Rare $K \to \pi\nu\bar{\nu}$ Decays

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We present a concise review of the theoretical status of rare $K \to \pi\nu\bar{\nu}$ decays in the standard model (SM). Particular attention is thereby devoted to the recent calculation of the next-to-next-to-leading order (NNLO) corrections to the charm quark contribution of $K^+ \to \pi^+\nu\bar{\nu}$, which removes the last relevant theoretical uncertainty from the $K \to \pi\nu\bar{\nu}$ system.

1. INTRODUCTION

The rare processes $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ play an outstanding role in the field of flavor changing neutral current transitions. The main reason for this is their unmatched theoretical cleanliness and their large sensitivity to short-distance (SD) effects arising in the SM and its innumerable extensions. As they offer a very precise determination of the unitarity triangle (UT) [1], a comparison of the information obtained from the $K \to \pi\nu\bar{\nu}$ system with the one from B-decays provides a completely independent and therefore critical test of the Cabibbo-Kobayashi-Maskawa (CKM) mechanism. Even if these K- and B-physics predictions agree, the $K \to \pi\nu\bar{\nu}$ transitions will play a leading, if not the leading part in discriminate between different extensions of the SM [2], as they allow to probe effective scales of new physics operators of up to a several TeV or even higher in a pristine manner.

2. BASIC PROPERTIES OF $K \to \pi\nu\bar{\nu}$

The striking theoretical cleanliness of the $K \to \pi\nu\bar{\nu}$ decays is linked to the fact that, within the SM, these processes are mediated by electroweak (EW) amplitudes of $\mathcal{O}(G_F^2)$, which exhibit a hard Glashow-Iliopoulos-Maiani cancellation of the form

$$\mathcal{A}_q(s \to d\nu\bar{\nu}) \propto \lambda_q m_q^2 \propto \begin{cases} m_t^2(\lambda^5 + i\lambda^5)\,, & q = t\,, \\ m_c^2(\lambda + i\lambda^5)\,, & q = c\,, \\ \Lambda_{\rm QCD}^2 \lambda\,, & q = u\,, \end{cases} \tag{1}$$

where $\lambda_q = V_{qs}^* V_{qd}$ denotes the relevant CKM factors and $\lambda = |V_{us}| = 0.2248$ is the Cabibbo angle. This peculiar property implies that the corresponding rates are SD dominated, while long-distance (LD) effects are highly suppressed. A related important feature, following from the EW structure of the SM amplitudes as well, is that the $K \to \pi\nu\bar{\nu}$ modes are governed by a single effective operator, namely

$$Q_\nu = (\bar{s}_L \gamma_\mu d_L)\,(\bar{\nu}_L \gamma^\mu \nu_L)\;, \tag{2}$$

which consists of left-handed fermion fields only. The required hadronic matrix elements of Q_ν can be extracted, including isospin breaking corrections [3], directly from the well measured leading semileptonic decay $K^+ \to \pi^0 e^+ \nu$.

After summation over the three lepton families the SM branching ratios for $K \to \pi\nu\bar{\nu}$ can be written as [4–7]

$$\begin{aligned} \mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}) &= (5.04 \pm 0.17)\left[\left(\frac{{\rm Im}\lambda_t}{\lambda^5} X\right)^2 + \left(\frac{{\rm Re}\lambda_t}{\lambda^5} X + \frac{{\rm Re}\lambda_c}{\lambda}\,(P_c + \delta P_c)\right)^2\right] \times 10^{-11}\,, \\ \mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) &= (2.20 \pm 0.07)\,\left(\frac{{\rm Im}\lambda_t}{\lambda^5} X\right)^2 \times 10^{-10}\,. \end{aligned} \tag{3}$$

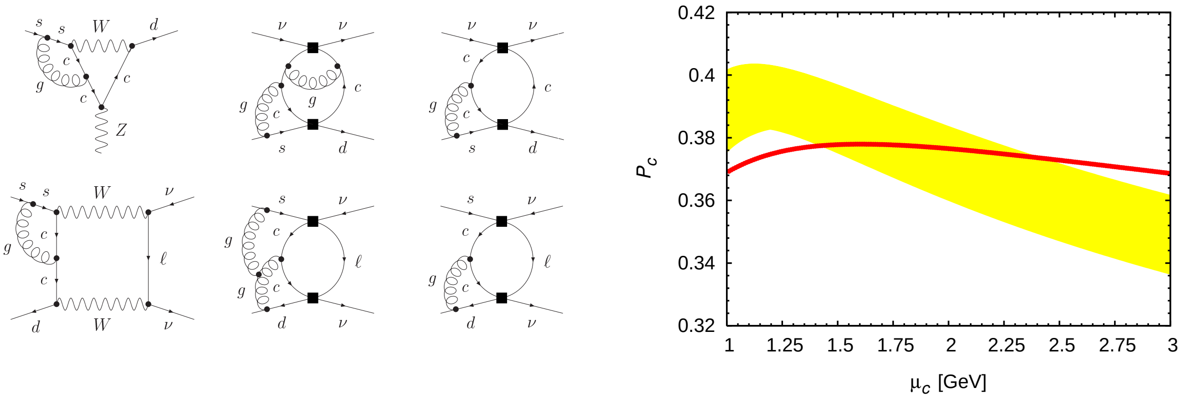

Figure 1: Left panel: Examples of diagrams appearing in the full SM (left column), describing the mixing of operators (center column) and the matrix elements (right column) in the Z-penguin (upper row) and the electroweak box (lower row) sector. Right panel: P_c as a function of μ_c at NLO (yellow band) and NNLO (red band). The width of the bands reflects the theoretical uncertainty due to higher order terms in α_s that arise in the calculation of $\alpha_s(\mu_c)$ from $\alpha_s(M_Z)$.

The top quark contribution $X = 1.464 \pm 0.041$ [7] accounts for 63% and almost 100% of the total rates. It is known through next-to-leading order (NLO) [5, 8], with a scale uncertainty of slightly less than 1%. In $K^+ \to \pi^+\nu\bar{\nu}$, corrections due to internal charm quarks and subleading effects, characterized by P_c and δP_c, amount to moderate 33% and a mere 4%. Both contributions are negligible in the case of the $K_L \to \pi^0\nu\bar{\nu}$ decay, which by virtue of Eq. (1) is purely CP violating in the SM.

3. RECENT DEVELOPMENTS IN $K^+ \to \pi^+\nu\bar{\nu}$

Two subleading effects, namely the SD contributions of dimension-eight charm quark operators and genuine LD corrections due to up quark loops have been calculated recently [6]. Both contributions can be effectively included by $\delta P_c = 0.04 \pm 0.02$ in Eq. (3). Numerically they lead to an enhancement of $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ by about 7%. The quoted residual error of δP_c can in principle be reduced by means of dedicated lattice QCD computations [9].

The main components of the state-of-the-art calculation of P_c [7], are i) the $\mathcal{O}(\alpha_s^2)$ matching corrections to the relevant Wilson coefficients arising at $\mu_W = \mathcal{O}(M_W)$, ii) the $\mathcal{O}(\alpha_s^3)$ anomalous dimensions describing the mixing of the dimension-six and -eight operators, iii) the $\mathcal{O}(\alpha_s^2)$ threshold corrections to the Wilson coefficients originating at $\mu_b = \mathcal{O}(m_b)$, and iv) the $\mathcal{O}(\alpha_s^2)$ matrix elements of some of the operators emerging at $\mu_c = \mathcal{O}(m_c)$. To determine the contributions of type i), iii) and iv) one must calculate finite parts of two-loop Green's functions in the full SM and in effective theories with five or four flavors. Sample diagrams for steps i) and iv) are shown in the left and right column of the left panel in Fig. 1. Contributions of type ii) are found by calculating the divergent pieces of three-loop Green's functions with operator insertions. Two examples of Feynman graphs with a double insertion of dimension-six operators are displayed in the center column of the left panel in Fig. 1.

Conceptual new features of this NNLO computation are a) the non-vanishing contribution from the vector component of the effective neutral-current coupling describing the interaction of neutrinos and quarks mediated by Z-boson exchange, b) the appearance of closed quark loops in gluon propagators, resulting in a novel dependence of P_c on the top quark mass and in non-trivial matching corrections at the bottom quark threshold, and c) the presence of anomalous triangle diagrams involving a top quark loop, two gluons and a Z-boson making it necessary to introduce a Chern-Simons operator in order to obtain the correct anomalous Ward identity of the axial-vector current. The inclusion of such a term is also required to cancel the anomalous contributions from triangle diagrams with a bottom quark loop. Since all these effects were absent in the NLO renormalization group analysis of P_c [4, 5], their actual size cannot be estimated from the magnitude of the residual scale uncertainties, but has to be determined by an explicit calculation.

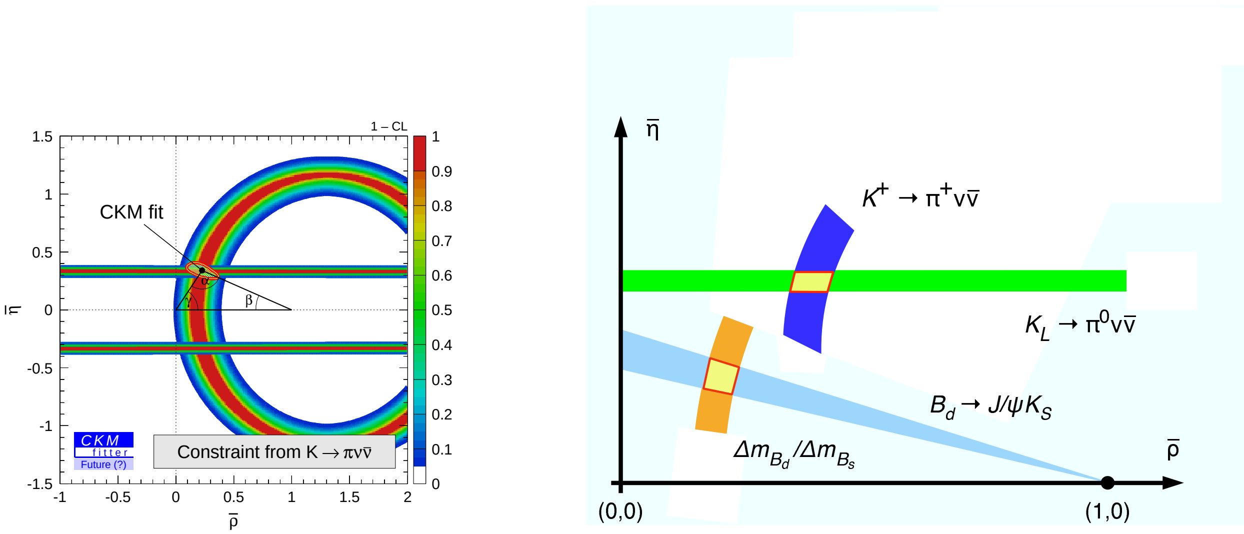

Figure 2: Left panel: UT from future measurements of $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ and $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$ with an accuracy of 10%. The constraint from the present global CKM fit is overlaid. Right panel: Schematic determination of the apex of the UT from either $B_{d,s}$–$\bar{B}_{d,s}$ oscillations and the mixing induced CP asymmetry in $B_d \to J/\psi K_s$ or from the $K \to \pi\nu\bar{\nu}$ system in the presence of non-minimal flavor violating new physics contributions.

The inclusion of the NNLO corrections removes essentially the entire sensitivity of P_c on the unphysical scale μ_c and on higher order terms in α_s that affect the evaluation of $\alpha_s(\mu_c)$ from $\alpha_s(M_Z)$. This is explicated by the plot in the right panel of Fig. 1 and by the theoretical errors of the latest SM predictions [7]

$$P_c = \begin{cases} 0.367 \pm 0.037_{\text{theory}} \pm 0.033_{m_c} \pm 0.009_{\alpha_s}\,, & \text{NLO}\,, \\ 0.371 \pm 0.009_{\text{theory}} \pm 0.031_{m_c} \pm 0.009_{\alpha_s}\,, & \text{NNLO}\,. \end{cases} \tag{4}$$

In obtaining these values the charm quark $\overline{\text{MS}}$ mass $m_c(m_c) = (1.30 \pm 0.05)\,\text{GeV}$ has been used. The residual error of P_c is now fully dominated by the parametric uncertainty from $m_c(m_c)$. A better determination of $m_c(m_c)$ is thus an important theoretical goal in connection with $K^+ \to \pi^+\nu\bar{\nu}$.

Taking into account all the indirect constraints from the global UT fit [10], the updated SM predictions of the two $K \to \pi\nu\bar{\nu}$ rates read

$$\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}) = (8.0 \pm 1.1) \times 10^{-11}\,, \qquad \mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) = (2.9 \pm 0.4) \times 10^{-11}\,. \tag{5}$$

Owing to our still limited knowledge of λ_t, the reduction of the theoretical error in P_c is at present not adequately reflected in the error of $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$. However, given the expected improvement in the extraction of the CKM elements and the foreseen theoretical progress in the determination of $m_c(m_c)$, the allowed ranges of the SM predictions for both $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ and $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$ should reach the 5% level, or better, by the end of the decade.

Experimentally the $K \to \pi\nu\bar{\nu}$ modes are in essence unexplored up to now. The AGS E787 and E949 Collaborations at Brookhaven observed the decay $K^+ \to \pi^+\nu\bar{\nu}$ finding three events [11], while there is only an upper limit on $K_L \to \pi^0\nu\bar{\nu}$, improved recently by the E391a experiment at KEK-PS [12]. The corresponding numbers read

$$\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}) = \left(14.7^{+13.0}_{-8.9}\right) \times 10^{-11}\,, \qquad \mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) < 2.86 \times 10^{-7} \;\; (90\%\,\text{CL})\,. \tag{6}$$

Within theoretical, parametric and experimental uncertainties, the observed value of $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ is fully consistent with the present SM prediction given in Eq. (5).

The impact of future accurate measurements of $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ and $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$ close to their SM predictions is shown in the left panel of Fig. 2. As can be seen the expected precision of this determination of $(\bar{\rho}, \bar{\eta})$ can easily compete with the one from the present global CKM fit [10]. A comparison of $\sin 2\beta$ determined from clean B-physics observables with $\sin 2\beta$ inferred from the $K \to \pi\nu\bar{\nu}$ system offers a very precise and highly non-trivial test of the CKM picture. Both determinations suffer from very small theoretical errors and any discrepancy between them would signal non-CKM physics, as illustrated by the hypothetical example in the right panel of Fig. 2. In particular, for $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ and $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$ close to their SM values, the reduction of the theoretical error in P_c from 10.1% down to 2.4% translates into the following uncertainties [7]

$$\frac{\sigma\left(|V_{td}|\right)}{|V_{td}|} = \begin{cases} \pm 4.1\%\,, & \text{NLO}\,, \\ \pm 1.0\%\,, & \text{NNLO}\,, \end{cases} \qquad \sigma\left(\sin 2\beta\right) = \begin{cases} \pm 0.025\,, & \text{NLO}\,, \\ \pm 0.006\,, & \text{NNLO}\,, \end{cases} \qquad \sigma\left(\gamma\right) = \begin{cases} \pm 4.9^\circ\,, & \text{NLO}\,, \\ \pm 1.2^\circ\,, & \text{NNLO}\,, \end{cases} \tag{7}$$

implying a very significant improvement of the NNLO over the NLO results. Here V_{td} is the element of the CKM matrix and β and γ are the angles of the UT. In obtaining these numbers we have used $\sin 2\beta = 0.724$ and $\gamma = 58.6^\circ$ [10] and included only the theoretical errors quoted in Eq. (4). Obviously the determination of the CKM parameters from the $K \to \pi\nu\bar{\nu}$ system will depend on the progress in the determination of $m_c(m_c)$ and the measurements of both branching ratios. Also a further reduction of the error in $|V_{cb}|$ would be very welcome in this respect.

4. CONCLUSIONS

An accurate measurement of $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$, either alone or together with one of $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$, will provide a very important extraction of the CKM parameters that compared with the information from B-decays will offer powerful and crucial tests of the CKM mechanism embedded in the SM and all its minimal flavor violating extensions. The drastic reduction of the theoretical uncertainty in P_c achieved by the recent NNLO computation will play an important role in these efforts and increases the power of the $K \to \pi\nu\bar{\nu}$ system in the search for new physics, in particular if $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ will not differ much from the SM prediction.

Acknowledgments

We are grateful to A. J. Buras, M. Gorbahn and U. Nierste for fruitful collaboration, and A. Höcker for providing the plot in the left panel of Fig. 2. This work was supported in part by the U.S. Department of Energy under contract No. DE-AC02-76CH03000.

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