Study of $V_L V_L \to t\bar{t}$ at the ILC Including $\mathcal{O}(\alpha_s)$ QCD Corrections

Stephen Godfrey
Ottawa Carleton Institute for Physics, Department of Physics, Carleton University, Ottawa K1S 5B6 Canada
Shou-hua Zhu
Institute of Theoretical Physics, School of Physics, Peking University, Beijing 100871, China

In the event that the Higgs mass is large or that the electroweak interactions are strongly interacting at high energy, top quark couplings to longitudinal components of the weak gauge bosons could offer important clues to the underlying dynamics. It has been suggested that precision measurements of $W_L W_L \to t\bar{t}$ and $Z_L Z_L \to t\bar{t}$ might provide hints of new physics. In this paper we present results for $\mathcal{O}(\alpha_s)$ QCD corrections to $V_L V_L \to t\bar{t}$ scattering at the ILC. We find that corrections to cross sections can be as large as 30% and must be accounted for in any precision measurement of $VV \to t\bar{t}$.

1. INTRODUCTION

Understanding the mechanism of electroweak symmetry breaking (EWSB) is a primary goal of the LHC and ILC [1]. While much effort has been devoted to the weakly interacting weak sector scenario the strongly interacting weak sector (SIWS) remains a possibility. Because the t-quark mass is the same order of magnitude as the scale of EWSB it has long been suspected that t-quark properties may provide hints about the nature of EWSB and the subprocess $V_L V_L \to t\bar{t}$ has been suggested as a probe. While $V_L V_L \to t\bar{t}$ can be studied at both hadron colliders and e^+e^- colliders the overwhelming QCD backgrounds will likely make it impossible to study the $V_L V_L \to t\bar{t}$ subprocess at the LHC [2].

In contrast, the ILC offers a much cleaner environment. The simplist approach is to study how the $t\bar{t}$ cross section varies with M_H [3]. A more general approach is to parametrize interactions in a nonlinearly realized electroweak chiral Lagrangian which is appropriate if the EWSB dynamics is strong with no Higgs bosons at low energies. A typical dimension five operator is $L_1^{eff} = (a_1/\Lambda)\bar{t}tW_\mu^+ W^{-\mu}$, where the coefficient a_1 is naively expected to be of order 1 when the cut-off of the theory is taken to be $\Lambda = 4\pi v = 3.1$ TeV. It is expected that a_1 can be measured to an accuracy of $\sim \pm 0.1$ at 95% C.L. [4]. But to be able to attach meaning to precision measurements it is necessary to understand radiative corrections, both electroweak and QCD. In this contribution we present the results of a study of $\mathcal{O}(\alpha_s)$ corrections to the tree level electroweak $V_L V_L \to t\bar{t}$ process in the SM at the ILC. Due to space limitations we point the interested reader to Ref. [5] for a more detailed account and a more complete set of references.

2. CALCULATIONS AND RESULTS

We are interested in the subprocesses $VV \to t\bar{t}$ which occur in the processes $e^+e^- \to \ell_1\ell_2 + VV \to \ell_1\ell_2 + t\bar{t}$ where $\ell_1\ell_2$ is $\nu\bar{\nu}$ for the $W^+W^- \to t\bar{t}$ subprocess and e^+e^- for the $ZZ \to t\bar{t}$ subprocess. The vector bosons are treated as partons inside the e^+ and e^- using the effective boson approximation [6, 7]. The total cross section is then obtained by integrating the W (of Z) luminosities with the subprocess cross section [8].

The $\mathcal{O}(\alpha_s)$ corrections for the processes $W^+W^- \to t\bar{t}$ $ZZ \to t\bar{t}$ are calculated using the FeynArts, FormCalc and LoopTools packages [9]. The QCD corrections to $W^+W^- \to t\bar{t}$ are shown in Fig. 1 (left side). The infrared singularity in the vertex corrections are cancelled by the soft contributions from the process $W^+W^- \to t\bar{t}g$ which are shown in Fig. 1 (right side). We regulate the IR-singularity by introducing a gluon mass which is equivalent to standard dimensional regularization for processes with no triple gluon vertex present. This approach has the

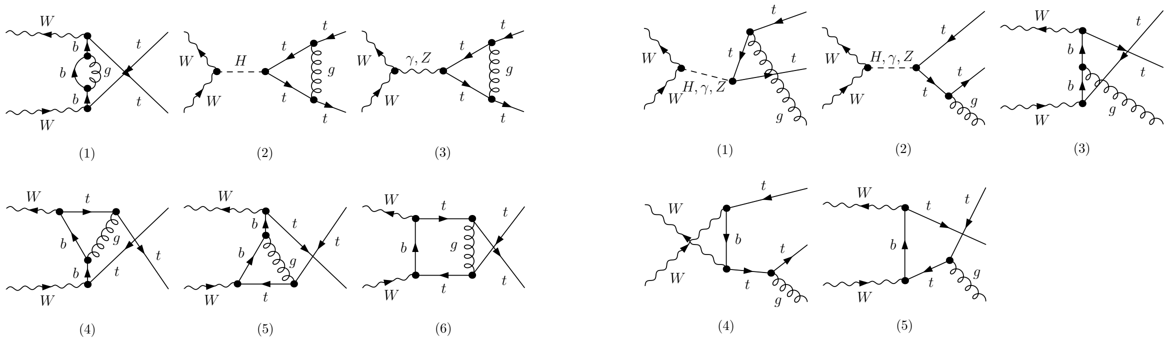

Figure 1: $\mathcal{O}(\alpha_s)$ QCD corrections to $W^+W^- \to t\bar{t}$. (a) Virtual QCD contributions to $W^+W^- \to t\bar{t}$. (b) Feynman diagrams for $W^+W^- \to t\bar{t} + g$.

additional benefit that varying the value of the gluon mass acts as a check of the numerical cancellations between the different contributions.

The cross-sections are calculated by replacing the Born matrix element squared by

$$|M_{Born}|^2 \to |M_{Born}|^2(1 + \delta_{soft}) + 2\mathrm{Re}(M^*_{Born}\delta M) \tag{1}$$

where δM is the sum of the one-loop Feynman diagrams and the corresponding counter-term diagrams and δ_{soft} is the soft-gluon correction factor coming from the $2 \to 3$ process.

To deal with renormalization associated with the ultraviolet divergences we adopt the on-mass shell renormalization scheme and use dimensional regularization. The software packages we used handle renormalization by employing numerical factors. Because our results must be independent of these factors we can vary them to check the consistency of our results. Likewise, we verified that the results are independent of the soft cutoff energy of the emitted gluon which divides the cross section into a piece with a soft gluon emitted and a piece with a hard gluon emitted. We estimate uncertainties due to scale dependence results in at most a $\sim 4\%$ uncertainty in the K-factor.

We include in our results the kinematic cuts $m_{t\bar{t}} > 400$ GeV and $p_T^{t,\bar{t}} > 10$ GeV. Since the longitudinal scattering cross section is much larger than the TT and TL cases and it is the longitudinal gauge boson processes which

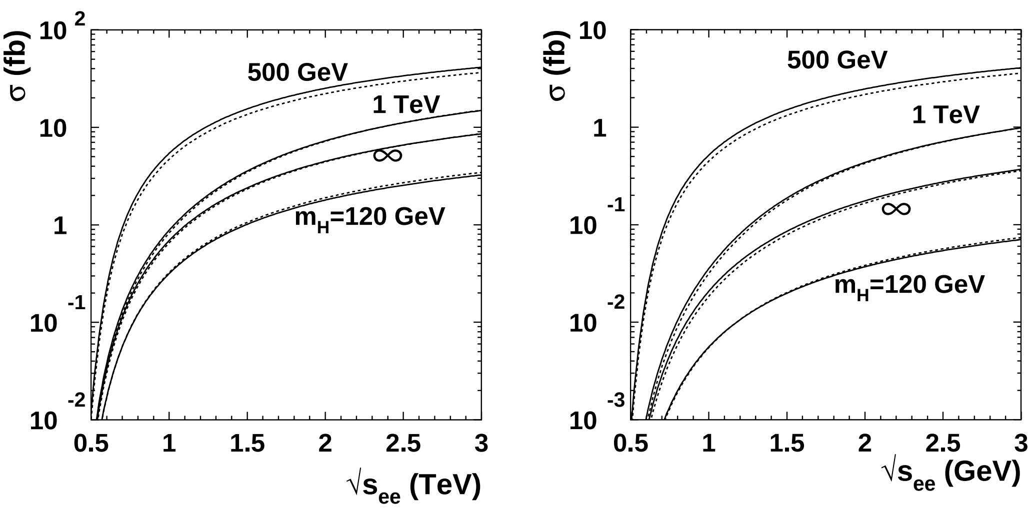

Figure 2: Cross sections as a function of $\sqrt{s}_{e^+e^-}$ for (a) $e^+e^- \to \nu\bar{\nu}t\bar{t}$ via $W_L^+W_L^-$ fusion and for (b) $e^+e^- \to e^+e^-t\bar{t}$ via Z_LZ_L fusion. In both cases the solid line is the $\mathcal{O}(\alpha_s)$ QCD corrected cross section and the dashed line is the electroweak tree level cross section.

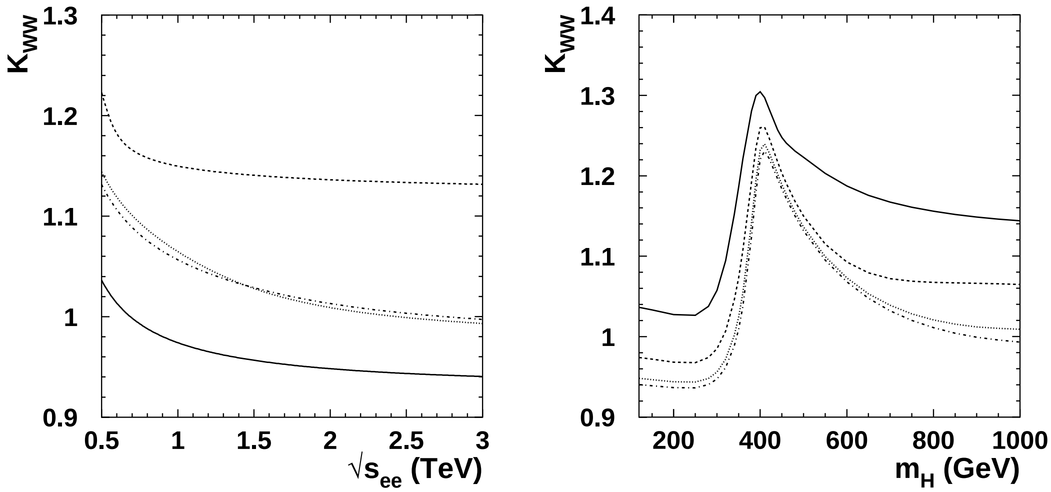

Figure 3: (left side) The K-factor as a function of $\sqrt{s}_{e^+e^-}$ for $e^+e^- \to \nu\bar{\nu}t\bar{t}$ (via $W_L^+W_L^-$ fusion). The solid line is for $M_H =$ 120 GeV, the dashed line for $M_H = 500$ GeV, the dotted line for $M_H = 1$ TeV, and the dot-dashed line for $M_H = \infty$ (LET). (right side) The K-factor as a function of M_H for $e^+e^- \to \nu\bar{\nu}t\bar{t}$ (via $W_L^+W_L^-$ fusion). The solid line is for $\sqrt{s}_{e^+e^-} = 500$ GeV, the dashed line for $\sqrt{s}_{e^+e^-} = 1$ TeV, the dotted line for $\sqrt{s}_{e^+e^-} = 2$ TeV, and the dot-dashed line for $\sqrt{s}_{e^+e^-} = 3$ TeV. See text for an explanation of the K-factor.

corresponds to the Goldstone bosons of the theory we will henceforth only include results for V_LV_L scattering. In Fig. 2 we show the cross section only including longitudinal W and Z scattering as a function of the e^+e^- centre of mass energy for several representative Higgs masses including the $M_H \to \infty$ case (corresponding to the LET).

The QCD corrections to longitudinal scattering are often presented as a K-factor, normally defined as the ratio of the NLO to LO cross sections. Because the $\mathcal{O}(\alpha_s)$ QCD corrections we calculated are LO corrections to a tree level electroweak result we take the K-factor to be the ratio of the cross section with the $\mathcal{O}(\alpha_s)$ QCD corrections and the tree level electroweak cross sections. The K-factors for $\sigma(e^+e^- \to \nu\bar{\nu}t\bar{t})$ which goes via $W_L^+W_L^-$ fusion and for $\sigma(e^+e^- \to e^+e^-t\bar{t})$ is shown in Fig. 3 (left side) as a function of $\sqrt{s}_{e^+e^-}$. The $\mathcal{O}(\alpha_s)$ QCD corrections are largest for $M_H = 500$ GeV with K-factors ranging from over 1.2 for $\sqrt{s}_{e^+e^-} = 500$ GeV to 1.15 for $\sqrt{s}_{e^+e^-} = 1$ TeV. The corrections decrease as $\sqrt{s}_{e^+e^-}$ increases. The variation of the K-factor with M_H is shown in Fig. 3 (right side). The fact that the K-factor is largest for $M_H = 500$ GeV in Fig. 3 (left side) and that it peaks at $M_H \simeq 400$ GeV in Fig. 3 (right side) is a threshold effect which is an artifact of the kinematic cut we imposed on the $t\bar{t}$ invariant mass. The important point is that the QCD corrections are not insignificant compared to the effects we might wish to study such as top Yukawa couplings or anomalous $VVt\bar{t}$ couplings.

3. FINAL COMMENTS

This paper concentrated on $\mathcal{O}(\alpha_s)$ QCD corrections. Other equally important considerations and contributions that we have not discussed here are the issue of backgrounds, electroweak radiative corrections for top quark production, and the accuracy of the effective W approximation. These are discussed in Ref. [5] along with relevant references.

In the event that the Higgs mass is heavy and the electroweak sector is strongly interacting it is quite possible that the underlying theory will manifest itself in the interactions between the top quark and longitudinal component of gauge bosons. In this paper we studied the $\mathcal{O}(\alpha_s)$ QCD corrections to the tree level electroweak process $VV \to t\bar{t}$. We found that they can be quite substantial, the same size as the effects we wish to study, so that they need to be taken into account when studying $t\bar{t}$ production.

Acknowledgments

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