Two-Loop Corrections to Heavy Quark Form Factors

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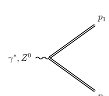
The vertex function of a heavy quark-antiquark pair coupled to an external (vector, axial vector, scalar or pseudoscalar) current is described by form factors. We describe the calculation of the two-loop QCD corrections to these form factors and discuss first applications. In particular, we derive the QCD corrections to the static $t\bar{t}Z$ and $t\bar{t}\gamma$ couplings, relevant for anomalous coupling studies.

1. INTRODUCTION

The international linear collider will produce large numbers of top-antitop quark pairs, thus allowing for precision studies of the properties of the top quark. These experimental precision studies require equally precise theoretical predictions, i.e. higher order corrections in perturbation theory. Up to now, the theoretical effort was focused on a precise description of top quark production at threshold (see [1] for a review), where QCD corrections to next-to-next-to-leading order (NNLO) in perturbation theory are known, while observables other than the total production cross section in the continuum are known only to next-to-leading order (NLO) accuracy. In this talk, we present results on the virtual two-loop corrections to vertex functions involving heavy quarks, which are an important ingredient to the NNLO corrections to top quark observables in the continuum. As a first application of these form factors, we derive their static limit, describing the effective couplings of top quarks to neutral gauge bosons, which are relevant for studies of anomalous couplings.

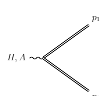
2. HEAVY QUARK FORM FACTORS

The vertex function coupling an on-shell heavy quark antiquark pair to an external current can be decomposed into so-called form factors, whose coefficients follow from Lorentz invariance and symmetry properies of the current. For the vector and axial vector current (electroweak gauge boson), the vertex function contains, within QCD, four form factors ($F_{1,2}$, $G_{1,2}$):



$$= (-i)\left(v_Q F_1(s,m^2)\gamma^\mu + v_Q \frac{1}{2m} F_2(s,m^2) i\sigma^{\mu\nu}(p_1+p_2)_\nu + a_Q G_1(s,m^2)\gamma^\mu\gamma_5 + a_Q\frac{1}{2m} G_2(s,m^2)\gamma_5 (p_1+p_2)^\mu\right) .$$

The coupling of heavy quarks to Higgs bosons of positive and negative parity contains the scalar and pseudoscalar form factors:



$$= -i\,\frac{m}{v}\,\left[S_Q F_S(s,m^2) + i P_Q F_P(s,m^2)\gamma_5\right]$$

Here s is the invariant momentum squared of the external current and m the heavy quark mass.

The two-loop corrections to the form factors are obtained by applying approproiate projections to the Feynman diagrams displayed in Figure 1. As a result, the form factors are expressed in terms of hundreds of different scalar integrals. These integrals are reduced to a small set of master integrals by means of the so-called Laporta algorithm [2] with the help of integration-by-parts identities [3] and Lorentz-invariance identities [4]. The master integrals

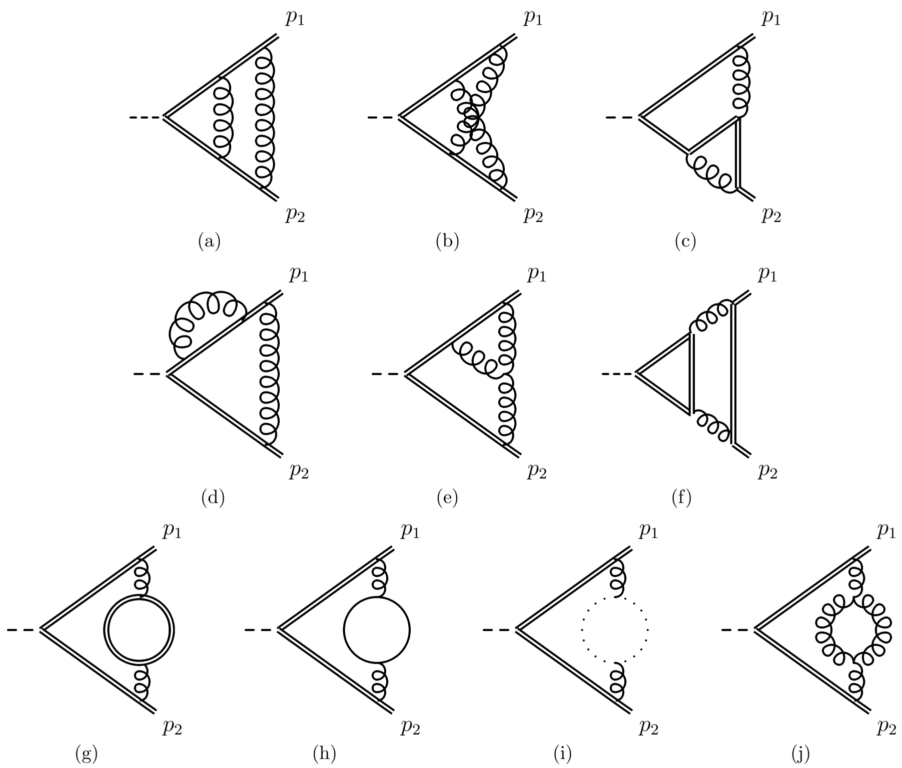


Figure 1: Feynman diagrams contributing to the two-loop QCD corrections to heavy quark form factors

themselves were evaluated with the method of differential equations [4–7] in [8, 9]. The master integrals, and thus the form factors are represented as series in the regularization parameter ϵ and expressed in terms of 1-dimensional harmonic polylogarithms up to weight 4 [10, 11].

We obtained the complete two-loop corrections for the renormalized vector [12], axial-vector [13, 14], scalar and pseudoscalar [15] form factors. Agreement was found with earlier partial results and with expansions around special kinematical points.

3. STATIC FORM FACTORS

An immediate physical application of the QCD corrections to the heavy quark form factors for arbitrary momentum transfer are predictions for form factors at zero recoil, the so-called static form factors.

Measurements of these static form factors for heavy quarks could allow indirect constraints on new physics scenarios. Recently, there has been considerable effort to determine the feasibility of such experimental measurements. Specifically, the couplings to photons and Z bosons have been studied in detail – both for heavy quark production at hadron colliders [16, 17] and at a future high-luminosity high-energetic linear electron-positron collider [18–20]. At this workshop, this issue was revisited in great depth especially in view of discriminating new physics scenarios [21].

The most prominent static form factor is the electromagnetic spin-flipping form factor: the anomalous magnetic moment, which is finite in the zero-recoil limit, and can be obtained from the results of the previous section [22]. It reads

$$F_{2,Q}(s=0) = \frac{\alpha_s}{2\pi}\,C_F + \left(\frac{\alpha_s}{2\pi}\right)^2 F_{2,Q}^{(2l)}\,,$$

with

$$F_{2,Q}^{(2l)} \;=\; C_F^2\left(-\frac{31}{4} + 2\,\zeta_2\,(5-6\,\ln(2)) + 3\,\zeta_3\right) + C_F\,C_A\left(\frac{317}{36} + 3\,\zeta_2\,(-1+2\,\ln(2)) - \frac{3}{2}\,\zeta_3\right)$$

	$t\ (\mu = m_t)$	$b\ (\mu = m_b)$	$b\ (\mu = m_Z)$
$(g-2)_Q^{\gamma,(1l)}/2$	$1.53 \cdot 10^{-2}$	$-1.52 \cdot 10^{-2}$	$-8.4 \cdot 10^{-3}$
$(g-2)_Q^{\gamma,(2l)}/2$	$4.7 \cdot 10^{-3}$	$-1.00 \cdot 10^{-2}$	$-6.6 \cdot 10^{-3}$
$(g-2)_Q^{\gamma}/2$	$2.00 \cdot 10^{-2}$	$-2.52 \cdot 10^{-2}$	$-1.50 \cdot 10^{-2}$
$(g-2)_Q^{Z,(1l)}/2$	$5.2 \cdot 10^{-3}$	$-1.87 \cdot 10^{-2}$	$-1.03 \cdot 10^{-2}$
$(g-2)_Q^{Z,(2l)}/2$	$1.6 \cdot 10^{-3}$	$-1.24 \cdot 10^{-2}$	$-8.1 \cdot 10^{-3}$
$(g-2)_Q^{Z}/2$	$6.8 \cdot 10^{-3}$	$-3.11 \cdot 10^{-2}$	$-1.85 \cdot 10^{-2}$

Table I: One- and two-loop QCD contributions, and their sums, to the anomalous magnetic and weak magnetic moments of the top and bottom quark, for different values of the renormalization scale μ.

$$+C_F\,T_F\left(\frac{119}{9} - 8\,\zeta_2\right) - \frac{25}{9}C_F\,T_F\,N_l + C_F\,\beta_0\,\ln(\mu^2/m_Q^2),$$

from which we can derive the static magnetic and weak magnetic form factor of a quark Q. We consider

$$\left(\frac{g-2}{2}\right)_Q^{\gamma,Z} \equiv F_{2,Q}^{\gamma,Z}(0) = v_Q^{\gamma,Z}\,F_{2,Q}(0)\ ,$$

which correspond to the anomalous magnetic (MDM) and weak magnetic (WMDM) moments of Q. (Notice that in the literature the WMDM is often associated with $F_{2,Q}^{Z}(s = m_Z^2)$.) Numerical values for t and b quarks are given in Table I.

For the b quark an upper bound on its magnetic moment was derived in [23] from an analysis of LEP1 data, which, in our convention, reads $\delta(g-2)_b^{\gamma}/2 < 1.5 \times 10^{-2}$ (68 % C.L.). Comparing it with Table I we see that the QCD-induced contributions to the b quark magnetic moment saturate this bound, which implies that there is limited room for new physics contributions to this quantity. At a future linear collider [19, 20], when operated at the Z resonance, the sensitivity to this variable could be improved substantially, either by global fits or by analyzing appropriate angular distributions in $b\bar{b}$ and $b\bar{b}\gamma$ events.

As to the static form factors of the top quark, no such tight constraints exist so far on possible contributions from new interactions (see [16] for a review). These quantities are particularly sensitive to the dynamics of electroweak symmetry breaking. For instance, in various models with a strongly coupled symmetry breaking sector one may expect contributions from this sector to the static t quark form factors at the 5 - 10 % level [21]. The QCD-induced anomalous magnetic moment and the QCD corrections to the axial charge of the top quark are of the same order of magnitude. Future colliders have the potential to reach this level of sensitivity.

4. OUTLOOK

In this talk, we presented the calculation of the two-loop QCD corrections to the heavy quark form factors. As a first application, we discussed the static form factors of t and b quark, especially the anomalous magnetic and weak magnetic moment. It was found that LEP limits on the anomalous magnetic moment of the b quark are almost saturated by the two-loop QCD contribution, thus leaving little room for new physics effects.

The form factors presented here have a number of applications, which will be addressed in future work. The vector and axial vector form factors contribute to the NNLO corrections to the forward-backward asymmetry for heavy quarks; they can also be used to compute the differential top quark pair production cross section at the ILC in the continuum, where top quark mass effects are still non-negligible. The scalar and pseudoscalar form factors enter the NNLO corrections to the decay of a Higgs boson into heavy quarks. These would become especially important for a very heavy Higgs boson, decaying into top quark pairs, where the form factors could be used for a differential description of the decay final state.

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