Two-loop QCD Corrections to the Heavy Quark Form Factors

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Introduction

Top quark pair production at ILC
- Differential distributions in threshold region understood at NNLO
  → talks of A. Hoang, T. Teubner
- Continuum description only at NLO, insufficient for precision studies
- Observables: $d\sigma_{t\bar{t}}/dp_T$, $A_{FB}^t$, ...

Anomalous couplings of top quarks at ILC
- Observables: $g - 2$ from $e^+e^- \rightarrow t\bar{t}\gamma$, weak axial vector charges from angular distributions, ...
  → talk of U. Baur
- require precise understanding of higher order effects on Standard Model couplings
Heavy Quark Form Factors

Vector and axial vector form factors

\[ \gamma^*, Z^0 \]

\[ (-i) \left( v_Q F_1(s, m^2) \gamma^\mu + v_Q \frac{1}{2m} F_2(s, m^2) i \sigma^{\mu\nu} (p_1 + p_2)_\nu ight) \]

\[ + a_Q G_1(s, m^2) \gamma^\mu \gamma_5 + a_Q \frac{1}{2m} G_2(s, m^2) \gamma_5 (p_1 + p_2)^\mu \]

Scalar and pseudoscalar form factors

\[ H, A \]

\[ -i \frac{m}{v} \left[ S_Q F_S(s, m^2) + i P_Q F_P(s, m^2) \gamma_5 \right] \]
Two-loop QCD corrections
Two-loop QCD Form Factors

Method of calculation
W. Bernreuther, R. Bonciani, R. Heinesch, T. Leineweber, P. Mastrolia, E. Remiddi, TG

- project form factors from Feynman diagrams
- reduce loop integrals to master integrals
  - integration-by-parts equations (K. Chetyrkin, F. Tkachov)
  - Lorentz invariance equations (E. Remiddi, TG)
  - Laporta algorithm (S. Laporta)
- evaluate master integrals using differential equations
  R. Bonciani, P. Mastrolia, E. Remiddi

Renormalisation: hybrid scheme
- heavy quark mass and wave function: on-shell \( Z_{m}^{OS}, Z_{2}^{OS} \)
- QCD coupling constant: \( \overline{MS} Z_{g}^{MS}, Z_{3}^{MS} \)
- Slavnov-Taylor identity: \( Z_{1F} = Z_{g}^{MS} Z_{2}^{OS} \sqrt{Z_{3}^{MS}} \)
Anomaly contributions

Consider four mass combinations: massive/massless for inside/outside quark.

Use $d$-dimensional $\gamma_5$ (S. Larin).

All mass combinations finite after renormalisation.

All mass combinations fulfil anomalous ward identities:

$$p^\mu \Lambda^R_{Q,\mu} = 2m\Lambda^R_Q - i\frac{\alpha_s}{4\pi} T_R F^R_Q$$

Recover axial contribution to $Z^0 \to \text{hadrons}$: $G_1(s, m, 0) - G_1(s, 0, 0)$

B. Kniehl, J. Kühn
Two-loop QCD Form Factors

Results

- Form factors expressed as function of
  \[ y = \frac{\sqrt{s} - \sqrt{s - 4m^2}}{\sqrt{s} + \sqrt{s - 4m^2}} \]

  in terms of rational factors and harmonic polylogarithms up to weight 4
  
  E. Remiddi, J. Vermaseren

- Form factors expanded at threshold in \( \beta = \sqrt{1 - 4m^2/s} \) and for asymptotic energy
  in \( r = s/m^2 \)

- Results checked using
  - Ward identities
  - Partial results in literature
    - A. Hoang, J. Kühn, T. Teubner;
    - A. Czarnecki, K. Melnikov; M. Beneke, A. Signer, V.A. Smirnov
Two-loop QCD Form Factors

Threshold and high energy expansion

- consider finite anomalous form factors

- threshold expansion requires at least $\beta^2$-terms; limited range of applicability
- asymptotic energy expansion converges better; larger range of applicability
Form factors at zero momentum transfer
(→ talk of U. Baur)

\[ F_1(s = 0) = 1 \quad \text{(current conservation)} \]
\[ v_{Q}^{\gamma,Z} F_2(s = 0) = (g - 2)^{\gamma,Z} \quad \text{(anomalous magnetic moment)} \]
\[ a_{Q}^{Z} G_1(s = 0) = a_{Q}^{Z,\text{eff.}} \quad \text{(effective axial vector charge)} \]

Affect search for anomalous couplings

<table>
<thead>
<tr>
<th>( (g - 2)_Q^{(1f)} )</th>
<th>( (g - 2)_Q^{(2f)} )</th>
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<th>( (g - 2)_Q^{(2f)} )</th>
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<tbody>
<tr>
<td>( (g - 2)_Q^{(1f)} )</td>
<td>( 1.53 \cdot 10^{-2} )</td>
<td>( -1.52 \cdot 10^{-2} )</td>
<td>( -8.4 \cdot 10^{-3} )</td>
</tr>
<tr>
<td>( (g - 2)_Q^{(2f)} )</td>
<td>( 4.7 \cdot 10^{-3} )</td>
<td>( -1.00 \cdot 10^{-2} )</td>
<td>( -6.6 \cdot 10^{-3} )</td>
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<tr>
<td>( (g - 2)_Q^{(1f)} )</td>
<td>( 5.2 \cdot 10^{-3} )</td>
<td>( -1.87 \cdot 10^{-2} )</td>
<td>( -1.03 \cdot 10^{-2} )</td>
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<tr>
<td>( (g - 2)_Q^{(2f)} )</td>
<td>( 1.6 \cdot 10^{-3} )</td>
<td>( -1.24 \cdot 10^{-2} )</td>
<td>( -8.1 \cdot 10^{-3} )</td>
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Experimental sensitivity

\[ \sqrt{s} = 500 \text{ GeV}: \]
\[ (g - 2)_t^{\gamma,Z} : \pm 1.1 \cdot 10^{-2} \]
\[ (G_{1,t} - 1) : \pm 1.6 \cdot 10^{-2} \]

\[ \sqrt{s} = 800 \text{ GeV}: \]
\[ (g - 2)_t^{\gamma,Z} : \pm 0.8 \cdot 10^{-2} \]
\[ (G_{1,t} - 1) : \pm 1.6 \cdot 10^{-2} \]
Forward-backward Asymmetry

Forward-backward asymmetry of heavy quarks

- is very sensitive on the Higgs mass
- displays $2.3\sigma$ deviation for $b$-quarks at LEP1
- at NNLO, receives contributions from 2, 3 and 4 parton final states
- two-parton final state infrared finite on its own

Two-parton contribution to

$$A_{FB}^{t\bar{t}} = \frac{\sigma_A}{\sigma_S}$$
computed two-loop QCD corrections to heavy quark form factors $\gamma^*, Z^0, H, A \rightarrow Q\bar{Q}$

crucial ingredient to $d\sigma^{tt}$ and $A^{tt}_{FB}$ in the continuum

tested convergence of threshold and large energy expansion

sizable two-loop effects on determination of anomalous couplings