2005 ALCPG & ILC Workshops - Snowmass, U.S.A.

Precision QCD at ILC: $e^+e^- \to 3$ Jets

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Jet production in electron-positron annihilation provides one of the most sensitive probes of QCD dynamics, turning into a precision measurement of QCD parameters, especially the strong coupling constant α_s. Present day measurements from LEP are largely limited by theoretical uncertainties from unknown higher order corrections in perturbative QCD. We discuss theoretical improvements required for precision QCD studies on existing LEP data and on future data expected from the ILC.

1. INTRODUCTION

The production of light quark-antiquark pairs in electron-positron annihilation gives rise to final states containing QCD jets. Depending on the amount of additional hard QCD radiation, one obtains final states with a certain number of jets: if only quark and antiquark are hard, two-jet final states are produced, one additional hard gluon yields a three-jet final state, two extra gluons or a secondary quark-antiquark pair can give rise to four-jet final states and so on. Studying these multi-jet final states, one can probe many aspects of perturbative QCD. Three-jet final states and related event shape observables were studied extensively at LEP in order to determine the strong coupling constant α_s, which controls the probability of radiating a hard gluon in these events. The measurement of four-jet-type observables at LEP established the gauge group structure of QCD. Five-jet and higher multiplicities were sometimes considered in new physics searches, where QCD-induced processes form a theoretically calculable background.

The LEP measurements of three-jet observables are of a very high statistical precision. The extraction of α_s from these data sets relies on a comparison of the data with theoretical predictions. Comparing the different sources of error in this extraction [1], one finds that the purely experimental error is negligible compared to the theoretical uncertainty. There are two sources of theoretical uncertainty: the theoretical description of the parton-to-hadron transition (hadronisation uncertainty) and the theoretical calculation of parton-level jet production (perturbative or scale uncertainty). Although the precise size of the hadronisation uncertainty is debatable and perhaps often underestimated, it is certainly appropriate to consider the scale uncertainty as the dominant source of theoretical error on the precise determination of α_s from three-jet observables. This scale uncertainty arises from truncating the perturbative QCD expansion of jet observables at the next-to-leading order (NLO) and can be improved considerably by computing next-to-next-to-leading order (NNLO) corrections.

Given the planned luminosity of the ILC, one expects that this collider will deliver jet-production data of a statistical quality similar to LEP. An attractive perspective of such a measurement at the ILC would be to determine the evolution of α_s over a wide energy range, which is potentially sensitive to new physics thresholds. Concerning uncertainties on such a determination, it is worthwhile to note that the hadronisation corrections become less important at higher energies, thus leaving the scale uncertainty as dominant source of theoretical error. Experimental aspects of such measurements, especially issues related to the beam energy profile (which were irrelevant at LEP) were not studied up to now, and certainly deserve further attention.

In the recent past, many steps towards the NNLO calculation of $e^+e^- \to 3$ jets have been accomplished (see [2] and references therein). Foremost, the relevant two-loop $1 \to 3$ matrix elements are now available. One-loop corrections to $1 \to 4$ matrix elements have been known for longer and form part of NLO calculations of $e^+e^- \to 4$ jets. These NLO matrix elements naturally contribute to $e^+e^- \to 3$ jets at NNLO if one of the partons involved becomes unresolved (soft or collinear). In this case, the infrared singular parts of the matrix elements need to be extracted and integrated over the phase space appropriate to the unresolved configuration to make the infrared pole structure explicit. As a final ingredient, the tree level $1 \to 5$ processes also contribute to $e^+e^- \to 3$ jets at NNLO. These contain double real radiation singularities corresponding to two partons becoming simultaneously soft and/or collinear. To compute the contributions from single unresolved radiation at one-loop and double real radiation at tree-level, one has to find subtraction terms which coincide with the full matrix elements in the unresolved limits and are still sufficiently simple to be integrated analytically in order to cancel their infrared pole structure with the two-loop virtual contributions. In the following, we present a new method, named antenna subtraction, to carry out NNLO calculations of jet observables and discuss its application to $e^+e^- \to 3$ jets.

2. ANTENNA SUBTRACTION

In electron-positron annihilation, an m-jet cross section at NLO is obtained by summing contributions from $(m+1)$-parton tree level and m-parton one-loop processes:

$$\mathrm{d}\sigma_{NLO} = \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^R_{NLO} - \mathrm{d}\sigma^S_{NLO}\right) + \left[\int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^S_{NLO} + \int_{\mathrm{d}\Phi_m} \mathrm{d}\sigma^V_{NLO}\right].$$

The cross section $\mathrm{d}\sigma^R_{NLO}$ is the $(m+1)$-parton tree-level cross section, while $\mathrm{d}\sigma^V_{NLO}$ is the one-loop virtual correction to the m-parton Born cross section $\mathrm{d}\sigma^B$. Both contain infrared singularities, which are explicit poles in $1/\epsilon$ in $\mathrm{d}\sigma^V_{NLO}$, while becoming explicit in $\mathrm{d}\sigma^R_{NLO}$ only after integration over the phase space. In general, this integration involves the (often iterative) definition of the jet observable, such that an analytic integration is not feasible (and also not appropriate). Instead, one would like to have a flexible method that can be easily adapted to different jet observables or jet definitions. Therefore, the infrared singularities of the real radiation contributions should be extracted using infrared subtraction terms. One introduces $\mathrm{d}\sigma^S_{NLO}$, which is a counter-term for $\mathrm{d}\sigma^R_{NLO}$, having the same unintegrated singular behaviour as $\mathrm{d}\sigma^R_{NLO}$ in all appropriate limits. Their difference is free of divergences and can be integrated over the $(m+1)$-parton phase space numerically. The subtraction term $\mathrm{d}\sigma^S_{NLO}$ has to be integrated analytically over all singular regions of the $(m+1)$-parton phase space. The resulting cross section added to the virtual contribution yields an infrared finite result. Several methods for constructing NLO subtraction terms systematically were proposed in the literature [3–6]. For some of these methods, extension to NNLO was discussed [8] and partly worked out. We focus on the antenna subtraction method [3, 4], which we extend to NNLO.

The basic idea of the antenna subtraction approach at NLO is to construct the subtraction term $\mathrm{d}\sigma^S_{NLO}$ from antenna functions. Each antenna function encapsulates all singular limits due to the emission of one unresolved parton between two colour-connected hard partons (tree-level three-parton antenna function). This construction exploits the universal factorisation of phase space and squared matrix elements in all unresolved limits, depicted in Figure 1. The individual antenna functions are obtained by normalising three-parton tree-level matrix elements to the corresponding two-parton tree-level matrix elements.

At NNLO, the m-jet production is induced by final states containing up to $(m+2)$ partons, including the one-loop virtual corrections to $(m+1)$-parton final states. As at NLO, one has to introduce subtraction terms for the $(m+1)$- and $(m+2)$-parton contributions. Schematically the NNLO m-jet cross section reads,

$$\begin{aligned}\mathrm{d}\sigma_{NNLO} = & \int_{\mathrm{d}\Phi_{m+2}} \left(\mathrm{d}\sigma^R_{NNLO} - \mathrm{d}\sigma^S_{NNLO}\right) + \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\sigma^S_{NNLO} \\ & + \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^{V,1}_{NNLO} - \mathrm{d}\sigma^{VS,1}_{NNLO}\right) + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^{VS,1}_{NNLO} \\ & + \int_{\mathrm{d}\Phi_m} \mathrm{d}\sigma^{V,2}_{NNLO}\,,\end{aligned}$$

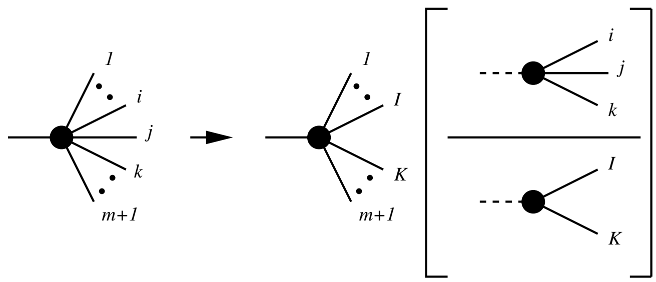

Figure 1: Illustration of NLO antenna factorisation representing the factorisation of both the squared matrix elements and the $(m+1)$-particle phase space. The term in square brackets represents both the antenna function and the antenna phase space.

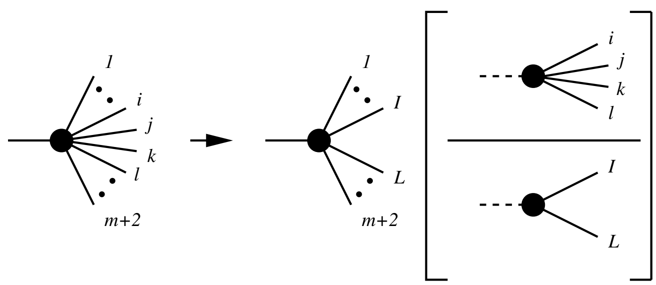

Figure 2: Illustration of NNLO antenna factorisation representing the factorisation of both the squared matrix elements and the $(m+2)$-particle phase space when the unresolved particles are colour connected. The term in square brackets represents both the antenna function and the antenna phase space.

where $\mathrm{d}\sigma^{S}_{NNLO}$ denotes the real radiation subtraction term coinciding with the $(m+2)$-parton tree level cross section $\mathrm{d}\sigma^{R}_{NNLO}$ in all singular limits [9]. Likewise, $\mathrm{d}\sigma^{VS,1}_{NNLO}$ is the one-loop virtual subtraction term coinciding with the one-loop $(m+1)$-parton cross section $\mathrm{d}\sigma^{V,1}_{NNLO}$ in all singular limits [10]. Finally, the two-loop correction to the m-parton cross section is denoted by $\mathrm{d}\sigma^{V,2}_{NNLO}$.

Both types of NNLO subtraction terms can be constructed from antenna functions. In $\mathrm{d}\sigma^{S}_{NNLO}$, we have to distinguish four different types of unresolved configurations: (a) One unresolved parton but the experimental observable selects only m jets; (b) Two colour-connected unresolved partons (colour-connected); (c) Two unresolved partons that are not colour connected but share a common radiator (almost colour-unconnected); (d) Two unresolved partons that are well separated from each other in the colour chain (colour-unconnected). Among those, configuration (a) is properly accounted for by a single tree-level three-parton antenna function like used already at NLO. Configuration (b) requires a tree-level four-parton antenna function (two unresolved partons emitted between a pair of hard partons) as shown in Figure 2, while (c) and (d) are accounted for by products of two tree-level three-parton antenna functions.

In single unresolved limits, the one-loop cross section $\mathrm{d}\sigma^{V,1}_{NNLO}$ is described by the sum of two terms [10]: a tree-level splitting function times a one-loop cross section and a one-loop splitting function times a tree-level cross section. Consequently, the one-loop single unresolved subtraction term $\mathrm{d}\sigma^{VS,1}_{NNLO}$ is constructed from tree-level and one-loop three-parton antenna functions, as sketched in Figure 3. Several other terms in $\mathrm{d}\sigma^{VS,1}_{NNLO}$ cancel with the results from the integration of terms in the double real radiation subtraction term $\mathrm{d}\sigma^{S}_{NNLO}$ over the phase space appropriate to one of the unresolved partons, thus ensuring the cancellation of all explicit infrared poles in the difference $\mathrm{d}\sigma^{V,1}_{NNLO} - \mathrm{d}\sigma^{VS,1}_{NNLO}$.

Finally, all remaining terms in $\mathrm{d}\sigma^{S}_{NNLO}$ and $\mathrm{d}\sigma^{VS,1}_{NNLO}$ have to be integrated over the four-parton and three-parton antenna phase spaces. After integration, the infrared poles are rendered explicit and cancel with the infrared pole terms in the two-loop squared matrix element $\mathrm{d}\sigma^{V,2}_{NNLO}$.

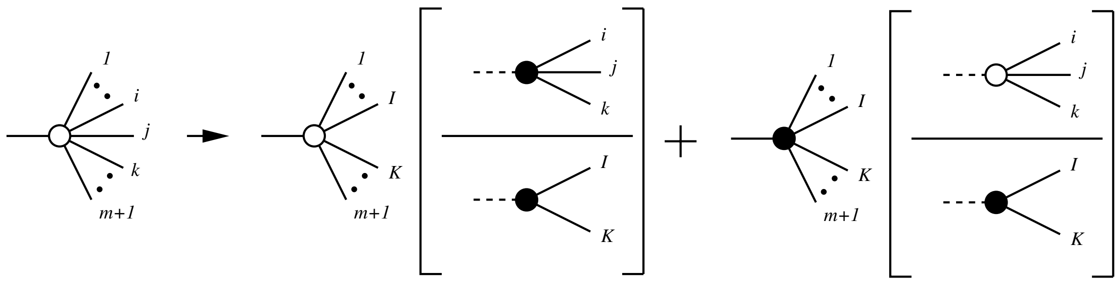

Figure 3: Illustration of NNLO antenna factorisation representing the factorisation of both the one-loop "squared" matrix elements (represented by the white blob) and the $(m+1)$-particle phase space when the unresolved particles are colour connected.

3. DERIVATION OF ANTENNA FUNCTIONS

The subtraction terms $\mathrm{d}\sigma^{S}_{NLO}$, $\mathrm{d}\sigma^{S}_{NNLO}$ and $\mathrm{d}\sigma^{VS,1}_{NNLO}$ require three different types of antenna functions corresponding to the different pairs of hard partons forming the antenna: quark-antiquark, quark-gluon and gluon-gluon antenna functions. In the past [3, 4], NLO antenna functions were constructed by imposing definite properties in all single unresolved limits (two collinear limits and one soft limit for each antenna). This procedure turns out to be impractical at NNLO, where each antenna function must have definite behaviours in a large number of single and double unresolved limits. Instead, we derive these antenna functions in a systematic manner from physical matrix elements known to possess the correct limits. The quark-antiquark antenna functions can be obtained directly from the $e^+e^- \to 2j$ real radiation corrections at NLO and NNLO [11]. For quark-gluon and gluon-gluon antenna functions, effective Lagrangians are used to obtain tree-level processes yielding a quark-gluon or gluon-gluon final state. The antenna functions are then obtained from the real radiation corrections to these processes. Quark-gluon antenna functions were derived [12] from the purely QCD (i.e. non-supersymmetric) NLO and NNLO corrections to the decay of a heavy neutralino into a massless gluino plus partons [13], while gluon-gluon antenna functions [14] result from the QCD corrections to Higgs boson decay into partons [15].

All tree-level three-parton and four-parton antenna functions and three-parton one-loop antenna functions are listed in [2], where we also provide their integrated forms, obtained using the phase space integration techniques described in [16].

4. APPLICATION TO $e^+e^- \to 3$ JETS

In [2, 17] we derived the $1/N^2$-contribution to the NNLO corrections to $e^+e^- \to 3$ jets. This colour factor receives contributions from $\gamma^* \to q\bar{q}ggg$ and $\gamma^* \to q\bar{q}q\bar{q}g$ at tree-level [18], $\gamma^* \to q\bar{q}gg$ and $\gamma^* \to q\bar{q}q\bar{q}$ at one-loop [19] and $\gamma^* \to q\bar{q}g$ at two-loops [20]. The four-parton and five-parton final states contain infrared singularities, which need to be extracted using the antenna subtraction formalism.

In this contribution, all gluons are effectively photon-like, and couple only to the quarks, but not to each other. Consequently, only quark-antiquark antenna functions appear in the construction of the subtraction terms.

Starting from the program `EERAD2` [3], which computes the four-jet production at NLO, we implemented the NNLO antenna subtraction method for the $1/N^2$ colour factor contribution to $e^+e^- \to 3j$. `EERAD2` already contains the five-parton and four-parton matrix elements relevant here, as well as the NLO-type subtraction terms.

The implementation contains three channels, classified by their partonic multiplicity: (a) in the five-parton channel, we integrate $\mathrm{d}\sigma^{R}_{NNLO} - \mathrm{d}\sigma^{S}_{NNLO}$; (b) in the four-parton channel, we integrate $\mathrm{d}\sigma^{V,1}_{NNLO} - \mathrm{d}\sigma^{VS,1}_{NNLO}$; (c) in the three-parton channel, we integrate $\mathrm{d}\sigma^{V,2}_{NNLO} + \mathrm{d}\sigma^{S}_{NNLO} + \mathrm{d}\sigma^{VS,1}_{NNLO}$. The numerical integration over these channels is carried out by Monte Carlo methods.

By construction, the integrands in the four-parton and three-parton channel are free of explicit infrared poles.

In the five-parton and four-parton channel, we tested the proper implementation of the subtraction by generating trajectories of phase space points approaching a given single or double unresolved limit. Along these trajectories, we observe that the antenna subtraction terms converge locally towards the physical matrix elements, and that the cancellations among individual contributions to the subtraction terms take place as expected. Moreover, we checked the correctness of the subtraction by introducing a lower cut (slicing parameter) on the phase space variables, and observing that our results are independent of this cut (provided it is chosen small enough). This behaviour indicates that the subtraction terms ensure that the contribution of potentially singular regions of the final state phase space does not contribute to the numerical integrals, but is accounted for analytically.

As a final point, we noted in [2] that the infrared poles of the two-loop (including one-loop times one-loop) correction to $\gamma^* \to q\bar{q}g$ are cancelled in all colour factors by a combination of integrated three-parton and four-parton antenna functions. This highly non-trivial cancellation clearly illustrates that the antenna functions derived here correctly approximate QCD matrix elements in all infrared singular limits at NNLO. They also outline the structure of infrared cancellations in $e^+e^- \to 3j$ at NNLO, and indicate the structure of the subtraction terms in all colour factors.

5. SUMMARY

In this talk, we discussed the theoretical prerequisites for performing precision QCD studies on existing LEP data and at the ILC. In particular, the precise extraction of the strong coupling constant α_s requires improved theoretical predictions to reduce the scale error inherent to calculations in perturbative QCD. At present, this extraction relies on the calculation of $e^+e^- \to 3$ jets at NLO accuracy, and we reported on progress towards the NNLO calculation.

This calculation requires a new method for the subtraction of infrared singularities which we call antenna subtraction. We introduced subtraction terms for double real radiation at tree level and single real radiation at one loop based on antenna functions. These antenna functions describe the colour-ordered radiation of unresolved partons between a pair of hard (radiator) partons. All antenna functions at NLO and NNLO can be derived systematically from physical matrix elements.

Using this method, we implemented the NNLO corrections to the subleading colour contribution to $e^+e^- \to 3$ jets into a flexible parton level event generator program, and are currently proceeding with the implementation [21] of the full set of colour factors relevant to the NNLO corrections to $e^+e^- \to 3$ jets.

Acknowledgments

This research was supported in part by the Swiss National Science Foundation (SNF) under contracts PMPD2-106101 and 200021-101874 and by the UK Particle Physics and Astronomy Research Council.

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