

Anomalous Couplings of the Higgs Boson

Tao Han

Univ. of Wisconsin–Madison

(ILC Workshop, Snowmass, Aug. 17, 2005)

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Standard Model as An Effective Theory

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Parameterize Physics Beyond the SM

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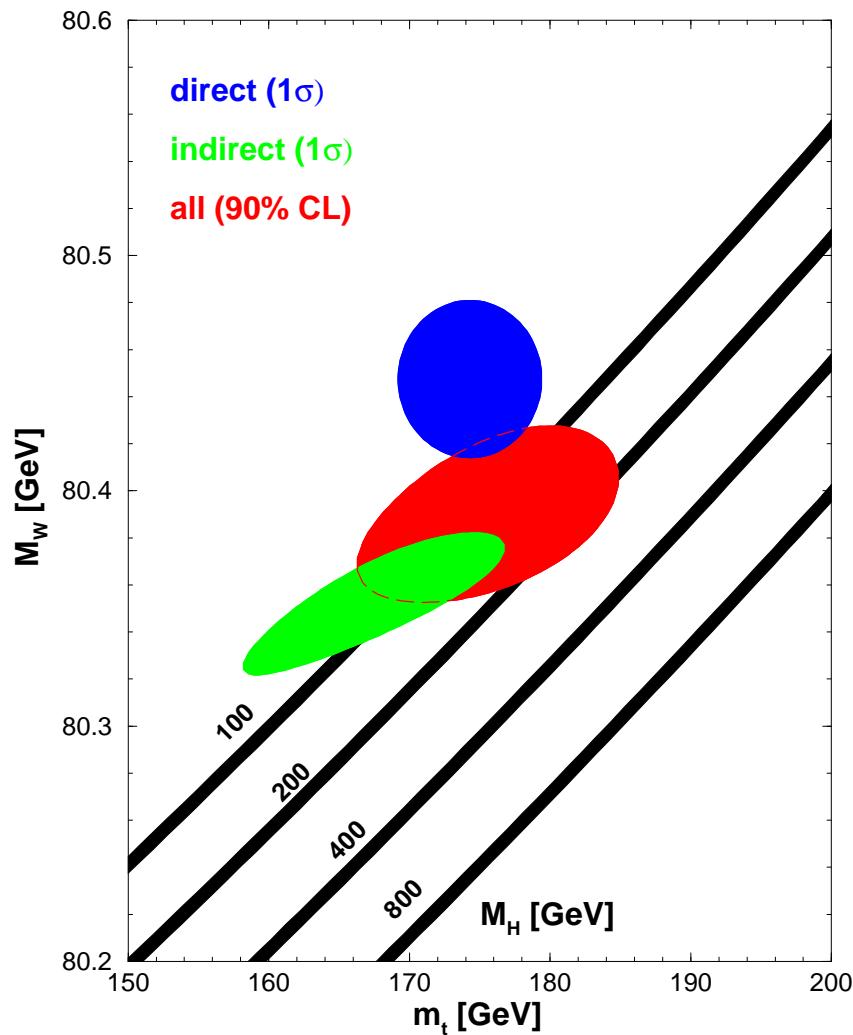
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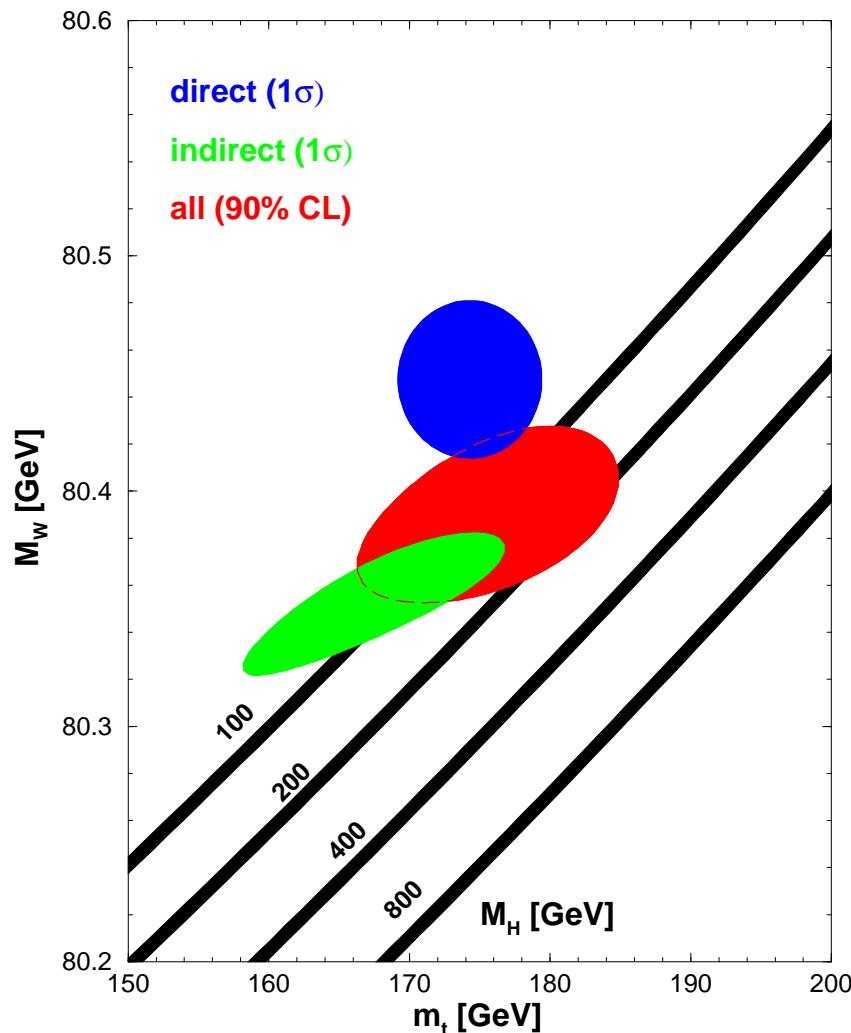
Collider Sensitivities

Concluding Remarks

SM with a light Higgs ?



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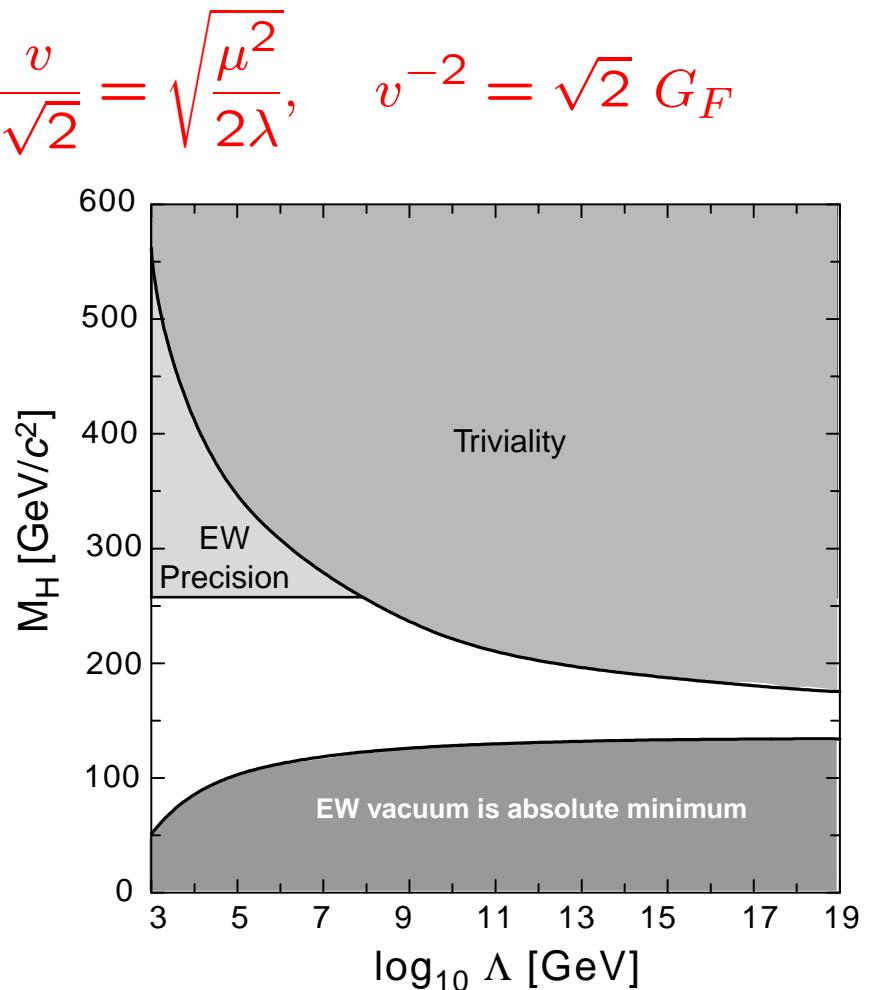
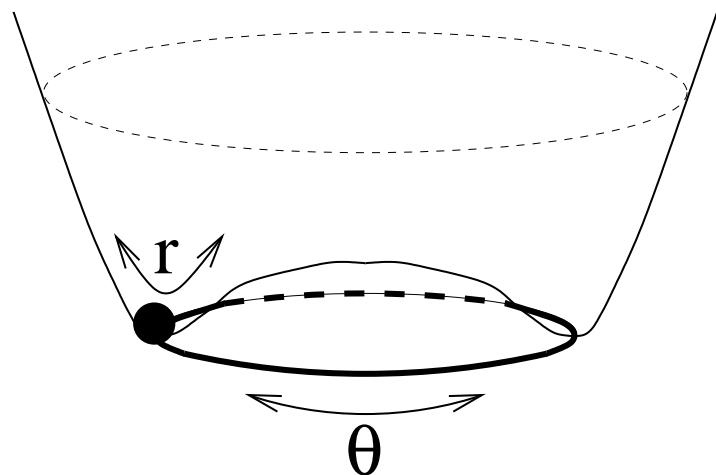
EW precision data: $m_H < 186$ GeV at 95% CL;*

*Recent fit with new m_t .

SM as an effective theory ?

$$V = -\mu^2 \Phi^2 + \lambda \Phi^4, \quad \langle \Phi \rangle = \frac{v}{\sqrt{2}} = \sqrt{\frac{\mu^2}{2\lambda}}, \quad v^{-2} = \sqrt{2} G_F$$

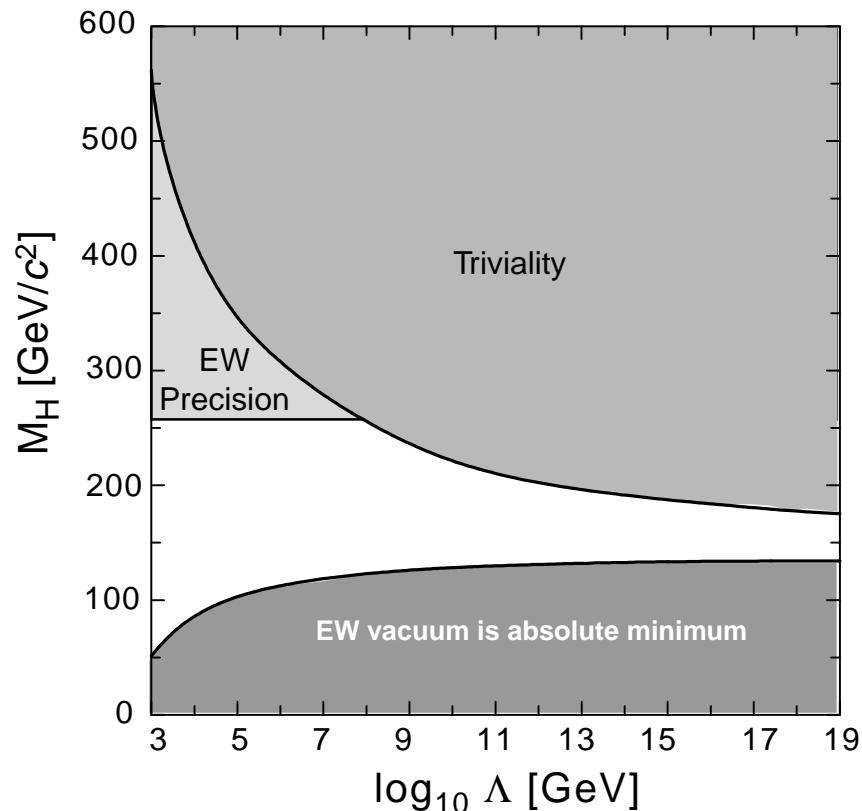
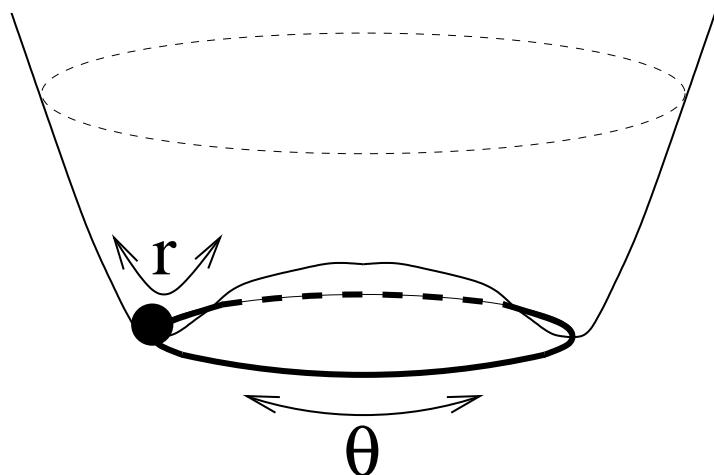
$$m_H = \sqrt{2\lambda} v$$



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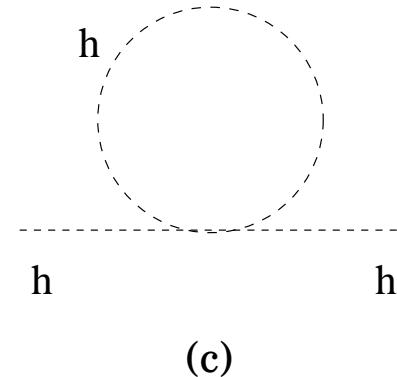
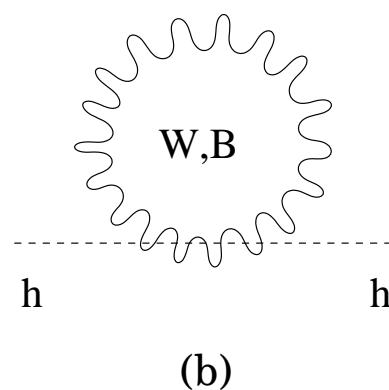
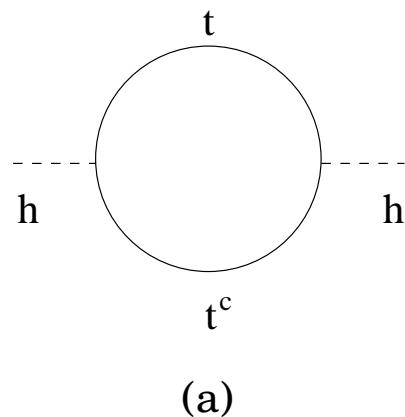


SM with a light H could be an effective theory to $\Lambda \sim M_{pl}$.

- a stable vacuum;
- non-trivial interactions;
- renormalizability ...

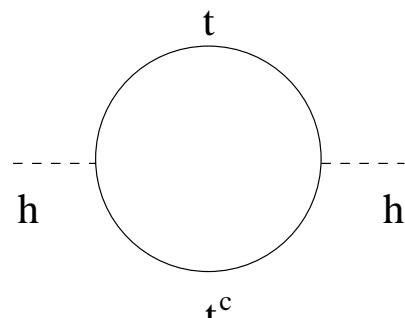
Unnatural ?

Due to quantum corrections, the Higgs mass is quadratically sensitive to the new physics (cutoff) scale: $\sim \Lambda^2$.

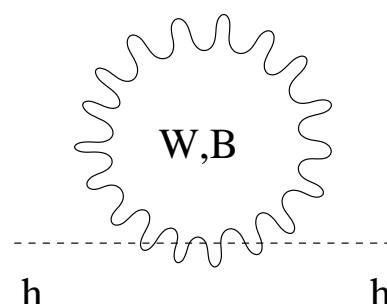


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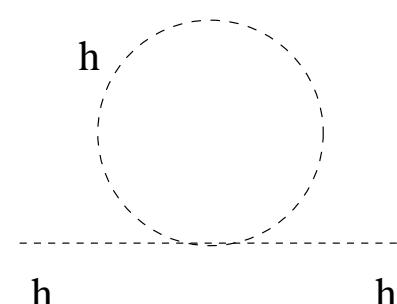
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(a)



(b)

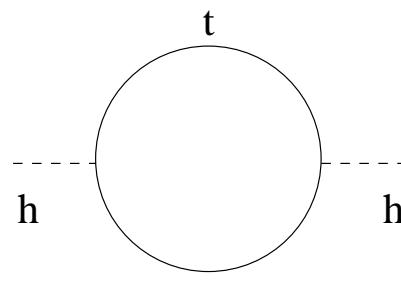


(c)

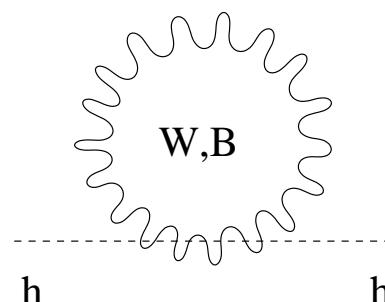
$$m_H^2 = m_{H0}^2 - \frac{3}{8\pi^2} y_t^2 \Lambda^2 + \frac{1}{16\pi^2} g^2 \Lambda^2 + \frac{1}{16\pi^2} \lambda^2 \Lambda^2$$

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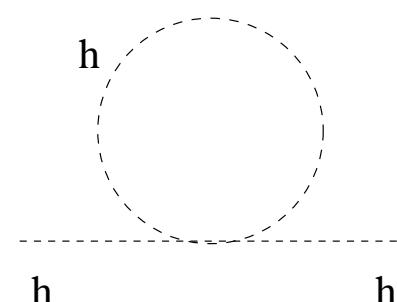
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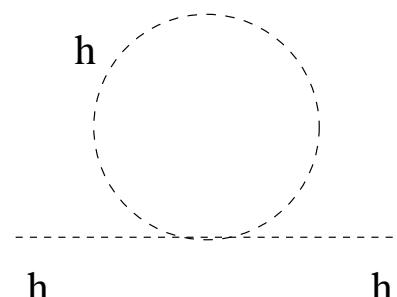
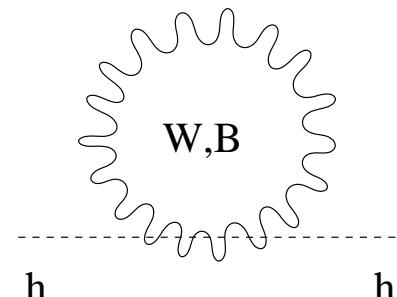
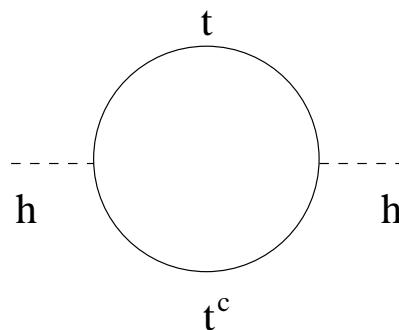
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$$(200 \text{ GeV})^2 = m_{H0}^2 + [-(2 \text{ TeV})^2 + (700 \text{ GeV})^2 + (500 \text{ GeV})^2] \left(\frac{\Lambda_{t,W,H}}{10 \text{ TeV}} \right)^2$$

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Naturalness requirement: less than 90% cancellation on m_H^2

$$\Lambda_t \lesssim 3 \text{ TeV} \quad \Lambda_W \lesssim 9 \text{ TeV} \quad \Lambda_H \lesssim 12 \text{ TeV}$$

Beyond the SM ϕ^4 Theory

Below a new physics scale Λ , the effective interactions at the NLO

$$\mathcal{L}_{\text{eff}}^{\text{dim-6}} = \sum_n \frac{f_n}{\Lambda^2} \mathcal{O}_n,$$

where f_n 's are dimensionless “anomalous couplings”.

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There are eleven SM gauge-invariant bosonic operators;*
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Class I

Among those, two of them contribute to two-point functions:

$$\begin{aligned}\mathcal{O}_{BW} &= \Phi^\dagger \hat{B}_{\mu\nu} \hat{W}^{\mu\nu} \Phi \implies S, \\ \mathcal{O}_{\Phi,3} &= (D_\mu \Phi)^\dagger \Phi^\dagger \Phi (D^\mu \Phi) \implies T.\end{aligned}$$

Tight constraints, at 95% CL:[†]

$$-1 < \frac{f_{BW}}{(\Lambda/\text{TeV})^2} < 8, \quad -0.07 < \frac{f_{\Phi,3}}{(\Lambda/\text{TeV})^2} < 0.6.$$

will not pursue them further.

*K. Hagiwara et al., PRD48, 2182 (1993); ...

[†]M.C. Gonzalez-Garcia, hep-ph/9902321.

Class II

Four of them lead to $V - H$ interactions:

$$\begin{aligned}\mathcal{O}_{WW} &= \Phi^\dagger \hat{W}_{\mu\nu} \hat{W}^{\mu\nu} \Phi, \\ \mathcal{O}_{BB} &= \Phi^\dagger \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} \Phi, \\ \mathcal{O}_W &= (D_\mu \Phi)^\dagger \hat{W}^{\mu\nu} (D_\nu \Phi), \\ \mathcal{O}_B &= (D_\mu \Phi)^\dagger \hat{B}^{\mu\nu} (D_\nu \Phi).\end{aligned}$$

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as

$$\begin{aligned}\mathcal{L}_{\text{eff}}^H = & g_{H\gamma\gamma} H A_{\mu\nu} A^{\mu\nu} + g_{HZ\gamma}^{(1)} A_{\mu\nu} Z^\mu \partial^\nu H + g_{HZ\gamma}^{(2)} H A_{\mu\nu} Z^{\mu\nu} \\ & + g_{HZZ}^{(1)} Z_{\mu\nu} Z^\mu \partial^\nu H + g_{HZZ}^{(2)} H Z_{\mu\nu} Z^{\mu\nu} \\ & + g_{HWW}^{(1)} (W_{\mu\nu}^+ W^{-\mu} \partial^\nu H + \text{h.c.}) + g_{HWW}^{(2)} H W_{\mu\nu}^+ W^{-\mu\nu},\end{aligned}$$

with

$$\begin{aligned}g_{H\gamma\gamma} &= -\frac{s^2(f_{BB} + f_{WW})}{2} K, \quad g_{HZ\gamma}^{(1)} = \frac{s(f_W - f_B)}{2c} K, \quad g_{HZ\gamma}^{(2)} = \frac{s(s^2 f_{BB} - c^2 f_{WW})}{c} K, \\ g_{HZZ}^{(1)} &= \frac{c^2 f_W + s^2 f_B}{2c^2} K, \quad g_{HZZ}^{(2)} = -\frac{s^4 f_{BB} + c^4 f_{WW}}{2c^2} K, \\ g_{HWW}^{(1)} &= \frac{1}{2} f_W K, \quad g_{HWW}^{(2)} = -f_{WW} K,\end{aligned}$$

where $s \equiv \sin \theta_W$, $c \equiv \cos \theta_W$ and $K = gm_W/\Lambda^2 \approx 0.053 \text{ TeV}^{-1} \approx 1/(19 \text{ TeV})$.

Current constraints not strong:

By precision electroweak data and one-loop calculations:

$$\begin{aligned} -4 < \frac{f_B}{(\Lambda/\text{TeV})^2} < 2, \quad -6 < \frac{f_W}{(\Lambda/\text{TeV})^2} < 5, \\ -17 < \frac{f_{BB}}{(\Lambda/\text{TeV})^2} < 20, \quad -5 < \frac{f_{WW}}{(\Lambda/\text{TeV})^2} < 6. \end{aligned}$$

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- LEP II Higgs search bound, but for $m_H < 113$ GeV;
- Triple gauge boson coupling constraints weaker — will improve;
- Partial wave unitarity bounds weaker as well.

Class III

The last two are pure H terms:

$$\begin{aligned}\mathcal{O}_{\Phi,1} &= \frac{1}{2} \partial^\mu (\Phi^\dagger \Phi) \partial_\mu (\Phi^\dagger \Phi), \\ \mathcal{O}_{\Phi,2} &= \frac{1}{3} (\Phi^\dagger \Phi)^3.\end{aligned}$$

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In terms of the canonically normalized Higgs field (via $a_1 = f_{\Phi,1} v^2 / \Lambda^2$) and the physical Higgs mass (via a_1, a_2),[‡]

$$\begin{aligned}\mathcal{L}_{VH} &= (M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu) \left((1 - \frac{a_1}{2}) \frac{2H}{v} + (1 - a_1) \frac{H^2}{v^2} \right), \\ \mathcal{L}_{H^3} &= -\frac{m_H^2}{2v} \left((1 - \frac{a_1}{2} + \frac{2a_2}{3} \frac{v^2}{m_H^2}) H^3 - \frac{2a_1 H \partial_\mu H \partial^\mu H}{m_H^2} \right), \\ \mathcal{L}_{H^4} &= -\frac{m_H^2}{8v^2} \left((1 - a_1 + \frac{4a_2 v^2}{m_H^2}) H^4 - \frac{4a_1 H^2 \partial_\mu H \partial^\mu H}{m_H^2} \right).\end{aligned}$$

[‡]V. Barger et al., hep-ph/0301097.

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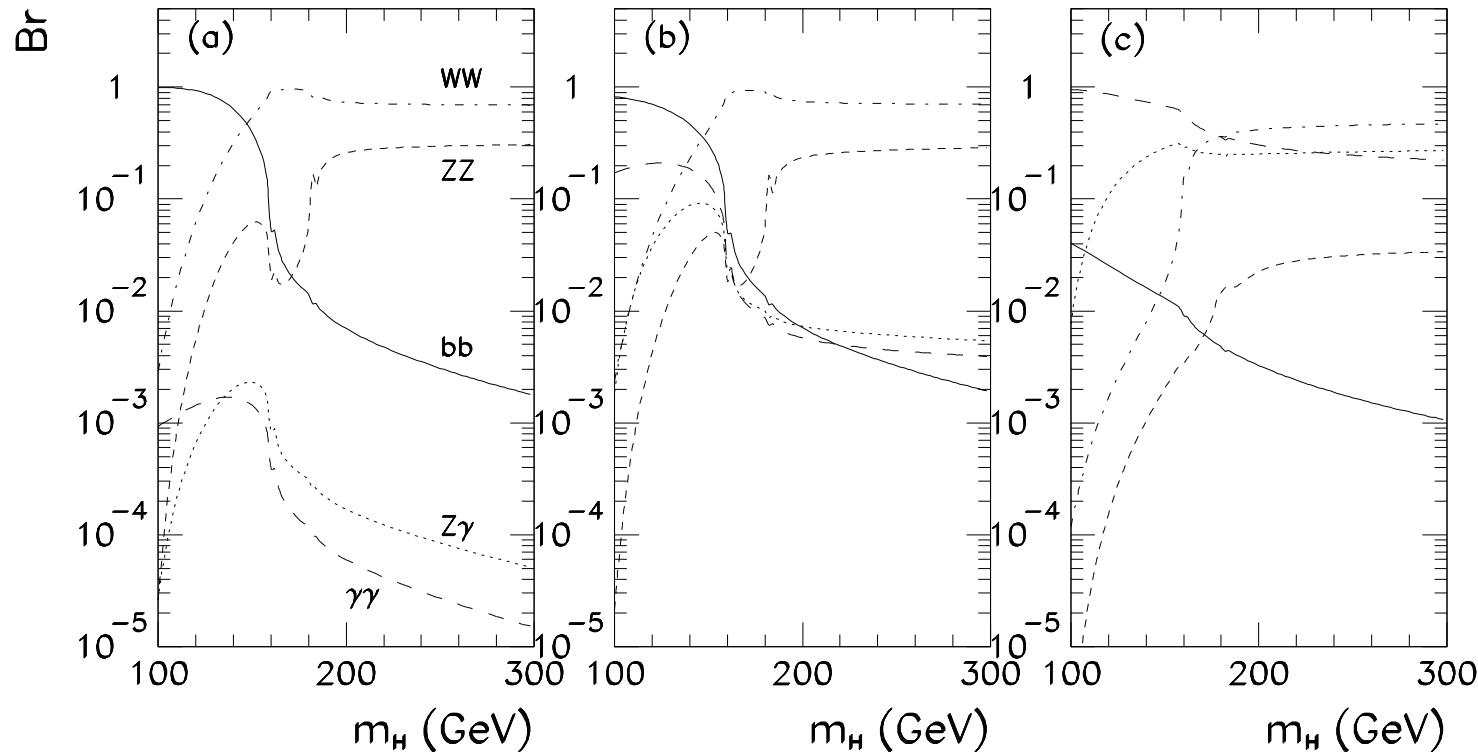
- Gauge-invariant formulation important !
- Not only a change in Higgs potential, but also kinetic corrections !
- VVH probe $f_{\Phi,1}$; H^3 (related to H^4) probes $f_{\Phi,2}$ ($f_{\Phi,1}$ as well).

[†]V. Barger et al., hep-ph/0301097.

Collider Sensitivity to the anomalous Couplings

In the case of Class II: g_{HVV}

H decays modified:



where (a)=SM; (b)= $f/\Lambda^2 = 10/\text{TeV}^2$; (c)= $f/\Lambda^2 = 100/\text{TeV}^2$.

H Production Changed:

$$\begin{aligned} pp(\bar{p}) &\rightarrow jjH \rightarrow jj\ \gamma\gamma, \\ &\rightarrow jjH \rightarrow jj\ Z\gamma, \\ &\rightarrow \gamma\gamma\gamma, \ \gamma\gamma Z, \\ &\rightarrow \gamma\gamma + \cancel{E}_T. \end{aligned}$$

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 \end{aligned}$$

Tevatron with 1 fb⁻¹ [10 fb⁻¹]

m_H (GeV)	f/Λ^2 (TeV ⁻²)
COMBINED	
100	(-7.6 , 19)[-3 , 5.6]
120	(-7.4 , 18)[-3.3 , 5.9]
140	(-9.1 , 20)[-4.0 , 8.7]
160	(-9.9 , 22)[-5.1 , 13]
180	(-24 , 33)[-16 , 24]
200	(-32 , 39)[-17 , 23]
220	(-42 , 45)[-19 , 26]

LHC $WW \rightarrow WW$, §

$$-1.4 < \frac{f_W}{\Lambda^2} \leq 1.2, \quad -2.2 \leq \frac{f_{WW}}{\Lambda^2} < 2.2.$$

§B. Zhang, Y.-P. Kuang et al., Phys.Rev.**D67**, 114024 (2003).

$$\begin{aligned}
e^+ e^- &\rightarrow jjH \rightarrow jj \gamma\gamma, \\
&\rightarrow jjH \rightarrow jj Z\gamma, \\
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&\rightarrow W^+W^-\gamma, ZZ\gamma.
\end{aligned}$$

ILC with $\sqrt{s} = 500$ GeV 100 fb^{-1}

$m_H(\text{GeV})$	$f/\Lambda^2(\text{TeV}^{-2})$	
	$e^+e^- \rightarrow W^+W^-\gamma$ at NLC	$e^+e^- \rightarrow Z^0Z^0\gamma$ at NLC
170	(-2.3 , 3.7)	(— , —)
200	(-3.2 , 4.0)	(-2.6 , 3.9)
250	(-4.3 , 4.8)	(-3.2 , 4.3)
300	(-6.3 , 6.3)	(-4.7 , 5.2)
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$$\gamma\gamma \rightarrow ZZ$$

For $\sqrt{s_{ee}} = 500$ GeV ($m_H = 115 - 200$ GeV) :

$$-0.65 \text{ TeV}^{-2} < f/\Lambda^2 < 1.7 \text{ TeV}^{-2} \quad \text{at } 2\sigma.$$

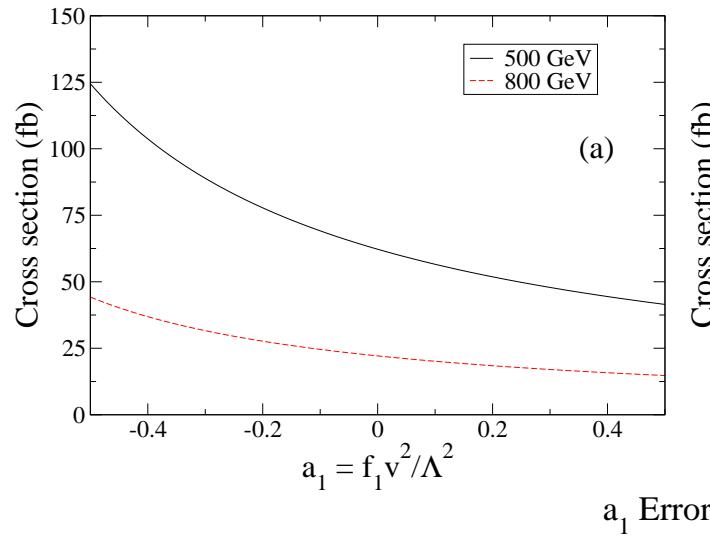
M.C. Gonzalez-Garcia, hep-ph/9902321.

B.Zhang, Y.-P. Kuang, et al. in progress.

In the case of Class III

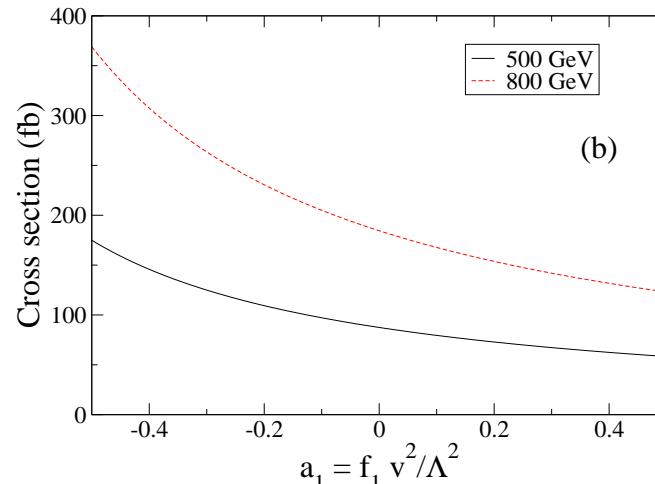
$$\partial^\mu (\Phi^\dagger \Phi) \partial_\mu (\Phi^\dagger \Phi) : \quad (M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu) \frac{2H}{v} (1 - \frac{a_1}{2})$$

$e^+ e^- \rightarrow Z H$

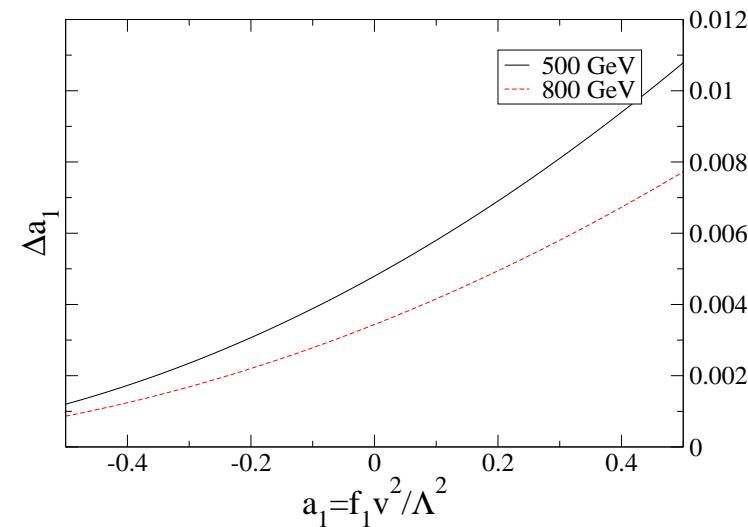


(a)

$e^+ e^- \rightarrow v \bar{v} H$



(b)



a_1 determined better than 2%.

Genuine self-couplings:

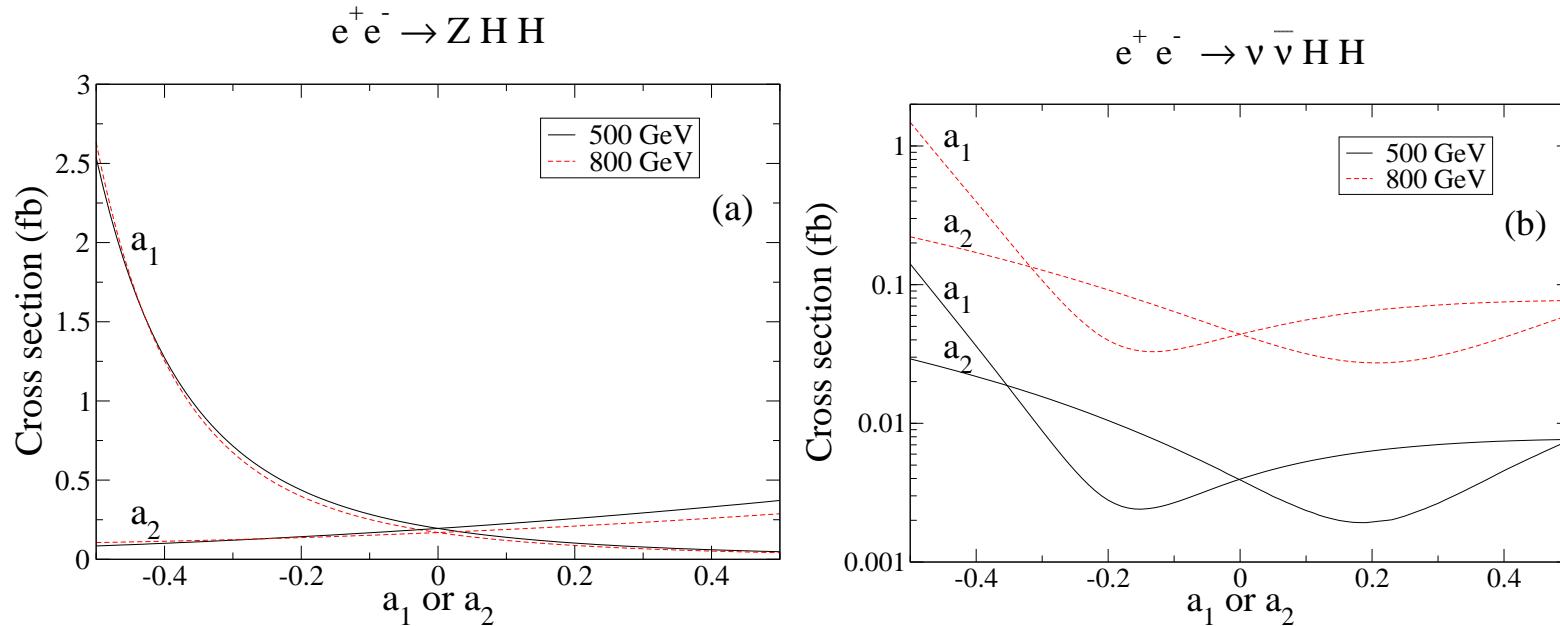
$$(\Phi^\dagger \Phi)^3 : \quad (M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu) \frac{H^2}{v^2} (1-a_1) - \frac{m_H^2}{2v} \left((1-\frac{a_1}{2} + \frac{2a_2}{3} \frac{v^2}{m_H^2}) H^3 - \frac{2a_1 H \partial_\mu H \partial^\mu H}{m_H^2} \right)$$

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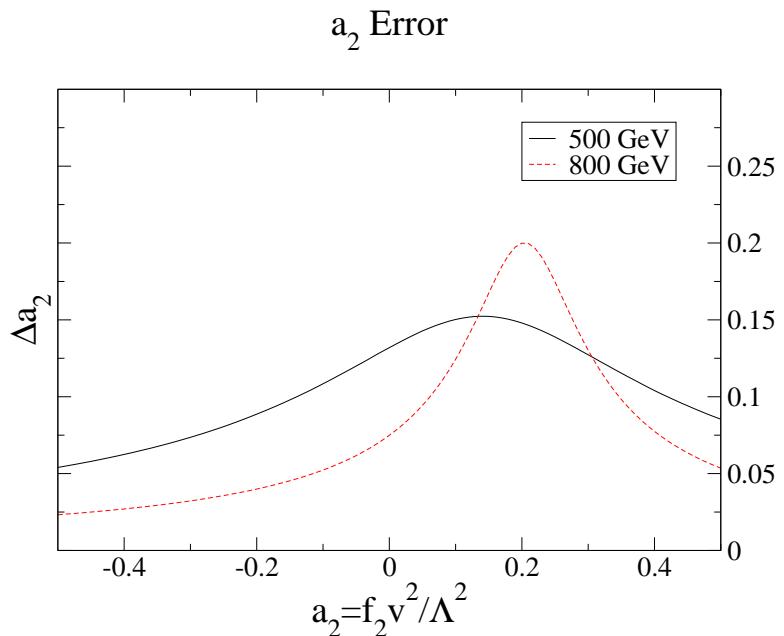
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At the LHC: very hard to observe HH , unless $H \rightarrow WW, ZZ$.*

At the ILC, only event rate matters. $m_H = 120$ GeV:



*U.Baur, T.Plehn, D.Rainwater, Phys.Rev.D67, 033003 (2003);
ibid. D68, 033001 (2003).



Translate to the effective triple coupling:

$$\frac{\delta g_{HHH}}{g_{HHH}} = \frac{2v^2 \delta a_2}{3m_H^2 + 2v^2 a_2} \approx 2.8 \Delta a_2.$$

SM corresponds to $g_{HHH}|_{a_2=0}$,
so at the ILC with $\sqrt{s} = 500$ GeV $m_h = 120$ GeV, **

Luminosity	500 fb ⁻¹	1 ab ⁻¹	2 ab ⁻¹
$\delta g_{HHH}/g_{HHH}$	42%	30%	20%
Δa_2	0.15	0.11	0.073

**C. Castanier, P. Gay, P. Lutz and J. Orloff, [hep-ex/0101028](#).

Concluding Remarks

- Consider only a light Higgs below $\Lambda \lesssim 4\pi v$:
BSM \implies “anomalous Higgs coupling”.
- Emphasize the gauge-invariant formulation in $1/\Lambda^2$ -expansion:
Unify our language !

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- Fermionic operators should be included in further studies.

Still more to do ...