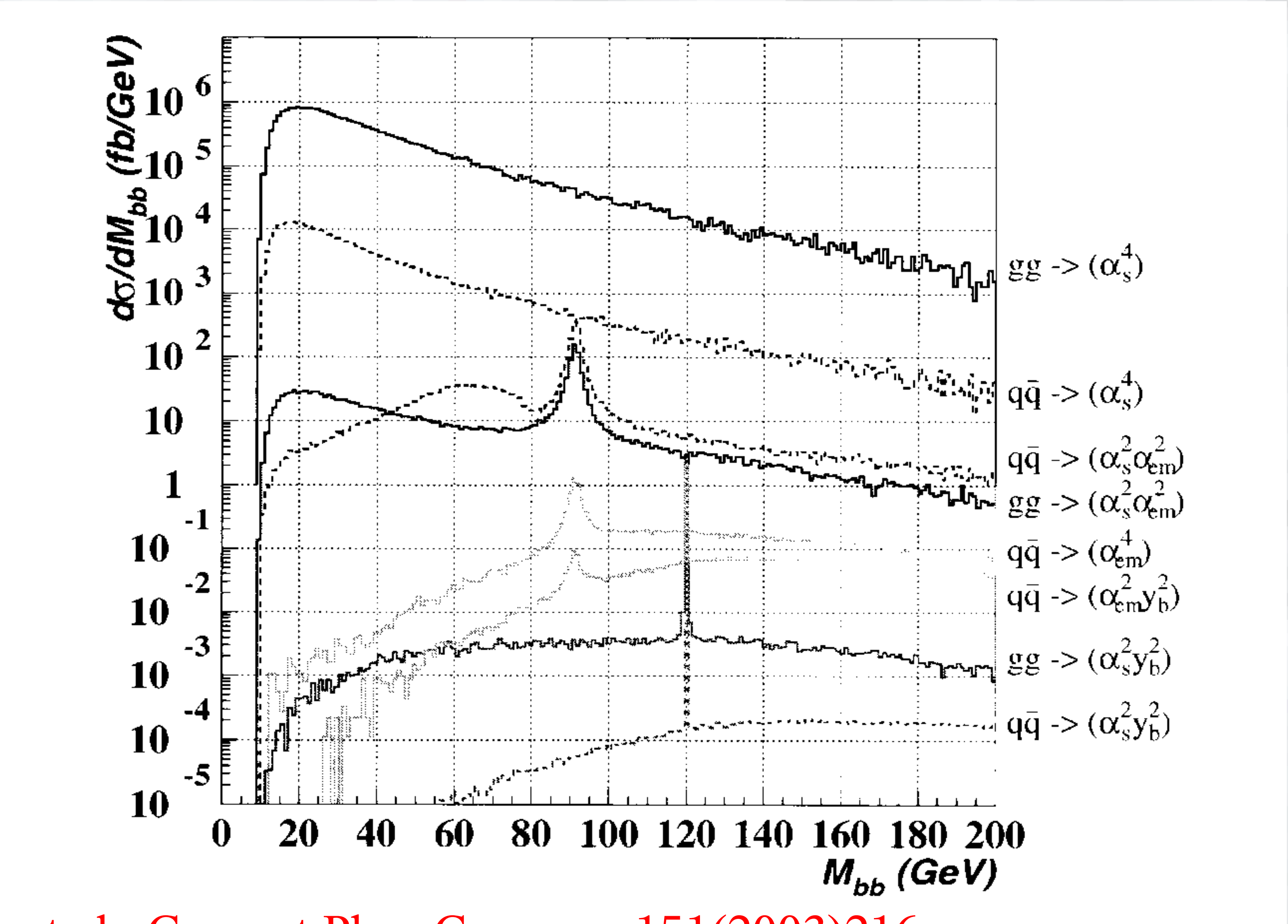
Obviously not enough!

We need

NLO Event generator!

QCD-event generator @ Tree-level

pp → bbbb, TEVATRON/LHC, GR@PPA_4b, S. Tsuno



S. Tsuno et al., Comput.Phys.Commun.151(2003)216

pp $\rightarrow$ many, TEVATRON/LHC, GR@PPA_ALL,

S. Tsuno

- W + jets (up to 4 jets) with the subsequent W decay to a fermion pair,
- Z + jets (up to 4 jets) with the subsequent Z decay to a fermion pair,
- Four bottom quarks via Z and Higgs-boson mediated processes as well as those from pure QCD interactions (same as GR@PPA_4b),
- top-quark pair with the subsequent decay to W and b, and the W decay to a fermion pair,
- di-boson (WW, WZ and ZZ) with the subsequent W/Z decay to a fermion pair.

Loop Calc. by GRACE-loop in ELWK proc. (Higgs production)

- ***Single Higgs production***
  - $e^+e^- \to$ *ZH (full number of graphs = 341)*
  - $e^+e^- \to \nu\ \nu$ *H (1,350)* *Phys.Lett. B559 (2003) 252-262*

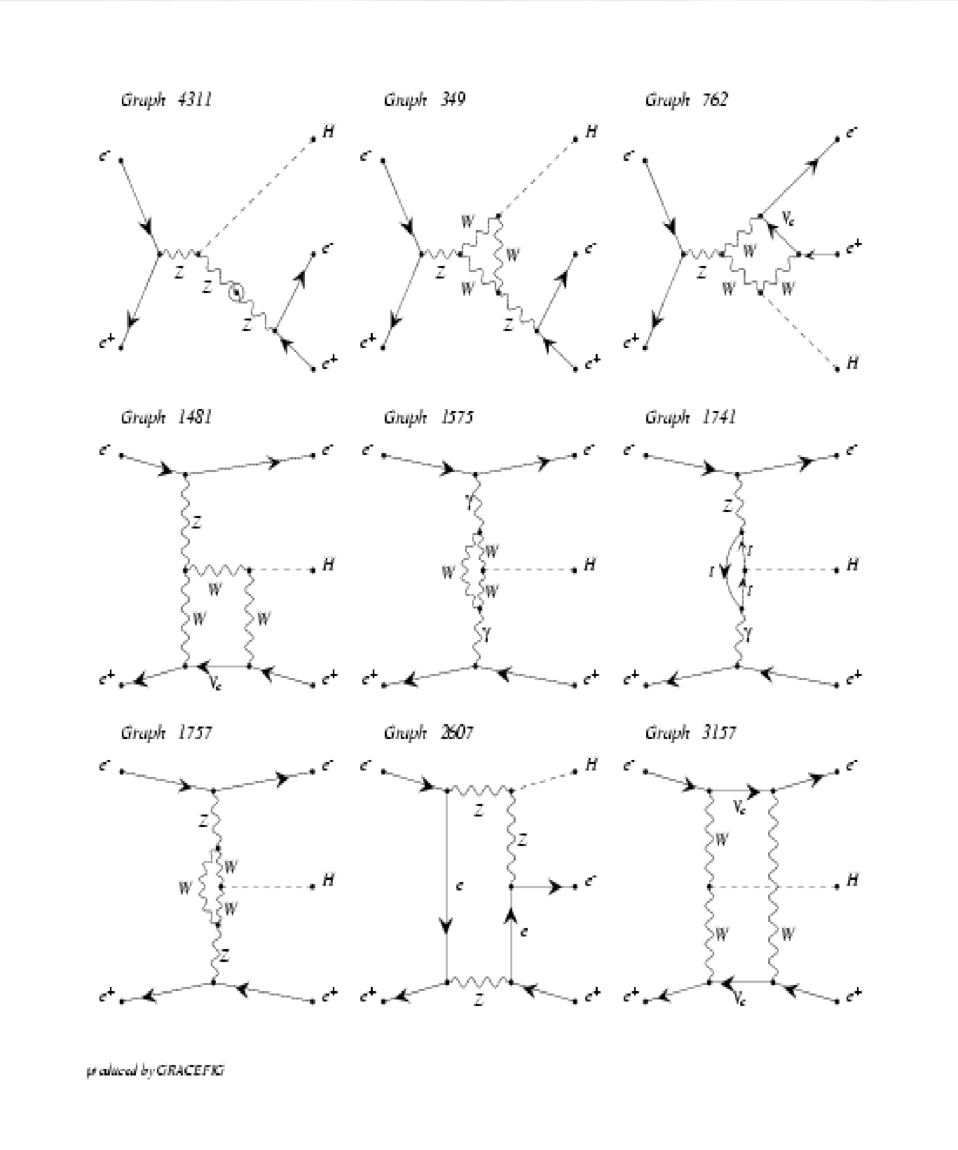
    A.Denner et.al. PLB 560(2003)196, NPB 660(2003)289
  - $e^+e^- \to e^+e^-$ *H (4,470)* *Phys.Lett.B600 (2004) 65-76*
- ***top Yukawa***
  - $e^+e^- \to t\,t$ *H (2,327)* *Phys.Lett. B571 (2003) 163-172*

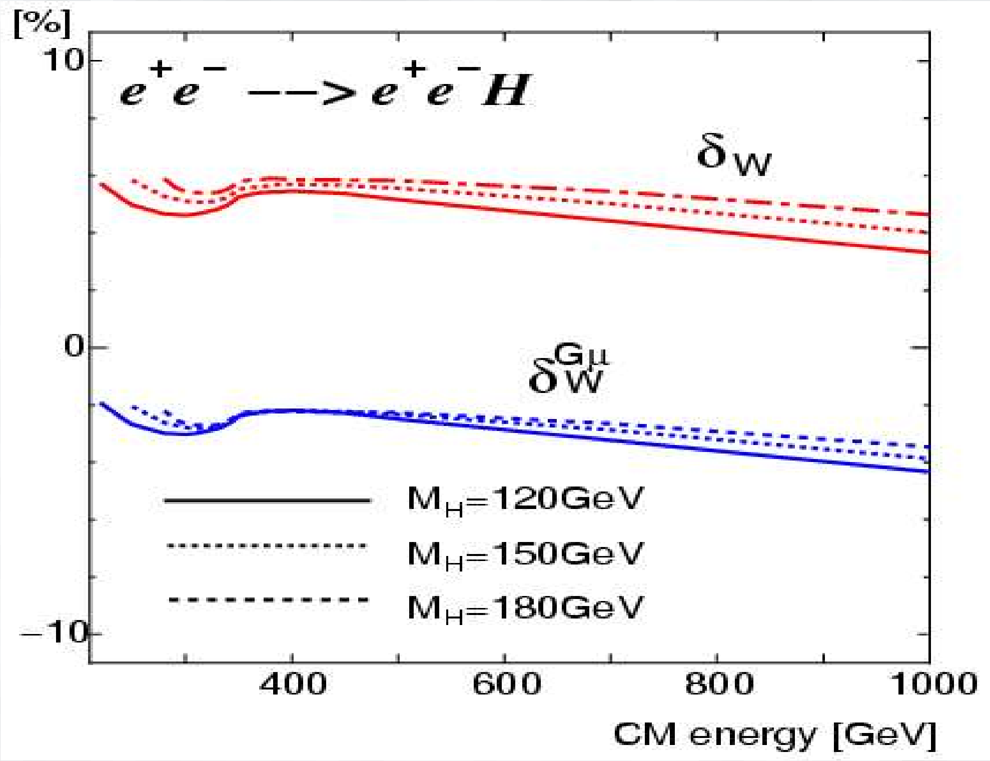
    Y.You et.al.PLB 571(2003)85

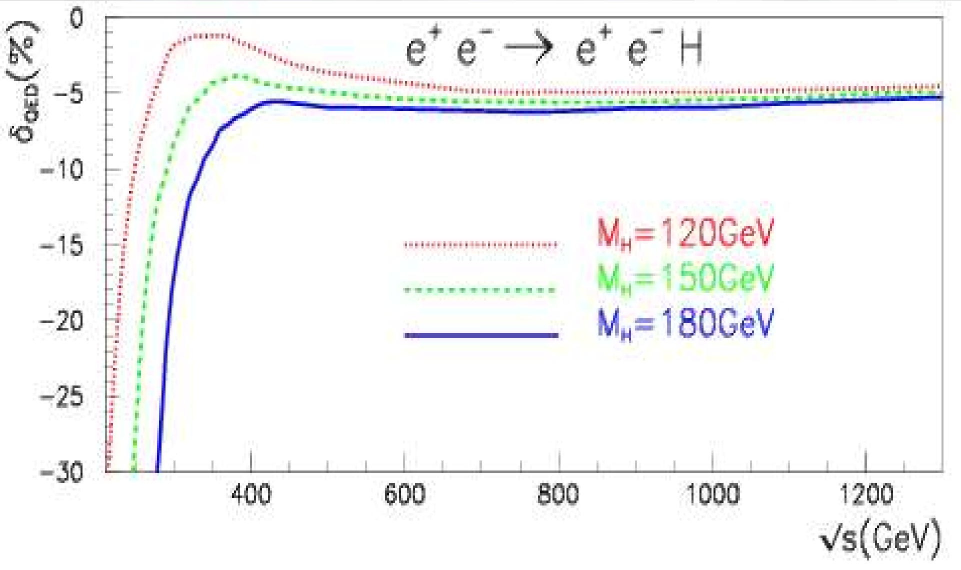
    A.Denner et.al. PLB 575(2003)290, NPB 680 (2004)85
- ***Multi Higgs production***
  - $e^+e^- \to$ *ZHH (5,417)* *Phys.Lett. B576 (2003) 152-164*

    R.Zhang et.al.PLB(2004)349
  - $e^+e^- \to \nu_e \nu_e$ ***HH (19,638)*** $\Rightarrow$ ***Preliminary***

$e^+e^- \to \quad e^+e^- H$

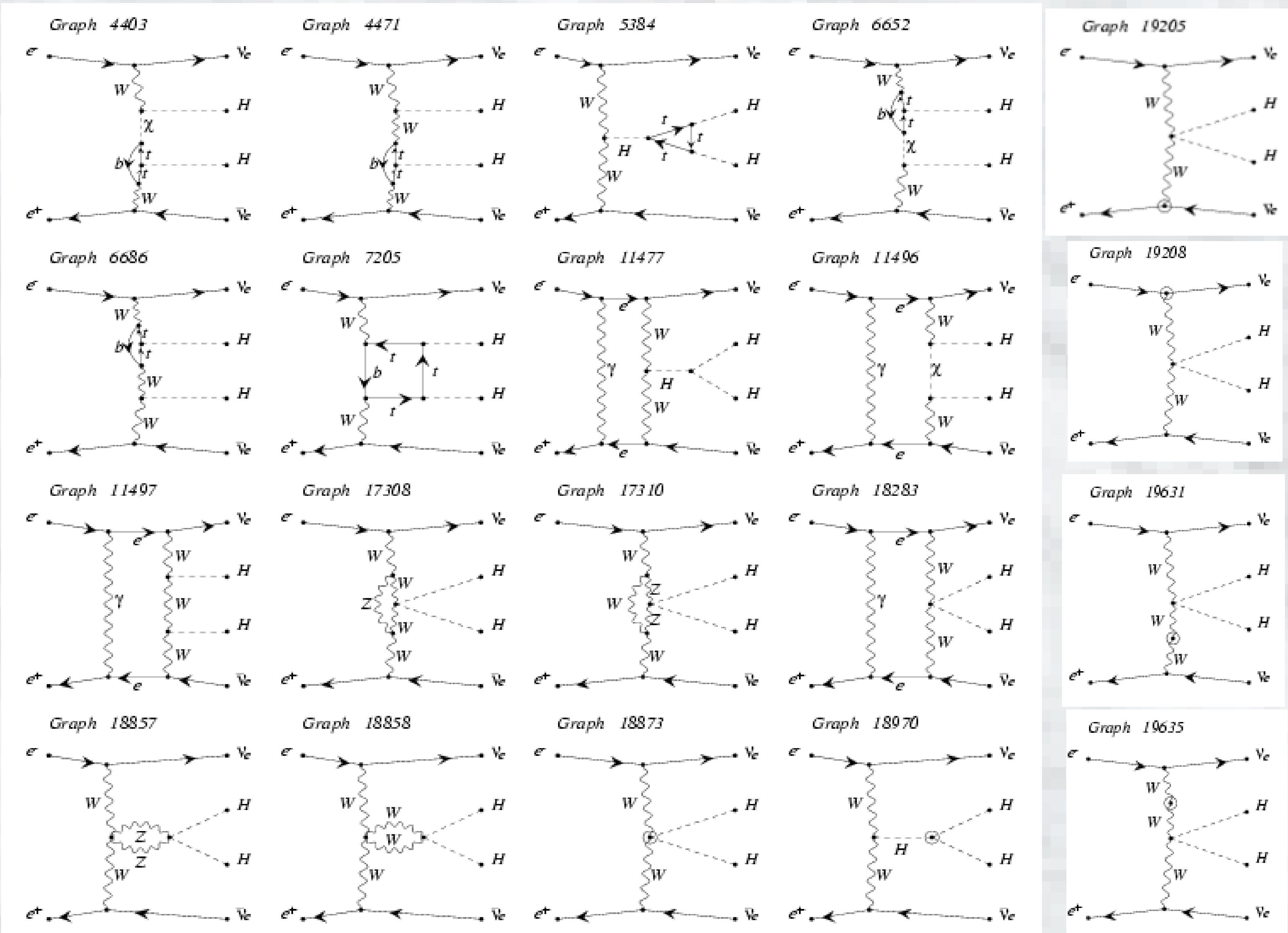






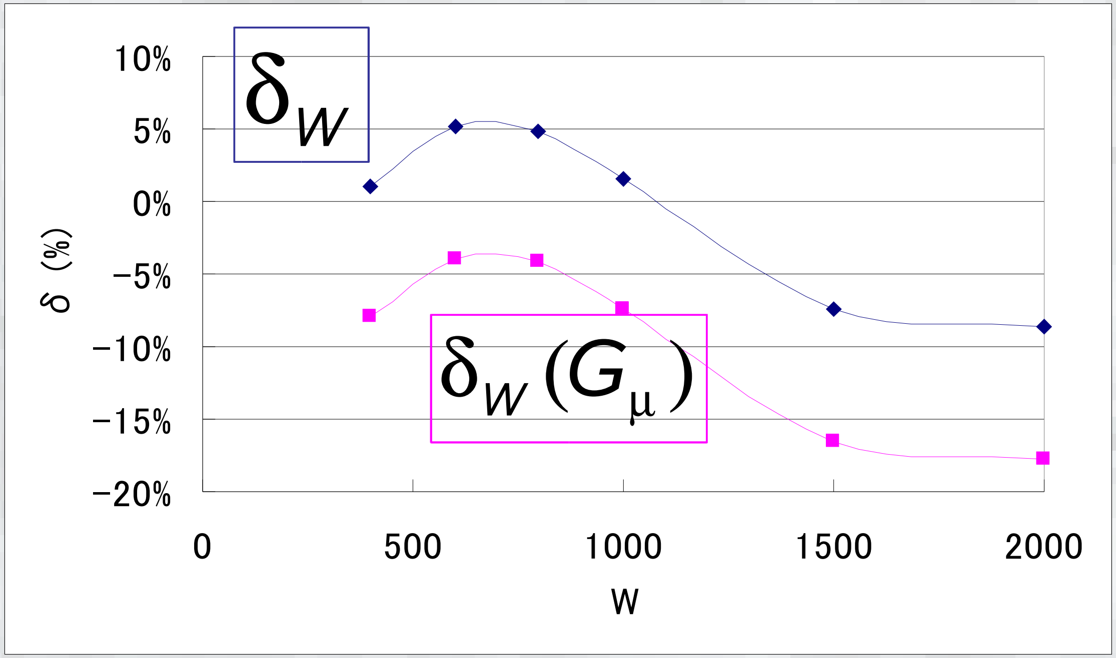
GRACE ***Phys.Lett. B559 (2003) 252-262***

$e^+e^- \rightarrow \nu_e\nu_e HH$ (final 4-body process)



$e^+e^- \to \nu_\mu \bar{\nu}_\mu HH$

$\delta = \sigma(O(\alpha))/\sigma(\text{tree})-1$



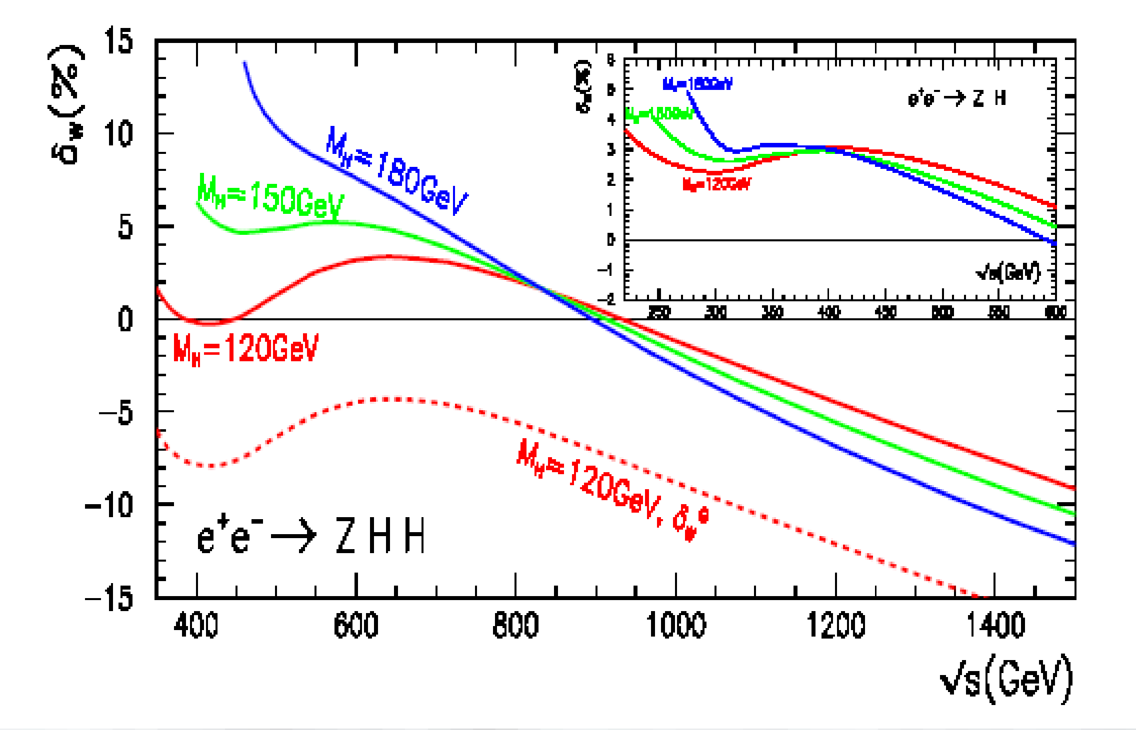
MH=120GeV

mt = 180GeV

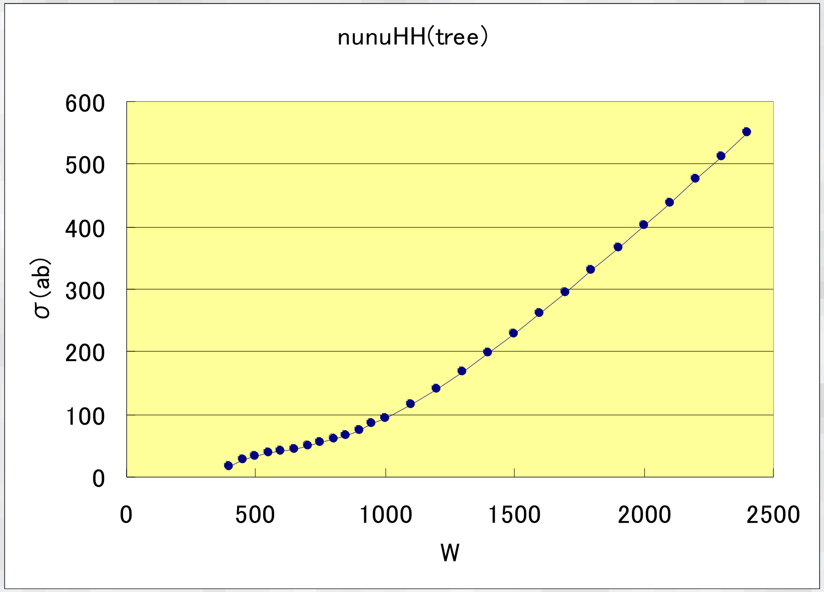
"s-channel"

$e^+e^- \to ZHH$

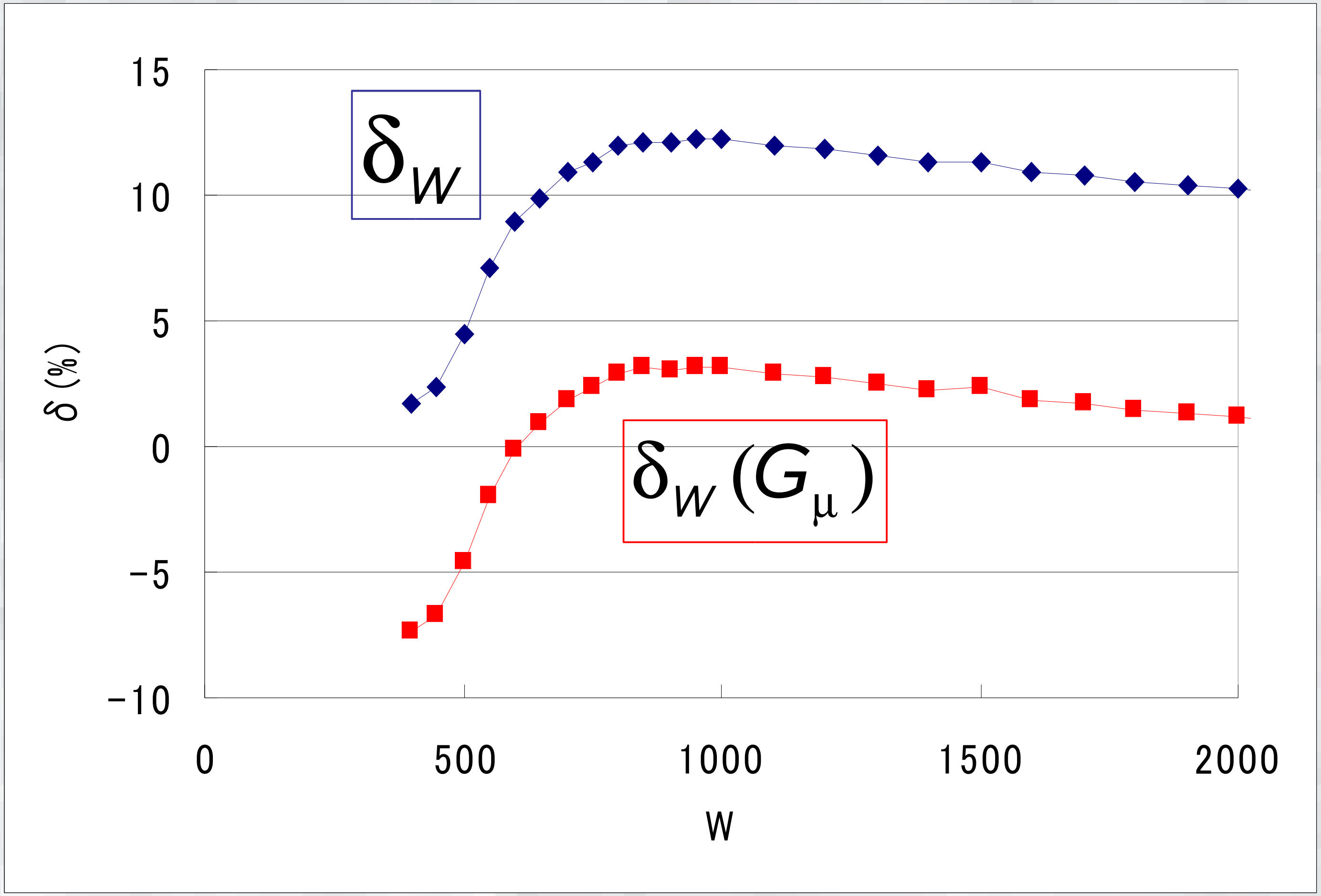
Phys.Lett.B 576(2003)152.
mt = 174GeV



$$e^+e^- \to \nu\bar{\nu}HH$$



MH=120GeV



Internal consistency check

For EW part:

System passes succesfully the usual checks:

- ultraviolet finiteness (better than 20 digits)
- infrared finitenesss (better than 20 digits)
- NLG dependence (better than 20 digits)
- k_c dependence consistent with MonteCarlo statistical error (0.02%)

1. One random phase space point
2. Full set of diagrams
3. Quadruple precision

New feature of NLO-QCD issues in GRACE

<u>QCD-tree : OK</u>
<u>ELWK 1-loop : OK</u>

- PDF/PS ↔ Real emission Double counting

→ Virtuality ordering/LL-subtraction

(YK et al.Nucl. Phys. B654 (2003) 301)

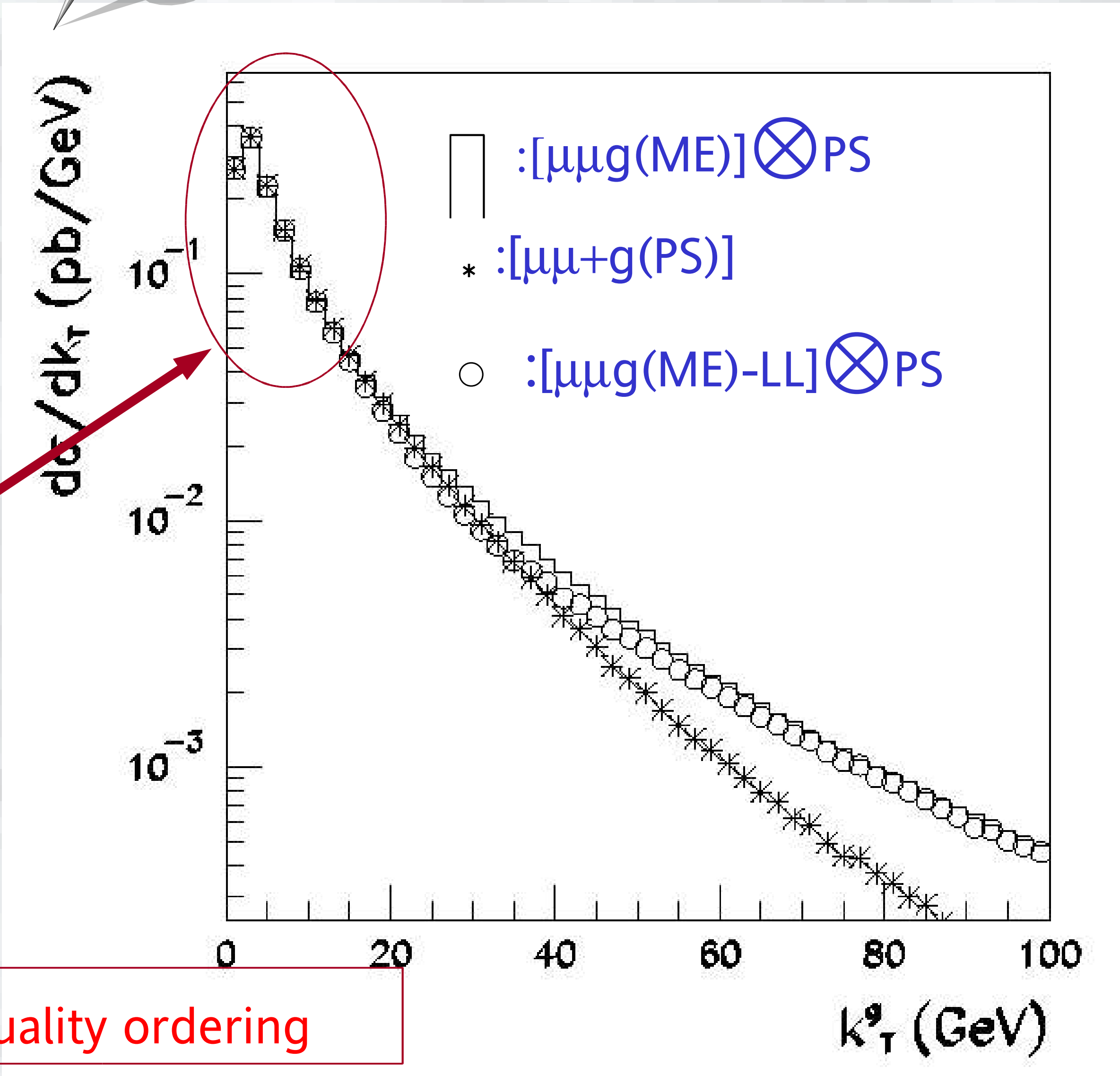
- Dimensional regularization in loop integrals for IR (fictitious mass in photon in ELWK)

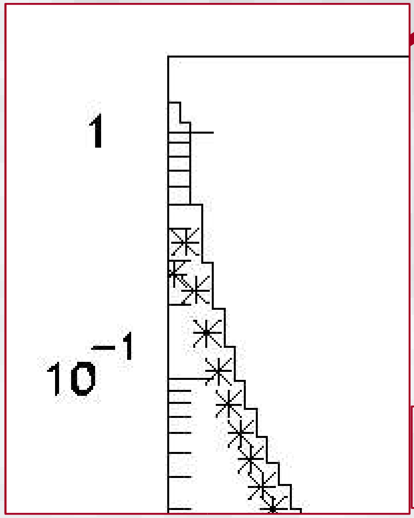
- IR (soft/collinear) approximation terms

Drell-Yan process

- Process : $u\bar{u} \to \mu^+\mu^-$(+gluon) in $p\bar{p}$ collision
- Cuts: $\sqrt{s_{\mu\mu}}$>40GeV, k_T^g> 1 GeV

k^g_T Test





w/o virtuality ordering

Box Integral

$$J_{(4)}(s,t;p_1^2,p_2^2,p_3^2,p_4^2;n_x,n_y,n_z) = \frac{\Gamma(2-\varepsilon_{IR})}{(4\pi)^2\left(4\pi\mu_R^2\right)^{\varepsilon_{IR}}}\int_0^1 dx\int_0^{1-x} dy\int_0^{1-x-y} dz\,\frac{x^{n_x}y^{n_y}z^{n_z}}{D^{2-\varepsilon_{IR}}},$$

$$\begin{aligned} D &= -s\,xz - t\,yw - p_1^2\,xy - p_2^2\,yz - p_3^2\,zw - p_4^2\,xw - i0, \\ w &= 1-x-y-z, \\ s &= (p_1+p_2)^2, \\ t &= (p_1+p_4)^2. \end{aligned}$$

YK hep-ph/0504251

All on-shell (massless) external legs

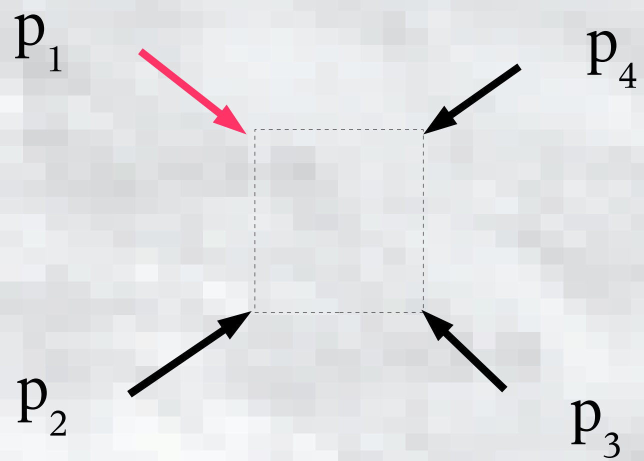
$$J_4(s,t;0,0,0,0;n_x,n_y,n_z) = \frac{1}{(4\pi)^2 s\, t} B(n_x+\varepsilon_{IR}, n_y+n_z+\varepsilon_{IR}) n_x! \Gamma(\varepsilon_{IR})\Gamma(1-\varepsilon_{IR})$$
$$\times \left[\left(\frac{-\tilde{t}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} \left(\frac{-t}{s}\right)^{n_x} \frac{B(1+n_z, n_x+n_y+\varepsilon_{IR})}{\Gamma(n_x+\varepsilon_{IR})} \right.$$
$$\times\ {}_2F_1\left(1+n_x, n_x+n_y+\varepsilon_{IR}, 1+n_x+n_y+n_z+\varepsilon_{IR}, -\frac{\tilde{u}}{\tilde{s}}\right)$$
$$+ \left(\frac{-\tilde{s}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} \sum_{l=0}^{n_x} \left(\frac{-s}{t}\right)^l \frac{(-1)^l}{\Gamma(l+\varepsilon_{IR})(n_x-l)!} B(1+n_y, l+n_z+\varepsilon_{IR})$$
$$\left. \times\ {}_2F_1\left(1+l, l+n_z+\varepsilon_{IR}, 1+l+n_y+n_z+\varepsilon_{IR}, -\frac{\tilde{u}}{\tilde{t}}\right)\right],$$

Scalar Integral

$$J_{(4)}(s,t;0,0,0,0;0,0,0) = \frac{1}{(4\pi)^2 s\, t} \frac{B(\varepsilon_{IR},\varepsilon_{IR})\Gamma(1-\varepsilon_{IR})}{\varepsilon_{IR}}$$
$$\times \left[\left(\frac{-\tilde{s}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} {}_2F_1\left(1,\varepsilon_{IR},1+\varepsilon_{IR},-\frac{\tilde{u}}{\tilde{t}}\right) + \left(\frac{-\tilde{t}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} {}_2F_1\left(1,\varepsilon_{IR},1+\varepsilon_{IR},-\frac{\tilde{u}}{\tilde{s}}\right)\right]$$

This result is compared with G. Duplančić, B. Nižić, Eur. Phys. J. C **20**, 357 (2001)

One off-shell box integral



$$J_4(s,t;p_1^2,0,0,0;n_x,n_y,n_z) =$$
$$\frac{\Gamma(2-\varepsilon_{IR})}{(4\pi)^2\left(4\pi\mu_R^2\right)^{\varepsilon_{IR}}}\int_0^1 dx\int_0^{1-x} dy\int_0^{1-x-y} dz \frac{x^{n_x}y^{n_y}z^{n_z}}{\left(-xzs-y(1-x-y-z)t-p_1^2xy-i0\right)^{2-\varepsilon_{IR}}}$$

$$\begin{aligned}
= \quad & \frac{1}{(4\pi)^2 s\ t} B(n_x+\varepsilon_{IR}, n_y+n_z+\varepsilon_{IR}) n_x! \Gamma(\varepsilon_{IR})\Gamma(1-\varepsilon_{IR}) \\
\times \quad & \left[\left(\frac{-\tilde{t}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}}\left(\frac{-t}{s}\right)^{n_x}\frac{B(1+n_z,n_x+n_y+\varepsilon_{IR})}{\Gamma(n_x+\varepsilon_{IR})}\mathcal{I}^{(1)}\right. \\
+ \quad & \left.\left(\frac{-\tilde{s}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}}\sum_{l=0}^{n_x}\frac{(-1)^l B(1+n_y,l+n_z+\varepsilon_{IR})}{\Gamma(l+\varepsilon_{IR})(n_x-l)!}\mathcal{I}_l^{(2)}\right]
\end{aligned}$$

$$\mathcal{I}^{(1)} = B(1+n_z, n_x+n_y+\varepsilon_{IR})\ {}_2F_1\left(1+n_x, n_x+n_y+\varepsilon_{IR}, 1+n_x+n_y+n_z+\varepsilon_{IR}, -\frac{\tilde{u}}{\tilde{s}}\right)$$

$$\begin{aligned}
\mathcal{I}_l^{(2)} &= \sum_{k_1=0}^{n_z} {}_{n_z}C_{k_1} \left(\frac{s}{p_1^2 - s}\right)^{n_y+k_1} \sum_{k_2=0}^{n_y+k_1} {}_{n_y+k_1}C_k\ (-1)^{n_y+k_2} \left(\frac{-t}{s}\right) \\
&\times \int_0^1 dw\ \left(1+\frac{\tilde{u}}{\tilde{s}}w\right)^{-(l+1)} \left(1+\frac{\tilde{t}+\tilde{u}}{\tilde{s}}w\right)^{k_2+l-1+\varepsilon_{IR}} \\
&= \sum_{k_1=0}^{n_z} \sum_{k_2=0}^{n_y+k_1} {}_{n_z}C_{k_1}\ {}_{n_y+k_1}C_k\ (-1)^{k_1+k_2} \left(\frac{s}{p_1^2 - s}\right)^{n_y+k_1} \frac{1}{l+k_2+\varepsilon_{IR}} \left(1+\frac{u}{t}\right)^l \\
&\times \left[\ {}_2F_1\left(1+l, l+k_2+\varepsilon_{IR}, 1+l+k_2+\varepsilon_{IR}, -\frac{\tilde{u}}{\tilde{t}}\right)\right. \\
&- \left.\left(\frac{\tilde{p_1}^2}{\tilde{s}}\right)^{l+k_2+\varepsilon_{IR}}\ {}_2F_1\left(1+l, l+k_2+\varepsilon_{IR}, 1+l+k_2+\varepsilon_{IR}, -\frac{\tilde{u}\tilde{p_1}^2}{\tilde{t}\tilde{s}}\right)\right],
\end{aligned}$$

Scalar Integral

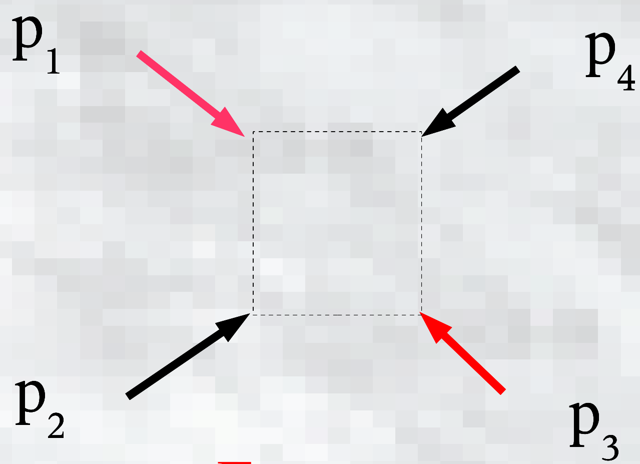
$$J_4(s,t;p_1^2,0,0,0;0,0,0) = \frac{1}{(4\pi)^2 s\, t}\,\frac{B(\varepsilon_{IR},\varepsilon_{IR})\Gamma(1-\varepsilon_{IR})}{\varepsilon_{IR}}$$

$$\times \left[\left(\frac{-\tilde{s}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} {}_2F_1\left(1,\varepsilon_{IR},1+\varepsilon_{IR},-\frac{\tilde{u}}{\tilde{t}}\right) + \left(\frac{-\tilde{t}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} {}_2F_1\left(1,\varepsilon_{IR},1+\varepsilon_{IR},-\frac{\tilde{u}}{\tilde{s}}\right)\right.$$

$$\left. - \left(\frac{-\tilde{p_1}^2}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} {}_2F_1\left(1,\varepsilon_{IR},1+\varepsilon_{IR},-\frac{\tilde{u}\tilde{p_1}^2}{\tilde{t}\tilde{s}}\right)\right],$$

This result is compared with G. Duplančić, B. Nižić, Eur. Phys. J. C **20**, 357 (2001)

Two off-shell box integral



Easy case

$$J_4(s,t;p_1^2,0,p_3^2,0;n_x,n_y,n_z) = \frac{\Gamma(2-\varepsilon_{IR})}{(4\pi)^2\left(4\pi\mu_R^2\right)^{\varepsilon_{IR}}}$$

$$\int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{x^{n_x} y^{n_y} z^{n_z}}{\left(-xzs - y(1-x-y-z)t - p_1^2 xy - p_3^2 z(1-x-y-z) - i0\right)^{2-\varepsilon_{IR}}}$$

$$= \frac{1}{(4\pi)^2 (s-p_3^2)(t-p_3^2)} B(n_x+\varepsilon_{IR}, n_y+n_z+\varepsilon_{IR}) n_x! \Gamma(\varepsilon_{IR}) \Gamma(1-\varepsilon_{IR})$$

$$\times \left[\left(-\frac{\tilde{t}-p_3^2}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} \left(-\frac{t-p_3^2}{s-p_3^2}\right)^{n_x} \frac{1}{\Gamma(n_x+\varepsilon_{IR})} \mathcal{I}^{(1)} + \left(\frac{-\tilde{s}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} \sum_{l=0}^{n_x} \frac{(-1)^l}{\Gamma(l+\varepsilon_{IR})(n_x-l)!} \mathcal{I}_l^{(2)}\right]$$

$$\mathcal{I}^{(1)} = \frac{1}{n_x+\varepsilon_{IR}} \sum_{k_1=0}^{n_x} \sum_{k_2=0}^{n_y+k_1} {}_{n_x}C_{k_1} \; {}_{n_y+k_1}C_{k_2} \, (-1)^{k_1+k_2} \left(\frac{p_3^2-s}{u}\right)^{n_y+k_1} (1-\alpha)^{k_2-n_x-1}$$

$$\times \left[\left(1+\frac{\bar{p}_3^2}{\bar{t}-\bar{p}_3^2}\right)^{n_x+\varepsilon_{IR}} {}_2F_1\left(1+n_x-k_2, n_x+\varepsilon_{IR}, 1+n_x+\varepsilon_{IR}, \frac{\bar{u}/(\bar{s}-\bar{p}_3^2)+\alpha}{\alpha-1}\right) \right.$$

$$\left. - \left(\frac{\bar{p}_3^2}{\bar{t}-\bar{p}_3^2}\right)^{n_x+\varepsilon_{IR}} {}_2F_1\left(1+n_x-k_2, n_x+\varepsilon_{IR}, 1+n_x+\varepsilon_{IR}, \frac{\alpha}{\alpha-1}\right) \right]$$

$$\mathcal{I}_l^{(2)} = \sum_{k_1=0}^{n_x} \sum_{k_2=0}^{n_y+k_1} {}_{n_x}C_{k_1} \; {}_{n_y+k_1}C_{k_2} \, (-1)^{k_1+k_2} \left(\frac{s}{s-p_1^2}\right)^{n_y+k_1} \frac{1}{l+k_2+\varepsilon_{IR}} \left(\frac{1}{1-\beta}\right)^{l+1} \left(\frac{t-p_3^2}{t+u-p_3^2}\right) \left(\frac{s}{s-p_3^2}\right)^{l}$$

$$\times \left[{}_2F_1\left(1+l, l+k_2+\varepsilon_{IR}, 1+l+k_2+\varepsilon_{IR}, \frac{\beta}{\beta-1}\right) \right.$$

$$\left. - \left(\frac{\bar{p_1}^2}{\bar{s}}\right)^{l+k_2+\varepsilon_{IR}} {}_2F_1\left(1+l, l+k_2+\varepsilon_{IR}, 1+l+k_2+\varepsilon_{IR}, \frac{\beta}{\beta-1}\frac{\bar{p}_1^2}{\bar{s}}\right) \right]$$

$$\alpha = \frac{\bar{p}_3^2}{\bar{t}-\bar{p}_3^2}\frac{\bar{u}}{\bar{s}-\bar{p}_3^2}, \quad \beta = \frac{\bar{u}}{\bar{s}-\bar{p}_3^2}\frac{\bar{s}}{\bar{t}+\bar{u}-\bar{p}_3^2}$$

Numerical calculation (IR finite case)

For $l, m, n \in N$,

$$
\begin{aligned}
&{}_2F_1(l, m+1, n+m+2; z) \\
&= \frac{1}{B(m+1,n+1)} \int_0^1 \tau^m (1-\tau)^n (1-z\tau)^{-l} d\tau \\
&= \sum_{k_1=0}^{n} \sum_{k_2=0}^{m+k_1} (-1)^{k_1+k_2} \frac{{}_nC_{k_1}\, {}_{m+k_1}C_{k_2}}{B(m+1,n+1)} \frac{1}{z^{m+k_1}} \int_0^1 (1-z\tau)^{-l+k_2} d\tau.
\end{aligned}
$$

$$
\int_0^1 (1-z\tau)^{-l+k_2} d\tau = \begin{cases} -\frac{\ln(1-z)}{z} & k_2 - l + 1 = 0, \\ \frac{1}{k_2-l+1} \frac{(1-z)^{k_2-l+1}-1}{-z} & k_2 - l + 1 \neq 0. \end{cases}
$$

Numerical check: mathematica ↔ our FORTRAN program

more than ten digit agreement

Numerical calculation (IR divergent case)

$$
\begin{aligned}
\mathcal{I}_{l,m,n} &\equiv \int_0^1 \tau^{l+n-1+\varepsilon_{IR}}(1-\tau)^m(1-z\tau)^{-(l+1)}d\tau, \\
&= B(1+m, l+n+\varepsilon_{IR})\ \underline{{}_2F_1\left(1+l, l+n+\varepsilon_{IR}, 1+l+n+m+\varepsilon_{IR}, z\right)} \\
&= \sum_{j=-1}^{\infty} \mathcal{F}^{(j)}_{l,m,n}(z)\varepsilon^j_{IR},
\end{aligned}
$$

Expansion w.r.t. ε_{IR}

$$
\tilde{F}^{(n)}_{j_1,j_2}(z) \equiv \frac{(-1)^n}{n!}\int_0^1 d\tau\ \tau^{j_1-1}(1-z\tau)^{-(j_2+1)}\ln^n\tau,
$$

When n=1

$$\tilde{F}^{(1)}_{1,0}(z) = \frac{\mathrm{Li}_2(z)}{z},$$

$$\tilde{F}^{(1)}_{1,1}(z) = -\frac{\ln(1-z)}{z}$$

$$\tilde{F}^{(1)}_{1,j_2+1}(z) = \frac{j_2}{j_2+1}\tilde{F}^{(1)}_{1,j_2} + \frac{(1-z)^{-j_2}-1}{j_2(j_2+1)z}$$

$$\tilde{F}^{(1)}_{j_1,j_2}(z) = \frac{1}{z^{j_1-1}}\sum_{k=0}^{j_1-1}(-1)^k\ {}_{j_1-1}C_k\ \tilde{F}^{(1)}_{1,j_2-k}(z)$$

When n=2

$$\tilde{F}^{(2)}_{1,0}(z) = \frac{\mathrm{Li}_3(z)}{z}$$

$$\tilde{F}^{(2)}_{1,1}(z) = \frac{\mathrm{Li}_2(z)}{z}$$

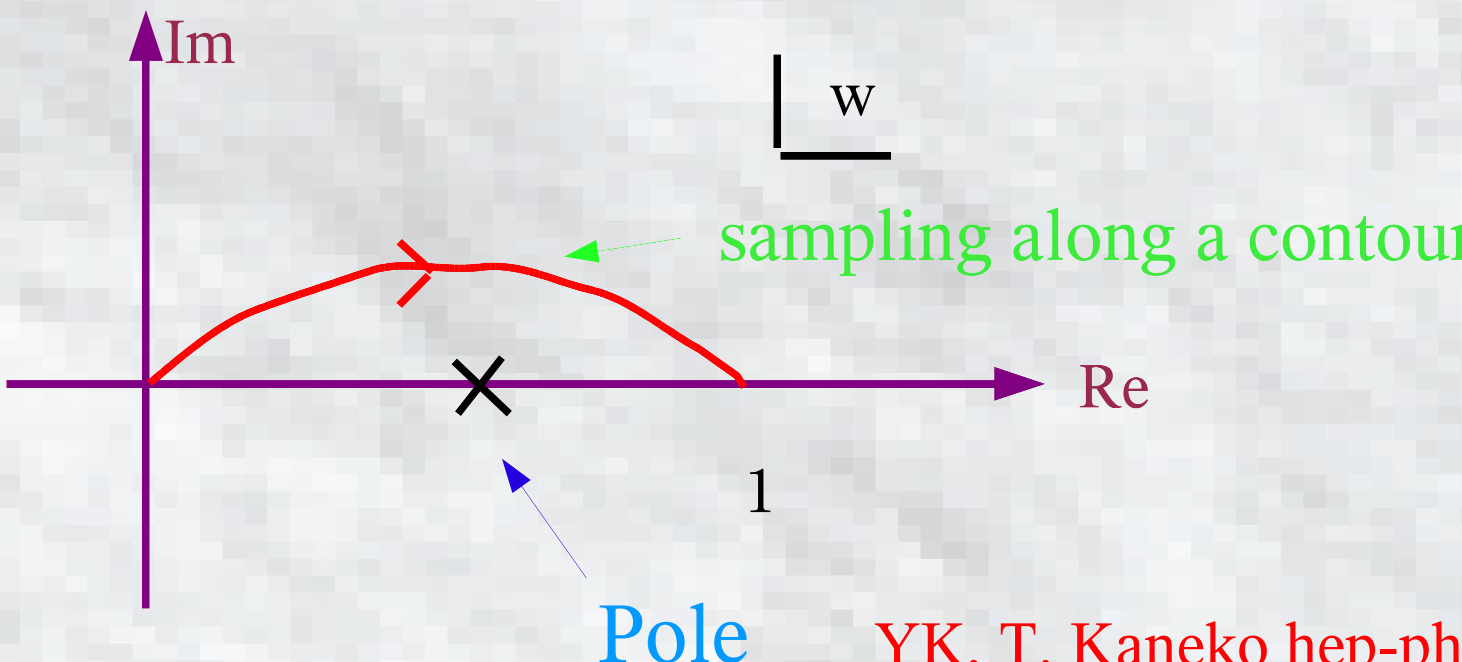
$$(j_2+1)\tilde{F}^{(2)}_{1,j_2+1}(z) - j_2\tilde{F}^{(2)}_{1,j_2}(z) - \tilde{F}^{(1)}_{1,j_2}(z) = 0$$

$$\tilde{F}^{(2)}_{j_1,j_2}(z) = \frac{1}{z^{j_1-1}}\sum_{k=0}^{j_1-1}(-1)^k \; {}_{j_1-1}C_k \; \tilde{F}^{(2)}_{1,j_2-k}(z)$$

Numerical test for Infrared finite case

$$
\begin{aligned}
J_4 &= \left(4\pi\mu^2\right)^{-\varepsilon_{IR}} \int_0^1 dr\; r^{-1+n_x+\varepsilon_{IR}}(1-r)^{-1+n_y+n_z+\varepsilon_{IR}} \\
&\times \int_0^1 dv \int_0^1 dw \frac{w^{n_y}(1-w)^{n_z} v^{n_x}}{(-s\; v(1-w) - t\;(1-v)w - i0)^{2-\varepsilon_{IR}}} \\
&= \left(4\pi\mu^2\right)^{-\varepsilon_{IR}} B(n_x+\varepsilon_{IR}, n_y+n_z+\varepsilon_{IR}) \\
&\times \int_0^1 dv \int_0^1 dw \frac{w^{n_y}(1-w)^{n_z} v^{n_x}}{(-s\; v(1-w) - t\;(1-v)w - i0)^{2-\varepsilon_{IR}}}
\end{aligned}
$$

Numerical contour integral



YK, T. Kaneko hep-ph/0503003

$J_4(s,t;0,0,0,0;n_x,n_y,n_z)$

n_x	n_y	n_z	real/imag.	analytic	NCI
1	2	3	real	-2.15298×10^{-9}	-2.15297×10^{-9}
			imag.	-2.78647×10^{-9}	-2.78650×10^{-9}
2	0	2	real	9.74570×10^{-9}	9.74572×10^{-9}
			imag.	-3.22229×10^{-8}	-3.22230×10^{-8}

$J_4(s,t;p_1^2,0,0,0;n_x,n_y,n_z)$

n_x	n_y	n_z	real/imag.	analytic	NCI
1	2	3	real	-7.88683×10^{-10}	-7.88689×10^{-10}
			imag.	-1.95176×10^{-9}	-1.95176×10^{-9}
2	0	2	real	1.48133×10^{-8}	1.48133×10^{-8}
			imag.	-2.04318×10^{-8}	-2.04318×10^{-8}

s=123, t=-200, p_1^2=80

Numerical test for IR divergent case

A sector decomposition method can be used.

$$J_4(s,t;0,0,0,0;0,0,0)$$

	real/imag.	analytic	SD
$1/\varepsilon_{IR}^2$	real	$-1.029686826 \times 10^{-6}$	$-1.029686826 \times 10^{-6}$
	imag.	0	$\mathcal{O}(10^{-16})$
$1/\varepsilon_{IR}$	real	$-5.205325212 \times 10^{-6}$	$-5.205325212 \times 10^{-6}$
	imag.	$1.617428283 \times 10^{-6}$	$1.617428283 \times 10^{-6}$
ε_{IR}^0	real	$-9.739160873 \times 10^{-6}$	$-9.739160872 \times 10^{-6}$
	imag.	$8.569648363 \times 10^{-6}$	$8.569648363 \times 10^{-6}$

s=123, t=-200

IR cancellation test

ex. Prompt photon production

@ one phase point

$\delta = a_2/\varepsilon_{IR}^2 + a_1/\varepsilon_{IR} + a_0$

BOX

a_1 = -18601.9993715016

a_2 = -3793.95539013131

S/C+vertex-(terms included in PDF)

a_1 = 18601.9993714494

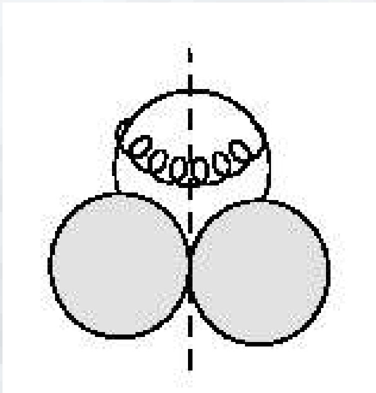
a_2 = 3793.95539013130

$1/\varepsilon_{IR}^2$: Soft/Coll. + Loop~$O(10^{-15})$

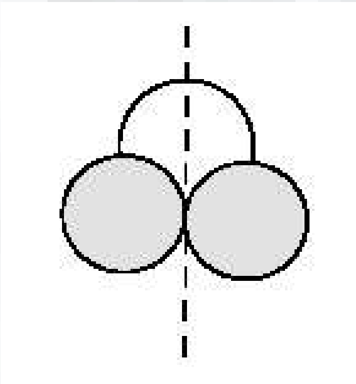
$1/\varepsilon_{IR}$: Soft/Coll. + Loop~$O(10^{-12})$

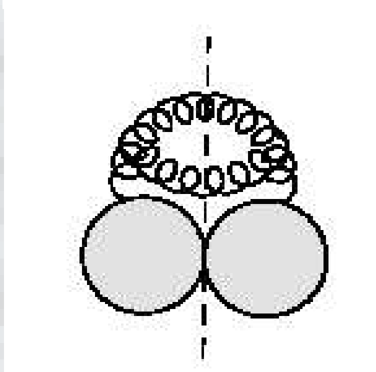
Soft/Collinear Approximation

In axial gauge,

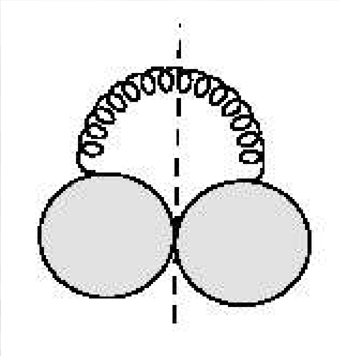


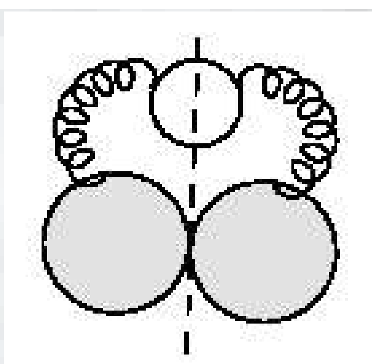
$$= f_{q \to qg} \times$$



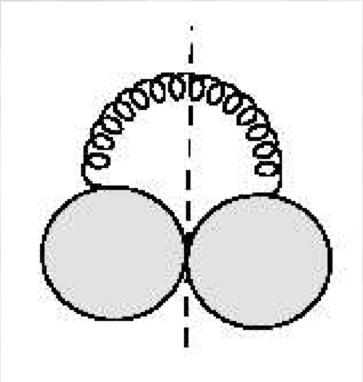


$$= f_{g \to gg} \times$$



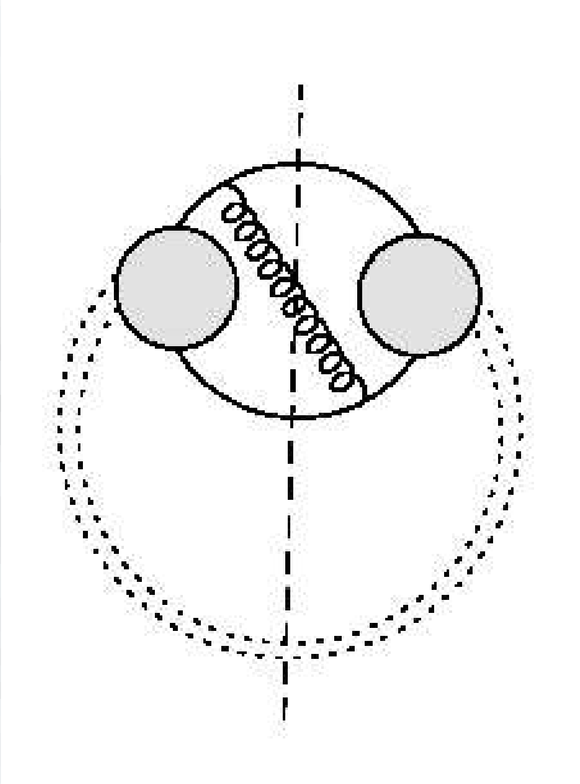


$$= f_{g \to qq} \times$$

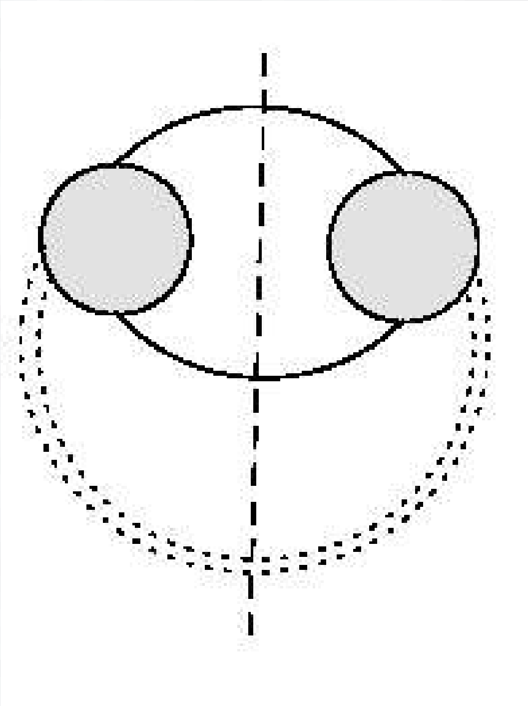


Soft/Collinear Approximation

In axial gauge,



$$= f^{int}_{q \to qg} \times$$



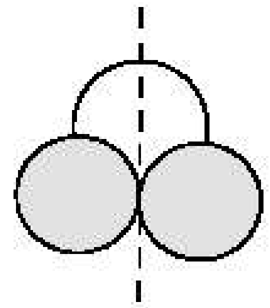
$$f^{int}_{q \to qg} = 0 \quad \text{in collinear region}$$
$$\neq 0 \quad \text{in soft region}$$

All f's will be implemented in the system soon.

Soft/Collinear Approximation

$$\boxed{f_{q_{in}\to q_{in} g_{out}}}$$

$$\sigma_{coll} = \frac{1}{(2p_1^0)(2p_2^0)v_{rel}} \int_{\Omega_{full}} d\Phi_{N+1}^{(d)} \left| f_{q\to qg} \times \right.$$



$$\left.\right|$$

$$\begin{aligned}
&= \left(\frac{s}{4\pi\mu^2}\right)^{\varepsilon_{IR}} \frac{B(\varepsilon_{IR},\varepsilon_{IR})}{2\Gamma(1+\varepsilon_{IR})} f_c \frac{\alpha_s}{2\pi} \int_0^1 dx \sigma_0(xs) \mathrm{P}(x) \left(\frac{1-x}{x}\right)^{2\varepsilon_{IR}} \\
&= \sigma_0(s) \frac{\alpha_s}{2\pi} f_c \left[\frac{1}{\varepsilon_{IR}^2} + \frac{2L-3}{2\varepsilon_{IR}} - \frac{\pi^2}{4} + \frac{L^2}{2}\right] \\
&+ \int_0^1 dx \sigma_0(xs) \phi(x,\varepsilon_{IR}) \\
&+ f_c \frac{\alpha_s}{2\pi} \int_0^1 dx \sigma_0(xs) \left[L \frac{1+x^2}{(1-x)_+} + \frac{(1+x^2)\ln(1-x)}{(1-x)_+} - \frac{1+x^2}{1-x} \ln x\right]
\end{aligned}$$

$$\begin{aligned}
\phi(x,\varepsilon_{IR}) &= \frac{1}{\varepsilon_{IR}} f_c \frac{\alpha_s}{2\pi} \mathrm{P}(x) = \frac{1}{\varepsilon_{IR}} f_c \frac{\alpha_s}{2\pi} \frac{1+x^2}{(1-x)_+} \\
L &= \ln(s/\mu^2).
\end{aligned}$$

Status of GRACE NLO Generator

Already tested
- Drell-Yan process
- W production
- Prompt photon

Under development
- V+1 jet

Future plan
- V+2 jets,.....
- VV+jet,2jets,....

Summary

(1) Automatic loop calculation in ELWK is established.

(2) QCD-NLO Matrix Elements
- ○Automatic generation by GRACE

(3) Loop integral
- ○Numerical loop-integration library

(4) Soft/Collinear treatment
- ○Basic tools of NLL-PS is available (under implementation)
- ○LL-subtraction method
- ○General calculation recipe

(5) Application
- ○Drell-Yan process/w production
- ○ prompt photon
- ○ V+1jet,2jet, VV+1 jet