EXTREME NEUTRON STARS

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Extreme Magnetism:

- B ~ 10^{8-9} G (Low-mass X-ray binaries, millisecond pulsars)
- B ~ 10^{15-16} G (Magnetars)
- Giant X-ray/γ-ray Flares (Soft Gamma Repeaters)
- Timing Noise: weak (millisecond pulsars);
  strong (SGRs)

Extreme Rotation (P > 1.6 msec observed):

- Low-Mass X-ray Binaries; Proto-Magnetars (?)
- Compositional Changes from Accretion
  (Direct URCA cooling; Quark Matter cores)
- Evidence for Gravity Wave Torques (?)

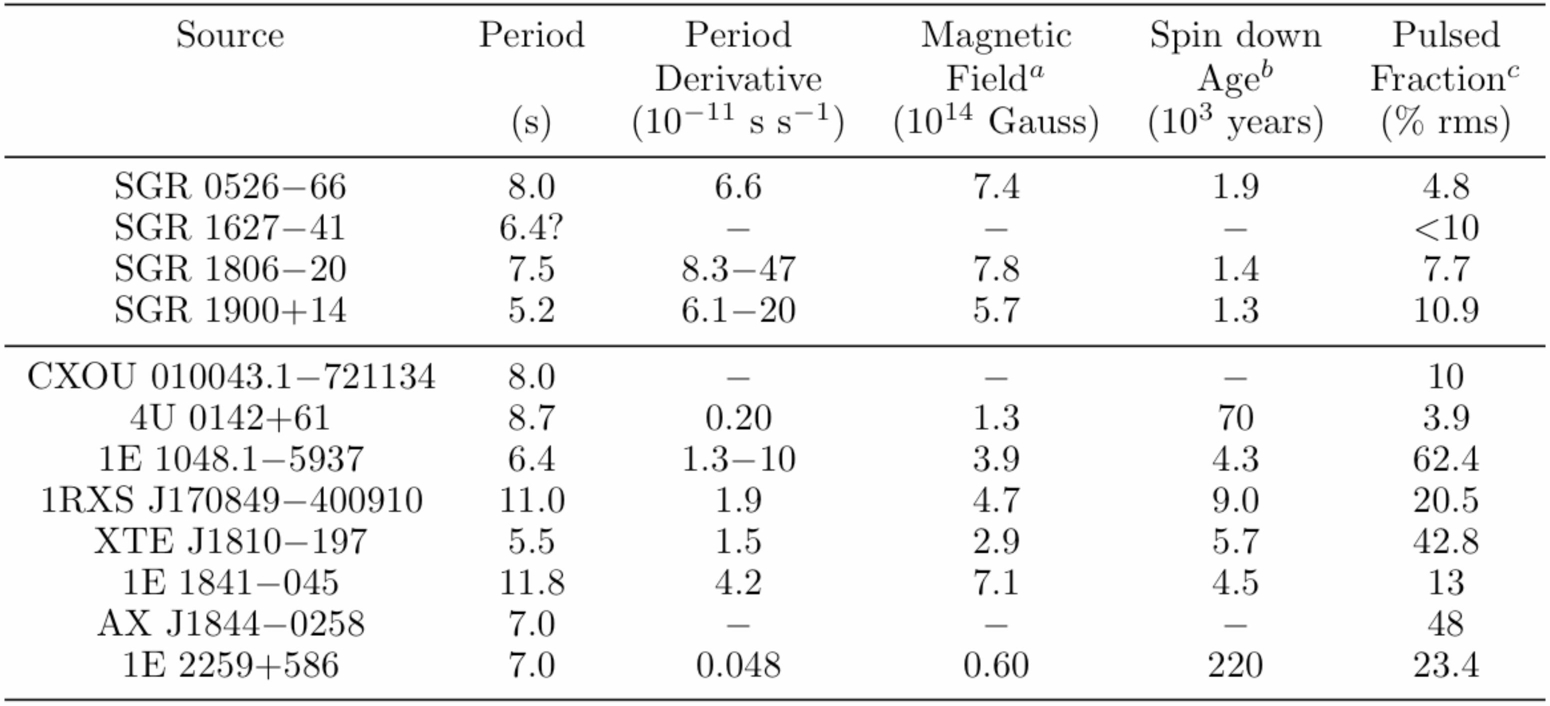
Known Galactic Population of Magnetars

Two basic classes, discovered by independent methods:

$$\left|\frac{\partial}{\partial t}\int \frac{B^2}{8\pi}\,dV\right| \gg \left|I_{\rm ns}\Omega_{\rm ns}\dot{\Omega}_{\rm ns}\right|$$

Soft Gamma Repeaters

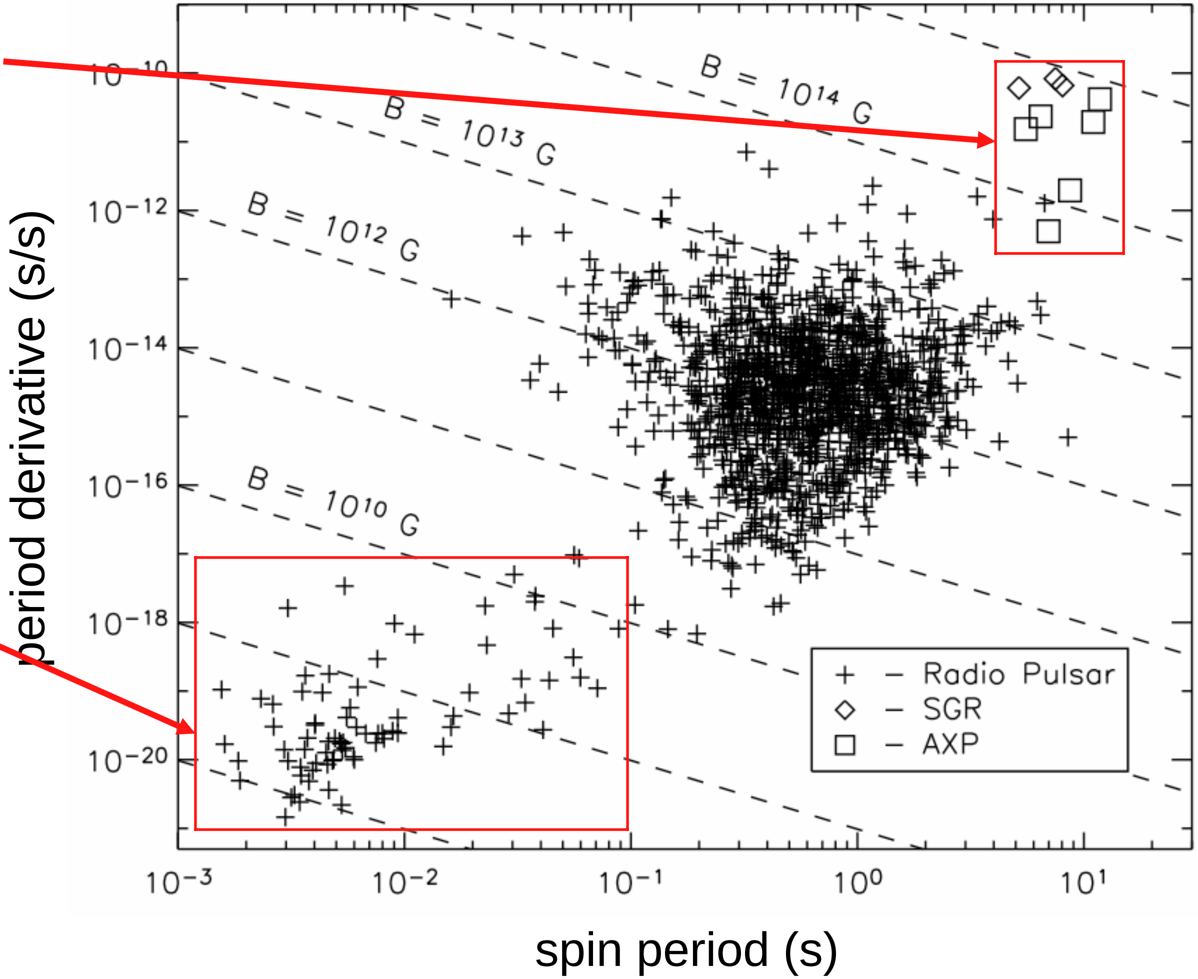
Anomalous X-ray Pulsars



Source	Period (s)	Period Derivative (10^{-11} s s^{-1})	Magnetic Field^a (10^{14} Gauss)	Spin down Age^b (10^3 years)	Pulsed Fraction^c (% rms)
SGR 0526−66	8.0	6.6	7.4	1.9	4.8
SGR 1627−41	6.4?	–	–	–	<10
SGR 1806−20	7.5	8.3−47	7.8	1.4	7.7
SGR 1900+14	5.2	6.1−20	5.7	1.3	10.9
CXOU 010043.1−721134	8.0	–	–	–	10
4U 0142+61	8.7	0.20	1.3	70	3.9
1E 1048.1−5937	6.4	1.3−10	3.9	4.3	62.4
1RXS J170849−400910	11.0	1.9	4.7	9.0	20.5
XTE J1810−197	5.5	1.5	2.9	5.7	42.8
1E 1841−045	11.8	4.2	7.1	4.5	13
AX J1844−0258	7.0	–	–	–	48
1E 2259+586	7.0	0.048	0.60	220	23.4

Recent review: Woods & Thompson (astro-ph/0406133)

Spinning-down Neutron Stars (non-accreting)



magnetars

`recycled' pulsars

Woods & Thompson 2004 (astro-ph/0406133)

27 Dec 2004
Giant Flare
SGR 1806-20

E ~ 4×10^{46} erg

(no beaming)

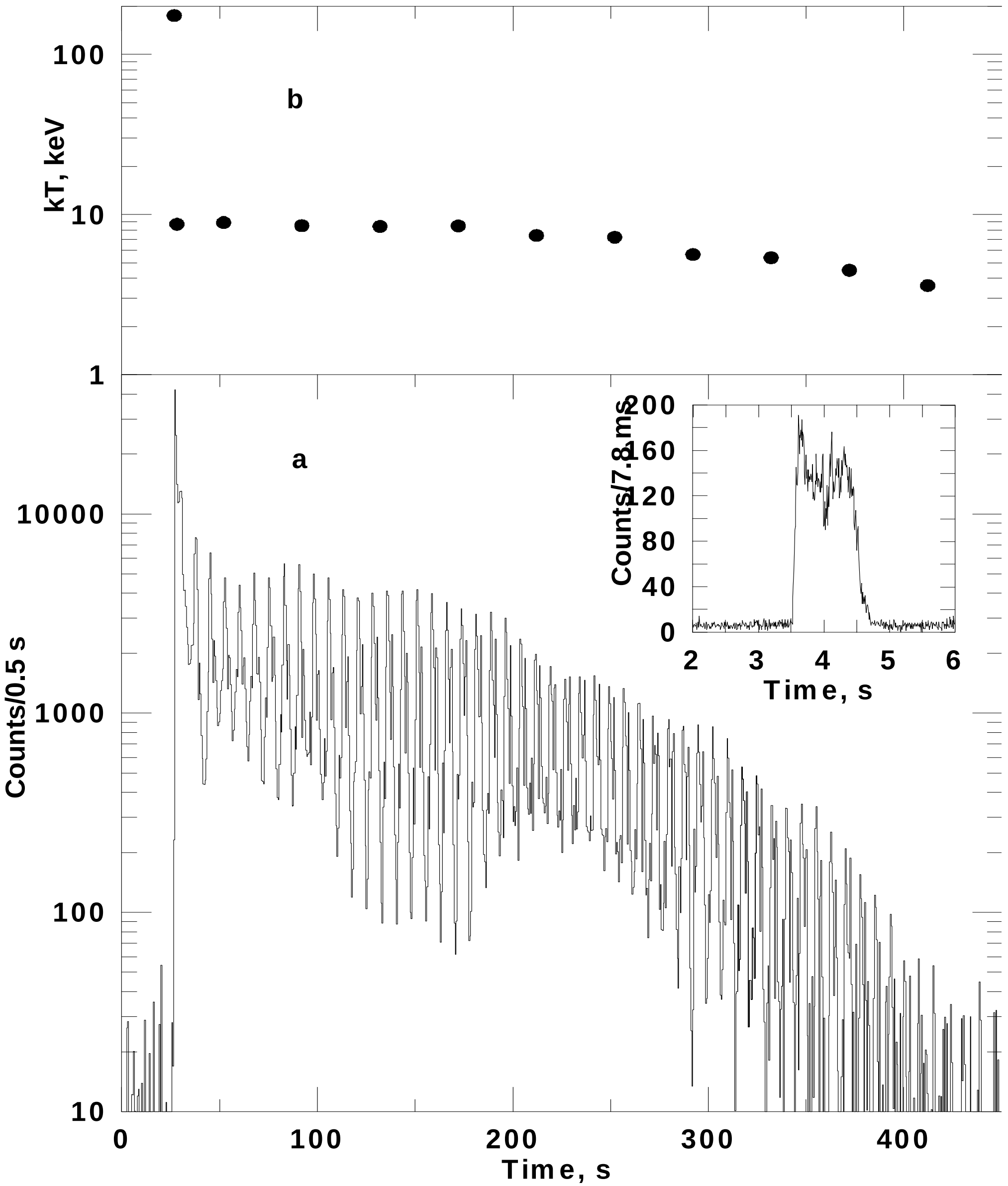
⇒ $B > 5 \times 10^{15}$ G $(N/100)^{1/2}$

crustal yield strain

> 0.01-0.03

(untwisting motion //

magnetic flux surfaces)



Hurley et al. 2005, Nature, 434, 1098

Relaxation Behavior in
Bursting Soft Gamma Repeaters

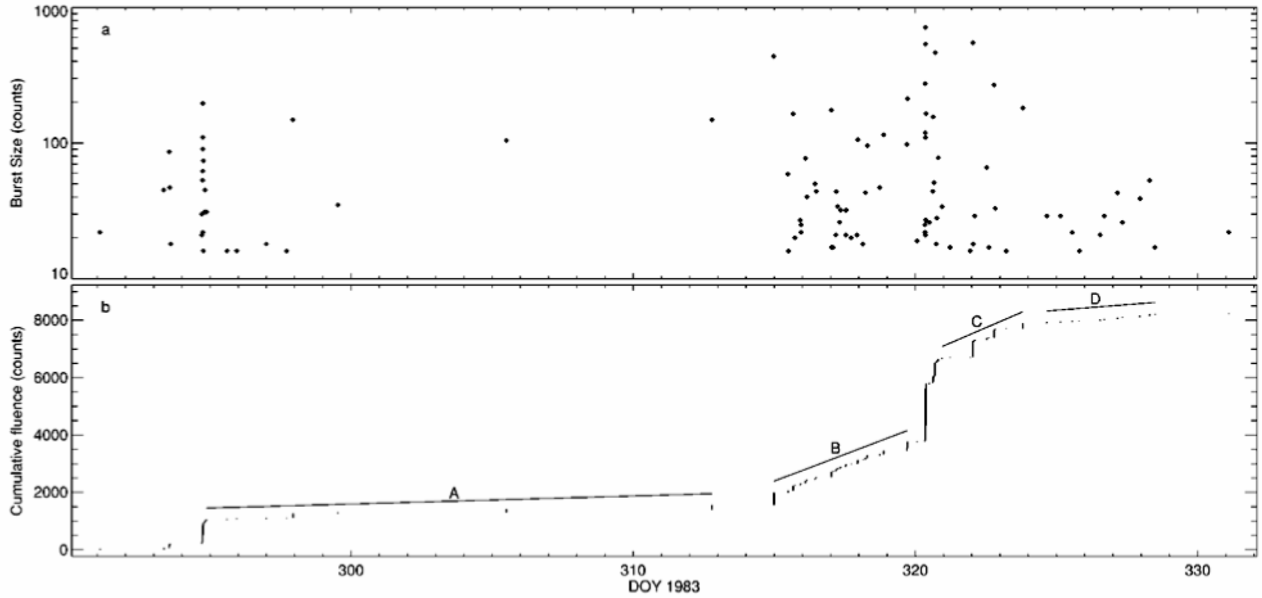
$dN/dE \propto E^{-1.6}$ (earthquakes; solar flares)

Something's Creeping:

SGR 1806-20 (continuous coverage 40 days 1983)

many
bursts

cumulative
fluence ↑



Palmer 1999, ApJ, 512, L113

Magnetars from Supernova Collapse

- Violent convection extends close to ν-sphere:

$$B^2/4\pi \sim \rho V^2 \;\Rightarrow\; B \sim 10^{16}\ \mathrm{G}$$

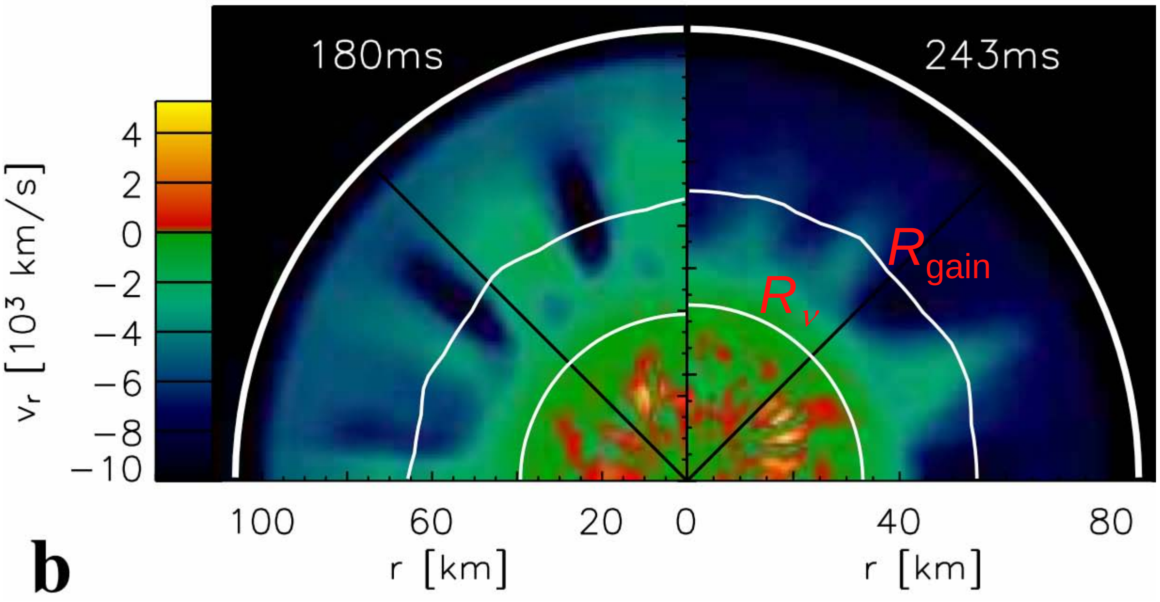
$R/V \sim 3$ ms $\Rightarrow$ Helical dynamo when $P < 5$ ms

(Helicity needed to stabilize B-field: Braithwaite & Spruit 2004)

- Magnetorotational instability $R_\nu < r < R_{\rm gain}$

  Disordered B-field (? no sunspots at high Solar latitude)

Buras et al. 2005
(astro-ph/0507135)

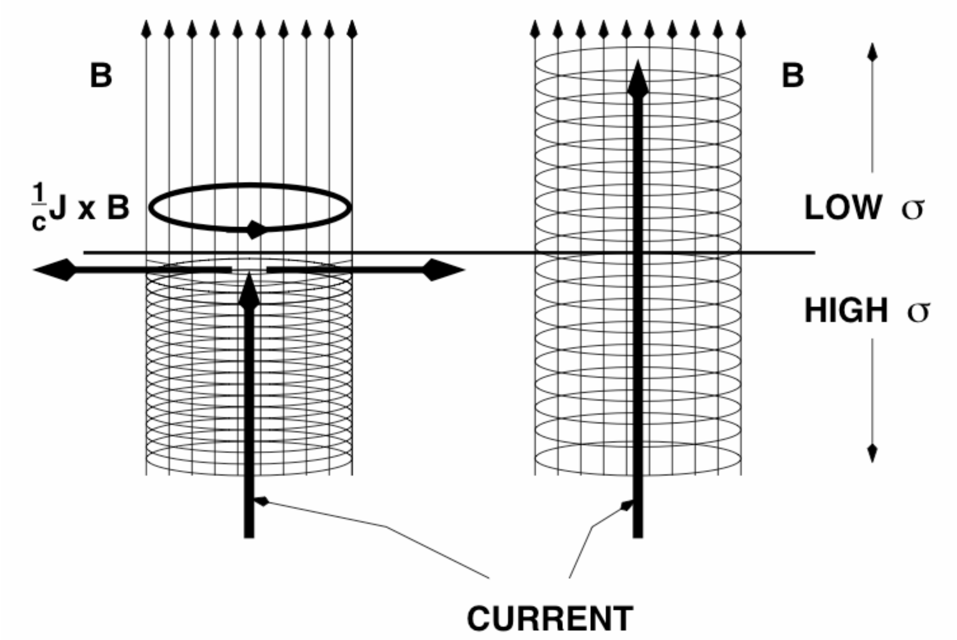


Helicity Injection into the Magnetosphere

Actively bursting magnetars show:

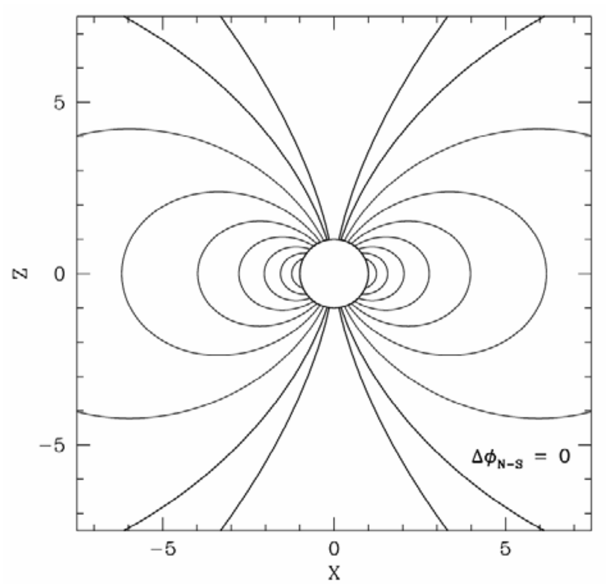
- Strong non-thermal X-ray emission when not bursting
- Long term (up to years) stable variations in X-ray pulse profile and spindown torque after outbursts

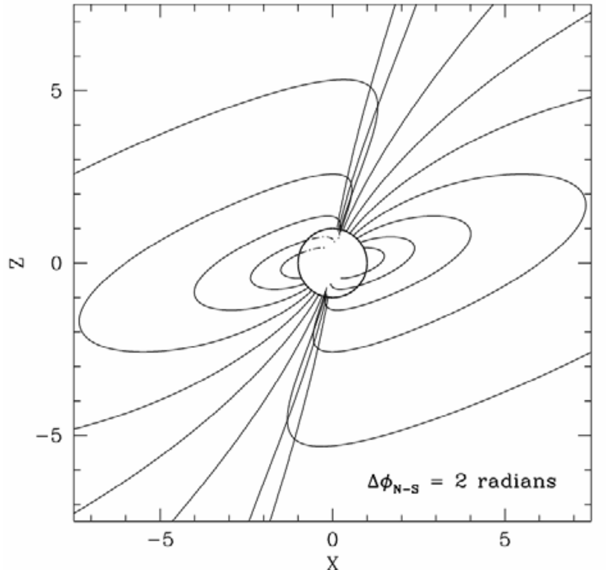
Helicity $\int \mathbf{A} \cdot \mathbf{B} \, dV$ decays slowly, on a resistive timescale, in a confined magnetized plasma



Helical Magnetosphere

Thompson, Lyutikov, & Kulkarni 2002, ApJ, 574 332





$$\mathbf{J} \times \mathbf{B} = 0$$

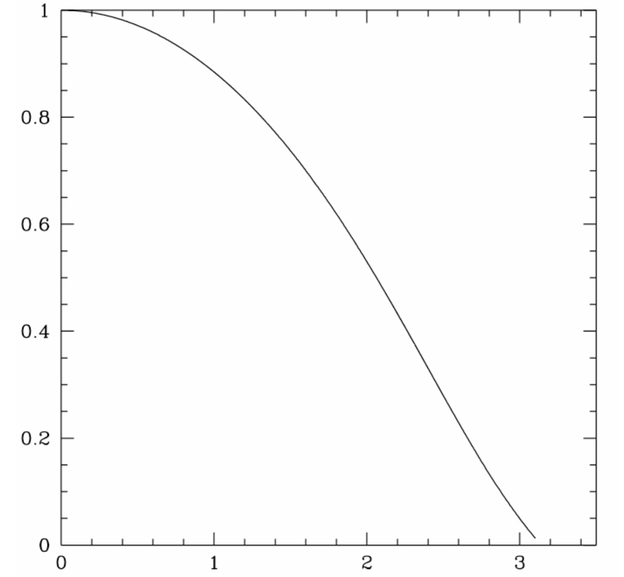
self-similar:

$$\mathbf{B} \propto r^{-(2+p)}$$

$$\Delta\phi_{\mathrm{N-S}} > 0$$

$$\Rightarrow \quad p < 3$$

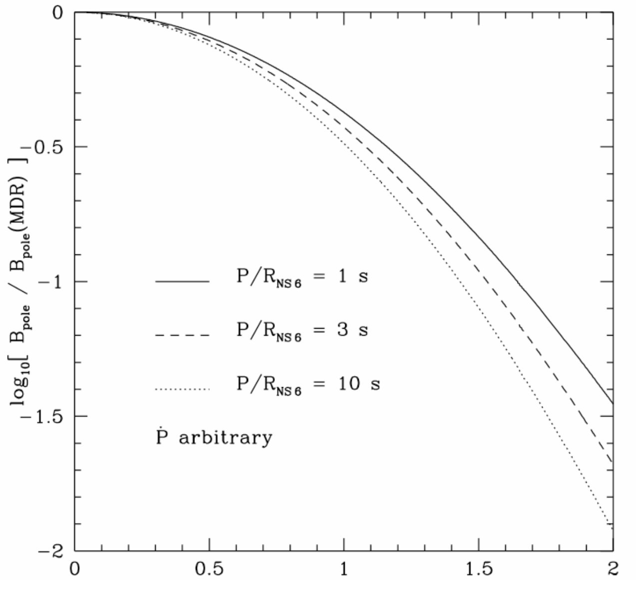
(stronger open-field current and persistent, accelerated spindown)



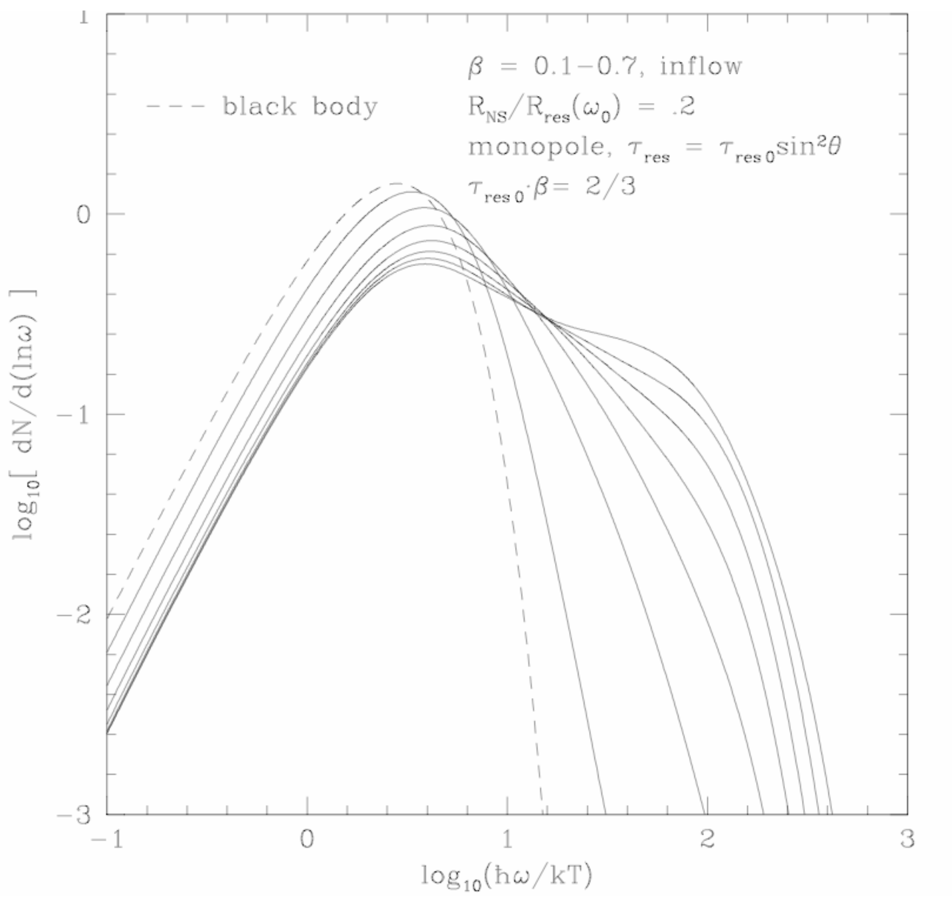
p

$\Delta\phi_{\mathrm{N-S}} \longrightarrow$ (rad)

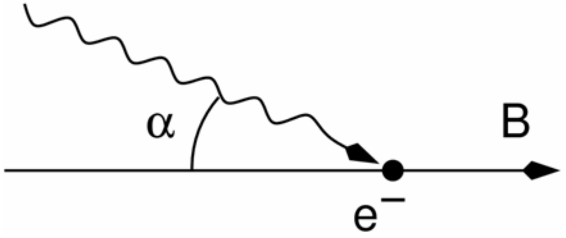
(relative twist of N/S poles)



Mutiple Resonant Cyclotron Scattering



Fernandez and Thompson 2005



$$\sigma = \frac{\pi^2 e^2}{m_e c}\delta\left(\omega - \frac{eB}{m_e c}\right)(1+\cos^2\alpha)$$

$$\mathbf{J} \simeq \frac{c\mathbf{B}}{4\pi R}\sin^2\theta$$

twisted dipole

$\Rightarrow$ optical depth

$$\boxed{\int\left(\frac{J}{e\beta c}\right)\sigma dl \sim \frac{\Delta\phi_{\mathrm{N-S}}}{\beta}}$$

QED Processes in Strong B-fields

$$\left(B > B_{\rm QED} = \frac{m_e c^3}{e\hbar} = 4.4 \times 10^{13}\ {\rm G} \right)$$

Vacuum is birefringent

Photons are linearly polarized

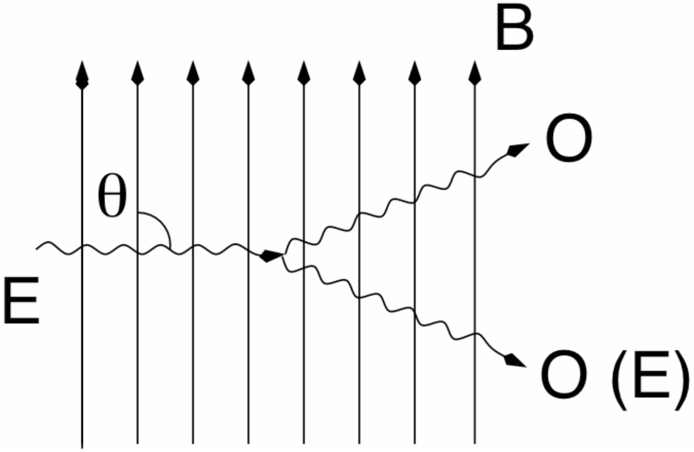
E-mode: $\delta \mathbf{E} \cdot \mathbf{B}_0 = 0$

O-mode: $\delta \mathbf{B} \cdot \mathbf{B}_0 = 0$

$$\boxed{\sigma_E(\omega) \simeq \left(\frac{m_e c\,\omega}{eB} \right)^2 \sigma_O}$$

enhanced transparency of E-mode

$n_O > n_E$ ⇒ thermal X-ray photons can split and merge



$$\boxed{\Gamma_{\rm split} = \frac{\alpha_{\rm em}^3}{2160\pi^2} \sin^6\theta \left(\frac{\hbar\omega}{m_e c^2} \right)^5 \frac{m_e c^2}{\hbar}}$$

$$\boxed{T > T_{\rm split} = 11 \left(\frac{\Delta R}{10\ {\rm km}} \right)^{-1/5} {\rm keV}}$$

Vacuum Polarization: Effects on Ion Cyclotron Features

$$\boxed{\hbar \frac{eB}{m_p c} = 6.3 \left(\frac{B}{10^{15}\ \mathrm{G}} \right) \ \mathrm{keV}}$$

resonant mode conversion:

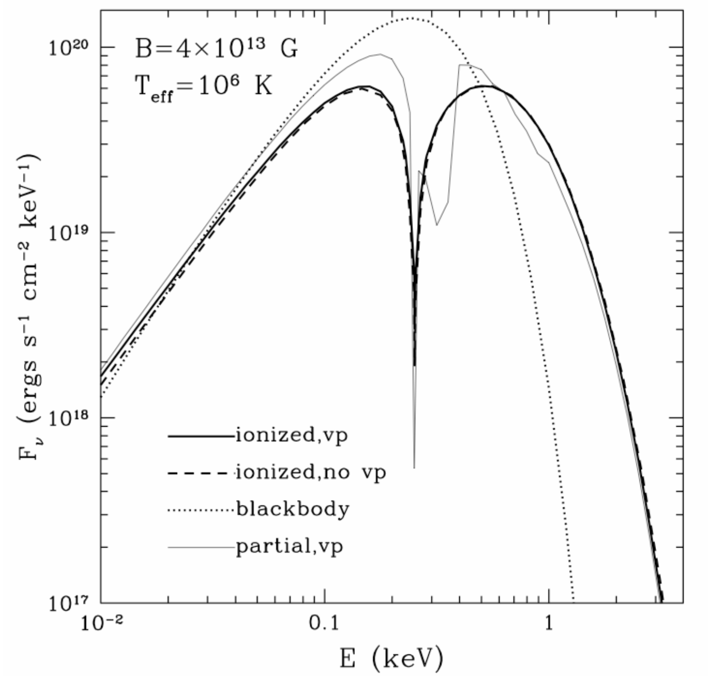
$$\rho_V \simeq 1\ Y_e^{-1} B_{14}^2 E_{\mathrm{keV}}^2 \quad \mathrm{g\ cm^{-3}}$$

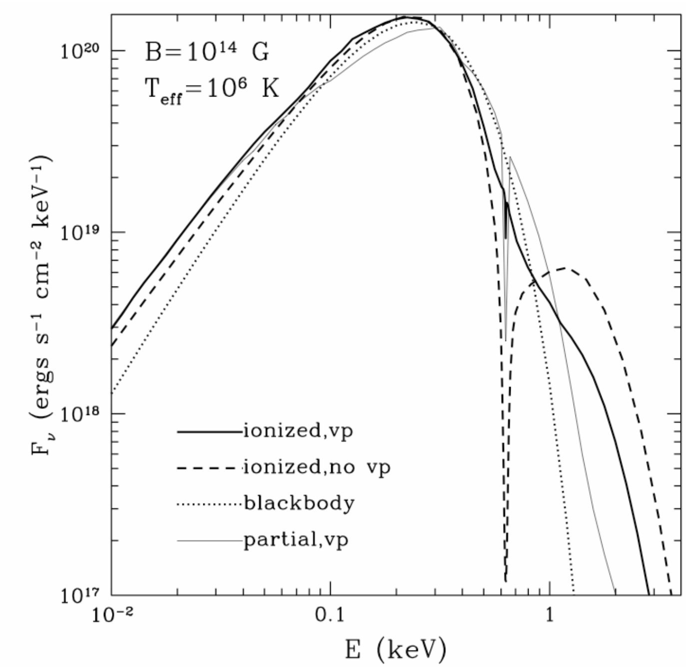
O-mode photosphere:

$$\rho_{O,\mathrm{novac}} \simeq 0.4\ T_6^{-1/2} E_{\mathrm{keV}}^{3/2} \quad \mathrm{g\ cm^{-3}}$$

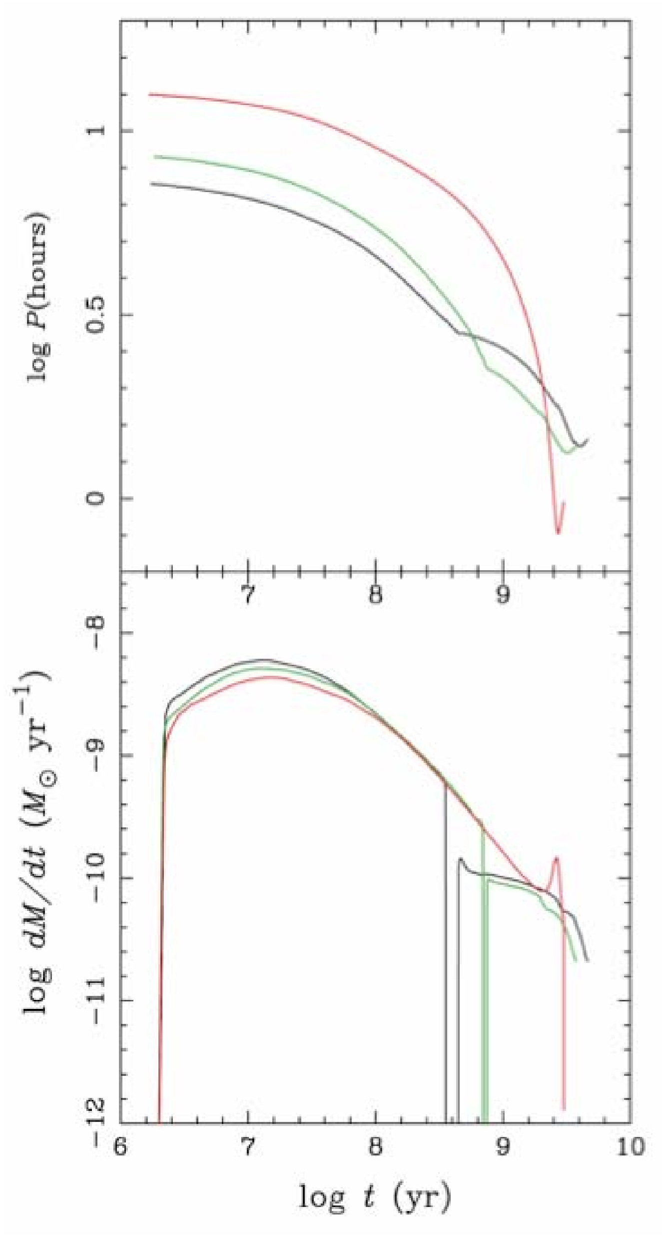
Ho & Lai 2004, ApJ 607, 420

$\rho_V > \rho_{O,\mathrm{novac}} \Rightarrow$ line feature erased





Accretion History of a Low-Mass X-ray Binary



Pfahl et al. 2002, ApJ, 565, 1107

net accreted mass can be $> 0.5\ M_\odot$

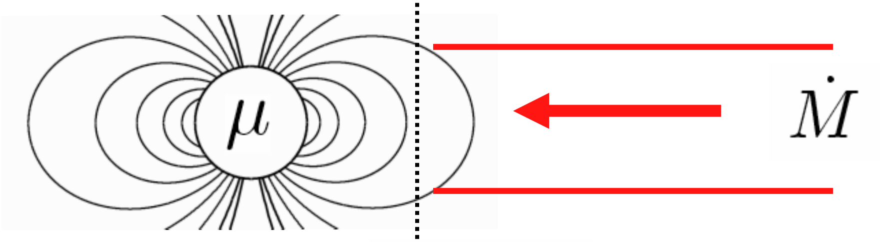
$\Rightarrow$

i) transition to direct URCA cooling $(M_{\rm ns} > 1.8\, M_\odot)$

ii) transition to mixed phase hadronic/quark matter core (Akmal et al. 1998, PRC, 58, 1804)

iii) crust material $(\sim 10^{-2}\, M_{\rm ns})$ is replaced multiple times

Spin-up of Accreting Neutron Star



$$\boxed{2\pi\Delta\nu \sim \frac{\Delta M}{I_{\rm ns}}\sqrt{GM_{\rm ns}R_{\rm Alfven}}}$$

$R_{\rm Alfven}$ - Alfven radius

$$\left(\frac{B_\phi B_\theta}{4\pi}\right) 2\pi R_{\rm Alfven}^2 \Delta r \sim \dot{M}(GM_{\rm ns}R_{\rm Alfven})^{1/2}$$

$$\Rightarrow R_{\rm Alfven} = \left(\frac{\mu^4}{2GM_{\rm ns}\dot{M}^2}\right)^{1/7} \qquad \mu = \langle B_{\rm ns}\rangle R_{\rm ns}^3$$

- magnetic moment

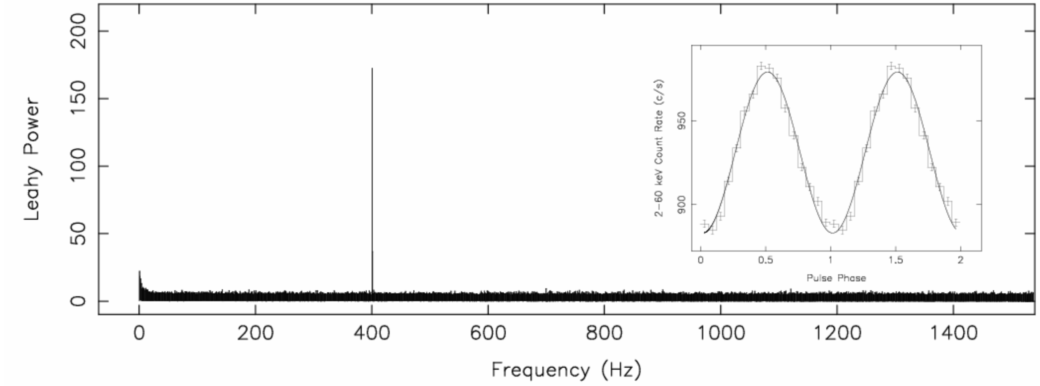
equilibrium

spin period

$$\boxed{\begin{aligned} P_{\rm eq} &= (4\pi^2 R_{\rm Alfven}^3/GM_{\rm ns})^{1/2} \\ &= 0.003\,\frac{(\mu/10^{27}\ {\rm G\ cm}^3)^{6/7}}{(M_{\rm ns}/1.4\ M_\odot)^{5/7}}\left(\frac{\dot{M}}{10^{-8}\ M_\odot/{\rm yr}}\right)^{-3/7}\ {\rm s}\end{aligned}}$$

Pulsations in LMXBs from RXTE

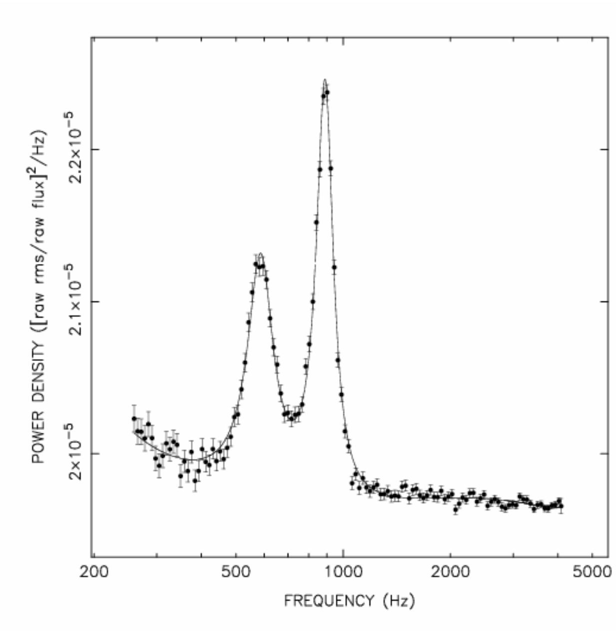
Wijnands & van der Klis 1998



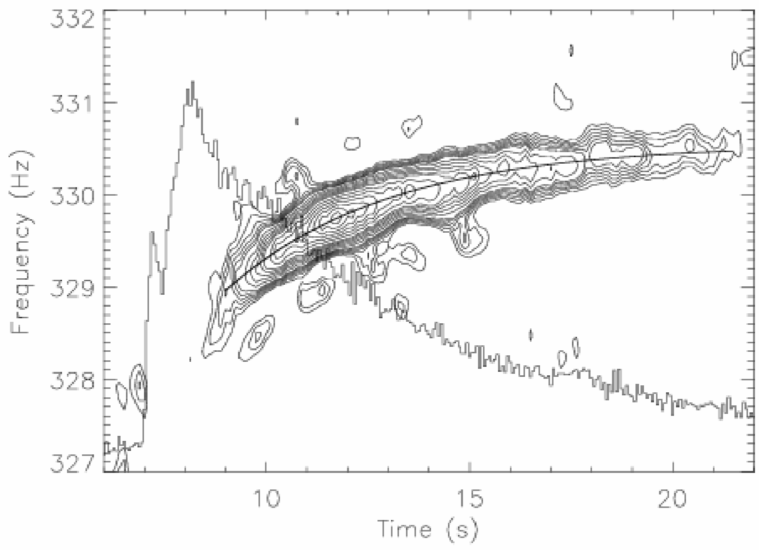
SAXJ1808.4-3658

Sco-X1 QPO

van der Klis et al. 1997



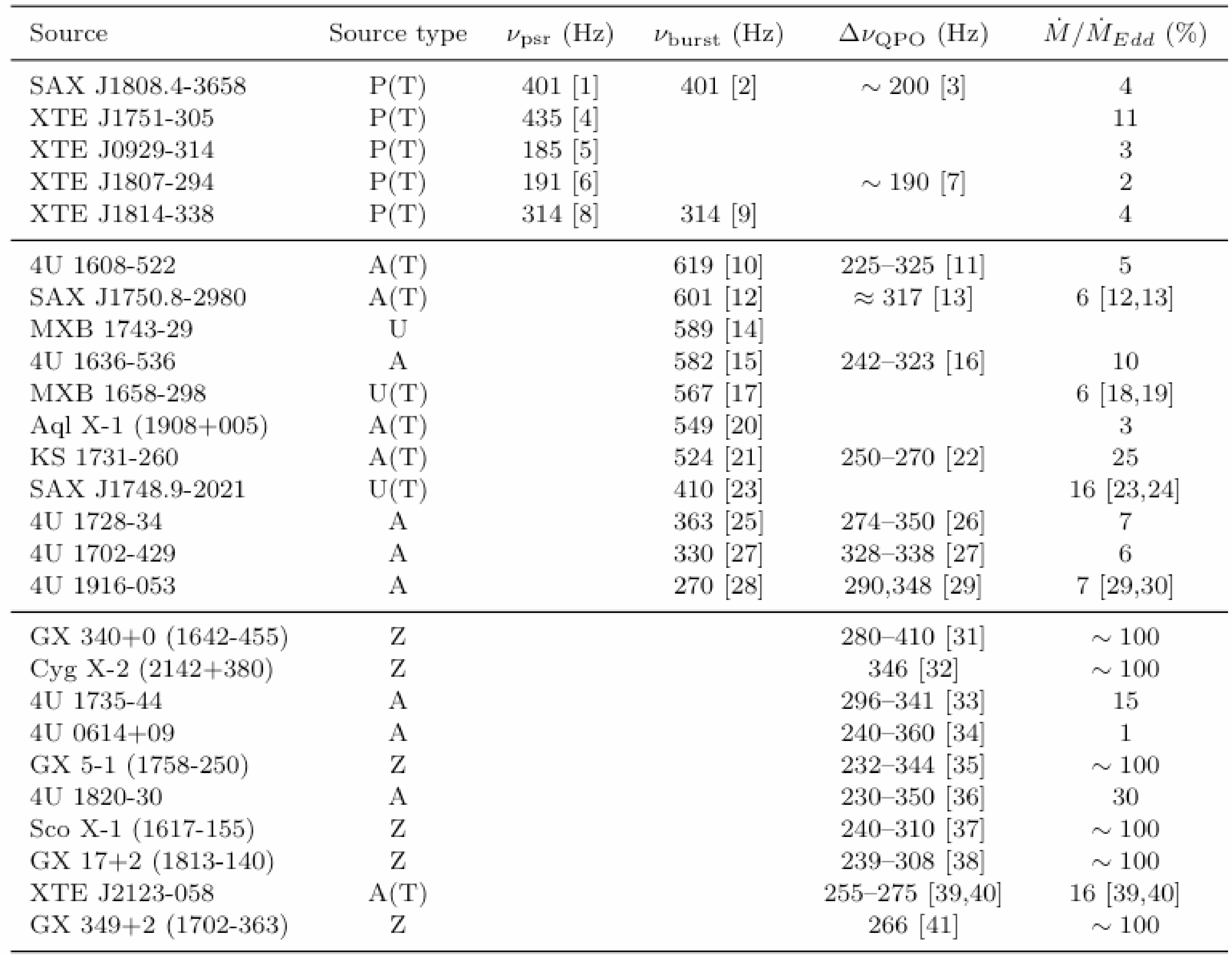
4U 1702-43
burst oscillation



Strohmayer & Markwardt 1999

Spin Frequencies and QPO Frequencies of LMXBs

Andersson et al. 2004 (astro-ph/0411717)



Source	Source type	ν_{psr} (Hz)	ν_{burst} (Hz)	$\Delta\nu_{\mathrm{QPO}}$ (Hz)	$\dot{M}/\dot{M}_{Edd}$ (%)
SAX J1808.4-3658	P(T)	401 [1]	401 [2]	$\sim$ 200 [3]	4
XTE J1751-305	P(T)	435 [4]			11
XTE J0929-314	P(T)	185 [5]			3
XTE J1807-294	P(T)	191 [6]		$\sim$ 190 [7]	2
XTE J1814-338	P(T)	314 [8]	314 [9]		4
4U 1608-522	A(T)		619 [10]	225–325 [11]	5
SAX J1750.8-2980	A(T)		601 [12]	$\approx$ 317 [13]	6 [12,13]
MXB 1743-29	U		589 [14]		
4U 1636-536	A		582 [15]	242–323 [16]	10
MXB 1658-298	U(T)		567 [17]		6 [18,19]
Aql X-1 (1908+005)	A(T)		549 [20]		3
KS 1731-260	A(T)		524 [21]	250–270 [22]	25
SAX J1748.9-2021	U(T)		410 [23]		16 [23,24]
4U 1728-34	A		363 [25]	274–350 [26]	7
4U 1702-429	A		330 [27]	328–338 [27]	6
4U 1916-053	A		270 [28]	290,348 [29]	7 [29,30]
GX 340+0 (1642-455)	Z			280–410 [31]	$\sim$ 100
Cyg X-2 (2142+380)	Z			346 [32]	$\sim$ 100
4U 1735-44	A			296–341 [33]	15
4U 0614+09	A			240–360 [34]	1
GX 5-1 (1758-250)	Z			232–344 [35]	$\sim$ 100
4U 1820-30	A			230–350 [36]	30
Sco X-1 (1617-155)	Z			240–310 [37]	$\sim$ 100
GX 17+2 (1813-140)	Z			239–308 [38]	$\sim$ 100
XTE J2123-058	A(T)			255–275 [39,40]	16 [39,40]
GX 349+2 (1702-363)	Z			266 [41]	$\sim$ 100

Table References

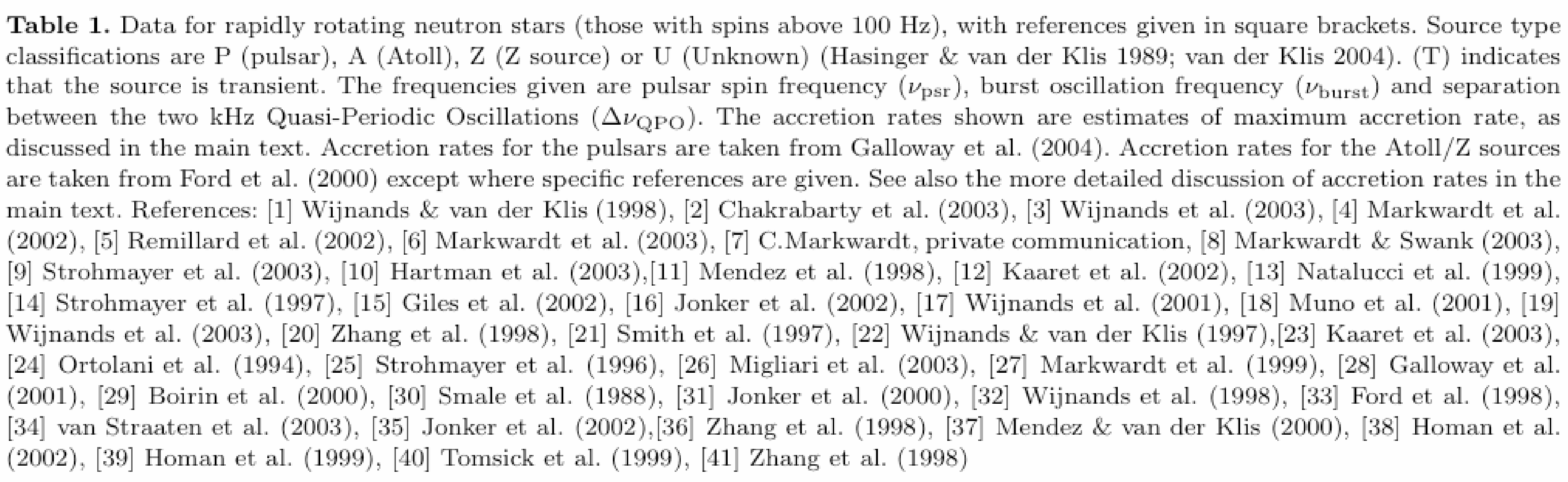


Table 1. Data for rapidly rotating neutron stars (those with spins above 100 Hz), with references given in square brackets. Source type classifications are P (pulsar), A (Atoll), Z (Z source) or U (Unknown) (Hasinger & van der Klis 1989; van der Klis 2004). (T) indicates that the source is transient. The frequencies given are pulsar spin frequency ($\nu_{\rm psr}$), burst oscillation frequency ($\nu_{\rm burst}$) and separation between the two kHz Quasi-Periodic Oscillations ($\Delta\nu_{\rm QPO}$). The accretion rates shown are estimates of maximum accretion rate, as discussed in the main text. Accretion rates for the pulsars are taken from Galloway et al. (2004). Accretion rates for the Atoll/Z sources are taken from Ford et al. (2000) except where specific references are given. See also the more detailed discussion of accretion rates in the main text. References: [1] Wijnands & van der Klis (1998), [2] Chakrabarty et al. (2003), [3] Wijnands et al. (2003), [4] Markwardt et al. (2002), [5] Remillard et al. (2002), [6] Markwardt et al. (2003), [7] C.Markwardt, private communication, [8] Markwardt & Swank (2003), [9] Strohmayer et al. (2003), [10] Hartman et al. (2003),[11] Mendez et al. (1998), [12] Kaaret et al. (2002), [13] Natalucci et al. (1999), [14] Strohmayer et al. (1997), [15] Giles et al. (2002), [16] Jonker et al. (2002), [17] Wijnands et al. (2001), [18] Muno et al. (2001), [19] Wijnands et al. (2003), [20] Zhang et al. (1998), [21] Smith et al. (1997), [22] Wijnands & van der Klis (1997),[23] Kaaret et al. (2003), [24] Ortolani et al. (1994), [25] Strohmayer et al. (1996), [26] Migliari et al. (2003), [27] Markwardt et al. (1999), [28] Galloway et al. (2001), [29] Boirin et al. (2000), [30] Smale et al. (1988), [31] Jonker et al. (2000), [32] Wijnands et al. (1998), [33] Ford et al. (1998), [34] van Straaten et al. (2003), [35] Jonker et al. (2002),[36] Zhang et al. (1998), [37] Mendez & van der Klis (2000), [38] Homan et al. (2002), [39] Homan et al. (1999), [40] Tomsick et al. (1999), [41] Zhang et al. (1998)

Andersson et al. 2004 (▪ astro-ph/0411717)

Endpoint of Spin-up

$\Delta M \sim 0.1\, M_\odot$ + FIELD DECAY needed to spin-up:

$$\boxed{\Delta\nu \sim 600 \left(\frac{I_{\rm ns}}{10^{45}\ {\rm g\ cm}^2}\right)^{-1} \left(\frac{\Delta M}{0.1\ M_\odot}\right)\ {\rm Hz}}$$

$$\left.\begin{array}{r} \mu = \langle B_{\rm ns}\rangle R_{\rm ns}^3 \sim 3\times 10^{26}\ {\rm G\ cm}^3 \\ \dot M \sim 10^{-8}\, M_\odot/{\rm yr} \end{array}\right\}\quad \nu_{\rm eq} \sim 900\ {\rm Hz}$$

PUZZLES:

i) observed spin frequencies 200 - 600 Hz

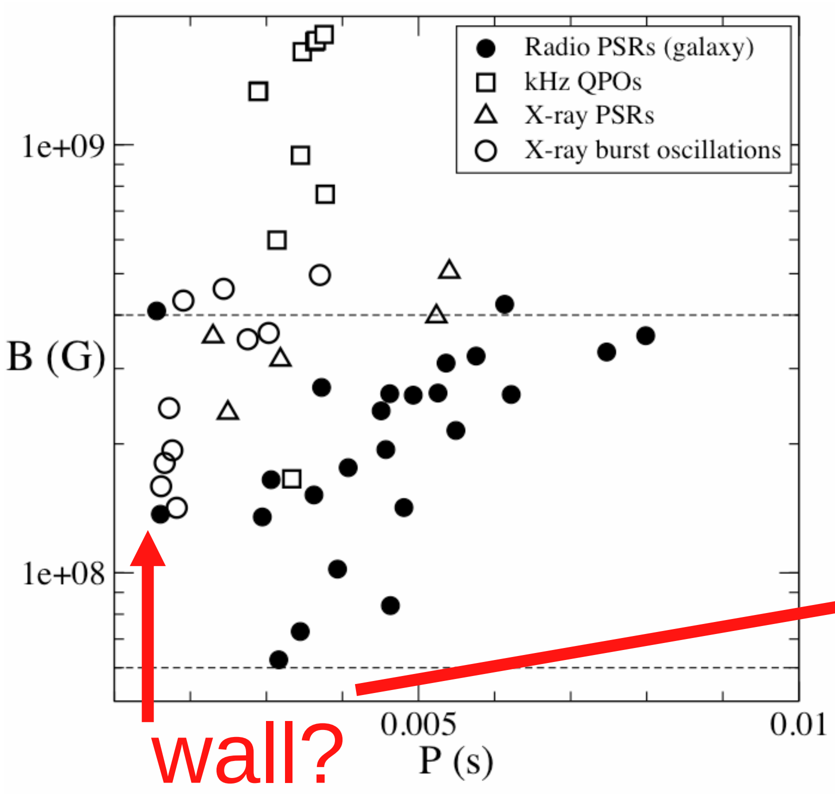
ii) possible frequency `wall' at ~ 600 Hz

iii) no measured persistent spin modulation

except in transient LXMB's

Competition between Magnetic Torque and Gravity Wave Torque:

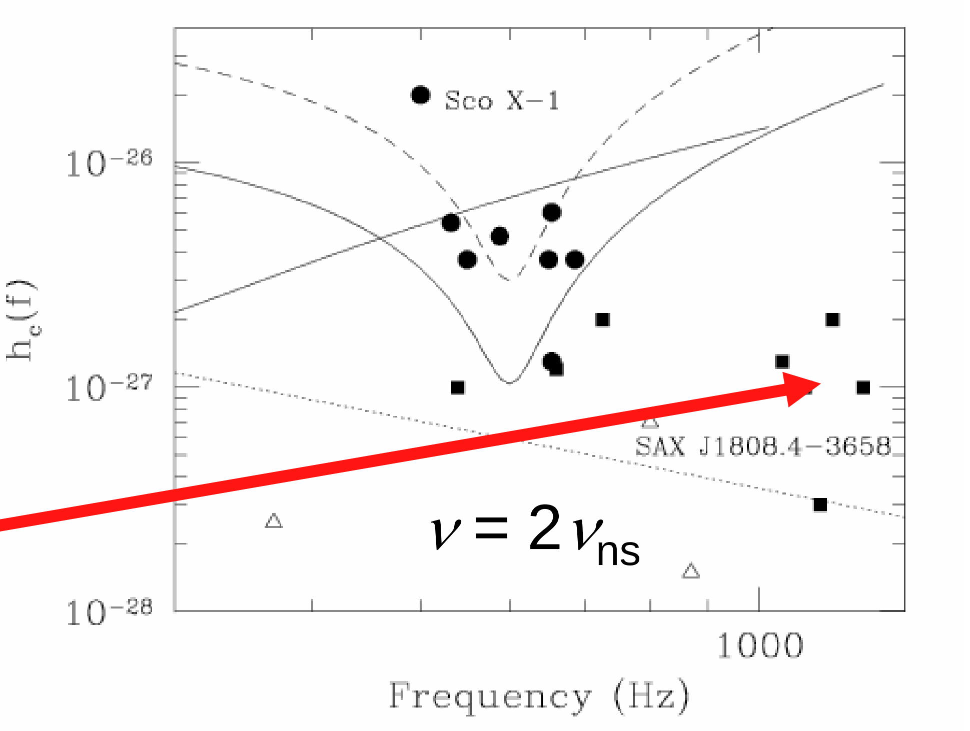
predicted B_{dipole}:



Andersson et al. (astro-ph/0411747)

predicted gravity wave strain:

$$h_c \approx 1.3 \times 10^{-27} \frac{R_6^{3/4}}{M_{1.4}^{1/4}} \left(\frac{F}{10^{-9}\ \mathrm{erg\ cm^{-2}\ s^{-1}}}\right)^{1/2} \left(\frac{300\ \mathrm{Hz}}{\nu_s}\right)^{1/2}$$



Bildsten (astro-ph/0212004)

RESOLUTION #1

Very stiff EOS - not likely (large accreted mass)

RESOLUTION #2

Magnetic torques limit spin frequency:

i) LMXB spends most time at $\langle \dot{M} \rangle \sim 10^{-10}\text{-}10^{-9}\, M_\odot/\mathrm{yr}$

(e.g. high-$\dot{M}$ LMXBs like Sco X-1 are long-period transients)

ii) magnetic moment is aligned with neutron star spin

(e.g. Rapid Burster)

RESOLUTION #3

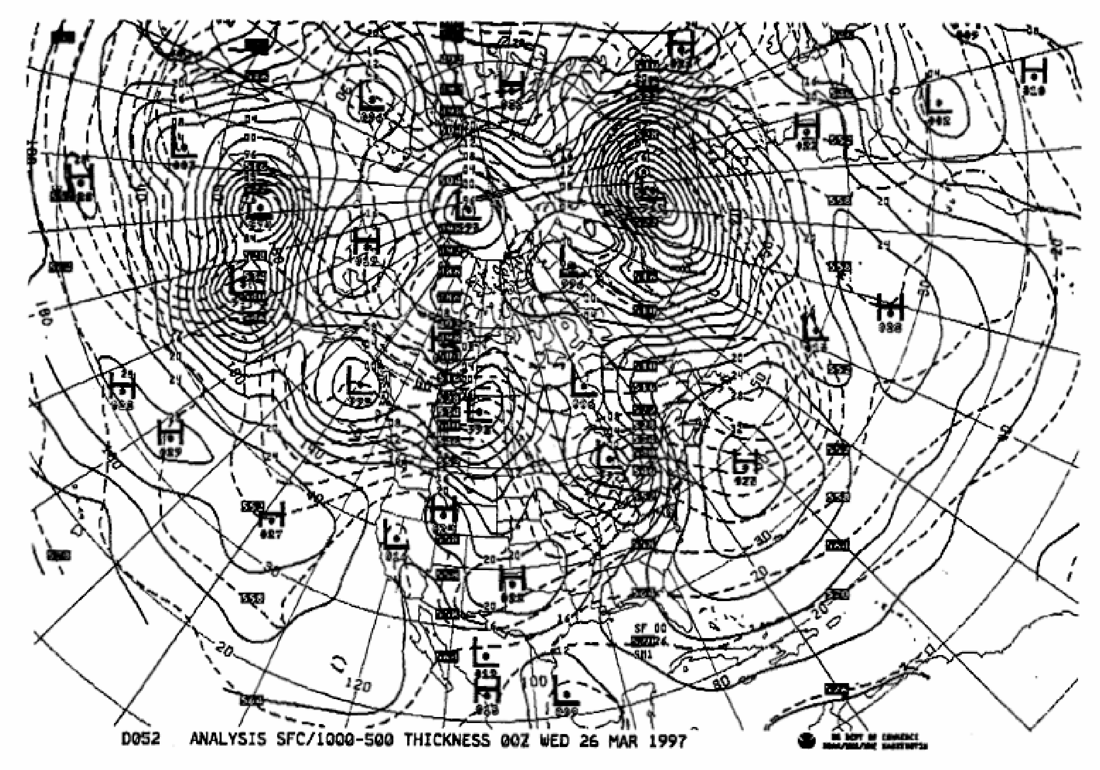
Gravity wave emission:

i) persistent quadrupole (Bildsten)

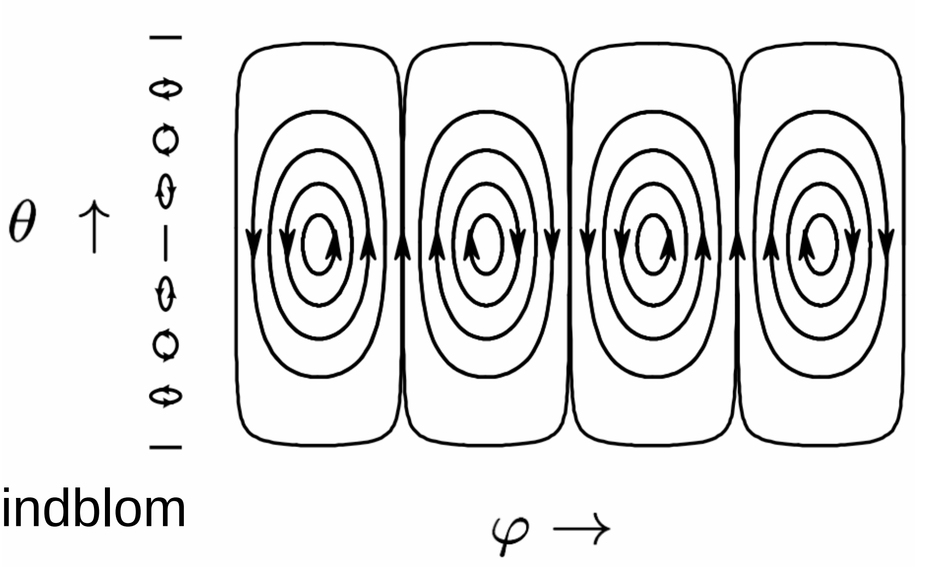
ii) self-excited mode (Rossby wave)

(Wagoner; Levin; Heyl;
Reisenegger & Bonacic)

Rossby Waves



Earth



Lindblom

m = l = 2

(fastest-growing mode on N.S.)

Rossby Wave Self-Excitation in a Rotating Star

Chandrasekhar
1970, PRL, 24, 611
Friedmann & Shutz
1978, ApJ, 222, 281
Andersson
1998, ApJ 502, 708

Gravity-wave emission by oscillatory mass currents

$$\mathbf{v} = \alpha\,(2\pi\nu_{\rm ns}R_{\rm ns})\left(\frac{r}{R_{\rm ns}}\right)^{\ell}\frac{r\vec{\nabla}\times(r\vec{\nabla}Y_{\ell m})}{\sqrt{\ell(\ell+1)}}e^{i\omega t}$$

Coriolis force is restoring force $\Rightarrow$ driving at all $\nu_{\rm ns}$

l = *m* = *2* is fast-growing mode
(at lower $\nu_{\rm ns}$ than quadrupolar f-mode, a.k.a. bar mode)

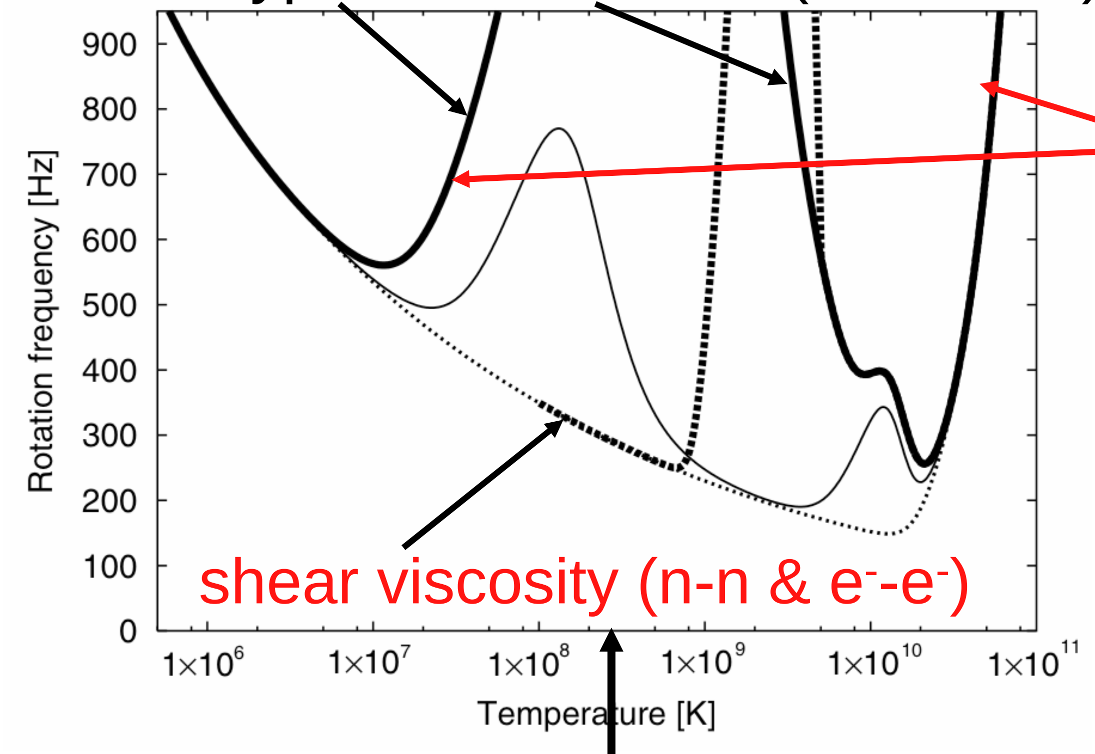
Pattern speed: $\sigma = -\frac{\omega}{m} = \left(\frac{2}{3}\right) 2\pi\nu_{\rm ns}$

Retrograde in rotating frame: $\sigma' = \sigma - 2\pi\nu_{\rm ns} < 0$

$\Rightarrow$ negative *J* mode

Gravity wave extracts positive *J* $\Rightarrow$ GROWTH!

Rossby Wave Damping in a Neutron Star



hyperon-rich matter (Λ^0, Σ^-, …)

bulk viscosity

shear viscosity (n-n & e^--e^-)

lowest curve:
n-p-e matter

Reisenegger & Bonacic
2003, PRL 91, 1103

internal temperature of LMXB
(modified URCA cooling)

excitation at ν > 300 Hz !!
($\Rightarrow$ limit cycle behavior, Levin)

Hyperons provide strong bulk viscosity (Jones) and allow equilibrium R-mode excitation (Wagoner 2002, ApJ, 578, L63; R&B)

Reality is probably more non-linear:

1. Saturation of R-mode by
   3-wave coupling to damped inertial modes of the star

$$\boxed{\text{limiting mode amplitude } \alpha \sim 10^{-4}}$$

(Arras et al. 2003, ApJ, 591, 1129; Brink et al. 2004, PRD, 70, 121501)

2. Strong magnetic shear layer between NS crust & core
   Increase of toroidal field energy:

$$\frac{dE_B}{dt} \sim \alpha^2 \left(2\pi\nu_{\rm ns}\right) \frac{B_\phi B_{\rm Poloidal}}{8\pi} \left(\frac{4\pi}{3} R_{\rm ns}^3\right)$$

Rezzolla et al. 2001,
PRD, 64, 104013

Balance with increase of mode energy (rotating frame):

$$\frac{dE_{\rm R-mode}}{dt} \sim 10^{-3}\, \alpha^2\, \frac{G M_{\rm ns}^2 R_{\rm ns}^6}{c^7} \left(2\pi\nu_{\rm ns}\right)^8$$

$$\Rightarrow \quad \boxed{B_\phi B_{\rm Poloidal} \sim 10^{27}\, (\nu/600\ {\rm Hz})^7\ {\rm G}^2}$$

damping by
crust cracking (?)

Implications for Isolated Neutron Stars

competition between
gravity wave torque and
magnetic dipole torque:

$$\boxed{\begin{aligned} \frac{dE_{\mathrm{GW}}}{dt} &= \frac{32 G Q^2 (2\pi\nu_{\mathrm{ns}})^6}{5c^5} \\ \frac{dE_{\mathrm{MDR}}}{dt} &= \frac{2}{3}\mu^2 \frac{(2\pi\nu_{\mathrm{ns}})^4}{c^3} \end{aligned}}$$

rigid quadrupole: $Q = \epsilon I_{\mathrm{ns}} = I_{\mathrm{ns}\,\|} - I_{\mathrm{ns}\,\perp}$

toroidal B-field: $\epsilon = -10^{-6}\left(\frac{B_\phi}{10^{15}\ \mathrm{G}}\right)^2$

Cutler 2002
PRD, 66, 4025

R-mode: $Q \to \alpha\,\frac{2\pi\nu_{\mathrm{ns}}}{R_{\mathrm{ns}}c}\int_0^{R_{\mathrm{ns}}} dr\,\rho(r)\,r^6 = 0.02\,\alpha\left(\frac{2\pi\nu_{\mathrm{ns}}R_{\mathrm{ns}}}{c}\right) M_{\mathrm{ns}}R_{\mathrm{ns}}^2$

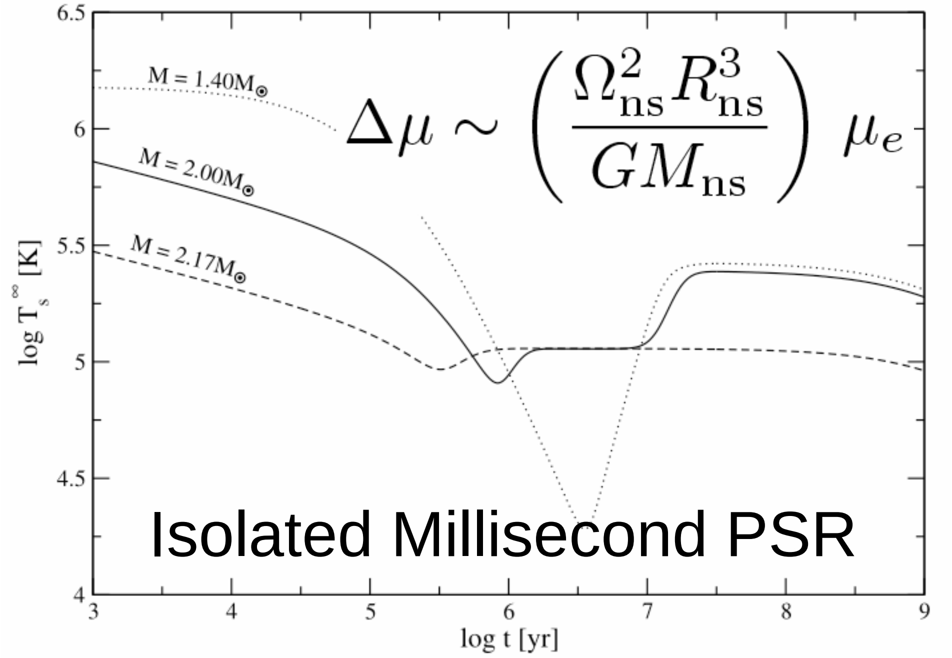
$$\boxed{\frac{dE_{\mathrm{GW}}/dt}{dE_{\mathrm{MDR}}/dt} = 3\times10^{-8}\,\varepsilon_{-6}^2\left(\frac{\mu}{10^{33}\ \mathrm{G\ cm}^3}\right)^{-2}\left(\frac{\nu_{\mathrm{ns}}}{\mathrm{kHz}}\right)^2}$$

$\alpha \sim 10^{-4}$
(saturation)

B_φ quadrupole is unstable to flipping over and radiating
(may require additional frictional force, e.g. from orbiting torus)

Internal Heating

Fernandez & Reisengger 2005, ApJ, 625, 291



$$\Delta\mu \sim \left(\frac{\Omega_{\rm ns}^2 R_{\rm ns}^3}{GM_{\rm ns}}\right)\mu_e$$

Isolated Millisecond PSR

chemical potential imbalance:

$$\Delta\mu = \mu_e + \mu_p - \mu_n$$

Magnetar

$$\Delta\mu \sim \frac{B^2}{8\pi n_e}$$

delayed pairing transition of core superfluid neutrons

($T_{cn} < 6\text{x}10^8$ K)

Arras, Cumming, & Thompson 2004, ApJ, 608, L49