QCD Radiation in the Production of High-$\hat{s}$ Final States

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In the production of very heavy final states — high Mandelstam $\hat{s}$ — extra QCD radiation can play a significant role. By comparing several different parton shower approximations to results obtained with fixed–order perturbation theory, we quantify the degree to which these approaches agree (or disagree), focussing on initial state radiation above $p_\perp = 50$ GeV, for top pair production at the Tevatron and at the LHC, and for SUSY pair production at the LHC. Special attention is paid to ambiguities associated with the choice of the maximum value of the ordering variable in parton shower models.

1. INTRODUCTION

Hadron colliders like the Tevatron and the LHC are ideal machines for producing strongly interacting states. To the extent that any new such state is related to a WIMP type explanation for the dark matter problem, a decay chain generally exists by which the coloured particle decays to jets plus missing $E_\perp$ (a stable WIMP), possibly accompanied by one or more leptons. A prime example of such a type of collider phenomenology is given by supersymmetry (SUSY), which simultaneously provides not only a possible solution to the dark matter problem, but also the maximal space–time symmetry, an elegant solution to the ultraviolet instability of the Higgs mass (the hierarchy problem), radiative breaking of electroweak symmetry, and gauge coupling unification. As such, the phenomenology of supersymmetric particle production at colliders has been a topic of ever increasing interest over the last few decades.

In the case of supersymmetry, the relevant hadron collider processes are pair production of squarks and gluinos [1–3], followed by cascade decays down to the Lightest Supersymmetric Particle (LSP), usually the $\tilde{\chi}_1^0$, which, if R–parity is conserved, has WIMP-like properties and escapes the detector unobserved.

Since gluino decays produce more jets than squark decays, the jet multiplicity in the event can be used to separate squark- and gluino-enriched samples. Also, since the LSP escapes detection, the decay kinematics cannot be fully reconstructed. Instead, the masses involved in the decay chain can be determined from kinematic edges in the invariant mass spectra of the observed particles and jets [4, 5]. As such, any extra jet activity in the event, e.g. from QCD bremsstrahlung, will introduce a source of combinatorial error, hence our interest in studying the production of extra QCD radiation in association with these processes. Note that most of our conclusions should be applicable, in general, to the production of any high-mass strongly interacting states.

At currently accessible collider energies, the QCD coupling strength, α_s, is large, ranging from the non–pertubative, at scales $\leq \Lambda_{\mathrm{QCD}} \sim 100$ MeV, to $\alpha_s \sim \mathcal{O}(0.1)$ at the highest energy scales relevant for the LHC. Thus, even in the perturbative domain of QCD, higher–order corrections are typically large, and one has to exercise caution in estimating the uncertainties coming from uncalculated contributions, whether these be due to higher orders or to other sources.

In this paper, we focus especially on the production of very heavy states, or more precisely processes with large factorisation scales, at the Tevatron and at the LHC. Following some general comments in Section 2, we take a closer look at three specific high-mass processes in Section 3: $\mathrm{t\bar{t}}$ at the Tevatron, $\mathrm{t\bar{t}}$ at the LHC, and SUSY pair production at the LHC. Finally, in Section 4 we present a summary and outlook. This report closely follows the studies reported in [6].

2. FIXED ORDER VS. RESUMMATION

There are essentially two widespread and somewhat complementary methods to calculate perturbative QCD amplitudes; fixed–order matrix elements and resummation / parton shower approaches.

Computing fixed–order matrix elements, we include the full dynamical, helicity, interference, and phase space structure, up to a given order in the coupling constants (here only α_s will be relevant). This is a procedure which by now is highly automated (for a brief overview, see e.g. [7]). We can also include virtual corrections, i.e. quantum loops; although complete results beyond one loop (NLO) are still scarce. Two problems occur in this approach, firstly that the complexity rapidly increases, both as a function of the number of legs and, even more rapidly, as a function of the number of loops, and secondly; bremsstrahlung corrections contain singularities which, for soft and collinear radiation, renders a truncation of the perturbative series unstable at any fixed order in certain regions of phase space.

On the other hand, in the collinear limit we are in fact able to sum the perturbative series to infinite order in the coupling constant, hence curing the truncation problem. Parton showers are examples of such approaches. The main virtue is that these descriptions should work well at low $p_\perp$, and (squared) amplitudes for final states with an arbitrary number of partons can be built out of relatively simple expressions. In addition, due to a combination of factorisation and universality, these descriptions can also be more easily macthed onto hadronisation models than fixed-order approaches. Nonetheless, since we are working in a particular *limit* of QCD, uncertainties and ambiguities appear as soon as we try to extrapolate away from that limit. Common problems for parton shower models include a simplified treatment of helicity structure, ambiguities in the size of the radiation phase space, and unknown corrections from contributions which vanish in the collinear limit.

Especially for hard radiation, large differences may exist between different shower algorithms. In PYTHIA [8, 9], two qualitatively different shower algorithms are implemented: one Q^2-ordered [10–13], and the other $p_\perp$-ordered [14]. Due to the large final state masses and since we force the tops and gluinos to be stable here, we are mainly exploring the properties of the initial-state showers, for which the crucial parameter is the starting scale of the shower. Nominally, this scale is identical to the factorization scale, μ_F, where the parton densities are convoluted with the matrix elements.

For the $p_\perp$-ordered shower, μ_F can be used directly as the maximum $p_\perp$. Below, we refer to this choice as the $p_\perp$-ordered 'wimpy shower'. Allowing the parton shower to populate the full phase space, with the maximum $p_{\perp j} = \sqrt{s}/2$, regardless of μ_F, we refer to as the $p_\perp$-ordered 'power shower' — strictly speaking in conflict with the factorization assumption, but with interesting phenomenological consequences, as we shall see.

The case of a Q^2-ordered shower is not so simple. The starting scale here is $Q^2_{\max} = \min\left(C\mu_F^2, s\right)$, where $C \geq 1$ parameterizes the translation from ${p_\perp}^2$ to Q^2. We refer to $C = 1$ as the Q^2-ordered wimpy shower, $C = 4$ as Tune A [15, 16], and $C \to \infty$ as power shower — with the same caveat concerning factorization as for the $p_\perp$-ordered version.

For the fixed-order results, we use tree-level matrix elements as implemented in the new supersymmetric version [17] of the event generator MadEvent [18, 19]. The factorization scale is set to the average final state mass, as is the renormalization scale for the heavy pair. The renormalization scale for additional jet radiation is $p_{\perp j}$.

3. RESULTS

In Tab. I we first show the inclusive production cross sections obtained in fixed-order perturbation theory for top and gluino production plus zero to two hard jets with $p_{\perp j} > 50$ GeV and $\Delta R_{jj} > 0.4$ (for the gluinos we use the SUSY parameter point SPS1a [20] with $m_{\tilde{g}} = 608$ GeV). We also compare with a toy top quark T with a mass of 600 GeV, to test the universality of the behaviour.

For top pairs at the Tevatron, we see that extra hard jets are not alarmingly frequent; for 50 GeV jets, the $\mathrm{t\bar{t}}j$ cross section is roughly 10% of the inclusive $\mathrm{t\bar{t}}$ one. Going to $\mathrm{t\bar{t}}$ at the LHC, the production of extra jets increases dramatically, not only due to the increase in phase space, but also to the more gluon-dominated and hence more

Table I: Cross sections for the production of $t\bar{t}$ pairs at the Tevatron and at the LHC, and for gluinos (in SPS1a) as well as a toy-model 600 GeV top quark T at the LHC. We show fixed-order matrix element results with 0,1,2 additional hard jets, with $p_{\perp,j}^{\min} > 50$ GeV, $|y_j| < 5$ and $R_{jj} > 0.4$.

		Tevatron	LHC		
	$\sigma_{\rm tot}\,$[pb]	$t\bar{t}$	$t\bar{t}$	$\tilde{g}\tilde{g}$	$T\bar{T}$
$p_{\perp j} > 50$ GeV	σ_{0j}	5.13	461	4.83	1.30
	σ_{1j}	0.45	273	5.90	1.50
	σ_{2j}	0.04	127	4.17	1.21

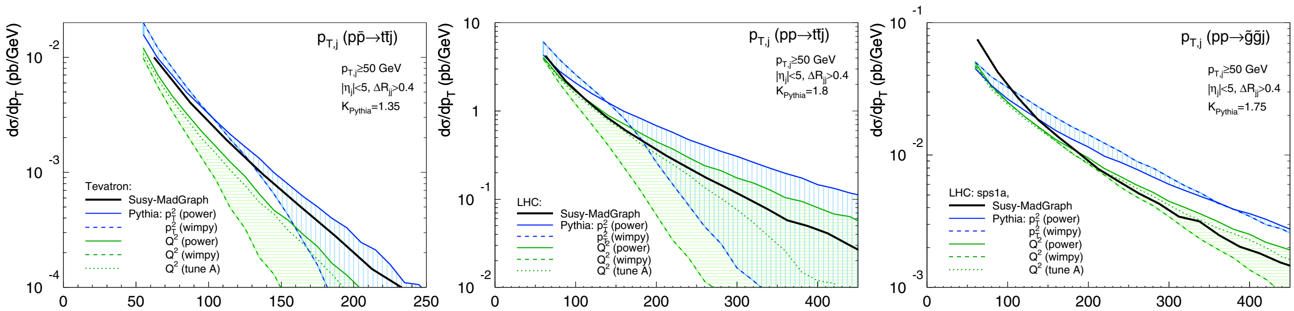

Figure 1: Jet $p_\perp$ distributions for $t\bar{t} + j$ at the Tevatron and at the LHC, and for $\tilde{g}\tilde{g} + j$ (in SPS1a) at the LHC. Comparison between fixed–order matrix elements (solid thick line) and parton shower models (blue: $p_\perp$-ordered, green: Q^2-ordered) for wimpy (thin dashed) and power (thin solid) showers. The dotted green line indicates 'Tune A' of the Q^2-ordered shower.

active initial state, and to the top quarks here being produced at larger relative velocities. A more detailed study of radiation in top events can be found in [21, 22].

For the production of 50 GeV jets in association with even higher-mass objects, here gluinos and our heavy toy quarks T, we see that fixed-order perturbation theory reaches its limit; the $1j$ and $2j$ cross sections are not noticeably smaller than the $0j$ inclusive one, hence a truncation at any finite order is not likely to give a trustworthy answer.Only by considering jets much harder than 50 GeV would the stability of the pertubative series be recovered.

What is happening is that the soft and collinear singularities of QCD radiation are introducing logarithmic enhancements of the form $(\alpha_s \log^2(Q^2_{\rm hard}/p_{\perp}{}_j^2))^N$ at all orders N. If the argument of the logarithm becomes large enough to counterbalance the α_s suppression, then these corrections cannot be neglected, and a truncated calculation will give a meaningless answer.

In Fig. 1 we show the jet transverse-momentum distributions for $t\bar{t} + j$ at the Tevatron and the LHC, and for $\tilde{g}\tilde{g} + j$ at the LHC. We compare the fixed-order calculation (thick black line) to the 5 different parton shower models described above.

For $t\bar{t}$ at the Tevatron, we concluded above that there is no reason not to trust the fixed-order results (thick solid line), for the range of $p_{\perp j}$ values we consider here. The interesting feature, then, is that the power showers (thin blue and green lines) do surprisingly well. While the agreement with the fixed-order result is not perfect, one could have expected a collinear-based approximation to do worse. The wimpy showers (dashed blue and green), on the other hand, drop off rapidly around the factorisation scale, due to the presence of the explicit phase space cutoff, as has also previously been noted e.g. for Drell–Yan production [23–27]. (The crossover at ~ 100 GeV between the two $p_\perp$-ordered showers illustrates the effect of changing the renormalization scale from $p_\perp/2$ in the wimpy shower to $3p_\perp$ in the power shower. The Q^2-ordered showers all use $\mu_R = p_{\perp j}$.)

For $t\bar{t}$ at the LHC, we have already noted that the total jet rates are larger, hence the slope of the $p_\perp$ spectrum is here much gentler than at the Tevatron (note the change in $p_\perp$ scale). This means that a correct description of harder jets becomes relatively more important, and hence the cataclysmic drop of the wimpy showers potentially more serious. Still, the asymptotic slopes of the power showers are not greatly different from that of the matrix element. If anything, the rate of hard jets is overestimated by the power showers (probably indicating that the neglected terms, which can be roughly classified as interference terms, are negative).

The tendency of the power showers to overestimate the hard jet tail is seen also in the last plot, $\tilde{g}\tilde{g}+j$ production. In addition, the drop of the wimpy showers is here not as catastrophic, ironically due to the larger masses involved. With a factorisation scale of ~ 600 GeV, the presence or not of a phase space cutoff at that scale does not produce a large impact for the $p_\perp$ range we consider here. On the other hand, at low $p_\perp$, we see that the fixed-order approximation starts breaking down already around 100 GeV, while the parton shower results converge to a common value.

4. CONCLUSIONS

We show that fixed-order QCD calculations predict a large number of hard jets associated with the production of heavy coloured states at the LHC. This additional radiation should be taken into account in studies of the separation of squark and gluino event samples, as well as for cascade decay reconstruction.

Comparisons of jet $p_\perp$ spectra between matrix elements and parton showers show that, for the radiation of one extra jet, conventional (wimpy) parton showers with a phase space cutoff at the factorization scale give a reasonable approximation up to $p_{\perp j} \sim \mu_F/2$, above which they rapidly break down. Removing the phase space cut tends to yield somewhat harder radiation spectra than produced by the matrix elements.The tendency of these 'power showers' to overestimate the rate of hard jets may be useful for future studies for which parton shower approximations are applied in hard regions of phase space. This is especially relevant for processes where the correct higher higher order matrix elements are not known, such as would be the case for many exotic BSM physics scenarios.

For fairly soft jets, we see that in the production of high-mass gluinos the breakdown of fixed-order perturbation theory caused by logarithmic corrections can occur already at jet transverse momenta of as high as 100 GeV.

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