# Top Threshold Physics

# . . . and Other Applications

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thanks to T. Teubner, A. Manohar, I. Stewart, C. Reisser, C. Farrell, P. Ruiz-Femenia

# Outline

- Physics at the Top Threshold at the ILC
- Theoretical Issues
- Effective Theory: stable top quarks
- Effective Theory: unstable top quarks
- Status of Computations $\sigma(e^+e^- \to t\bar{t})$
- Other Applications:
  - $e^+e^- \to t\bar{t}H$
  - $e^+e^- \to \tilde{q}\bar{\tilde{q}}$

# Top Physics and the ILC

- $e^+e^-$ collider: $E_{\rm cm} = 350$ GeV – $1$ TeV
- Luminosity: $10^{34}$–$10^{35}\,{\rm cm}^{-2}{\rm s}^{-1}$ $\rightarrow$ $100$–$1000\,{\rm fb}^{-1}$/year

LC $\sim 10^5$ $t\bar{t}$ pairs [$\sigma_{\rm tot} < 1\,{\rm pb}$] ($e^+e^- \rightarrow t\bar{t}$)

LHC $\sim 10^8$ $t\bar{t}$ pairs [$\sigma_{\rm tot} \approx 850\,{\rm pb}$] ($gg \rightarrow t\bar{t}$)

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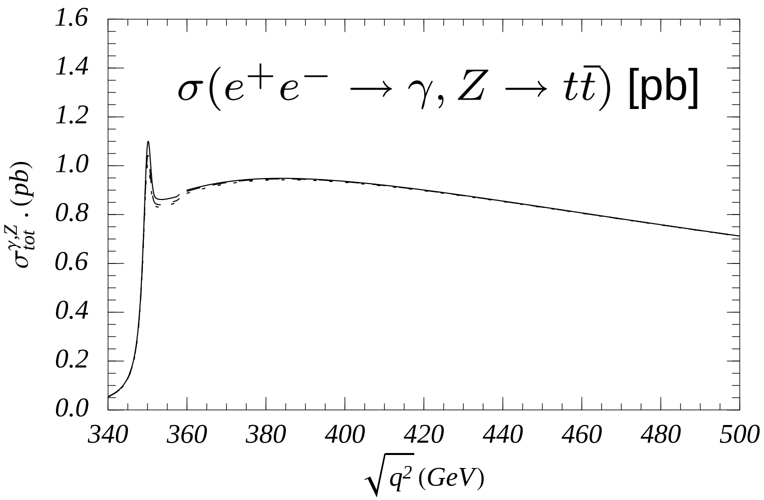
- Initial state tunable and very well known
  - Centre of mass energy variable $\to$ threshold & continuum




(Luminosity spectrum ignored)

  - Electron beam polarizable: $\to e^-$: 80%
  - $\gamma\gamma$, $\gamma e$ options: $e^+e^- \to t\bar{t}\,(^3S_1)$, $\gamma\gamma \to t\bar{t}\,(^1S_0)$

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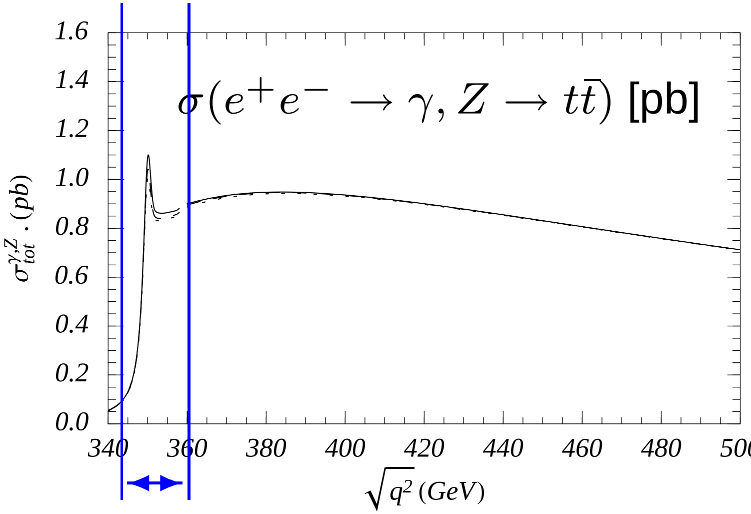


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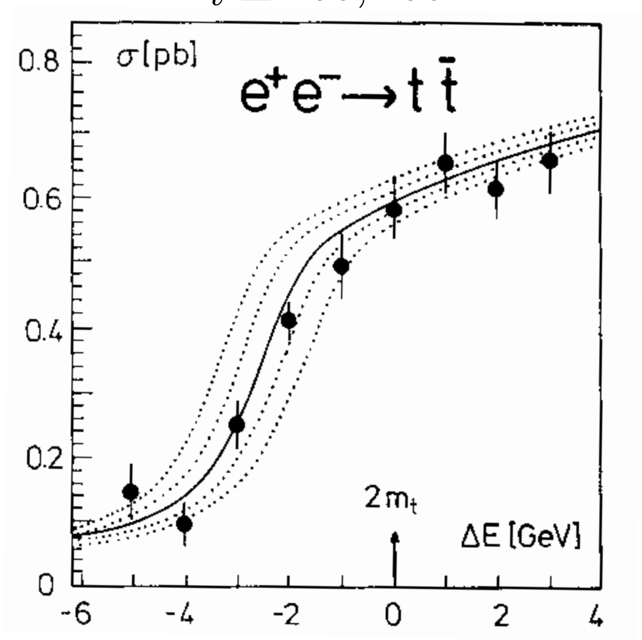
# Threshold Measurements

- Top quark mass $\longrightarrow$ threshold scan
  - ▷ count number of $t\bar{t}$ events
  - ▷ color singlet state
  - ▷ backbround is non-resonant
  - ▷ measured mass is unambiguous

  $\longrightarrow$ threshold masses (e.g. $m_t^{1S}$)

  to avoid artificial perturb. ambiguities

$m_t \pm 200, 400$ MeV



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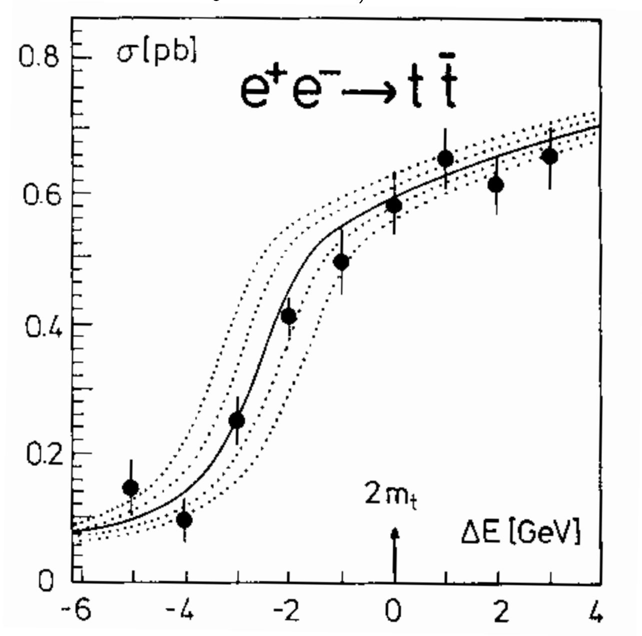


Simulations $\mathcal{L} = 300\,\mathrm{fb}^{-1}$, $9+1$ scan points Peralta, Martinez, Miquel

$\boxed{(\delta m_t)^{\rm stat} \sim 20 \text{ MeV}}$

$(\delta\lambda_t/\lambda_t)^{\rm stat} = 15 - 50\%$

$(\delta\alpha_s(M_Z))^{\rm stat} = 0.001$

$(\delta\Gamma_t)^{\rm stat} = 50$ MeV

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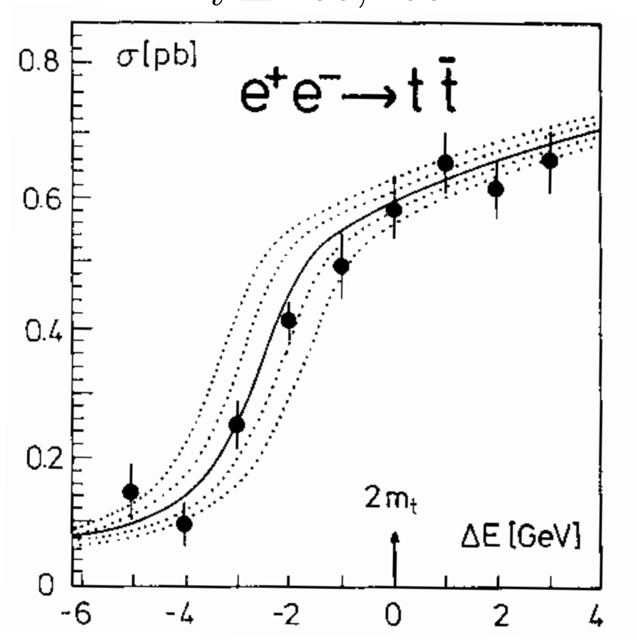
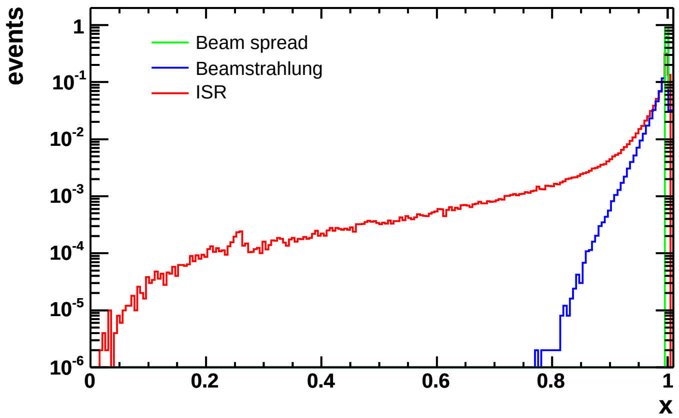
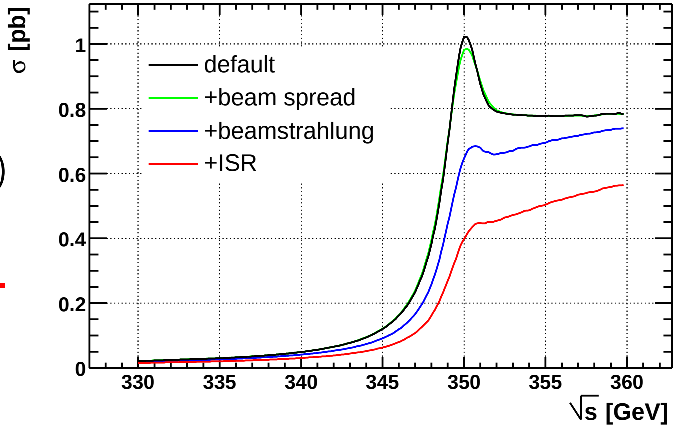



Simulations Influence of Luminosity spectrum Boogert




$$\sigma(s) = \int_0^1 dx L(x) \sigma^0(x^2 s)$$



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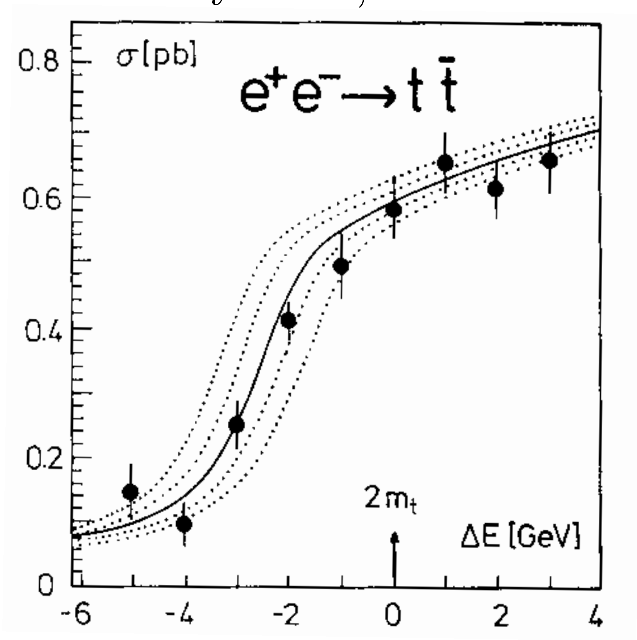



Simulations & theory goal: $\boxed{(\delta\sigma/\sigma)^{\rm theo} \le 3\,\%}$

$\boxed{(\delta m_t)^{\rm stat} \sim 20 \text{ MeV}}$ $\boxed{(\delta m_t)^{\rm syst} = 50 \text{ MeV}}$ $\boxed{(\delta m_t)^{\rm theo} = 100 \text{ MeV}}$

$(\delta\lambda_t/\lambda_t)^{\rm stat} = 15 - 50\%$ $(\delta\lambda_t/\lambda_t)^{\rm syst} = ?$ $(\delta\lambda_t/\lambda_t)^{\rm theo} \sim ?$

$(\delta\alpha_s(M_Z))^{\rm stat} = 0.001$ $(\delta\alpha_s(M_Z))^{\rm syst} = 0.002$ $(\delta\alpha_s(M_Z))^{\rm theo} \sim ?$

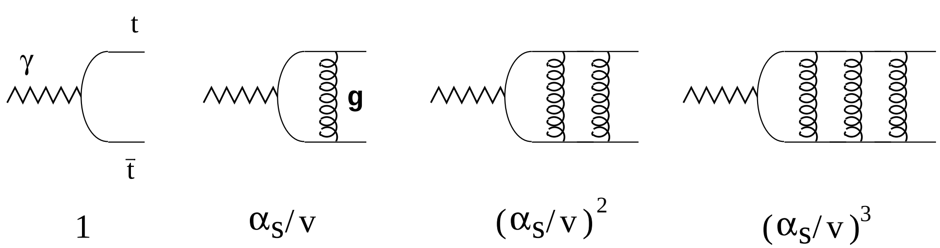
$(\delta\Gamma_t)^{\rm stat} = 50$ MeV $(\delta\Gamma_t)^{\rm syst} = 15$ MeV $(\delta\Gamma_t)^{\rm theo} \sim ?$

# Theory Issues

$$\boxed{m_t \text{ (hard)} \quad \gg \quad \mathbf{p} \sim m_t v \text{ (soft)} \quad \gg \quad E \sim m_t v^2 \text{ (ultrasoft)}}$$

“multi-scale problem”

- perturbation theory in $\alpha_s$ breaks down $\qquad (\alpha_s/v)^n$




“Coulombsingularities”

$\longrightarrow$ Schrödinger Equation

- perturbation theory in $\alpha_s$ breaks down $\longrightarrow$ large logs $(\alpha_s \ln v)^n$

$m_t = 175$ GeV, $\mathbf{p} \sim 25$ GeV, $E \sim 4$ GeV $\quad \Rightarrow \ln\left(\frac{m_t^2}{E^2}\right) = 8 \quad \longrightarrow$ RGE’s

# Theory Issues

- $\Gamma_t \approx 1.5\,\text{GeV} \gg \Lambda_{\rm QCD} \quad \Rightarrow \quad v = \sqrt{\frac{E}{m}} \;\longrightarrow\; v_{\rm eff} = \sqrt{\frac{E+i\Gamma_t}{m}}$

  (Fadin,Khoze)

  $\Rightarrow\; m_t \gg p = m v_{\rm eff} \gg E = m v_{\rm eff}^2 \gtrsim \Lambda_{\rm QCD}$ always true !

  $\Rightarrow$ top threshold entirely perturbative ! $\longrightarrow$ “Schrödinger theory”

- top quarks are always produced off-shell !

  $\longrightarrow$ methods for on-shell production do not apply for

  - theoretical computations
  - experimental analysis

“theory for unstable particles”

# EFT - Shopping List

$$\sigma_{t\bar{t}} \sim v \sum_{n,m} \left(\frac{\alpha_s}{v}\right)^n \left(\alpha_s \ln v\right)^m \left[1,\ \{v, \alpha_s\},\ \{v^2, \alpha_s v, \alpha_s^2\},\ \ldots\right]$$

LL NLL NNLL

- Action from Lagrangian in terms of fi eld operators
  - ▷ Manifest power counting in $v$ at all times
    consistently obeye $E \ll p \ll m$ and $\Gamma_t \sim E \sim p^2/m$
  - ▷ Symmetries: gauge, spin, ...
  - ▷ Regulator independence $\rightarrow$ matching in different schemes
- Renormalization of the theory $\rightarrow$ anomalous dimensions, summation of $(\alpha_s \ln v)^m$
- Dimensional regularization

# Different EFT's (for QCD)

Classic "NRQCD": ('92-'95) Bodwin, Braaten, Lepage

Separate $p > m$ from $p < m$ [Production, Decays, Lattice]

"NRQCD-pNRQCD" ('97-'99) Pineda, Soto, Brambilla, Vairo

Separate also $p > mv$ from $p < mv$ [$\bar{c}c$, $\bar{b}b$]

Combined 1- and 2-step matching, running Pineda ('01)

Power counting not uniform in different parts of the theory

"vNRQCD" ('99) Luke, Manohar, Rothstein, Stewart, A.H.

Separation of all degrees of freedom at m [$t\bar{t}$]

1-step matching and running, uniform power counting

Correlation of $mv$ and $mv^2$

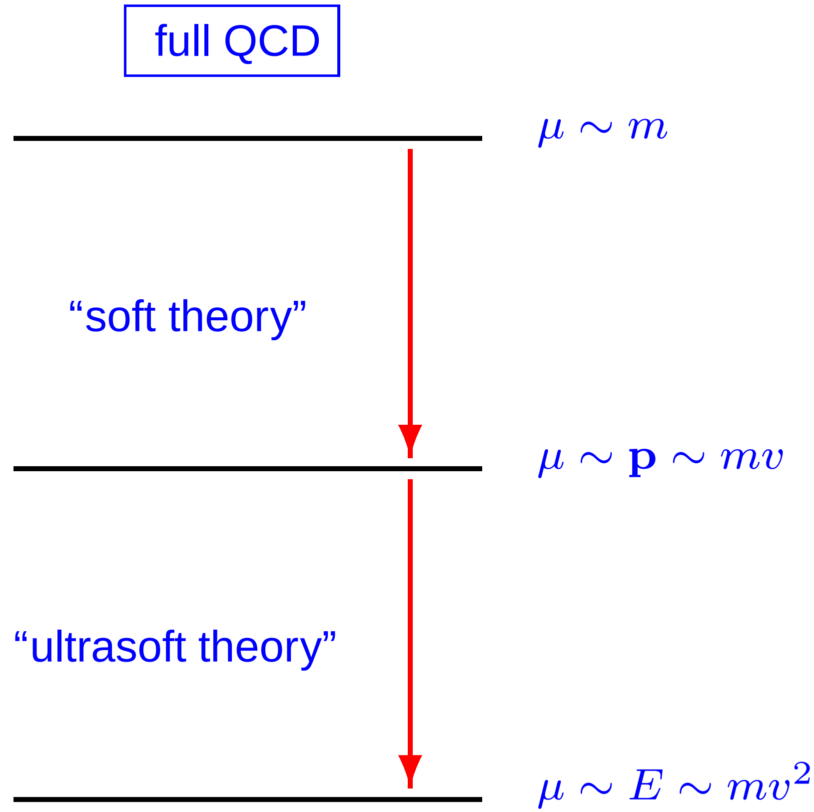
(needed in general for $\Lambda_{\mathrm{QCD}} \ll mv^2$: e. g. $c_1$, also in QED)

# Degrees of Freedom

naive picture:

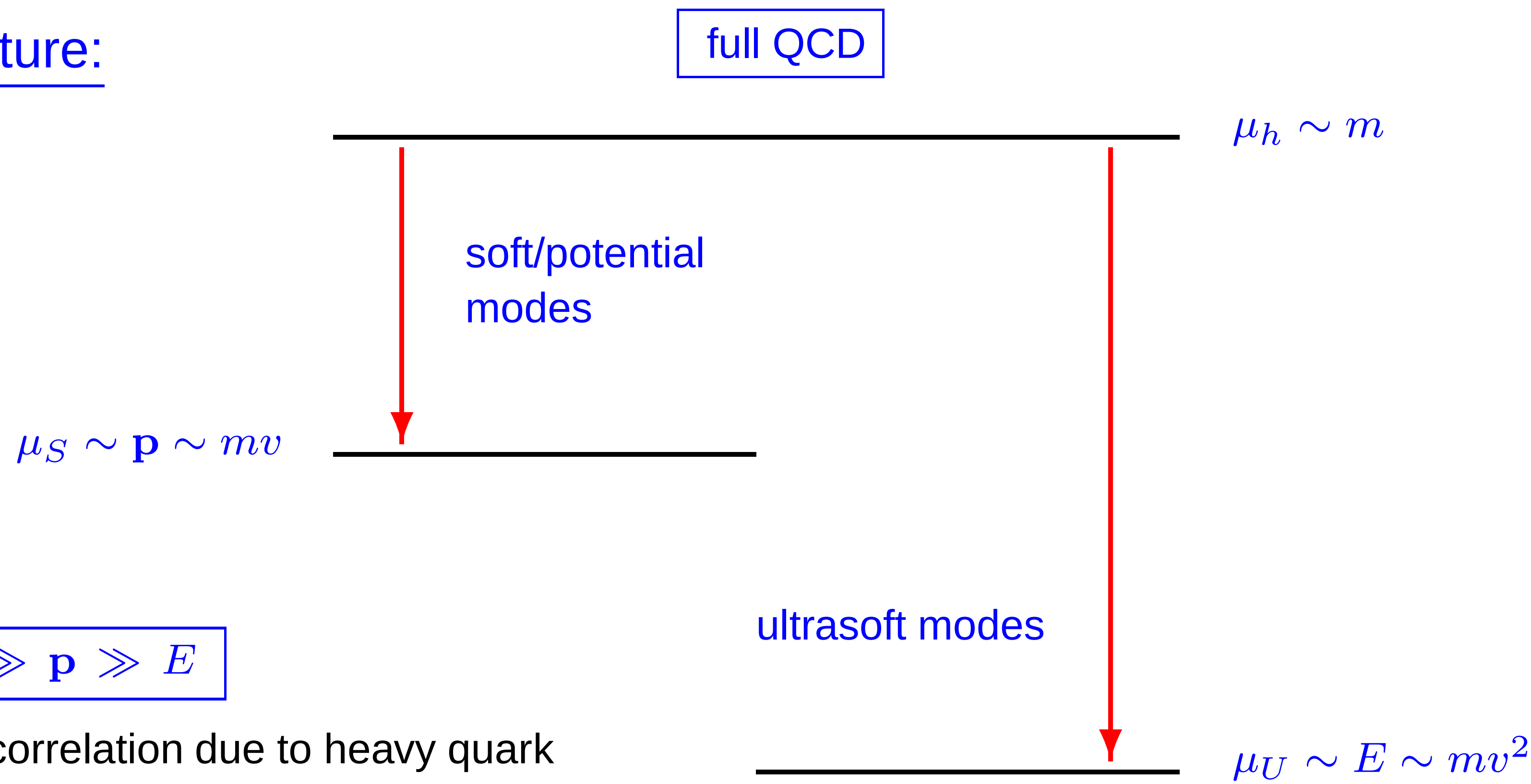
$\boxed{m_t \gg \mathbf{p} \gg E}$

- sucessively integrated out d.o.f.'s
- sequence of effective theories



# Degrees of Freedom

proper picture:




$\boxed{m_t \gg \mathbf{p} \gg E}$

and scale correlation due to heavy quark

equation of motion: $\boxed{E = \mathbf{p}^2/m}$

- soft and ultrasoft modes exist at the same time
- $\mu_U = \mu_S^2/m \quad \rightarrow \quad \mu_S = m\nu\,, \;\; \mu_U = m\nu^2$

# Degrees of Freedom

- fields for degrees of freedom that can resonate for the quark-antiquark system

potential quarks $\psi_p, \chi_p$: $(k_0, \mathbf{k}) \sim (mv^2, mv)$

soft gluons ${A_q}^\mu$: $(k_0, \mathbf{k}) \sim (mv, mv)$

ultrasoft gluons $A^\mu$: $(k_0, \mathbf{k}) \sim (mv^2, mv^2)$

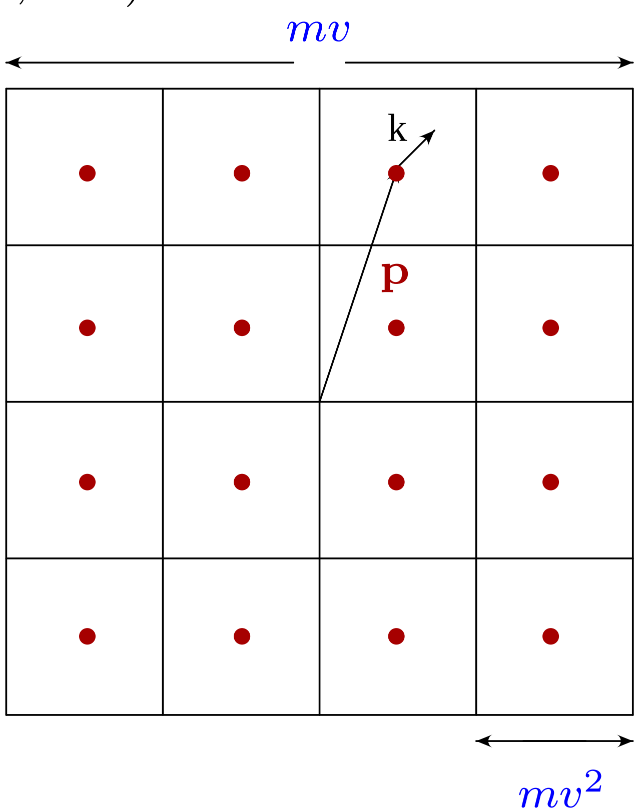
- vNRQCD label formalism:

$$(P^0, \mathbf{P}) = (0, \mathbf{p}) + (k^0, \mathbf{k})$$

soft component label

ultrasoft component dynamic variable

$$\psi_{\rm QCD}(x) \to \sum_{\mathbf{p}} e^{i\mathbf{p}.\mathbf{x}}\, \psi_{\mathbf{p}}(x)$$



# vNRQCD (stable quarks)

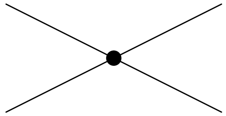
$$\underline{\mathcal{L} = \mathcal{L}_{\text{usoft}} + \mathcal{L}_{\text{potential}} + \mathcal{L}_{\text{soft}}}$$

Luke, Manohar, Rothstein, Stewart, A.H.

$\mathcal{L}_{\text{usoft}}$ :

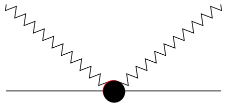
$$\psi^\dagger_{\mathbf{p}}(x)\left\{iD^0 - \frac{(\mathbf{p}-i\mathbf{D})^2}{2m_t} - \delta m_t\right\}\psi_{\mathbf{p}}(x)$$

$\mathcal{L}_{\text{potential}}$ :



$$\left\{\frac{V_c(\nu)}{(\mathbf{p}-\mathbf{p}')^2} + \cdots\right\}\psi^\dagger_{\mathbf{p}'}\psi_{\mathbf{p}}\chi^\dagger_{-\mathbf{p}'}\chi_{-\mathbf{p}}$$

$\mathcal{L}_{\text{soft}}$ :



$$U_{\mu\nu}(\nu)\ \psi^\dagger_{\mathbf{p}'}A^\mu_q A^\nu_{q'}\psi_{\mathbf{p}}$$

$$V = \left[\frac{\mathcal{V}_c(\nu)}{\mathbf{k}^2} + \frac{\mathcal{V}_k(\nu)\pi^2}{m|\mathbf{k}|} + \frac{\mathcal{V}_r(\nu)(\mathbf{p}^2+\mathbf{p}'^2)}{2m^2\mathbf{k}^2} + \frac{\mathcal{V}_2(\nu)}{m^2} + \frac{\mathcal{V}_s(\nu)}{m^2}\mathbf{S}^2 + \frac{\mathcal{V}_\Lambda(\nu)}{m^2}\Lambda + \frac{\mathcal{V}_t(\nu)}{m^2}T\right]$$

$$\mathbf{k} \equiv \mathbf{p} - \mathbf{p}'$$

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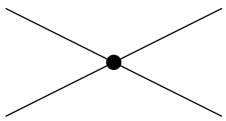
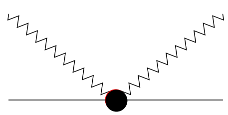
$$\psi_{\mathbf{p}}^\dagger(x)\left\{iD^0 - \frac{(\mathbf{p}-i\mathbf{D})^2}{2m_t} - \delta m_t\right\}\psi_{\mathbf{p}}(x)$$

$\mathcal{L}_{\rm potential}$ :



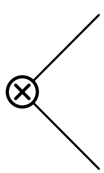
$$\left\{\frac{V_c(\nu)}{(\mathbf{p}-\mathbf{p}')^2} + \cdots\right\}\psi_{\mathbf{p}'}^\dagger\psi_{\mathbf{p}}\chi_{-\mathbf{p}'}^\dagger\chi_{-\mathbf{p}}$$

$\mathcal{L}_{\rm soft}$ :



$$U_{\mu\nu}(\nu)\;\psi_{\mathbf{p}'}^\dagger A_q^\mu A_{q'}^\nu\psi_{\mathbf{p}}$$

<u>external currents:</u> (production & annihilation)



$$\mathbf{O}_{\mathbf{p}} = C_{^3S_1}(\nu)\cdot(\psi_{\mathbf{p}}^\dagger\,\boldsymbol{\sigma}\,\tilde{\chi}_{-\mathbf{p}}^*) + \cdots \qquad t\bar{t}\ (^3S_1)$$

# Cross Section at NNLL Order

Schematic:

$$\sigma_{\rm tot} \quad \propto \quad \mathrm{Im}\left[\int \mathrm{d}^4\mathrm{x}\, \mathrm{e}^{-\mathrm{i}\hat{\mathrm{q}}\mathrm{x}} \left\langle\, 0 \,\middle|\, \mathrm{T}\, \mathbf{O}_{\mathbf{p}}^{\dagger}(0)\, \mathbf{O}_{\mathbf{p}'}(\mathrm{x}) \,\middle|\, 0 \,\right\rangle\right]$$

$$\propto \quad \mathrm{Im}\left[\mathrm{C}(\nu)^2\, \mathrm{G}(0,0,\sqrt{\mathrm{s}})\right]$$

$$\left(-\frac{\nabla^2}{m} - \frac{\nabla^4}{4m^3} + V(\mathbf{r}) - (\sqrt{s} - 2m - 2\delta m) - i\Gamma_t\right) G(\mathbf{r},\mathbf{r}') = \delta^{(3)}(\mathbf{r}-\mathbf{r}')$$

potentials fully known

at NNLL order

Pineda,Soto ’00-’01

Manohar,Stewart, AHH ’99-’02

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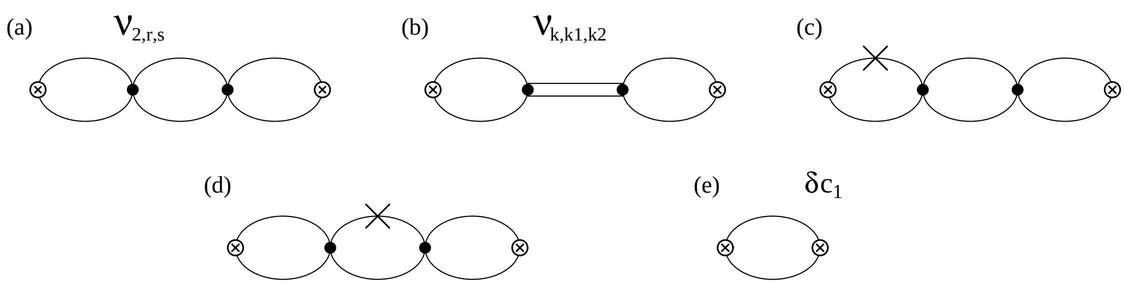
$$\sigma_{\rm tot} \;\propto\; \mathrm{Im}\left[\int d^4x\, e^{-i\hat{q}x} \left\langle 0 \middle| T\, \mathbf{O}_{\mathbf{p}}^{\dagger}(0)\, \mathbf{O}_{\mathbf{p}'}(x) \middle| 0 \right\rangle\right]$$

$$\propto\; \mathrm{Im}\left[C(\nu)^2\, G(0,0,\sqrt{s})\right]$$

NLL running of $C(\nu)$




$$\frac{d}{d\ln\nu}\ln C(\nu) = -\frac{\mathcal{V}_c(\nu)}{16\pi^2}\left(\frac{\mathcal{V}_c(\nu)}{4} + \mathcal{V}_2(\nu) + \mathcal{V}_r(\nu) + \mathbf{S}^2\mathcal{V}_s(\nu)\right) + \frac{\mathcal{V}_k(\nu)}{2}$$

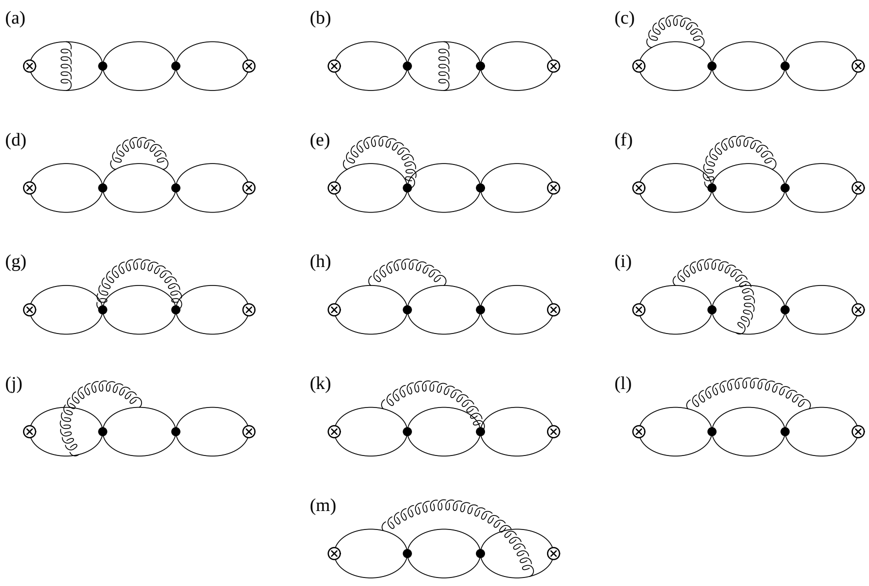
Luke,Manohar,Rothstein '99, Pineda '01
Stewart,AHH '02

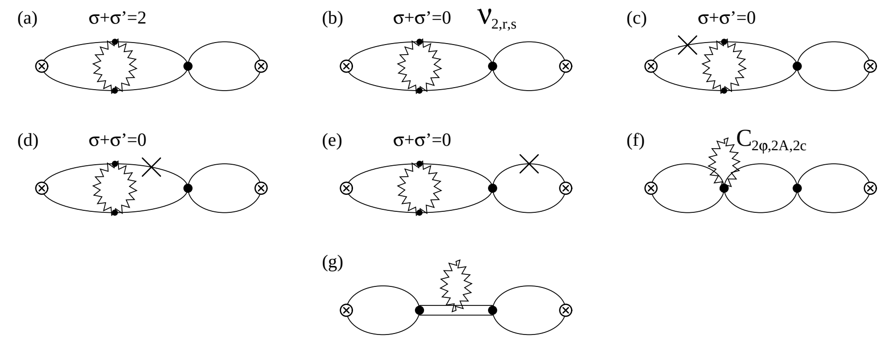
# Cross Section at NNLL Order

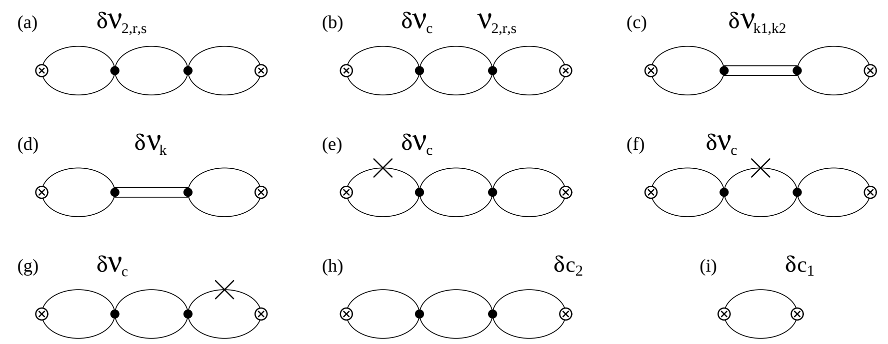
NNLL running of $C(\nu)$

(a) next-order running of couplings in NLL anom. dim. $\rightarrow$ ?

(b) genuine next-order diagrams AHH '03



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(b) genuine next-order diagrams AHH '03

$$\frac{\delta z_c^{\mathrm{NNLL},1}}{\epsilon} = \frac{\mathrm{const}}{\epsilon} + \frac{\mathrm{const}}{\epsilon} \times \ln\left(\frac{m_t\,\mu_U}{\mu_S^2}\right)$$

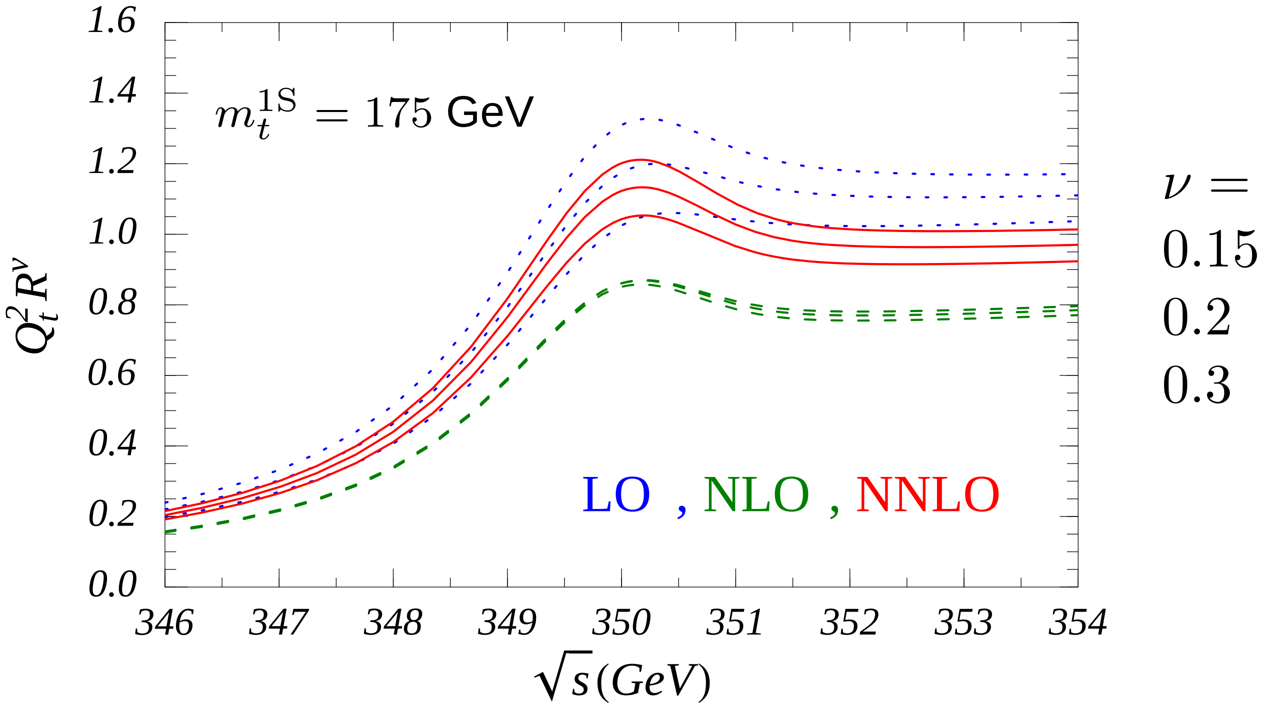
$\Rightarrow$ consistency under renormalization requires:

$$\mu_U = \mu_S^2/m_t$$

# Cross Section at NNLL Order

A.H.

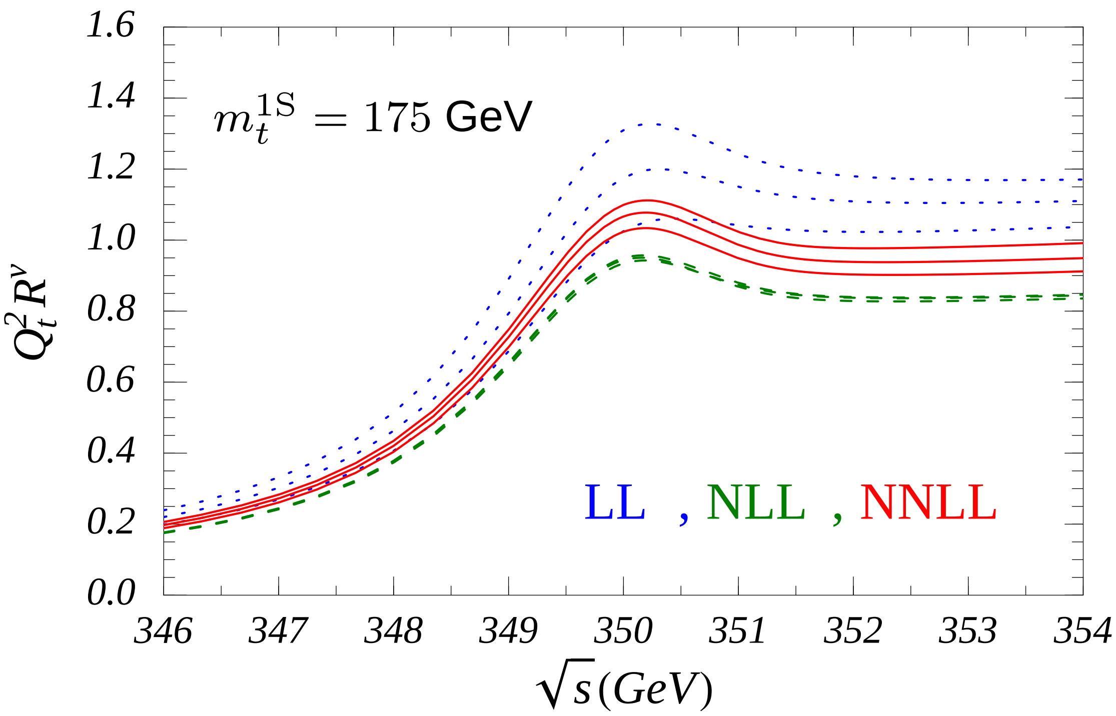
1S mass - fi xed order



# Cross Section at NNLL Order

A.H.

1S mass - RG-improved, with NNLL non-mixing terms




$\nu =$

0.15

0.2

0.3

- expansion shows better convergence
- theory error: $\delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} \sim \pm 6\%$

$\longrightarrow$ full NNLL running of $C(\nu)$ required $\rightarrow$ w.i.p.

# vNRQCD (unstable quarks)

"inclusive treatment"

⇒ Optical Theory: effective complex indices of refraction for absorptive processes

⇒ vNRQCD: contributions from real $Wb$ fi nal states included in EFT matching conditions to QCD+ew. theory (=SM)

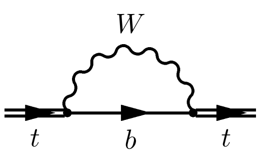
- complex matching conditions
- effective Lagrangian non-hermitian
- total rates through the optical theorem
- power counting maintained

# vNRQCD (unstable quarks)

quark bilinears: $(\rightarrow \mathcal{L}_{\text{usoft}})$

time dilatation correction

$\text{Im}\Sigma_t = \frac{1}{2}\Gamma_t$




$= i\Sigma_t \quad \Longrightarrow \quad \delta\mathcal{L} = \psi_{\mathbf{p}}^{\dagger}\left[ i\frac{\Gamma_t}{2}\left(1 - \frac{\mathbf{p}^2}{2m_t^2}\right)\right]\psi_{\mathbf{p}}$

- power counting: $\Gamma_t \propto m_t g^2 \sim m_t v^2 \quad \Rightarrow \quad \boxed{g \sim g' \sim v \sim \alpha_s}$
- fi nite lifetime is LL order, $E \rightarrow E + i\Gamma_t$ Fadin,Khoze

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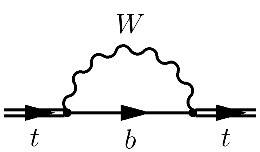


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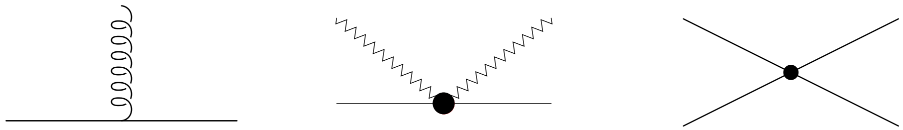
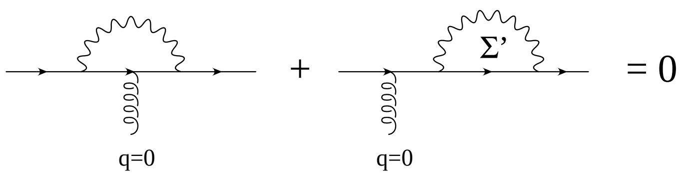
gluon interactions & potentials:



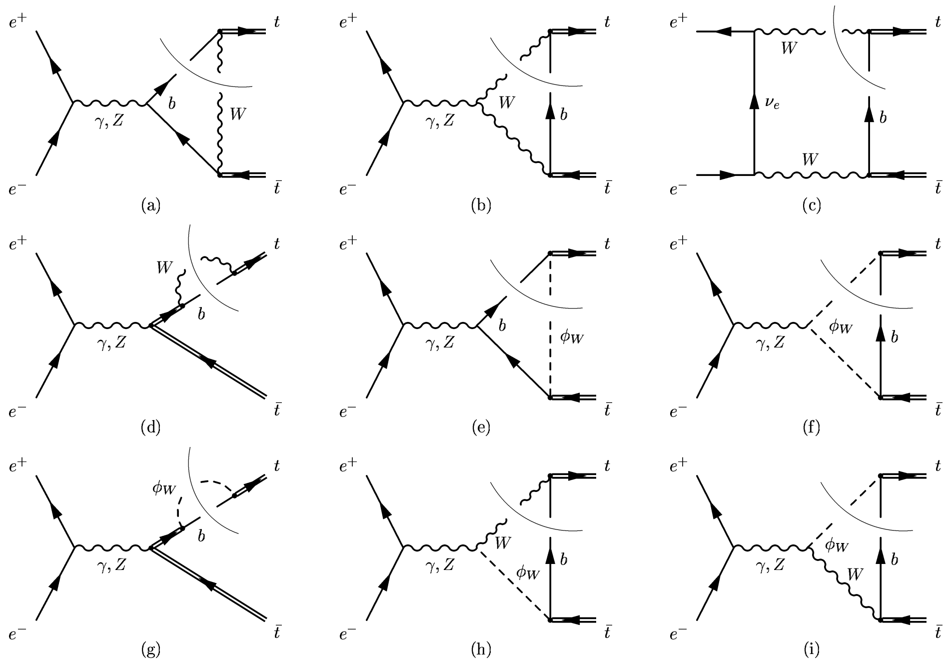



- electroweak corrections either beyond NNLL order or vanish due to gauge cancellations

$\longrightarrow$ ultrasoft gluon interference effects vanish at NLL [Khoze etal., Melnikov etal.] and NNLL order (new !)

# vNRQCD (unstable quarks)

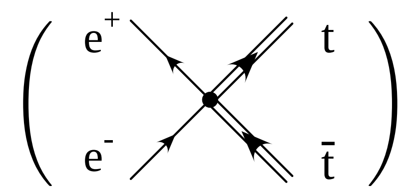
Currents:




real virtual electroweak corrections

$bW^+$ and $\bar{b}W^-$ cuts

$$\mathbf{O_p} = \left[\, C^{\rm LL} + C^{\rm NLL} + C^{\rm NNLL} + i C^{\rm NNLL}_{\rm abs} \ldots \,\right] \cdot$$

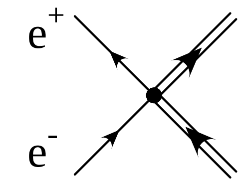



$$+ \cdots$$

# vNRQCD (unstable quarks)

Currents:

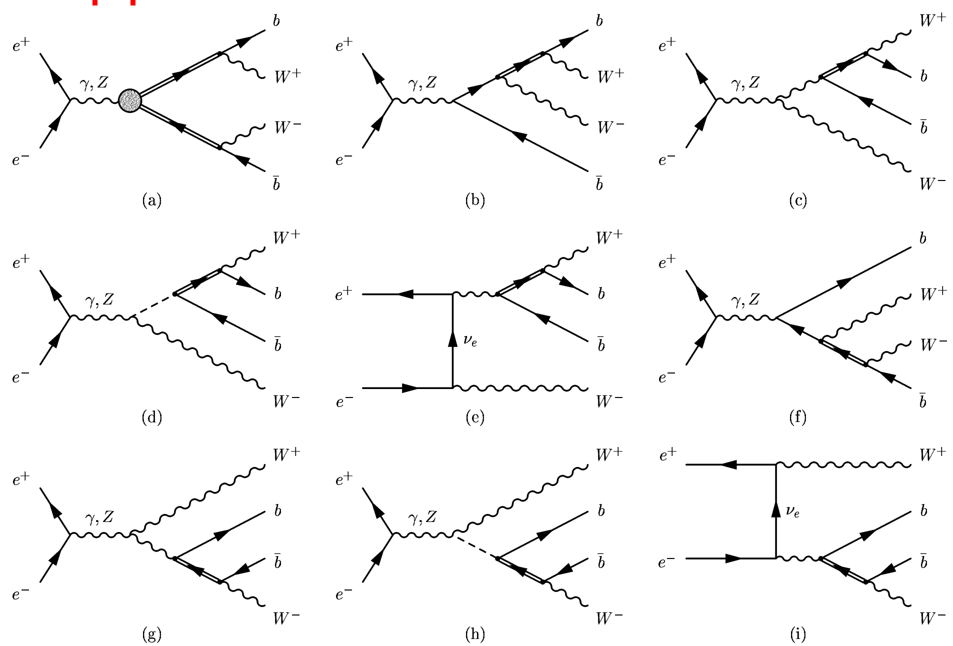
$$\mathbf{O_p} = \underbrace{\Big[ C^{\mathrm{LL}} + C^{\mathrm{NLL}} + C^{\mathrm{NNLL}} + iC^{\mathrm{NNLL}}_{\mathrm{abs}} \ldots \Big]} \cdot \Bigg(\qquad\Bigg) + \cdots$$



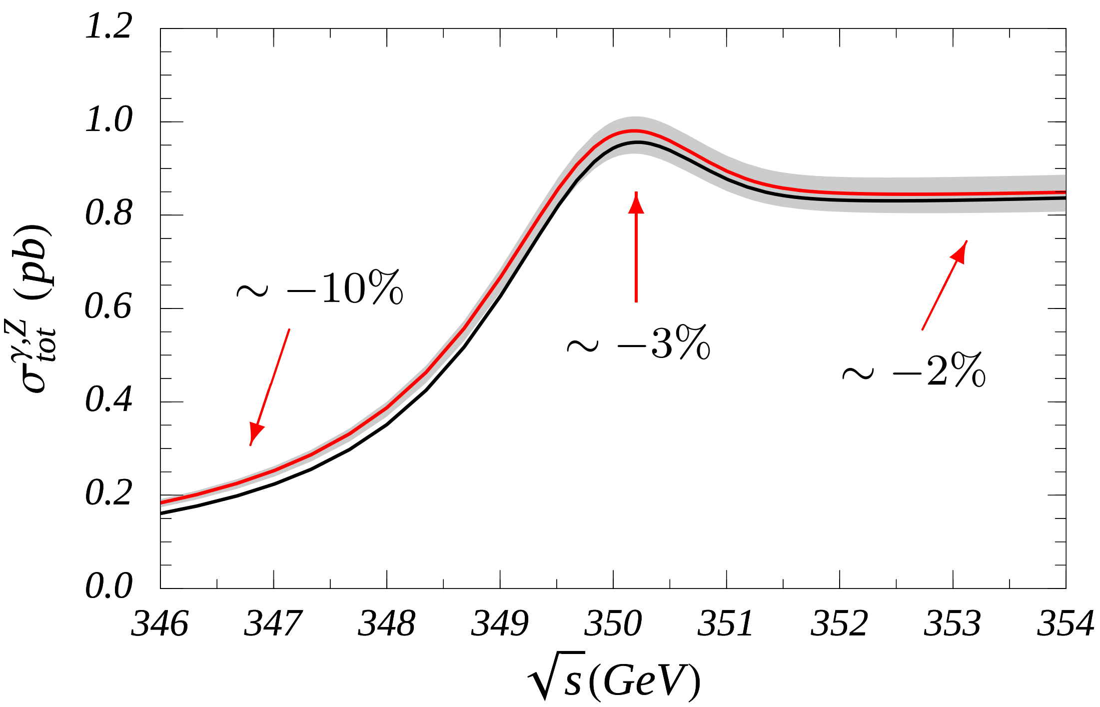

$$\sigma_{\mathrm{tot}} \;\propto\; \mathrm{Im}\left[\mathrm{C}(\nu)^2\,\mathrm{G}(0,0,\sqrt{\mathrm{s}}+\mathrm{i}\Gamma_{\mathrm{t}})\right]$$

- accounts for irreducible interference contributions:

  resonant $\leftrightarrow$ non-resonant $W^+W^-b\bar{b}$ final states



# Total Cross Section




Reisser, AHH

- also included summation of phase space logs $\sim (\alpha_s \ln v)^n$
- fi nite lifetime corrections comparable to NNLL QCD corrections
- shift in the peak position: $30 - 50$ MeV ($\delta m_t^{\rm ex} \approx 50$ MeV)

# What's left to do . . .

NNLL Total Cross Section:

(A) QCD corrections (RGE-improved)

- Current uncertainty: $(\delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}})^{\mathrm{QCD}} \sim \pm 6\%$

  $\rightarrow$ full NLL running of potentials

(B) electroweak corrections

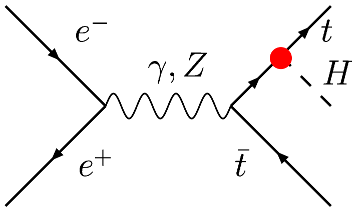
- experimental cuts
- QED effects: beam effects $\leftrightarrow$ QED dynamics

Differential Cross Sections, (multi-quark fi nal states): $\rightarrow$ effi ciencies

$\rightarrow$ more involved treatment of unstable particles

# Other Applications

$\boxed{e^+e^- \to t\bar{t}H}$ $\longrightarrow$ top-Yukawa coupling




- Theory Status: $\sigma(e^+e^- \to t\bar{t}H)$

  Born ✓ [Gaemers et al., Djouadi et al.]

  1-loop ew. ✓ [Denner et al., Belanger et al., You et al.]

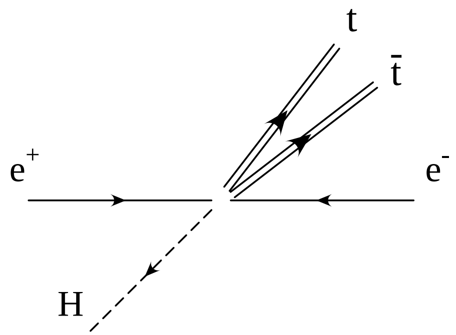
  $\mathcal{O}(\alpha_s)$ fixed-order ✓ [Dittmaier et al., Dawson et al.]

  NLL large-$E_H$ QCD endpoint corrections ✓ [C. Farrell, AHH]

# Other Applications

$\boxed{e^+e^- \longrightarrow t\bar{t}H}$

$\longrightarrow$ region of large Higgs energy




$\longrightarrow$ $t\bar{t}$ collinear

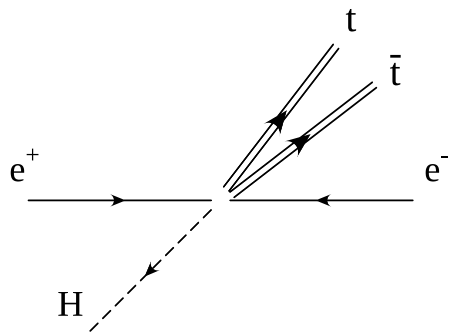
$\longrightarrow$ QCD effects localized in $t\bar{t}$ system

$\Rightarrow$ $t\bar{t}$ dynamics non-relativistic

# Other Applications

$\boxed{e^+e^- \longrightarrow t\bar{t}H}$

$\longrightarrow$ region of large Higgs energy

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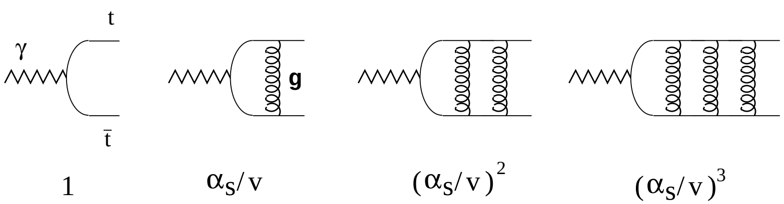
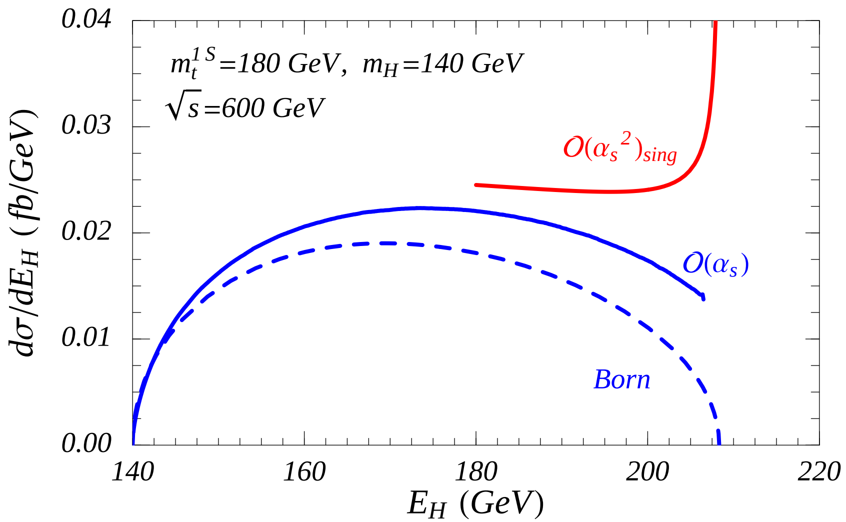



$\longrightarrow$ singularities: $\sim (\alpha_s/v)^n$,

$\sim (\alpha_s \ln v)^n$

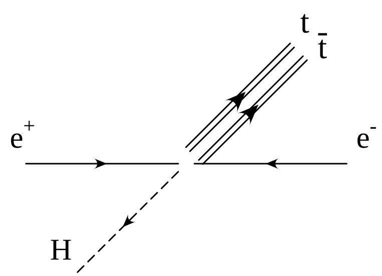
$\longrightarrow$ fixed order expansion breaks down

$\Rightarrow$ summation of singular terms

# Other Applications

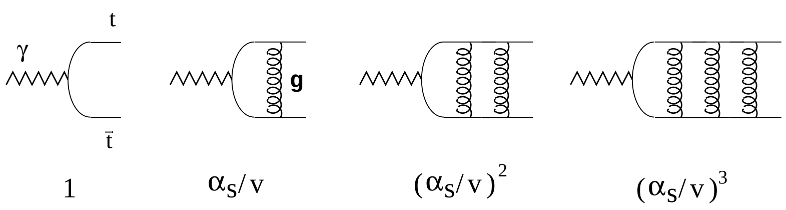
$\boxed{e^+e^- \longrightarrow t\bar{t}H}$ $\longrightarrow$ factorization formula

$$\left(\frac{d\sigma}{dE_H}\right)_{\mathrm{E_H \approx E_H^{max}}} \sim C^2(\mu, \sqrt{s}, m_t, m_H) \times \mathrm{Im}[G(0,0,v,\mu)]$$



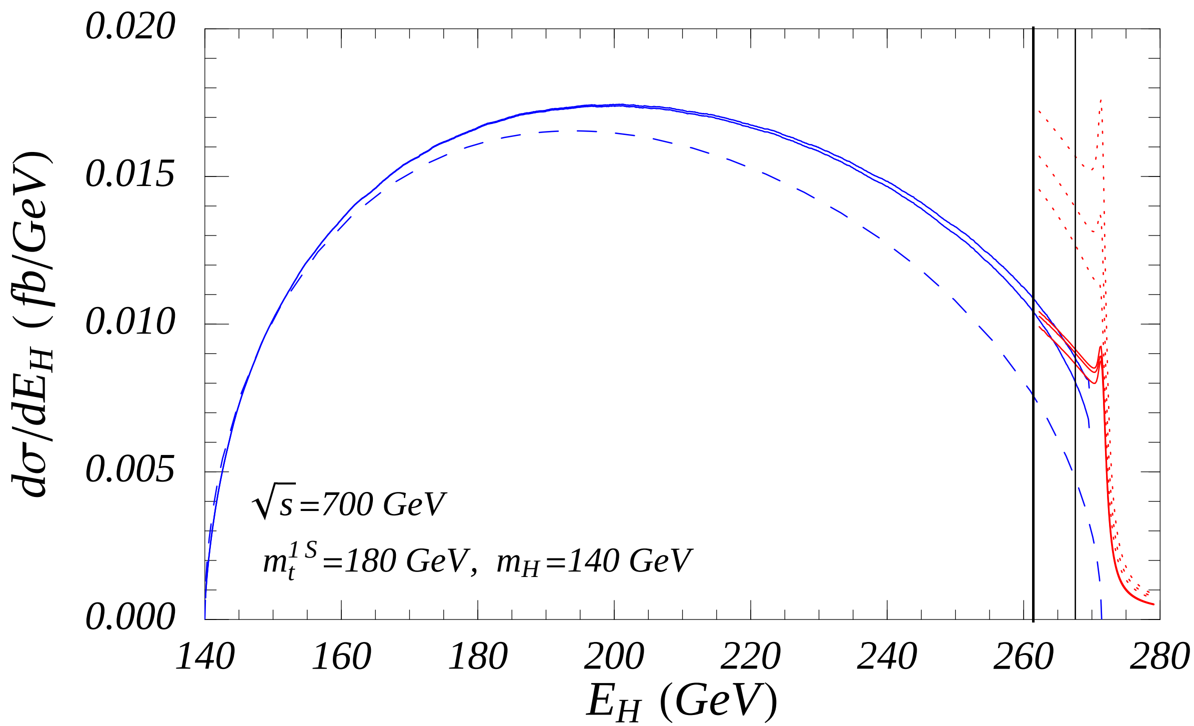

$+$ hard QCD corrections

soft & ultrasoft QCD corrections



# Other Applications

$\boxed{e^+e^- \to t\bar{t}H}$ $\to$ NLL Higgs energy spectrum

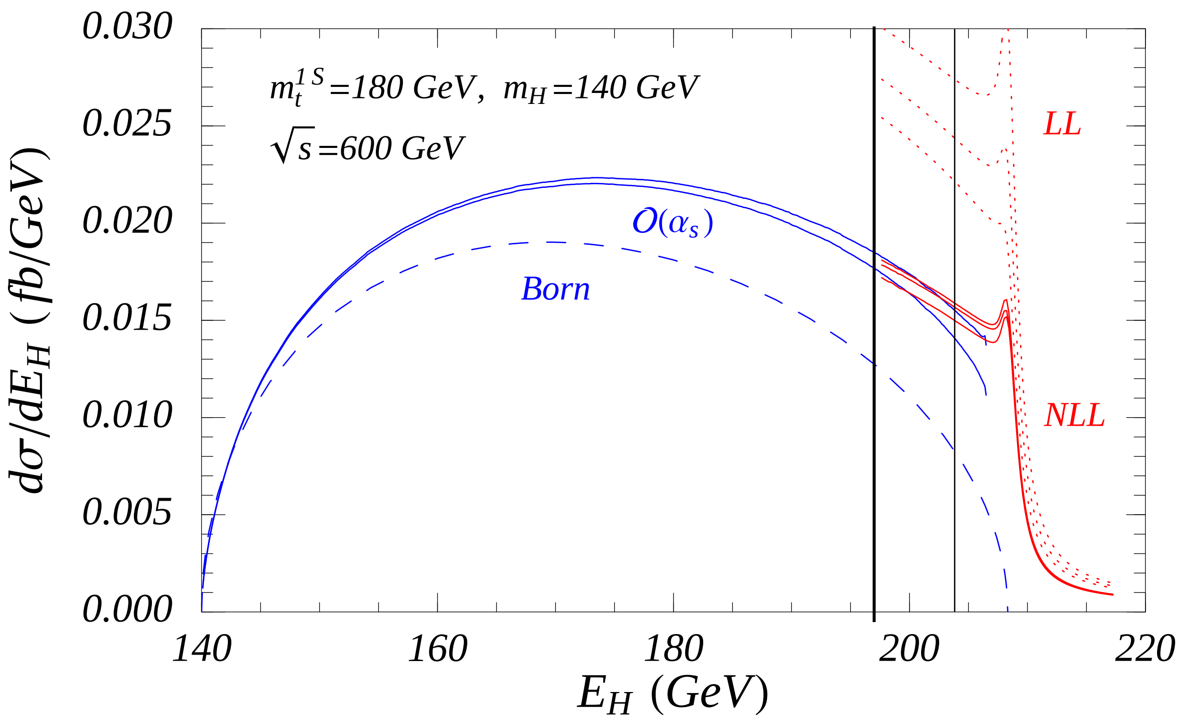



Farrell, AHH

- large $E_H$ endpoint regions increases for smaller $\sqrt{s}$ / larger $m_H$

# Other Applications

$\boxed{e^+e^- \to t\bar{t}H}$ $\to$ NLL Higgs energy spectrum

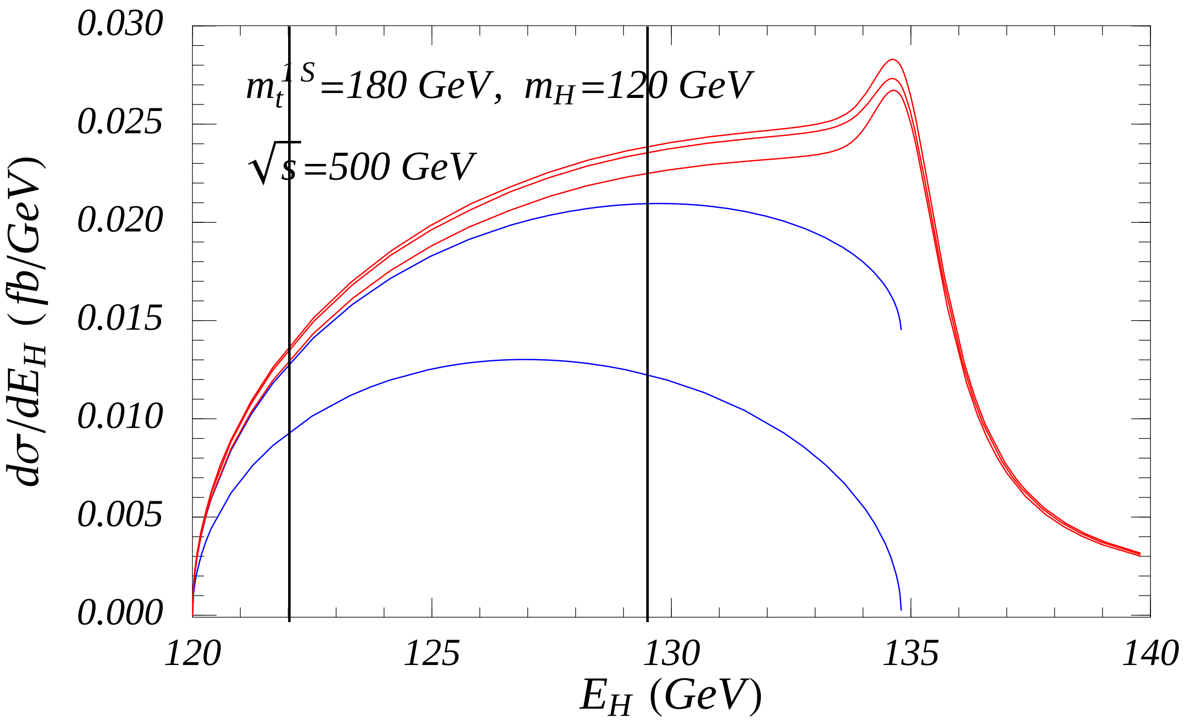



Farrell, AHH

- large $E_H$ endpoint regions increases for smaller $\sqrt{s}$ / larger $m_H$

# Other Applications

$\boxed{e^+e^- \to t\bar{t}H}$ $\quad\to$ NLL Higgs energy spectrum

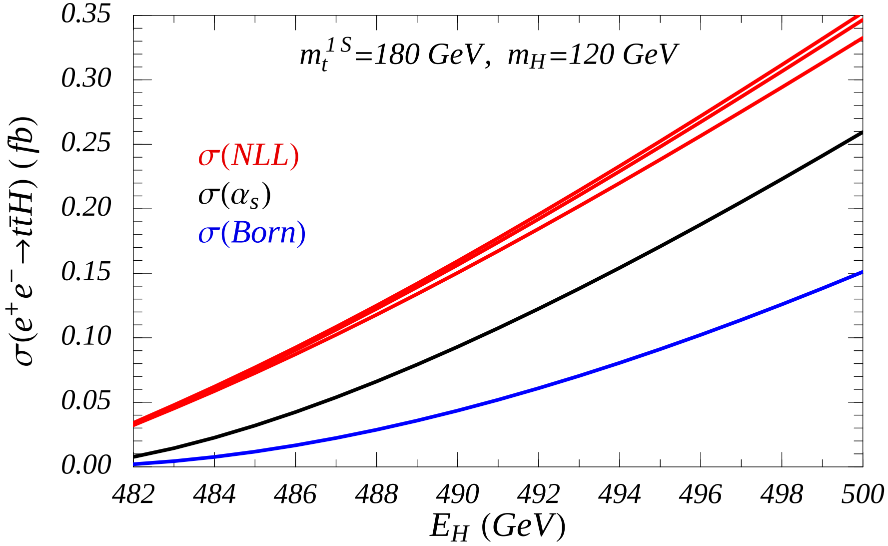


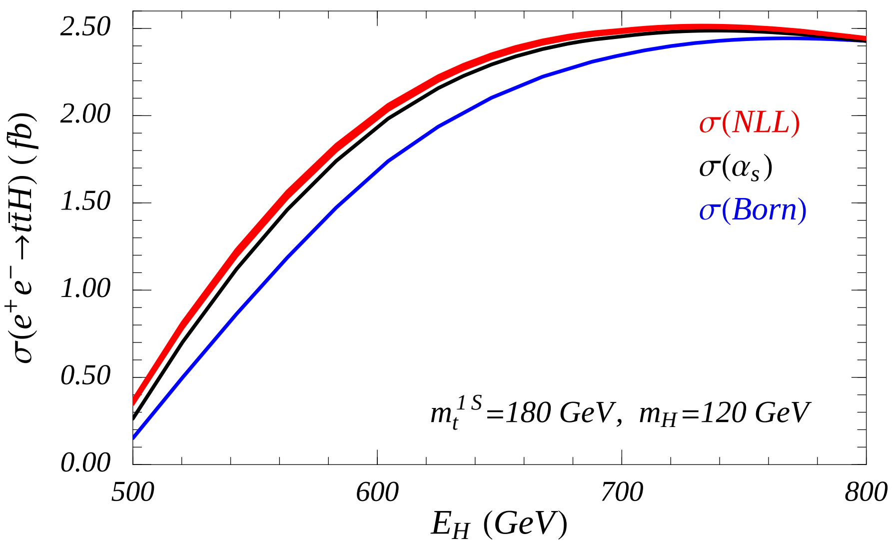

Farrell, AHH

- large $E_H$ endpoint regions increases for smaller $\sqrt{s}$ / larger $m_H$

# Other Applications

$\boxed{e^+e^- \to t\bar{t}H}$ $\to$ total cross section



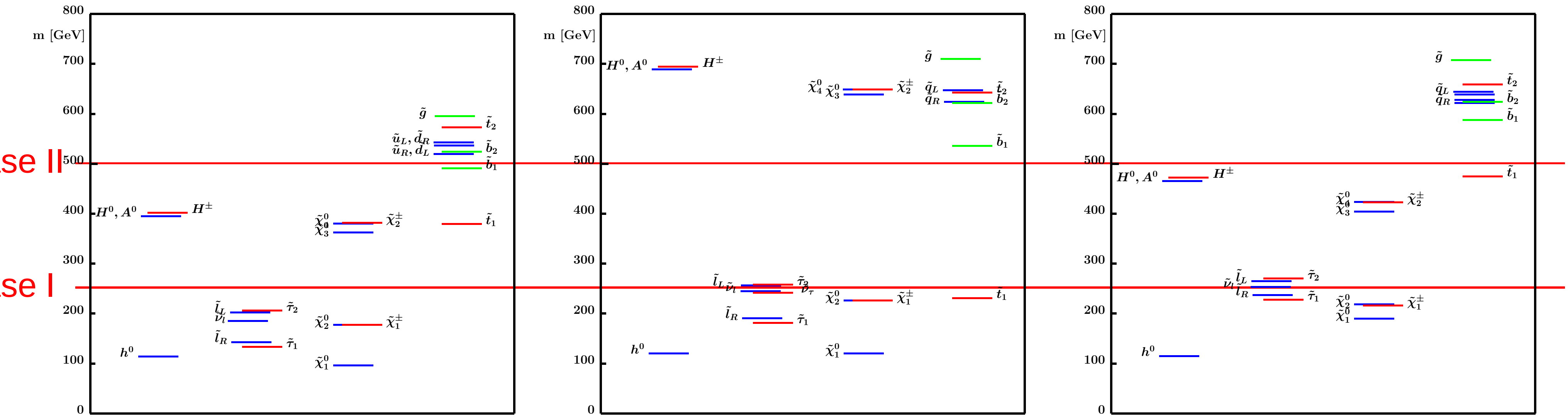

- signifi cant enhancement from summation of $(\alpha_s/v)^n$, $(\alpha_s \ln v)^n$ singularities
- essential for realistic studies for ILC (phase I)

# Other Applications

$$\boxed{e^+e^- \to \tilde{q}\bar{\tilde{q}}}$$

- in many models for SUSY breaking squark pair production is possible at the ILC

SPS 1a | SPS 5 | SPS 6




$m_{\tilde{t}_1} = 396$ GeV

$\tilde{t}_1 \to b\chi_1^+, \ldots$

$\Gamma_{\tilde{t}_1} = 1.92$ GeV

$m_{\tilde{t}_1} = 240$ GeV

$\tilde{t}_1 \to b\chi_1^+, c\chi_1^0$

$\Gamma_{\tilde{t}_1} = 0.04$ GeV

$m_{\tilde{t}_1} \simeq 490$ GeV

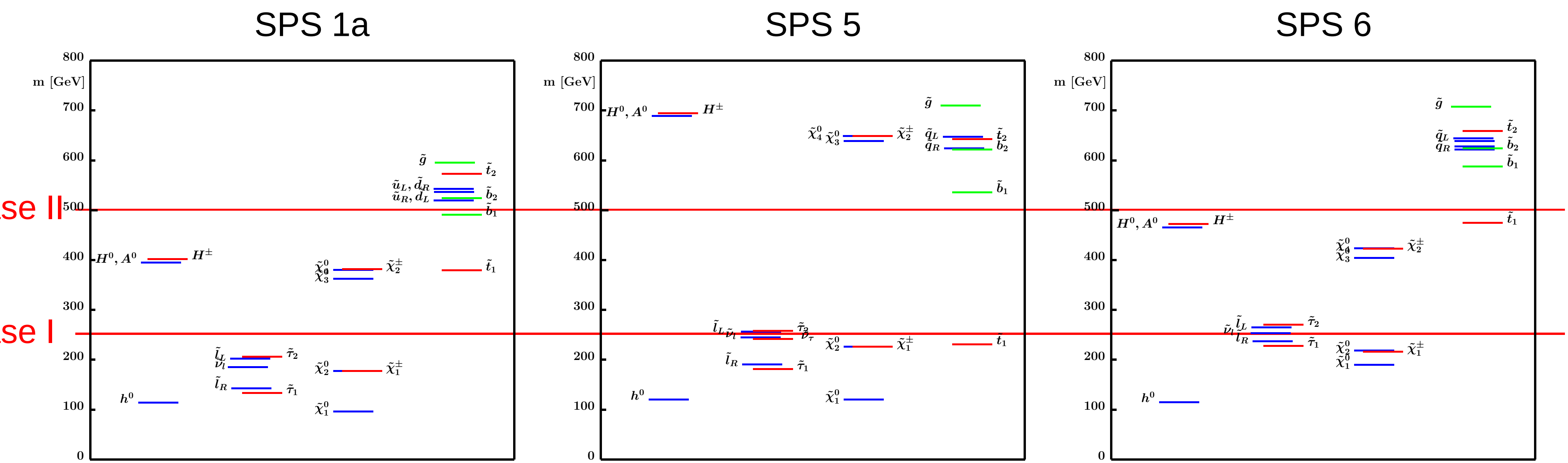
$\tilde{t}_1 \to b\chi_1^+, \ldots$

$\Gamma_{\tilde{t}_1} \simeq 3.2$ GeV

# Other Applications

$\boxed{e^+e^- \to \tilde{q}\bar{\tilde{q}}}$

- in many models for SUSY breaking squark pair production is possible at the ILC




- low-energy QCD dynamics very similar to $t\bar{t}$ threshold physics
- e.w. weak effects & phenomenology can differ significantly: $m_{\tilde{q}}, \Gamma, \ldots$
- no coherent theoretical analysis of QCD & electroweak effects exists

Ruiz-Femenia, Teubner, AHH

# Other Applications

$\boxed{e^+e^- \to \tilde{q}\bar{\tilde{q}}}$ $\to$ vNRQCD

quarks: $\Lambda = -i\frac{\mathbf{S}\cdot(\mathbf{p}'\times\mathbf{p})}{\mathbf{k}}$ , $T = \boldsymbol{\sigma}_1\boldsymbol{\sigma}_2 - \frac{3(\mathbf{k}\boldsymbol{\sigma}_1)(\mathbf{k}\boldsymbol{\sigma}_2)}{\mathbf{k}^2}$

$$V = \left[\frac{\mathcal{V}_c(\nu)}{\mathbf{k}^2} + \frac{\mathcal{V}_k(\nu)\pi^2}{m|\mathbf{k}|} + \frac{\mathcal{V}_r(\nu)(\mathbf{p}^2+\mathbf{p}'^2)}{2m^2\mathbf{k}^2} + \frac{\mathcal{V}_2(\nu)}{m^2} + \frac{\mathcal{V}_s(\nu)}{m^2}\mathbf{S}^2 + \frac{\mathcal{V}_\Lambda(\nu)}{m^2}\Lambda + \frac{\mathcal{V}_t(\nu)}{m^2}T\right]$$

$$\frac{d}{d\ln\nu}\ln C_{^3S_1}(\nu) = -\frac{\mathcal{V}_c(\nu)}{16\pi^2}\left(\frac{\mathcal{V}_c(\nu)}{4} + \mathcal{V}_2(\nu) + \mathcal{V}_r(\nu) + \mathbf{S}^2\mathcal{V}_s(\nu)\right) + \frac{\mathcal{V}_k(\nu)}{2}$$

$$C_{^3S_1}(\nu=1) = 1 - \frac{8}{3}\frac{\alpha_s(m_t)}{\pi}$$

# Other Applications

$\boxed{e^+e^- \to \tilde{q}\bar{\tilde{q}}}$ $\to$ scalar vNRQCD

squarks:

- no spin-dependent interactions
- P-wave production
- full NLL QCD running completed [P. Ruiz-Femenia, AHH]

$$V = \left[ \frac{\mathcal{V}_c(\nu)}{\mathbf{k}^2} + \frac{\mathcal{V}_k(\nu)\pi^2}{m|\mathbf{k}|} + \frac{\mathcal{V}_r(\nu)(\mathbf{p}^2+\mathbf{p}'^2)}{2m^2\mathbf{k}^2} + \frac{\mathcal{V}_2(\nu)}{m^2} \right]$$

$$\frac{d}{d\ln\nu} \ln C_{{}^1P_1}(\nu) = -\frac{\mathcal{V}_c(\nu)}{48\pi^2}\left(\frac{\mathcal{V}_c(\nu)}{4} + \mathcal{V}_2(\nu) + \mathcal{V}_r(\nu)\right) + \frac{\mathcal{V}_k(\nu)}{6}$$

$$C_{{}^1P_1}(\nu = 1) = 1 - \frac{8}{3}\frac{\alpha_s(m_t)}{\pi}$$

# Conclusion

- Top pair total rate predictions become more and more realistic
  - full set of electroweak corrections (w.i.p)
  - full NNLL QCD correction still unknown $\rightarrow$ $d\sigma/\sigma \sim \pm 6\%$
- reaching goals like $\delta m_t \sim 100$ MeV is no free lunch

  still considerable work has to be invested

  but prospects are very good

  (all remaining problems appear solvable)
- Many interesting applications of threshold physics exist.

# Colors

This is blue

This is red

This is brown

This is magenta

This is Dark Green

This is Dark Blue

This is Green

This is Cyan

Test how this color looks

Test how this color looks

Test how this color looks

Test how this color looks

Test how this color looks