Constraining New Models with Precision Electroweak Data

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EW Sector of the Standard Model

3 parameters in the gauge boson+Higgs sector: \(g, g'\) and \(v\). Plus masses of Higgs boson and fermions. \(\alpha, G_\mu\) and \(M_Z\) usually chosen as 3 inputs:

\[
g^2 = \frac{4\pi\alpha}{s_W^2}, \quad g'^2 = \frac{4\pi\alpha}{c_W^2}, \quad v^2 = \frac{1}{(\sqrt{2}G_\mu)^{\frac{1}{2}}}.
\]

Custodial symmetry \(SU(2)_c\) is present in the SM. It guarantees that at tree level

\[
\rho \equiv \frac{M_W^2}{M_Z^2 c_W^2} = 1.
\]

Then \(s_W\) is an output of the calculation:

\[
s_W^2 c_W^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu M_Z^2}.
\]

Tree-level relation \(\rho = 1\) remains correct for:

- SM extensions with extra generations of fermions
- MSSM
- etc.
If a new model predicts some deviation from the SM in the EW observables, it has to be small.
The Usual

\[ \mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{NEW}, \]

where

\[ \mathcal{L}_{NEW} = \Sigma_i c_i \frac{O_i}{\Lambda^2}. \]

Common assumptions:

1. Use the SM renormalization procedure with new particles in the loops.
2. As \( \Lambda \to \infty \), New Physics effects become small.

If \( \rho = 1 \) at tree level, this is correct.
SM Renormalization

At tree level:

\[ G_\mu = \frac{\pi \alpha}{\sqrt{2} s_W^2 c_W^2 M_Z^2} \rho, \quad \rho \equiv \frac{M_W^2}{M_Z^2 c_W^2} = 1. \]

One loop result (on-shell definition):

\[ M_W^2 = \frac{\pi \alpha}{\sqrt{2} s_W^2 G_\mu (1 - \Delta r_{SM})}, \quad s_W^2 = 1 - \frac{M_W^2}{M_Z^2}. \]

Here \( \Delta r_{SM} \) collects all radiative corrections:

\[ \Delta r_{SM} = -\frac{\delta G_\mu}{G_\mu} + \frac{\delta \alpha}{\alpha} - \frac{\delta s_W^2}{s_W^2} - \frac{\delta M_W^2}{M_W^2}. \]

The SM is non-trivially constrained because of custodial symmetry (\( \rho = 1 \)):

\[ \frac{\delta s_W^2}{s_W^2} = c_W^2 \left[ \frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right] \sim m_t^2. \]

\( M_W \) and \( s_W^2 \) are outputs. \( G_\mu, \alpha \) and \( M_Z + \) all fermion and Higgs boson masses are inputs.
SM Prediction for $M_W(m_t)$

In terms of 2-point functions

$$
\Delta r_{SM} = \frac{\Pi_{WW}(0) - \Pi_{WW}(M_W^2)}{M_W^2} + \Pi'_\gamma(0) + \frac{s_W}{c_W} \frac{\Pi_{\gamma Z}(0)}{M_Z^2}
- \frac{c_W^2}{s_W^2} \left[ \frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2} - \frac{\Pi_{WW}(M_W^2)}{M_W^2} \right].
$$

For large $m_t$ top quark-dependent part:

$$
\Delta r_{SM}^t \approx -\frac{3G_\mu}{\sqrt{28\pi^2 s_W^2}} m_t^2.
$$

A good approximation for $\Delta r$ is:

$$
\Delta r_{SM} \approx 0.067 + \Delta r_{SM}^t + \frac{\alpha}{\pi s_W^2} \frac{11}{48} \left( \ln \frac{M_H^2}{M_Z^2} - \frac{5}{6} \right) + \ldots
$$

LEP EW WG:
The Simplest Example: SM + Higgs Triplet


Add a real Higgs triplet to the SM:

\[ H = \left( \frac{1}{\sqrt{2}}(v + \phi_R^0 + i\phi_I^0) \right), \quad \Phi = \begin{pmatrix} \eta^+ \\ \frac{1}{2}v' + \eta^0 \\ -\eta^- \end{pmatrix}. \]

Assume that triplet does not couple to fermions.

The set of model parameters becomes:

- \( g, g', v \) and \( v' \)
- fermion masses and the masses of the Higgses:
  - \( m_{H^0}, m_{K^0} \) and \( m_{H^\pm} \).

In this model

\[ c_\theta^2 = \frac{g^2}{g^2 + g'^2}, \]
\[ M_Z^2 = \frac{1}{4}(g^2 + g'^2) v^2, \quad M_W^2 = \frac{1}{4}g^2(v^2 + v'^2). \]

And

\[ \rho = 1 + \frac{v'^2}{v^2} \neq 1. \]
Higgs bosons mix:
\[
\begin{pmatrix}
    H^0 \\
    K^0
\end{pmatrix} = \begin{pmatrix}
    c_\gamma & s_\gamma \\
    -s_\gamma & c_\gamma
\end{pmatrix} \begin{pmatrix}
    \phi_R^0 \\
    \eta^0
\end{pmatrix}, \quad \begin{pmatrix}
    G^\pm \\
    H^\pm
\end{pmatrix} = \begin{pmatrix}
    c_\delta & s_\delta \\
    -s_\delta & c_\delta
\end{pmatrix} \begin{pmatrix}
    \phi^+ \\
    \eta^\pm
\end{pmatrix}.
\]

For simplicity set \( \gamma = 0 \).

In terms of mixing angle \( c_\delta^2 = \frac{v^2}{v^2 + v'^2} \):
\[
M_W^2 = \frac{g^2 v^2}{4c_\delta^2}, \quad \rho \equiv \frac{M_W^2}{M_Z^2 c_\theta^2} = \frac{1}{c_\delta^2} \geq 1.
\]

\( \rho \approx 1 \) experimentally \( \Rightarrow v' \ll v \).

**Renormalization**

\( s_\theta^2 \) is no longer a calculable quantity. It has to be fixed by experiment. Choose effective leptonic mixing angle as input:

\[
L = -i\bar{e}(\nu_e + \gamma_5 a_e)\gamma^\mu e Z^\mu, \quad \nu_e = \frac{1}{2} - 2s_\theta^2, \quad a_e = \frac{1}{2}, \\
1 - 4s_\theta^2 \equiv \frac{\text{Re}(\nu_e)}{\text{Re}(a_e)}.
\]

LEP result: \( s_\theta^2 = 0.23150 \pm 0.00016 \).
Return to muon decay:

\[ G_\mu = \frac{\pi \alpha}{\sqrt{2}s_\theta c_\theta M_Z^2 \rho (1 - \Delta r_{TM})}, \]

where

\[ \Delta r_{TM} = -\frac{\delta G_\mu}{G_\mu} + \frac{\delta \alpha}{\alpha} - \frac{\delta M_W^2}{M_Z^2} - \frac{c_\theta^2 - s_\theta^2 \delta s_\theta^2}{c_\theta^2 s_\theta^2} - \frac{\delta \rho}{\rho}. \]

In SM the following relation was used to derive \( s_\theta^2 \) counterterm:

\[ \frac{\delta \rho}{\rho} = \frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2} + \frac{c_W^2 \delta s_W^2}{s_W^2} = 0. \]

Now this relation can only be used as an expression for \( \frac{\delta \rho}{\rho} \neq 0. \)

Do \( Z\overline{e}e \) vertex renormalization to calculate:

\[ \frac{\delta s_\theta^2}{s_\theta^2} = \frac{c_\theta}{s_\theta} \text{Re} \left[ \frac{\Pi_{\gamma z}(M_Z^2)}{M_Z^2} + \delta_{\text{self-energy+vertex}} \right]. \]

\[ \Pi_{\gamma z}(M_Z^2) \sim \ln \frac{m_t^2}{Q^2} \Rightarrow \frac{\delta s_\theta^2}{s_\theta^2} \sim \ln \frac{m_t^2}{Q^2}. \]
Top quark dependence in TM

Put it all together:

$$
\Delta r_{TM} = \frac{\Pi_{WW}(0) - \Pi_{WW}(M_W^2)}{M_W^2} + \Pi_{\gamma\gamma}'(0) + 2s_{\theta} \frac{\Pi_{\gamma Z}(0)}{c_{\theta} M_Z^2} \\
- \frac{c_{\theta} \Pi_{\gamma Z}(M_Z^2)}{s_{\theta} M_Z^2} + \delta_{\text{self-energy+vertex+box}}.
$$

W mass is calculated from

$$
M_W^2 = \frac{\pi \alpha}{\sqrt{2} G_{\mu} s_{\theta}^2 (1 - \Delta r_{TM})}.
$$

For large $m_t$: $M_W^2 \sim \ln \frac{m_t^2}{Q^2}$!
$M_W(Higgs)$

Higgs mass dependence is **quadratic** vs. logarithmic dependence in the SM!

If $M_{H^0} \approx M_{K^0} \approx M_{H^\pm}$, then

$$\Delta r_{TM}^{\text{scalars}} \rightarrow \frac{\alpha}{4\pi s^2_{\theta}} \left[ -\frac{1}{2} \left[ c^2_{\delta} \frac{M^2_{H^0}}{M^2_W} \ln \frac{M^2_{H^0}}{M^2_W} + 4s^2_{\delta} \frac{M^2_{K^0}}{M^2_W} \ln \frac{M^2_{K^0}}{M^2_W} \right] + s^2_{\delta} \frac{M^2_{H^\pm} M^2_Z}{M^4_W} \ln \frac{M^2_{H^\pm}}{M^2_W} \right]$$

$$+ \frac{5}{72} \left[ s^2_{\delta} \frac{M^2_{H^\pm} - M^2_{H^0}}{M^2_{H^0}} + 4c^2_{\delta} \frac{M^2_{H^\pm} - M^2_{K^0}}{M^2_{K^0}} \right].$$

If $M_{H^0} \ll M_{K^0} \approx M_{H^\pm}$, then

$$\Delta r_{TM}^{\text{scalars}} \rightarrow \frac{\alpha}{4\pi s^2_{\theta}} \left[ -\frac{1}{2} \left[ c^2_{\delta} \frac{M^2_{H^0}}{M^2_W} \ln \frac{M^2_{H^0}}{M^2_W} + 4s^2_{\delta} \frac{M^2_{K^0}}{M^2_W} \ln \frac{M^2_{K^0}}{M^2_W} \right] \right.$$

$$+ \left. s^2_{\delta} \frac{M^2_{H^\pm} M^2_Z}{M^4_W} \ln \frac{M^2_{H^\pm}}{M^2_W} \right]$$

$$- s^2_{\delta} \frac{M^2_{H^\pm} M^2_{H^0}}{2M^4_W} \ln \frac{M^2_{H^\pm}}{M^2_{H^0}} + \frac{5}{18} c^2_{\delta} \frac{M^2_{H^\pm} - M^2_{K^0}}{M^2_{K^0}}.$$

Take $M_{K^0} = M_{H^\pm} \gg M_{H^0}$ and assume $M_{H^0} \approx M_Z$:

$$\Delta r_{TM}^{\text{scalars}} \rightarrow -\frac{\alpha}{8\pi s^2_{\theta}} \left[ c^2_{\delta} \frac{M^2_{H^0}}{M^2_W} \ln \frac{M^2_{H^0}}{M^2_W} + 4s^2_{\delta} \frac{M^2_{K^0}}{M^2_W} \ln \frac{M^2_{K^0}}{M^2_W} \right].$$
Random scan over $300 \text{ GeV} < M_{H^0} < 600 \text{ GeV}$, $300 \text{ GeV} < M_{K^0} < 600 \text{ GeV}$, $300 \text{ GeV} < M_{H^\pm} < 600 \text{ GeV}$.

Random scan over $1 \text{ TeV} < M_{H^0} < 3 \text{ TeV}$, $1 \text{ TeV} < M_{K^0} < 3 \text{ TeV}$, $1 \text{ TeV} < M_{H^\pm} < 3 \text{ TeV}$.

$M_W(M_{H^0})$ FOR FIXED $M_{K^0}$ AND $M_{H^\pm}$.

EXPERIMENTAL ERROR IMPACT ON $M_W$ PREDICTION.
(Non)-Decoupling

Shouldn’t we recover the SM results when either
1. \( m_{K^0} \to \infty \) and \( m_{H^\pm} \to \infty \),
or 2. \( s_\delta \to 0 \) (no mixing between \( \eta^\pm \) and \( \phi^\pm \))?

As long as \( s_\delta \neq 0 \), no matter how small, custodial symmetry is broken and one needs an additional counterterm for \( \rho \): \( \frac{1}{\xi} \) is floating around otherwise! One does not have a smooth transition from 4 to 3 input parameters.

To understand what is going on, examine the tree level potential of TM:
\[
V(\Phi, H) = \mu_1^2 H^\dagger H + \frac{1}{2} \mu_2^2 \Phi^\dagger \Phi + \lambda_1 (H^\dagger H)^2 + \frac{1}{4} \lambda_2 (\Phi^\dagger \Phi)^2 \\
+ \frac{1}{2} \lambda_3 (H^\dagger H)(\Phi^\dagger \Phi) + \lambda_4 v \Phi_i H^\dagger \sigma_i H.
\]


Here
\[
\Phi_U = U^\dagger \Phi, \quad U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i & 0 \\ 0 & 0 & \sqrt{2} \\ -1 & -i & 0 \end{pmatrix}.
\]
Define \( t_\beta \equiv \frac{v'}{v} \). Minimization with respect to \( \phi_R^0 \) and \( \eta^0 \) yields:

\[
8 \mu_1^2 + 8 \lambda_1 v^2 + \lambda_3 t_\beta^2 v^2 - 4 \lambda_4 t_\beta v^2 = 0,
4 \mu_2^2 t_\beta + \lambda_2 t_\beta^2 v^2 + 2 \lambda_3 t_\beta v^2 - 4 \lambda_4 v^2 = 0.
\]

We have assumed that there is no mixing between neutral Higgs components \( \Rightarrow \frac{\partial^2 V(\Phi,H)}{\partial \eta^0 \partial \phi_R^0} \langle \rangle = 0 \). That implies

\[
2 \lambda_4 = \lambda_3 t_\beta.
\]

And then minimization equations become

\[
8 \mu_1^2 + 8 \lambda_1 v^2 - \lambda_3 t_\beta^2 v^2 = 0,
\]

\[
t_\beta \left( 4 \mu_2^2 + \lambda_2 t_\beta^2 v^2 \right) = 0.
\]

1. Let mixing between \( \eta^\pm \) and \( \phi^\pm \) vanish. That can be achieved by setting \( v' \to 0 \). But then custodial symmetry is restored and the 1-loop renormalization should be done as in the SM.

2. Let the mass of \( \eta^\pm \) go to \( \infty \). Then \( \frac{\partial^2 V(\Phi,H)}{\partial \eta^\pm^2} \langle \rangle \to \infty \):

\[
4 \mu_2^2 + \lambda_2 t_\beta^2 v^2 + 2 \lambda_3 v^2 \to \infty.
\]

\( v \) and \( t_\beta \) cannot get too large. In the TM

\[
v^2 + v'^2 = v_{SM}^2 = (246 \, \text{GeV})^2.
\]

\( \lambda_3 \) cannot get too large, otherwise \( \mu_1^2 \to \infty \) and also theory becomes non-perturbative.

Therefore \( 4 \mu_2^2 + \lambda_2 t_\beta^2 v^2 \to \infty \) is the only solution.
If \( 4\mu_2^2 + \lambda_2 t_\beta^2 v^2 \rightarrow \infty \), then minimization condition
\[
t_\beta (4\mu_2^2 + \lambda_2 t_\beta^2 v^2) = 0
\]
implies that \( t_\beta \rightarrow 0 \). But this case corresponds to the restoration of custodial symmetry, \( \rho = 1 \) at tree level.

There is one more option to eliminate the mixing between neutral components of triplet and doublet:
\[
\left. \frac{\partial^2 V(\Phi,H)}{\partial \eta_0^2} \right|_{\langle \rangle} \rightarrow \infty, \text{ while keeping } \left. \frac{\partial^2 V(\Phi,H)}{\partial \eta_0^0 \partial \phi_R^0} \right|_{\langle \rangle} \text{ fixed.}
\]
Then \( 4\mu_2^2 + 3\lambda_2 t_\beta^2 v^2 + 2\lambda_3 v^2 \rightarrow \infty \) and \( 2\lambda_4 - \lambda_3 t_\beta \) is finite. The minimization condition implies:
\[
t_\beta (4\mu_2^2 + \lambda_2 t_\beta^2 v^2) = 2v^2 (2\lambda_4 - \lambda_3 t_\beta).
\]

If one restores the custodial symmetry, \( t_\beta \rightarrow 0 \), then \( \mu_2 \rightarrow \infty \) can be accommodated. However, if \( t_\beta \neq 0 \), then \( \lambda_3 \) has to \( \rightarrow \infty \), and therefore \( \mu_1^2 \rightarrow \infty \) and theory becomes non-perturbative.

It is impossible to decouple the triplet without either restoring the custodial symmetry, or else blowing up the perturbative framework.
Including all electroweak observables in the analysis provides a better constraint on $m_t$. Example:

![Graph showing $\Gamma_Z$ vs. $m_t$](image)


Global fit is coming soon.
A (very incomplete) list of models with \( \rho \neq 1 \) at tree level (with even less complete list of references):

**Littlest Higgs**


\[ SU(2)_L \times SU(2)_R \text{ model} \]

J. C. Pati and A. Salam, Phys.Rev.D10:275-289, 1974,

**Warped extra dimensions without \( SU(2)_c \)**

Conclusions

- Models with $\rho \neq 1$ at tree level require careful treatment. SM renormalization is not applicable.

- Top quark mass constraint from electroweak precision measurements may be loosened.

- One can have massive Higgs bosons in the theory.

- Constraints on some models need to be reexamined.