Virtual Corrections to Bremsstrahlung with Applications to Luminosity Processes and Radiative Return

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We will describe radiative corrections to bremsstrahlung, focusing on applications to luminosity, fermion pair production, and radiative return at high-energy e^+e^- colliders. A precise calculation of the Bhabha luminosity process was essential at SLC and LEP, and will be equally important in ILC physics. We will review the exact results for two-photon radiative corrections to Bhabha scattering which led to the precision estimates for the BHLUMI MC. We will also compare the implementation of the virtual photon correction to bremsstrahlung for fermion pair production in the $\mathcal{KK}$ MC to similar exact expressions developed for other purposes, and discuss applications to radiative return in high energy e^+e^- colliders.

1. INTRODUCTION

In the 1990's S. Jadach, B.F.L. Ward and S.A. Yost calculated the two real photon[1] and with M. Melles, the real plus virtual photon corrections[2] to the small angle Bhabha scattering process. These corrections were used to bring the theoretical uncertainty in the luminosity measurement, as calculated by the BHLUMI Monte Carlo (MC) program [3], to within a 0.06% precision level for LEP1 parameters and 0.122% for LEP2 parameters[4].

A key component of the two-photon radiative corrections calculated for BHLUMI was the virtual photon contribution to hard photon bremsstrahlung. This radiative correction is also an important contribution to the fermion pair production process in e^+e^- annihilation implemented in the $\mathcal{KK}$ MC[13]. Comparisons of these results[8] to similar expressions obtained by other authors are reviewed. In particular, we focus attention on recent results for the virtual correction to hard photon radiation in radiative return applications[10, 11, 21].

2. BHABHA LUMINOSITY PROCESS

BHLUMI was developed into a high-precision tool for calculating the Bhabha luminosity process at SLC and LEP, and it can continue to be developed to meet the requirements of a future linear collider such as the ILC. A key advantage of the program is its exact treatment of the multi-photon emission phase space using a YFS-exponentiation procedure[5], so that IR singularities are canceled exactly to all orders and the leading soft photon effects are exponentiated, leaving only well-behaved YFS residuals to be calculated exactly to the order needed.

Table 1 shows a summary of the contributions to the theoretical uncertainty of the Bhabha luminosity process calculated by BHLUMI4.04, which includes the complete second order leading log ($\mathcal{O}(\alpha^2 L^2)$) photonic radiative corrections[4]. The LEP1 CMS energy is taken to be the Z mass, with an angular range between 1° and 3°, while the LEP2 result is calculated at a CMS energy of 176 GeV and an angular range of $3^\circ - 6^\circ$. The portion of the error budget of interest here is the missing photonic $\mathcal{O}(\alpha^2 L)$ contribution, which is due to all two-photon radiative corrections at next-to-leading log (NLL) order. For ILC physics, it is desirable to reach 0.01% precision. This is in reach for BHLUMI. Here, we will concentrate on the photonic contributions.

The photonic part of the error estimate in Table 1 comes from comparing to an exact $\mathcal{O}(\alpha^2)$ calculation. There are three contributions: a double real emission term[1] which can reach 0.012%, a real plus virtual photon term[2] which is bounded by 0.02%, and a two-loop pure virtual correction[4, 6] making up 0.014% in the LEP1 case. Adding these in quadrature gives the error quoted in Table 1. Since all of the exact $\mathcal{O}(\alpha^2)$ photonic radiative corrections are available, adding them to BHLUMI would remove almost all of the error quoted for these effects.

The only remaining exact two photon contribution would then be "up-down interference." The entire $\mathcal{O}(\alpha)$ up-down interference effect was 0.011% at 3° and 0.099% at 9°[12], and the $\mathcal{O}(\alpha^2)$ contribution to up-down interference would be supressed by an additional factor on the order of $\frac{\alpha}{\pi}L \approx 0.04$. This contribution may be neglected for small-angle Bhabha scattering. However, a complete calculation of $\mathcal{O}(\alpha^2)$ effects, including the full up-down interference contribution, could prove useful if the ILC requires a wider angular acceptance for the luminosity monitor. Some new $\mathcal{O}(\alpha^2)$ computational tools and results on Bhabha scattering have appeared recently[14–18]. Comparisons to these results will be useful in gauging the precision of different approaches to the higher order radiative corrections in Bhabha scattering.

Source of Uncertainty	LEP1	LEP2
Missing Photonic $\mathcal{O}(\alpha^2 L)$	0.027%	0.04%
Missing Photonic $\mathcal{O}(\alpha^3 L^3)$	0.015%	0.03%
Vacuum Polarization	0.04%	0.10%
Light Pairs	0.03%	0.05%
Z Exchange	0.015%	0.0%
Total	0.061%	0.122%

Table 1: Summary of theoretical uncertainties for a typical calorimetric detectorfor LEP1 and LEP2 parameters.

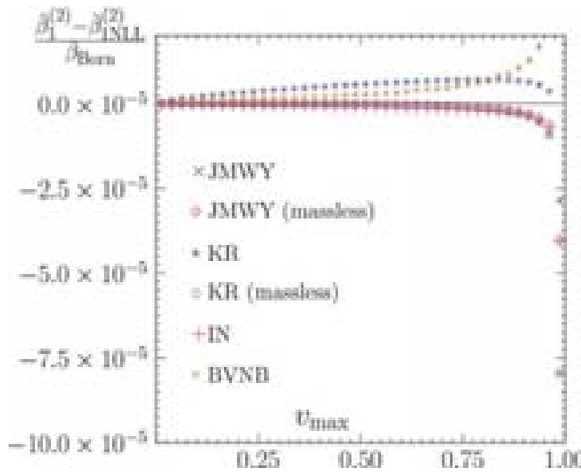

Figure 1: NNLL contribution to the integral of $\overline{\beta}_1^{(2)}$ for 10^8 events as a function of the cut on the fraction $v_{\max}$.

3. PAIR PRODUCTION AND RADIATIVE RETURN

Another important process at electron-positron colliders is fermion pair production. This process is calculated, for example, by the $\mathcal{KK}$ MC[13], and again, photonic radiative corrections are essential. In particular, we have presented explicit results real plus virtual photon emission from the initial or final state fermion line.

In the case of initial state radiation, emitting a single hard photon permits the final fermion pair creation process to be investigated over a wide range of effective CM energies $s' = s(1-v)$, where v is the energy fraction carried away by the hard photon. This is known as the "radiative return" method, and it can be used at a high energy collider to probe the Z resonance over a range of energies, or at a lower energy collider to measure the pion form factor.

Virtual photon emission is the most important radiative correction to radiative return. We have compared our results for this process in the context of the $\mathcal{KK}$ MC with several other results, including Ref. [7] (IN), which is fully differential, but lacking mass corrections, Ref. [6] (BVNB), which is differential only in v, but includes mass corrections, and Ref. [10] (KR), which is fully differential and includes mass corrections. The KR result was developed for calculating radiative return in the PHOKHARA MC, and is the newest available comparison.

We have compared these results by using them to calculate the YFS residual $\overline{\beta}_1^{(2)}$, which includes the IR-finite part of single hard bremsstrahlung including virtual photon corrections. We have shown earlier that our result (JMWY) agrees with the IN and BVNB results analytically to NLL order $((\frac{\alpha}{\pi})^2 L)$. We have recently shown similar analytical agreement for the KR result at NLL order[19, 20].

The NLL result is in fact very compact, and represents the exact results to high accuracy over most of the range of hard photon energy fraction v. Without mass corrections,

$$\overline{\beta}_{1\ \mathrm{NLL}}^{(2)} = L - 1 + 3\ln(1-r_1) + 2\ln r_2 \ln(1-r_1) - \ln^2(1-r_1) + 2\ \mathrm{Li}_2(r_1) + \frac{r_1(1-r_1)}{1+(1-r_1)^2} + (r_1 \leftrightarrow r_2) \qquad (1)$$

where $L = \ln(s/m_e^2)$, $r_i = 2p_i \cdot k/s$ with p_i the incoming $e^\pm$ momenta and k the hard photon momentum. This expression is taken as a baseline in comparing all of the exact expressions in a run of the $\mathcal{KK}$ MC shown in Fig. 1.

Fig. 1 compares the NNLL contributions of the various exact expressions in a MC run with 10^8 events at a CM energy of 200 GeV for a muon final state, using the YFS3ff generator (EEX3 option of $\mathcal{KK}$ MC). We find that the size of all of the NNLL contributions is less than 2×10^{-6} as a fraction of the Born cross section for $e^+e^- \to \mu^+\mu^-$, for photon energy cut $v_{\max} < 0.75$. For $v_{\max} < 0.95$, all of the results except BVNB, which is not fully differential, agree to within 2.5×10^{-6} in units of the Born cross section. For the final data point, $v_{\max} = 0.975$, the KR and JMWY results differ by 3.5×10^{-5} without mass corrections, or 5×10^{-5} with mass corrections. Comparisons have also been run at 1 GeV CMS energy with comparable results[20, 21].

These comparisons show that we have a firm understanding of the precision tag for an important part of the order α^2 corrections to fermion pair production in precision studies of the final LEP2 data analysis and future ILC physics.

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