The Cosmological Constant

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References: (I borrowed heavily from some of them)

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S. Perlmutter, Physics Today, April '03 p.53.

C. Deffayet, G. Dvali, G. Gabadaze, astro-ph/0105068.

G. Dvali and M. Turner, astro-ph/0301510.

R. Bousso and J. Polchinski, Sci. Am. 291 (2004) 60.

S. Kachru, R. Kallosh, A. Linde, S. Trivedi hep-th/0301240.

E. Silverstein, hep-th/0405068.

M. Douglas, hep-th/0303194.

+ others cited in talk ...

Introduction

Let us begin with a poem:

> As I was going up the stair,
> I met a man who wasn't there.
> He wasn't there again today,
> I wish, I wish he'd stay away.

Hughes Mearns

[As quoted in this context by
S. Weinberg, Rev Mod Phys, 1989]

Of course, things have changed since 1989. We now know that the man is there; he is just extremely tiny.

Since the dark energy constitutes (by one natural measure) 70% of the Universe, but only (1/48)th of the lectures at SSI, my coverage will necessarily be sporadic & idiosyncratic.

Plan:

I. Experimental evidence

II. Some testable alternatives to Λ

III. Ideas in string theory related to Λ

But first!

Lightning review of what Λ is.

Einstein's field equations:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

In a homogeneous, isotropic Universe with Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t) R_0^2 \left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right]$$

and assuming $T_{\mu\nu}$ is sourced by a perfect fluid of energy density ρ, pressure p:

$$T_{\mu\nu} = (p+\rho) U_\mu U_\nu + p g_{\mu\nu}$$

one finds the Friedmann equations:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho - \frac{k}{a^2 R_0^{\,2}}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

In the presence of an energy density of "empty space" $\sim \int d^4x \sqrt{-g}\, M_P^2 \Lambda$, these equations are modified:

$$H^2 = \frac{8\pi}{3} G\rho + \frac{\Lambda}{3} - \frac{k}{a^2 R_0^{\,2}}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3} G(\rho + 3p) + \frac{\Lambda}{3}$$

At "critical density" (after sensibly absorbing Λ in "ρ",

$$\rho_{crit} \equiv \frac{3H^2}{8\pi G}$$

one obtains a <u>flat</u> Universe. It is conventional to describe the ingredients of our Universe in terms of the fractions

$$\Omega_i \equiv \frac{\rho_i}{\rho_{crit}}$$

where "i" labels the various ingredients.

If a given ingredient has equation of state:

$$p_i = w_i \rho_i$$

you can show:

$$\rho_i \sim (a(t))^{-n_i}$$

$$n_i \equiv 3[1 + w_i]$$

source	w_i	n_i
matter	0	3
radiation	1/3	4
curvature	-1/3	2
Λ	-1	0

Intuitively, we expect gravity to slow the expansion rate of the Universe. So it is conventional to discuss the "deceleration parameter"

$$q = -\frac{\ddot{a}\,a}{\dot{a}^2}$$

$$= \sum_i \frac{(n_i - 2)}{2}\,\Omega_i$$

Note that $\Lambda \Rightarrow$ negative deceleration -- it accelerates the expansion.

The great problem with Λ is its order of magnitude. Natural sizes of radiative corrections / energy releases in phase transitions in modern particle theories :

$$M_{Planck} \sim 10^{18}\ GeV$$
$$M_{GUT} \sim 10^{16}\ GeV$$
$$M_{EW} \sim 10^{3}\ GeV$$
$$\Lambda_{QCD} \sim 200\ MeV$$

} All far larger than Λ in our world

This is the "cosmological constant problem."
How do we explain a world with $\Lambda \simeq 0$ without SUSY? [$\not{S}\!U\!SY \geq M_{EW}$]

For years, many expected $\Lambda = 0$ exactly.
A deep, unknown principle of physics would be discovered to explain this.
(Note the total wishful thinking here).
However ...

I. Experimental Evidence

There are now at least two strong, independent lines of experimentation that seem to indicate $\Lambda > 0$.

i) IA Supernovae as standard candles

In principle, the expansion history of the Universe can be obtained using "standard candles" - objects of known intrinsic brightness that can be identified over a wide range of distances.

The redshift z of radiation from a co-moving object is related to $a(t)\big|_{\text{emission}}$ by :

$$a = \frac{1}{1+z}$$

The <u>time</u> elapsed between today and the emission from an object at redshift z_* is

$$t_0 - t_* = \int_{t_*}^{t_0} dt = \int_{\frac{1}{1+z_*}}^{1} \frac{da}{a\,H(a)}$$

$$\left[\text{where} \quad H(a) = \left(\sum_i \Omega_{i0}\, a^{-n_i} \right)^{1/2} \cdot H_0 \right]$$

E.g. pure matter → if $\Omega = \Omega_m = 1$,

$$t_0 = \frac{2}{3} \frac{1}{H_0}$$

- age <u>decreases</u> as Ω_m is increased (expansion was faster in past)
- age <u>increases</u> as Ω_Λ is increased (accelerating expansion → was slower in the past)

So in particular:

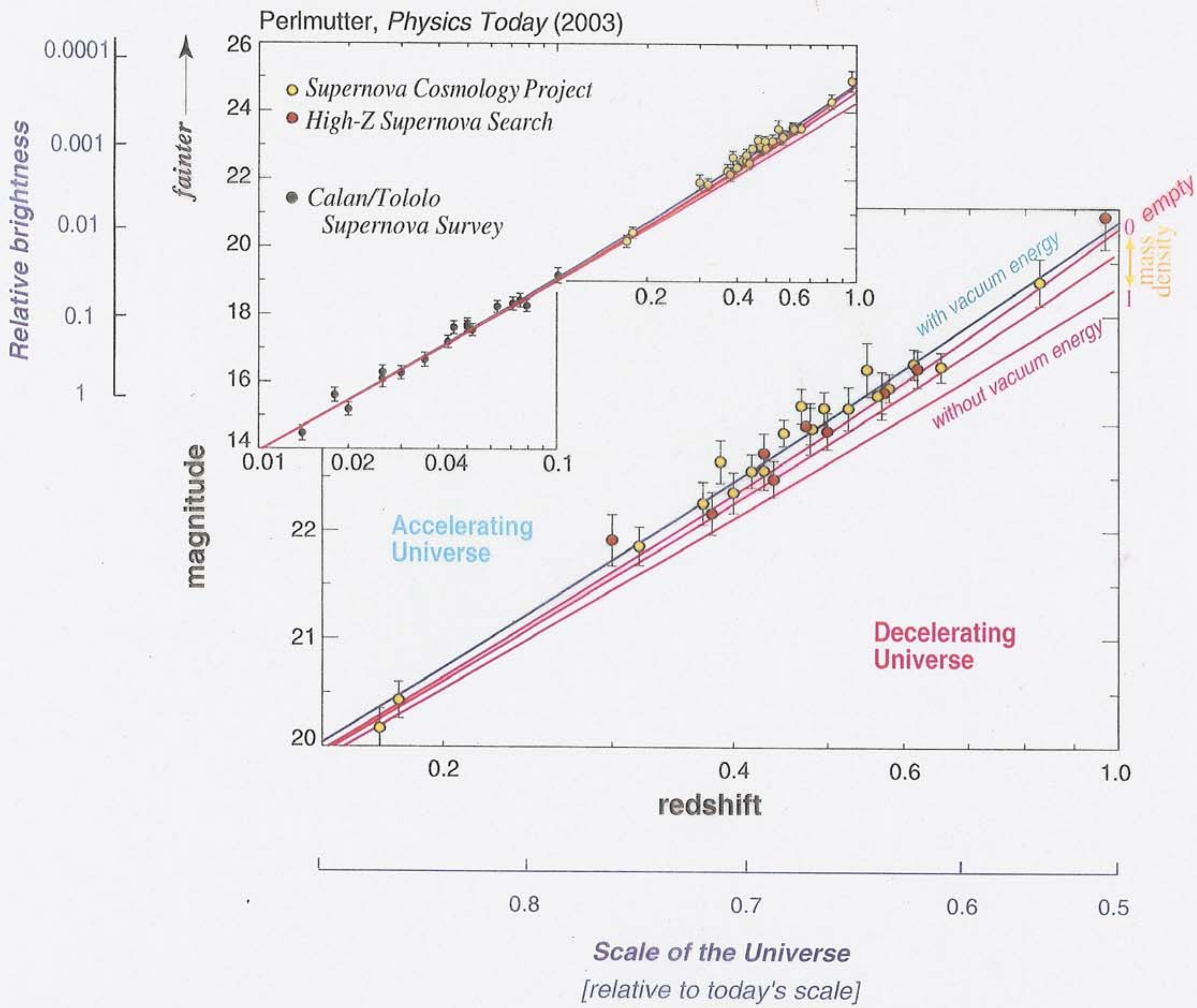
> In an accelerating Universe, standard candles at a given z_* should look <u>dimmer</u> than in a decelerating one (as the light has had time to disperse over a wider area).

The results of the supernova cosmology project and the High z supernova team both support a picture where today

$$\Omega_M \sim 0.3, \quad \underline{\underline{\Omega_\Lambda \sim 0.7}} \quad !!$$

See figure.

Type Ia Supernovae



ii) CMB data combined w/ LSS surveys

My summary here will be even more cartoonish. The location in l-space of the first Doppler peak seen by WMAP, roughly satisfies:

$$l_{peak} \approx 200\ \Omega^{-1/2}$$

$\Rightarrow$ CMB data supports $\boxed{\Omega_{tot} = 1.}$

On the other hand, attempts to "weigh" the matter in the Universe (apply virial thm to cluster dynamics to get M, compute mass to light ratio M/L, & extrapolate) $\Rightarrow$

$$\boxed{\Omega_M \sim 0.3}$$

c.f. Turner astro-ph/0106035

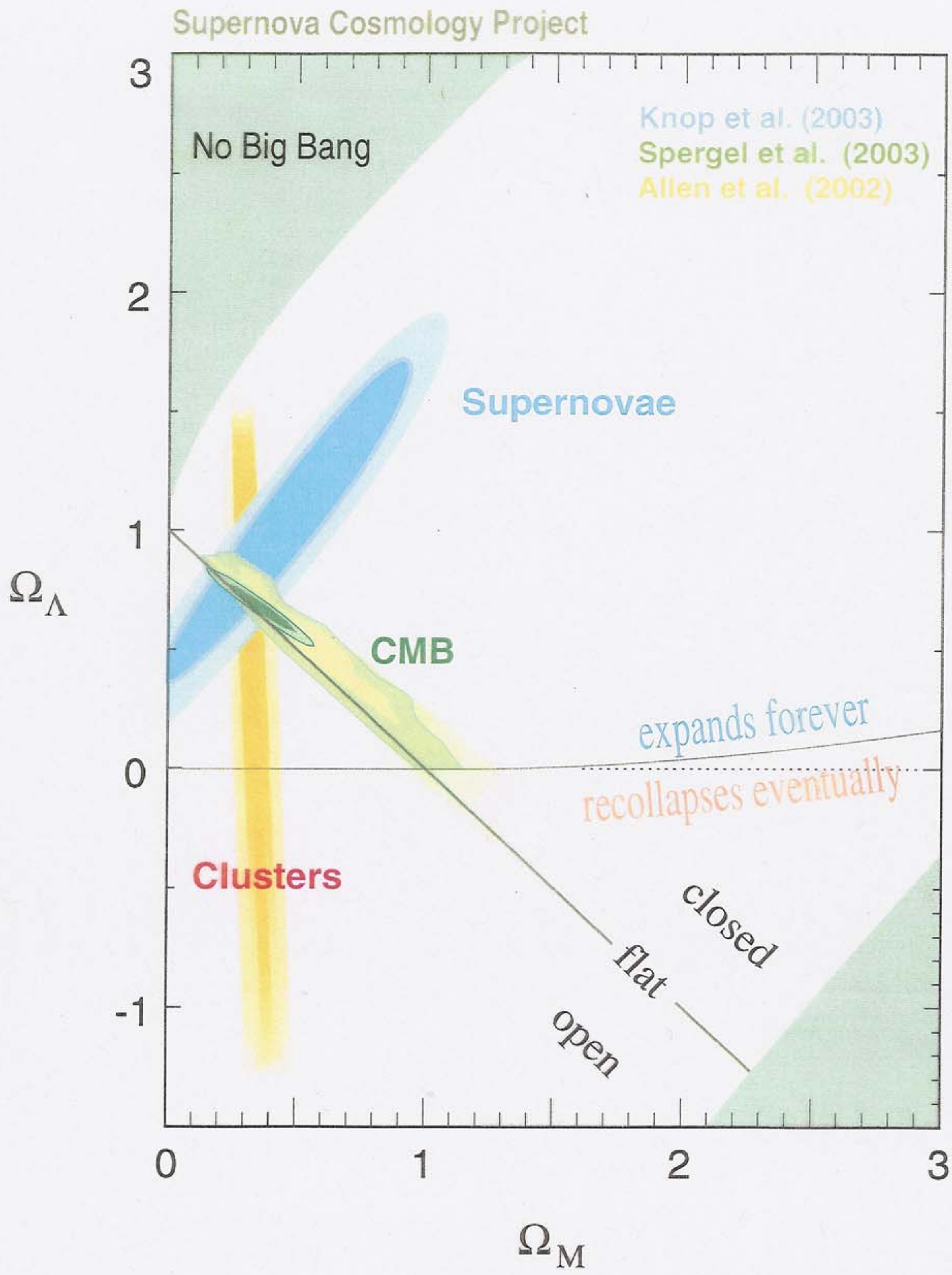
So WMAP + budget of matter, supports $\Omega_\Lambda \approx 0.7$ even <u>without</u> considering the Supernova data.

See figure.

II. Is it really Λ ?

Clearly, it has not been shown beyond reasonable doubt that $w = -1$ for this new dark energy we see. [Current bounds $\to$ $w \lesssim -.7$]. $\exists$ proposals to try and constrain $w(z)$ more precisely.

Alternatives to Λ? Here I mention two.


Supernova Cosmology Project
No Big Bang
Knop et al. (2003)
Spergel et al. (2003)
Allen et al. (2002)
Supernovae
CMB
expands forever
recollapses eventually
Clusters
closed
flat
open
Ω_Λ
Ω_M
3
2
1
0
-1
0
1
2
3

i) "Quintessence"

Consider

$$\mathcal{L} = \int d^4x \sqrt{-g} \left[M_P^2 R + \tfrac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$

- If ϕ is in a vacuum with $\frac{\partial V}{\partial \phi} = 0$, $V'' > 0$, then $V|_{vacuum} \sim$ cosmological term.

- But what if ϕ is still rolling down, with

$$w_{eff} = \frac{T - V}{T + V} \lesssim -.7 \ ?$$

Such a late stage of inflation could → accelerated expansion, $w(z) \neq -1$.

Issues :

- Does not solve CC problem (small value of V today a mystery)
- Must understand how ϕ can be light enough to vary only on time scale $\frac{1}{H_0}$

$$\Rightarrow \qquad M_\phi \lesssim 10^{-33}\,\text{eV}.$$

- Must understand why ϕ doesn't couple to matter + gauge fields in such a way as to induce new long range forces, variation of α with time, etc.

Probably the most reasonable candidates are pseudo-Goldstone bosons with an approximate shift symmetry $\phi \to \phi + \beta$.

E.g. Axions

For an axion, in general

$$V(\phi) \sim \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f_\phi}\right) \right]$$

Λ = dynamical scale of gauge sector whose instantons generate ϕ pot'l

f_ϕ = axion decay constant

In many UV theories (eg string theory)

- Λ naturally exponentially small → $\Lambda \sim 10^{-3}$ eV is plausible
- $f_\phi \lesssim M_P$ is natural ⟹ $m_\phi \gtrsim 10^{-33}$ eV

} Only a small fine tune to get a few e-folds out of ϕ

Such an axion might be detectable!
One of the few allowed couplings to matter + gauge fields is:

$$\mathcal{L} \supset \cdots + \frac{\phi}{M_*} F_{\mu\nu} \tilde{F}^{\mu\nu} + \cdots$$

S. Carroll (1998) has pointed out that such a coupling, if ϕ varies, would rotate the direction of polarization of light from distant radio sources -- potentially observable (and bounds coef. of that operator in $\mathcal{L}$).

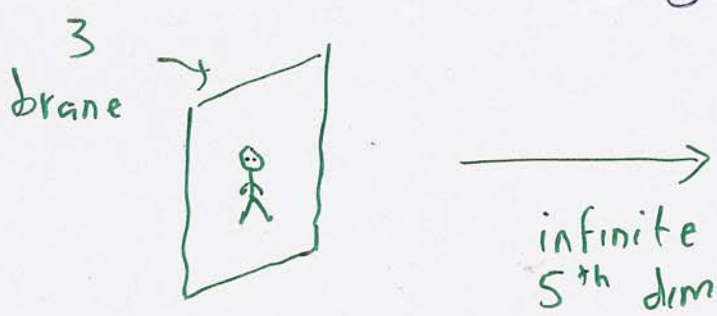
(ii) IR modifications of gravity (c.f. Dvali, Gabadadze, Porrati)

Here, the idea is modify General Relativity at distances $\sim 1/H_0$. Again, I am not aware of any such framework that explains why Λ is so small; but given this tune, such models can make predictions for e.g. $W(z)$ that differ from a hard C.C.

DGP Model

$$S = \frac{M_5^3}{2} \int d^5X \sqrt{-\tilde{g}}\, \tilde{R} + \frac{M_P^2}{2} \int d^4x \sqrt{-g}\, R$$

$$g_{\mu\nu}(x) \equiv \tilde{g}_{\mu\nu}(x, y=0)$$



They and others (Deffayet, ...) argue that if one defines the length scale:

$$r_c = \frac{M_P^2}{2 M_5^3}$$

and sets $r_c \sim \frac{1}{H_0}$ $(\to M_5 \sim 10 \text{ MeV})$

then one gets an interesting "self-accelerated" cosmology.

In this model, the Friedmann equations get modified! Simplest example of a phenomenological approach espoused by Dvali & Turner.

- Assume single crossover scale r_c
- Assume corrections to Friedmann eq. $\sim H^\alpha$

$$H^2 - \frac{H^\alpha}{r_c^{2-\alpha}} = \frac{8\pi}{3} G \rho_M$$

$\alpha = 0 \to$ C.C. ; $\alpha = 1 \to$ DGP model ; ...

These models lead to distance vs redshift curves similar to C.C. for $r_c \sim \frac{1}{H_0}$ and have

$$w_{eff} \cong -1 + .3\,\alpha \qquad (\text{for } z \lesssim 2)$$

Of course, could always cook up quintessence models with same $w(z)$ behavior.

Recent claim: (c.f. Ishak, Upadhye, Spergel, astro-ph / 0507184)

Growth rate of large scale structure related to $H(z)$ by a simple diff. eq. in General Relativity; models of IR modified gravity change this → can distinguish them from conventional physics.

III. Ideas in string theory related to Λ

Of course, many conventional ideas (like quintessence) may be realized $\in$ strings. But I have in mind two kinds of ideas that are more deeply tied to our notions of fundamental physics.

i) UV miracles

Could it be that $\Lambda \cong 0$ because a deep fact about UV complete theory requires

$$\Lambda_{QCD}^4 \pm \Lambda_{EW}^4 \pm \Lambda_{GUT}^4 + \cdots = 0 \; ?$$

Of course this might be true. But concretely :

- We know of many solutions to string theory with $\Lambda \neq 0$ (in seemingly reasonable approximation schemes) $\rightarrow$ at least in strings, no such general deep principle is (yet) evident.

- Attempts to find vacua where UV miracles $\rightarrow$ small Λ
  - Moore, Atkin-Lehner symmetry
  - SK + Silverstein, special orbifolds

are both heavily tied to low orders of pert thy, and hard to exemplify (Dienes, D'Hoker & Phong).

ii) "Multiverse" or "Landscape"

In PRL (1987), Weinberg made the following striking observation.

Imagine $\exists$ many possible vacua of our UV theory, where Λ takes on different values ["many" $\to$ $> 10^{120}$].

Hold $\frac{\delta \rho}{\rho}, \ldots$ (all else) fixed for now.

Then requiring that structure (galaxies) can form before Λ domination makes it impossible $\Rightarrow$

$$\Omega_M (z_{gal}) \geq \Omega_\Lambda (z_{gal})$$

$$\Rightarrow \quad \frac{\Omega_{\Lambda 0}}{\Omega_{M0}} \lesssim \frac{1}{a_{gal}^3} = (1 + z_{gal})^3 \approx 125$$

Furthermore, he said, since the allowed $\Lambda \ll$ any fundamental scale we know of,

> Distribution of Λ values $\in$ acceptable range should be flat
> $\rightarrow$ we should see Λ not far from maximal allowed, in such a theory.

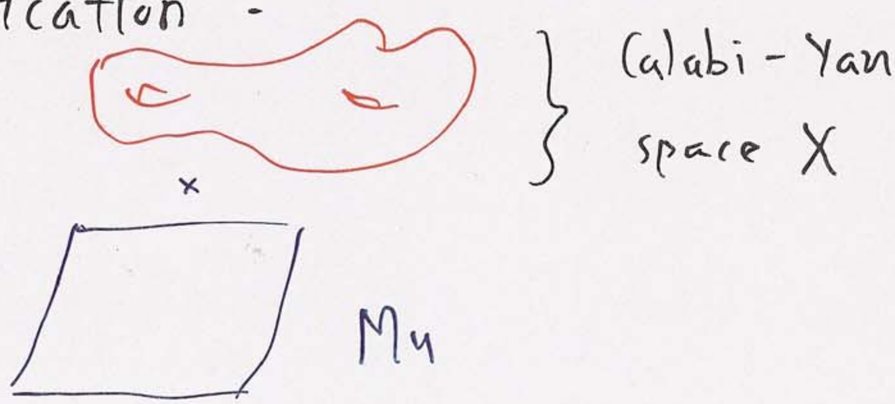
He was (rather explicitly) hoping that the next generation of experiments would prove $\Omega_{no} \lessapprox \Lambda_{mo}$ and start to "rule out" such 'anthropic' reasoning.

We all know what happened instead.

Now, why do I bring this up in the context of string theory? A needed condition for such reasoning is

UV theory should have
$N \gg 10^{100}$ possible vacua

It has been known since mid 1980s that one can get nice pseudo-realistic models of particle physics (e.g. SUSY GUTs) by compactification:

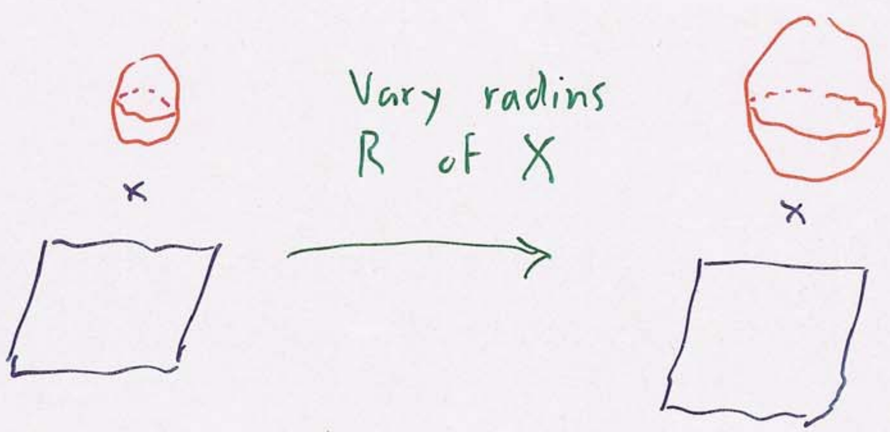


$$X \quad \text{solves} \qquad R_{\mu\nu}\left[g_{\mu\nu}(x) \right] = 0$$

But given one solution to $R_{\mu\nu} = 0$, often $\exists$ more which are continuously connected in a "moduli space of Ricci flat metrics on X" (Yau's theorem):

$$g_{\mu\nu} \to g_{\mu\nu} + \delta g$$
$$R_{\mu\nu}[g + \delta g] = 0$$

e.g.



By ~ mid 90s, clear that $\exists \gtrsim 10^4$ choices of topology of X, each with a moduli space whose typical dimension ~ 100s.

Now, <u>moduli</u> of metric g on X show up in 4d physics as scalar fields ϕ_i with <u>$V(\phi_i) = 0$</u>.

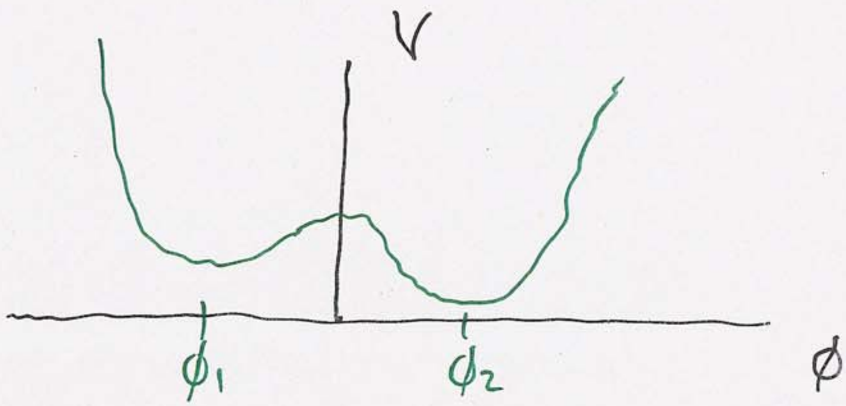
- While it was long suspected that SUSY → potential for moduli + discrete set of vacua, calculable examples were not known.

Work of last 5 years (relying heavily on discovery of D-branes + their associated gauge charges in mid 90s):

- ∃ calculable models where $V(\phi_i)$ is computable, → discrete set of stable vacua with various Λs, gauge groups, ...

Estimated $N_{vacua} > 10^{300}$ in some fairly simple examples

· ∃ known non-perturbative "instantons" that connect many of the vacua by bubbles of false vacuum decay :

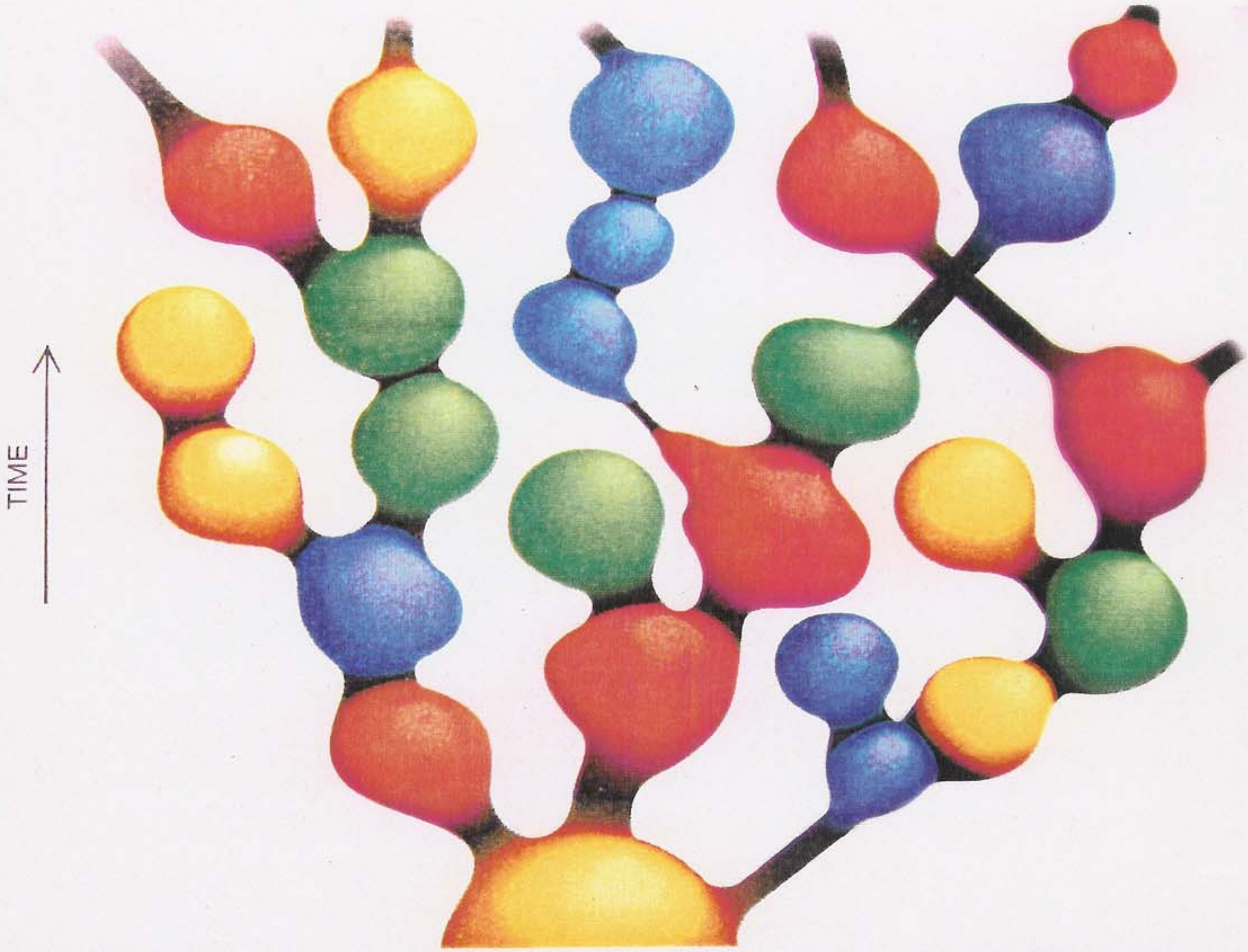


"Bubble of $\phi = \phi_2$" vacuum in $\phi = \phi_1$ will expand if

radius of bubble

$$T_{bubble} \cdot R^2 < \Delta V \cdot R^3$$

Resulting heuristic picture: Many vacua may be occupied, connected by tunneling events, and manifesting various values of Λ, ... (see picture)



SELF-REPRODUCING COSMOS appears as an extended branching of inflationary bubbles. Changes in color represent "mutations" in the laws of physics from parent universes. The properties of space in each bubble do not depend on the time when the bubble formed. In this sense, the universe as a whole may be stationary, even though the interior of each bubble is described by the big bang theory.

This large set of vacua with various values of Λ:

- May provide a natural home for Weinberg's argument, i.e., "anthropic" explanation of Λ

- Has mathematical structure that may, with deeper understanding, $\Rightarrow$ predictions of string theory (see Figure)

- Techniques yielding this have also $\Rightarrow$:
  - New, testable ideas about cosmic strings
  - New, testable ideas about inflation
  - New, testable ideas about EWSB

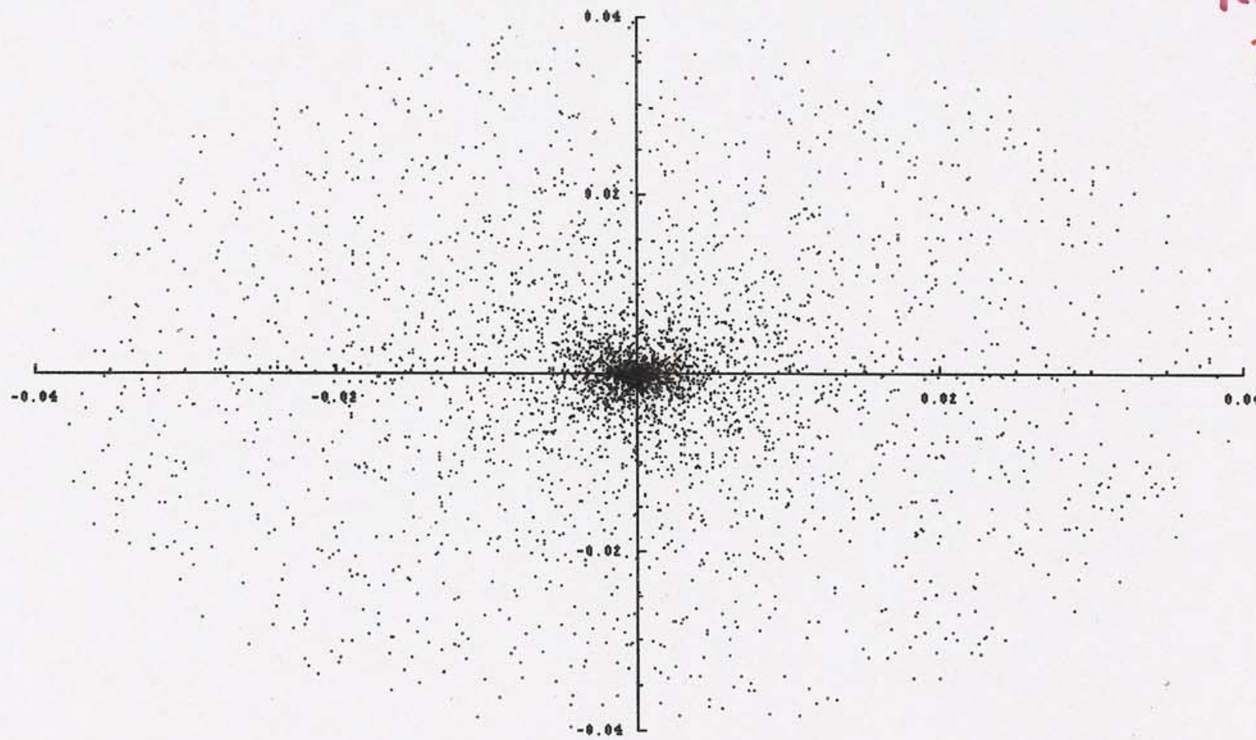
Attractor --

c.f. Dvali's work...

GKP +

Randall-Sundrum

Douglas, Denef, ...



Vacua near conifold $\psi = 1$

I want to close with a quote from Weinberg's classic review:

"... even if we are willing to suppose that the vacuum energy changes with time, a vacuum energy comparable with the present mass density seems very difficult to explain on other than anthropic grounds."

We should view this as our challenge!