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Unstable particle production near threshold with effective theory methods

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We illustrate the use of effective theory methods to describe resonant unstable particles. We outline the necessary ingredients to describe W-pair production close to threshold in e^-e^+ collisions.

1. INTRODUCTION

Physical processes at ongoing and upcoming colliders involve the production and subsequent decay of heavy, unstable particles (Z_0, $W^{+/-}$, t, $\dots$). Their resonant production allows a precise determination of particle properties. However, weak-coupling perturbation theory (PT) is known to fail in the resonant region [1]. The break-down of ordinary PT is due to the appearance of a second small parameter, besides the coupling constant: the width Γ of the unstable particle in unit of its mass M. A self-energy resummation allows one to take into account finite width effects, however this procedure introduces some arbitrariness often reflected in gauge-dependent results. Additionally, there is no clear prescription on how to improuve systematically the accuracy of the results.

We start from the observation that the presence of two small parameters is *the* characteristic feature of the problem, so that in a theory which formulates correctly the double expansion in the coupling and in Γ/M all other issues (gauge invariance, resummation) should follow automatically. As is typical in multi-scale problems, we use effective theory methods to formulated this expansion. This provides us with a computational scheme for performing calculations of resonant production of unstable particles at any accuracy.

2. W-PAIR PRODUCTION CLOSE TO THRESHOLD

In [2] we presented how an effective theory approach allows one to describe processes with resonant unstable particles in intermediate stages. We applied the method to the description of the resonant production of a scalar, heavy particle. This simple toy-model allowed us to study the problem of treating consistently finite width effects while keeping all technical difficulties to a minimum.

Here we consider W-pair production close to threshold at an e^+e^- collider. Elements of the calculation were first presented in [3]. We consider specifically the process

$$e^-(p_1)\, e^+(p_2) \to W^+(k_1)\, W^-(k_2) \to \mu^-(l_1)\, \bar{\nu}_\mu(l_2)\, u(l_3)\, \bar{d}(l_4)\,. \tag{1}$$

At threshold $k_{1/2} \sim \{M_W(1+v^2), \pm M_W\vec{v}\}$ and the counting is $\alpha_{ew} \sim \alpha_s^2 \sim v^2$. This process is crucial for the precise determination of the W mass and a lot of work has been carried out in the last years to improuve the accuracy of the description of W^+W^- mediated four-fermion final states. Recently the full $\mathcal{O}(\alpha_{ew})$ to e^+e^- to four-fermion has been completed [4]. Here we focus on obtaining results that are valid near threshold, where ordinary PT and the double pole approximation break down.

The first step in the construction of the effective theory is to integrate out hard fluctuations $k \sim M$ (so called factorizable corrections), for which there is no quantum-interference with resonant, slowly propagating particles. This gives rise to the hard matching coefficients of an effective theory where only soft, collinear or resonant modes are dynamical. The hard matching coefficients are determined by matching onshell Green functions in full theory to operators in the effective theory. Notice that the precise definition of what are the "hard" modes depends on the process and on the observable under consideration. The splitting between hard and dynamical does not involve a cut-off, instead in dimensional regularization this splitting can be achieved with the strategy of regions [5].

Once the matching coefficients and the effective operators have been calculated at the required order in PT, calculations can be done suitably in the effective theory framework. Here we will outline how to organize the lowest order calculation and the classes of terms contributing to the first order correction to it. The aim is to make transparent how the calculation can be systematically extended to higher orders.

2.1. Leading order

The leading order (LO) contribution is obtained by tree-level matching of the on-shell operators and by resuming onshell one-loop self-energies in the propagators. Specifically, at LO one needs

- tree level matching for the production vertex $e^+e^- \to W^+W^-$

$$\mathcal{L}_{\mathcal{P}}^{(0)} = \frac{2\pi\alpha_{ew}}{M_W^2}\left(\bar{e}_L\gamma^{[i}iD^{j]}e_L\right)\left(\Omega_-^{*i}\Omega_+^{*j}\right) ,$$

  where $\Omega_\pm$ denote the non-relativistic vector fields with mass dimension $3/2$ for the $W^\pm$ bosons. Notice that at LO only the $e_L^- e_R^+$ amplitude contributes;

- resummation of $\mathcal{O}(\alpha_{ew})$ onshell self-energies in the propagators

$$\mathcal{L}_{\mathcal{NR}}^{(0)} = \sum_\pm \Omega_\pm^{*i}\left(iD^0 + \frac{\vec{D}^2}{2M_W} - \frac{\Delta_1}{2}\right)\Omega_\mp^i ,$$

  here $\Delta \equiv (s - M_W^2)/M_W$ is the hard matching coefficient and Δ_1 denotes the leading contribution $\mathcal{O}(M_W\alpha_{ew})$. The W bosons are then described by a non-relativistic Lagrangian similar to NRQCD;

- tree level matching for the decay vertices $W \to l\bar{l}$

$$\mathcal{L}_{\mathcal{D}}^{(0)} = -\frac{g_{ew}}{\sqrt{2}}\Omega_-^i\bar{\mu}_L\gamma^i\nu_L - \frac{g_{ew}}{\sqrt{2}}\Omega_+^i\bar{u}_L\gamma^i d_L .$$

To obtain the LO amplitude, one derives the Feynman rules corresponding to these effective operators and combines the various elements. The diagram contributing at LO in the effective theory is shown in Fig. 1.

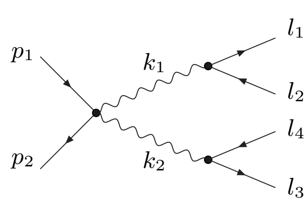

Figure 1: *Leading order Feynman diagram in the effective theory.*

2.2. $\mathcal{O}(\alpha_s, v, \alpha_{ew}/v)$ corrections

The first correction to the LO amplitude is obtained by including all corrections $\mathcal{O}(\alpha_s \sim v \sim \alpha_{ew}/v)$ to it. Since $\alpha_s \sim \alpha_{ew}^{1/2}$ we call this perturbative order $N^{1/2}LO$. The set of terms needed at this order is

- v corrections to production vertex

$$\begin{aligned}\mathcal{L}_p^{(1/2)} &= \frac{c_1}{M_W^3}\left(\bar{e}_L\gamma^j e_L\right)\left(\Omega_-^{*i}(-i)D^j\Omega_+^{*i}\right) + \frac{c_2}{M_W^3}\left(\bar{e}_L\gamma^{[i}e_L\right)\left(\Omega_-^{*i}(-i)D^{j]}\Omega_+^{*j}\right) \\ &+ \frac{c_3}{M_W^3}\left(\bar{e}_L\gamma^{[i}n^{j]}n^l e_L\right)\left(\Omega_-^{*i}(-i)D^l\Omega_+^{*j}\right) + \frac{c_4}{M_W^3}\left(\bar{e}_L\gamma^j\gamma^l\gamma^i e_L\right)\left(\Omega_-^{*i}(-i)D^l\Omega_+^{*j}\right) ,\end{aligned} \tag{2}$$

with the matching coefficients

$$c_1 = \pi\alpha_{ew}\frac{M_Z^2\sin^2\theta_w - 2M_W^2}{4M_W^2 - M_Z^2}\,; \qquad c_2 = \pi\alpha_{ew}\frac{M_Z^2(1-2\sin^2\theta_w)}{4M_W^2 - M_Z^2}\,; \qquad c_3 = 2\pi\alpha_{ew}\,; \qquad c_4 = \pi\alpha_{ew}\,. \tag{3}$$

Additionally, at $N^{1/2}LO$ there is a contribution from the $e_R^- e_L^+$ amplitude;

- two-loop $\alpha_s\alpha_{ew}$ corrections to the onshell propagator. These give rise to the matching coefficient $\Delta_{3/2}$, which can either be resummed, i. e. included in the effective operator, or included as perturbative interactions;
- α_s corrections to the decay stage, which cancel if one is inclusive on hadronic decay products. There are no $\mathcal{O}(v)$ corrections in the decay stage, the first non-trivial kinematical correction being $\mathcal{O}\left(v^2\right)$;
- the exchange of one potential photon ($q_0 \sim M_W v^2, \vec{q} \sim M_W v$), which is $\mathcal{O}(\alpha_{ew}/v)$

$$\mathcal{A}^{(1/2,c)} = -i\,(4\pi\alpha)\,\mathcal{A}^{(0)} \times \int \frac{d^Dq}{(2\pi)^D}\,\frac{1}{\vec{q}^{\,2}}\,\frac{1}{\left(E_1 - q^0 - \frac{(\vec{k}_1-\vec{q})^2}{2M_W} - \frac{\Delta^{(1)}}{2} + i\epsilon\right)}\,\frac{1}{\left(E_2 + q^0 - \frac{(\vec{k}_2+\vec{q})^2}{2M_W} - \frac{\Delta^{(1)}}{2} + i\epsilon\right)}, \tag{4}$$

where $\vec{k} \equiv \vec{k}_1 = \vec{l}_1 + \vec{l}_2$ and $\vec{k}_2 = \vec{l}_3 + \vec{l}_4 = -\vec{k}$. One obtains

$$\mathcal{A}^{(1/2,c)} = \mathcal{A}^{(0)}\,\frac{\alpha\,M_W}{|\vec{k}|} \times \arctan\frac{|\vec{k}|}{\sqrt{M_W(\Delta^{(1)} - E_1 - E_2) - i\epsilon}}, \tag{5}$$

in agreement with [6]. The exchange of one potential photon turns out to be the only contribution at $N^{1/2}LO$ which is not due to hard contributions. [1]

Combining these terms one obtains the $N^{1/2}LO$ amplitude in the effective theory. Similarly the calculation can be organized at higher orders. Higher order corrections come from

- matching of lower order effective operators at higher accuracy in the expansion in the couplings;
- kinematical corrections to the effective operators;
- matching of higher order effective operators;
- contribution from dynamical effective modes.

The power counting allows one to identify the terms needed at a given order prior to performing the calculation and therefore makes a systematic organization of the calculation straightforward. The only bottleneck in going to higher orders remains the (standard) complexity of multi-loop and multi-scale integrals, the same difficulties one encounters in the treatment of stable particles.

3. CONCLUSIONS

We considered the resonant W-pair production with effective theory methods. We outlined the calculation of the $\mathcal{O}(\alpha_s, v, \alpha_{ew}/v)$ corrections. Work in progress is the calculation of higher order corrections, though technically more difficult, no new conceptual issue arise.

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[1] Notice that in the similar $t\bar{t}$ threshold production case, a resummation of potential photons is necessary.

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