

Electron energy spectrum in muon decay

Kirill Melnikov

University of Hawaii

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In collaboration with C. Anastasiou and F. Petriello

Outline

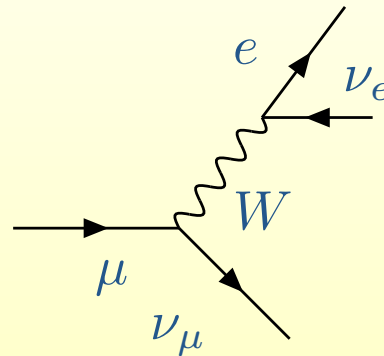
- Introduction
- QED corrections with the logarithmic accuracy
- QED corrections beyond the logarithmic accuracy
- Results
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Introduction

- Muon decay to electrons and neutrinos occupies a special place in high energy physics
 - $V - A$ current; the Fermi theory;
 - Kinoshita-Lee-Nauenberg theorem was conceived as the result of explicit computation of QED radiative corrections for $\mu \rightarrow e\nu_e\nu_\mu$ in particular;
 - One of the first complete one-loop calculations in the Standard Model;
 - The Fermi coupling constant, G_F , is extracted from the muon lifetime measurements; it is an input parameter for precision electroweak fits.
- Robust experimental program with long history and established tradition.
- Recent highlights
 - Calculation of $\mathcal{O}(\alpha^2)$ QED corrections to muon life-time within the Fermi Model Stuart, van Ritbergen
 - Ongoing measurements of the muon lifetime at μ LAN and FAST experiments at PSI; the expected precision is 10^{-6} which is a factor of seventeen better than the current precision.
 - Ongoing measurement of the electron energy spectrum by the TWIST collaboration. Expected to reach the precision of 10^{-4} .

Introduction

- The electron energy spectrum in muon decay offers an opportunity to test the $V - A$ structure of the charged weak current Michel, Bouchiat



- The electron energy spectrum in the decay of polarized muon is given by

$$\frac{d\Gamma}{dx d\cos\theta_e} = \frac{G_F^2 m}{4\pi^3} E_{\max}^5 \beta_e x^2 (F(x) - P_\mu \cos\theta_e G(x))$$
$$F(x) = x(1-x) + \frac{2}{9}\rho(4x^2 - 3x - x_0^2) + \eta x_0(1-x)$$
$$G(x) = \frac{1}{3}\xi\sqrt{x^2 - x_0^2} \left[1 - x + \frac{2\delta}{3} \left(4x - 3 + \sqrt{1 - x_0^2} - 1 \right) \right].$$

- $x = E_e/E_{\max}$, $E_{\max} = (m^2 + m_e^2)/(2m)$ and ρ, η, ξ, δ are Michel parameters.

Introduction

- In the Standard Model , Michel parameters

$$\rho = \frac{3}{4}, \quad \eta = 0, \quad \xi = 1, \quad \delta = \frac{3}{4}.$$

The direct consequence of the $V - A$ structure of weak interactions.

- Current measurements of the electron energy spectra imply

$$\underline{\rho = 0.751(1)} \quad \eta = -0.007(13), \quad \xi = 1.003(84), \quad \underline{\delta = 0.749(1)};;$$

any deviation from the Standard Model values implies New Physics.

TWIST collaboration, PDG 2004

- The standard example of New Physics is the left-right symmetric models with heavy W_R .

Herzceg, Langacker

- If TWIST reaches the expected precision, the bound on the mass of W_R becomes competitive with CDF and D0 bounds $M \geq 500 - 600$ GeV and the bound on the mixing angle becomes very tight $\zeta \sim 10^{-2}$.

QED corrections

- The description of the electron energy spectrum in muon decay in terms of Michel parameters is, of course, oversimplified.
- With the precision $\mathcal{O}(10^{-4})$, the QED radiative corrections have to be calculated; QED corrections change the **functional form** of $d\Gamma/dx$ and, hence, have to be subtracted before the fit to extract the Michel parameters is attempted.
- The total decay rate can be computed assuming that electron is massless; this approximation is invalid for the electron energy spectrum

$$\frac{d\Gamma}{dx} = \sum_{i=0}^{\infty} \sum_{j=0}^i \left(\frac{\alpha}{\pi}\right)^i \ln^j \frac{m}{m_e} f_{ij}(x).$$

- Since $\ln(m/m_e) \approx 5$, radiative corrections are important

$$\frac{\alpha}{\pi} \ln \frac{m}{m_e} \approx 1.2 \times 10^{-2}.$$

Computation of $\mathcal{O}(\alpha^2)$ corrections is required for the interpretation of TWIST results.

QED corrections in the logarithmic approximation

- If we are only interested in $\mathcal{O}(\alpha^2 \ln(m/m_e))$, computations can be simplified using the so-called perturbative fragmentation function **Mele and Nason**
- Similar to the familiar concept of fragmentation in QCD, we may write ($z = E_j/E_{\max}$) **Melnikov and Arbuzov**

$$\frac{d\Gamma}{dx} = \sum_j \int_x^1 \frac{dz}{z} \frac{d\Gamma_i^{\overline{\text{MS}}}(\mu_f, z)}{dz} D_i^e\left(\frac{x}{z}, \mu_f, m\right).$$

- The interpretation: “hard process” at distance scales $1/m$, followed by soft fragmentation at distances $1/m_e$. In the hard process, the dependence on the electron mass can be neglected.
- In contrast to QCD, the fragmentation function D_i^e is **fully computable from first principles**. Similar to QCD, the dependence on the factorization scale is governed by the QED analog of the DGLAP evolution equation

$$\frac{dD_i^e(\mu_f, x)}{d \ln \mu_f} = \frac{\alpha}{2\pi} \int_x^1 \frac{dz}{z} P_{ij}\left(\frac{x}{z}\right) D_j^e(\mu_f, z).$$

QED corrections with the logarithmic accuracy

- In the above formalism, the logarithms of the muon to electron mass ratio reside in the fragmentation function. Choosing $\mu_f = m$,

$$D_e^e(m, x) = \delta(1 - x) + \frac{\alpha}{2\pi} \left(P_{ee}(x) \ln \frac{m}{m_e} + D_e^{e,1}(x) \right) + \dots$$

- The $\mathcal{O}(\alpha^2 \ln(m/m_e))$ corrections to the electron energy spectrum can easily be computed provided that
 1. $\frac{d\Gamma_i^{\overline{\text{MS}}}}{dx}$ is known through $\mathcal{O}(\alpha)$ in the massless approximation ;
 2. $D_i^e(x)$ is known **exactly** through $\mathcal{O}(\alpha)$.
- The results in midpoint of the spectrum are
 1. $\mathcal{O}(\alpha^2 \ln^2 m/m_e)$ corrections are $\sim 7 \times 10^{-4}$
Arbuzov, Czarnecki, Gaponenko ;
 2. $\mathcal{O}(\alpha^2 \ln m/m_e)$ corrections are $\sim -3 \times 10^{-4}$
Melnikov, Arbuzov .
- Hence, to make full use of the TWIST precision, we require $\mathcal{O}(\alpha^2)$ corrections without the $\ln m/m_e$ enhancement.

QED corrections

- There are two ways to approach the calculation of $\mathcal{O}(\alpha^2)$ corrections;
- It is possible to generalize the method based on the fragmentation function. Then we require
 1. $\frac{d\Gamma_i^{\overline{\text{MS}}}}{dx}$ through $\mathcal{O}(\alpha^2)$ in the massless approximation
 2. $D^e(x)$ through $\mathcal{O}(\alpha^2)$ Melnikov and Mitov .
- The other option is to approach the calculation numerically, using techniques developed in the context of QCD computations for differential observables Anastasiou, Melnikov and Petriello .
- We choose to pursue the second option since
 1. It is more flexible; computing $d\Gamma/dz$ is equivalent to computing any differential distribution for $\mu \rightarrow e\nu_\mu\nu_e$;
 2. Extension of the method to massive particles, with potential applications to heavy quark decay spectra;
 3. Interesting to check if $m_e/m \approx 1/200$ is large enough, to permit a stable numerical evaluation.

Method: the basics

- Automated, numerical method for extracting and canceling the infra-red singularities.

- The NNLO decay rate:

$$d\Gamma_{\text{NNLO}} = d\Gamma_{VV} + d\Gamma_{RV} + d\Gamma_{RR}.$$

- For each component, obtain an expansion:

$$d\Gamma_{AB} = \sum_{j=j_{\min}}^{j=0} \frac{M_j^{\text{AB}}}{\epsilon^j},$$

where M_j^{AB} are ϵ -independent and integrable throughout the phase-space.

- M_j^{AB} can be computed **numerically**. Poles in ϵ cancel, when all $d\Gamma_{AB}$ are combined.
- In principle, the method deals with the **differential** cross-sections \Rightarrow **arbitrary cuts on the final states are allowed**.
- For the purposes of this talk, I focus on $d\Gamma/dx$.

Method: the sketch of the algorithm

- The method applies to VV, RV and RR, with minimal modifications. **I focus on RR since it is where the bottleneck usually is.**
- **The algorithm:**
 - map the differential phase-space onto the unit hypercube:

$$\int \prod_i \frac{dp_i}{2p_i^0} \delta^d \left(P_{\text{in}} - \sum p_i \right) \dots \Rightarrow \int_0^1 \prod_j dx_j x_j^{-a_j \epsilon} (1 - x_j)^{-b_j \epsilon} \dots$$

- use the “sector decomposition” to disentangle overlapping singularities;
Binoth, Heinrich, Denner, Roth.
- use “plus”-distribution expansion for book-keeping:

$$\frac{1}{x^{1+a\epsilon}} = -\frac{1}{a\epsilon} \delta(x) + \left[\frac{1}{x} \right]_+ - a\epsilon \left[\frac{\ln x}{x} \right]_+ + \dots$$

- **The outcome:** all the singularities from RR diagrams are extracted **without a single integration.**

Method: what changes with massive particles

- For massless particles, the double real emission correction is, usually, the most difficult to deal with. **This changes, if massive particles are involved.**
- Convenient parametrization of the phase-space is the key for the efficiency of the method; **trivially** achieved if fermions are massive since only **soft**, $\omega_\gamma \rightarrow 0$, photons contribute to the divergencies.
- With massless particles, many two-loop integrals are known analytically. With massive particles, this is not the case.
- Loops are amenable to sector decomposition.

Binoth, Heinrich, Denner, Roth.

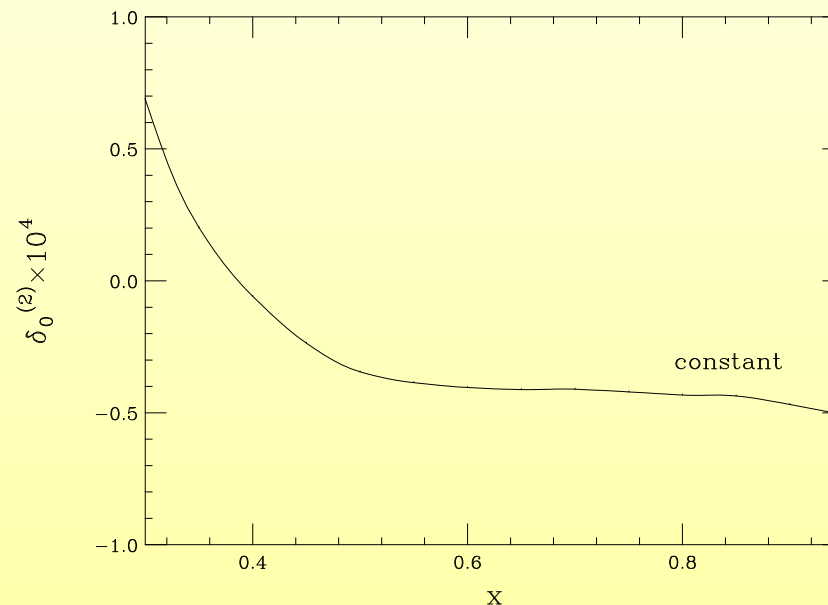
- Dealing with loop integrals for $\mu \rightarrow e\nu_e\nu_\mu$ is, in principle, straightforward. It is possible to deal numerically with the complete diagram **at once**; neither the Passarino-Veltman reduction nor the reduction to master integrals is required. Not quite trivial, but can be worked out.
- We do not use the fact that $m_e \ll m$; keeping m_e arbitrary is good for checks, for example $m_e \rightarrow m$ limit can be deduced from the known $\mathcal{O}(\alpha_s^2)$ corrections to $b \rightarrow c$ transitions at zero recoil. We achieve good numerical precision for $m_e/m \approx 200$.

QED corrections: results

- We write the second order correction to the electron energy spectrum as

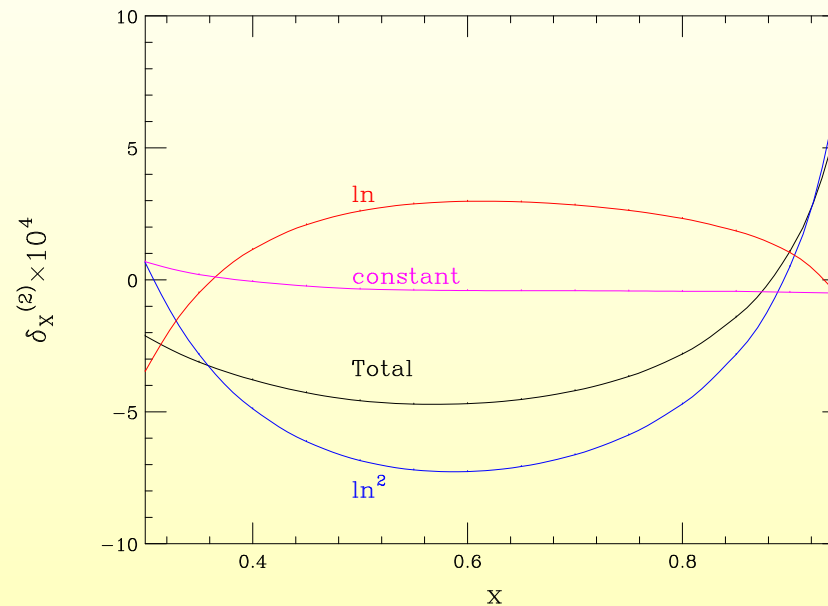
$$f^{(2)}(x) = f_2^{(2)}(x) \ln^2(m_\mu/m_e) + f_2^{(2)}(x) \ln(m_\mu/m_e) + f_0^{(2)}(x).$$

- The interesting piece is $f_0^{(2)}(x)$. Define $\delta_0^{(2)} = f_0^{(2)}(x)/f_0^{(0)}(x)$.



- QED corrections not enhanced by the logarithm of the muon to electron mass ratio are $\sim 0.5 \times 10^{-4}$, i.e. **one half of the expected experimental precision.**

QED corrections: results



- A hierarchy of $\ln^j(m/m_e)$, $j = 2..0$ corrections exists, but it does not quite follow the naive expectation.
- The leading \ln^2 -enhanced correction is good for estimates **within a factor 1.5 – 2**.
- The theory uncertainty in the prediction for the electron energy spectrum is **conservatively** estimated to be 5×10^{-6} using $\alpha^3 \ln^3(m/m_e) \sim \text{few} \times 10^{-6}$, hadronic vacuum polarization correction and finite W mass effects.

Arbuzov, Davydichev, Schilcher, Spiesberger

Conclusions

- First calculation of the energy spectrum of any charged particle through $\mathcal{O}(\alpha^2)$; for $\mu \rightarrow e \nu_e \nu_\mu$, almost half a century after the $\mathcal{O}(\alpha)$ corrections to the electron energy spectrum were obtained.
- $\mathcal{O}(\alpha^2)$ contribution to the electron energy spectra are in the range -5 to 8×10^{-4} depending on the value of x . The largest contribution comes from the $\ln m/m_e$ -enhanced corrections.
- The remaining theoretical uncertainty on the electron energy spectrum is estimated to be 5×10^{-6} , well below the requirements of the TWIST experiment.
- The calculation I just discussed applies to **unpolarized** muon decay only; to make full use of the TWIST measurement, the calculation has to be extended to include the muon polarization.
- The computational methods developed in the context of this calculation are applicable to decays of heavy particles (top, Higgs, bottom).