Precision SUSY and the GUT Scale

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SSI 2005, SLAC

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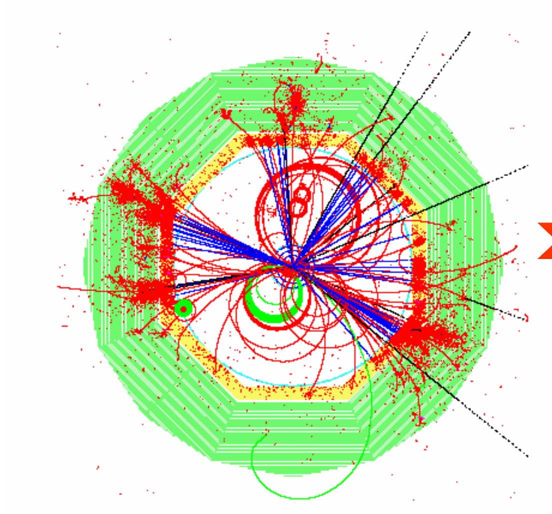
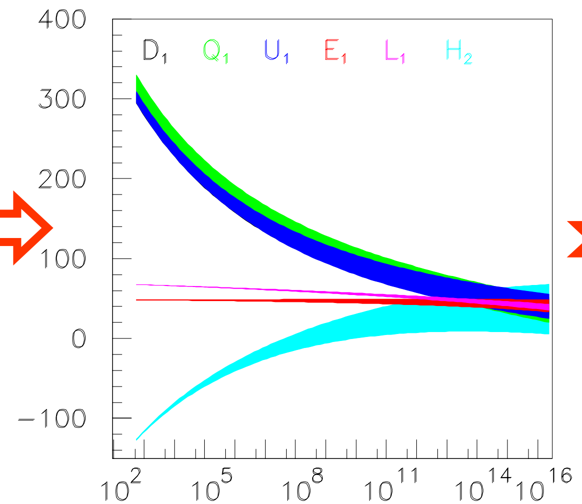
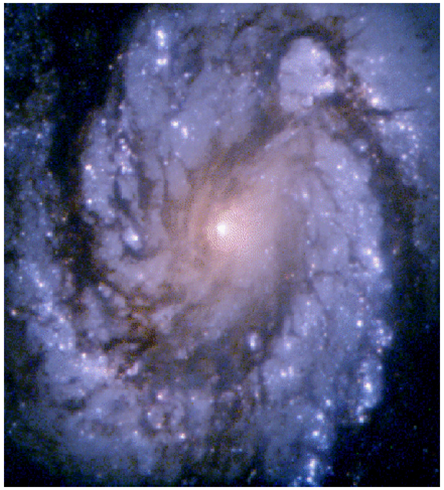






Overview

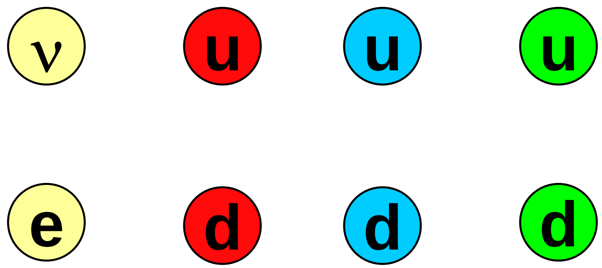
- Grand Unification
- Renormalsation Group Equations
- Unification of the couplings
- Symmetry breaking
- Supersymmetry (SUSY)
- Soft SUSY Breaking
- Experimental Determination of TeV scale parameters
- LHC and ILC
- Cosmological Connections
- Summary

The First Generation

Leptons

charge = 0

charge = - 1



Quarks with 3 "colours"

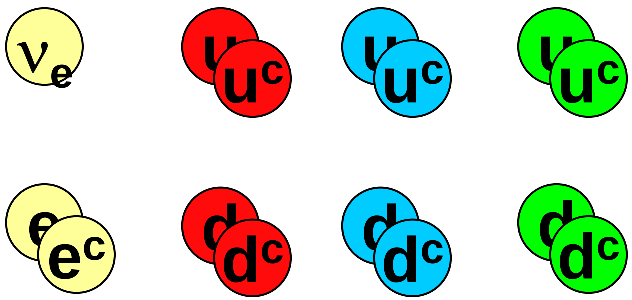
charge = + 2/3

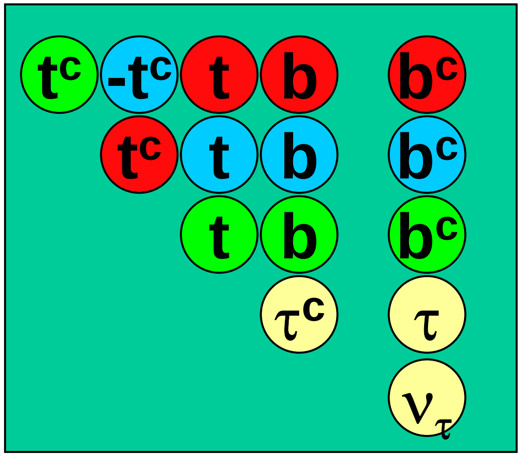
charge = - 1/3

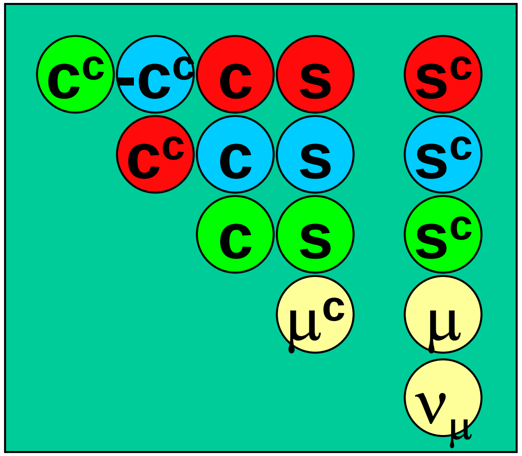
All these matter particles are spin-1/2 ➔ all are fermions

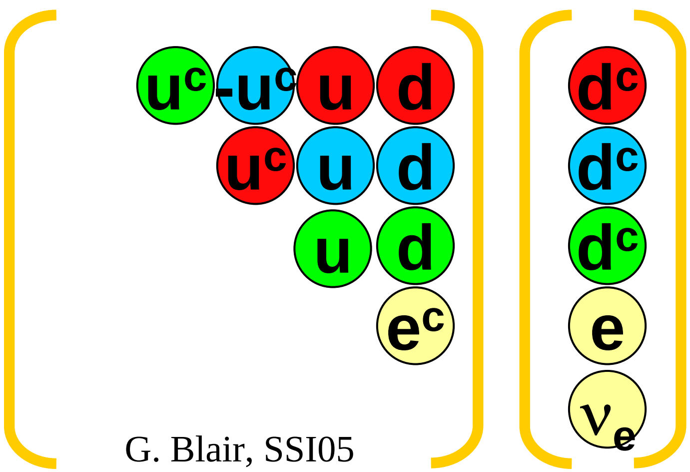
➔ 2 helicity states

The First Generations









Grand Unification ?

Running of Model Parameters

Bare Lagrangian:

Bare fermion mass

Bare gauge field

$$L_0 = i\overline{\psi}_0\gamma_\mu\partial^\mu\psi_0 - m_0\overline{\psi}_0\psi_0 + g_0 A_0^\mu\overline{\psi}_0\gamma_\mu\psi_0 + \ldots$$

Bare gauge coupling

Bare fermion field

At any arbitrary energy scale, E, the combined effects of the interactions in the theory can be taken into account:

$$L_0 = [L_0 - \delta L(E, g, m, \psi, A, \ldots)] + \delta L(E, g, m, \psi, A, \ldots)$$

Finite "renormalised" parameters

Running of Model Parameters

Invariance of the bare Lagrangian under the arbitrary choice of E leads to consistency-requirements on the renormalised parameters –
the Renormalisation Group Equations (RGEs)

e.g. for the strong coupling constant in the SM:

$$\frac{\partial g_3}{\partial \ln E^2} \equiv \beta(g_3) = -\frac{g_3^3}{16\pi^2}\left(\frac{11}{3}N_C - \frac{4}{3}\frac{N_F}{2}\right) + O(g^5)$$

Rate of evolution, including the sign, depends on masses and couplings in the ENTIRE theory

All parameters (masses, couplings, wave functions,…) will run with E in a manner that depends on the particle properties

Running of gauge couplings

$$\frac{\partial g_i}{\partial \ln E^2} \propto g_i^3 + O(g^5)$$

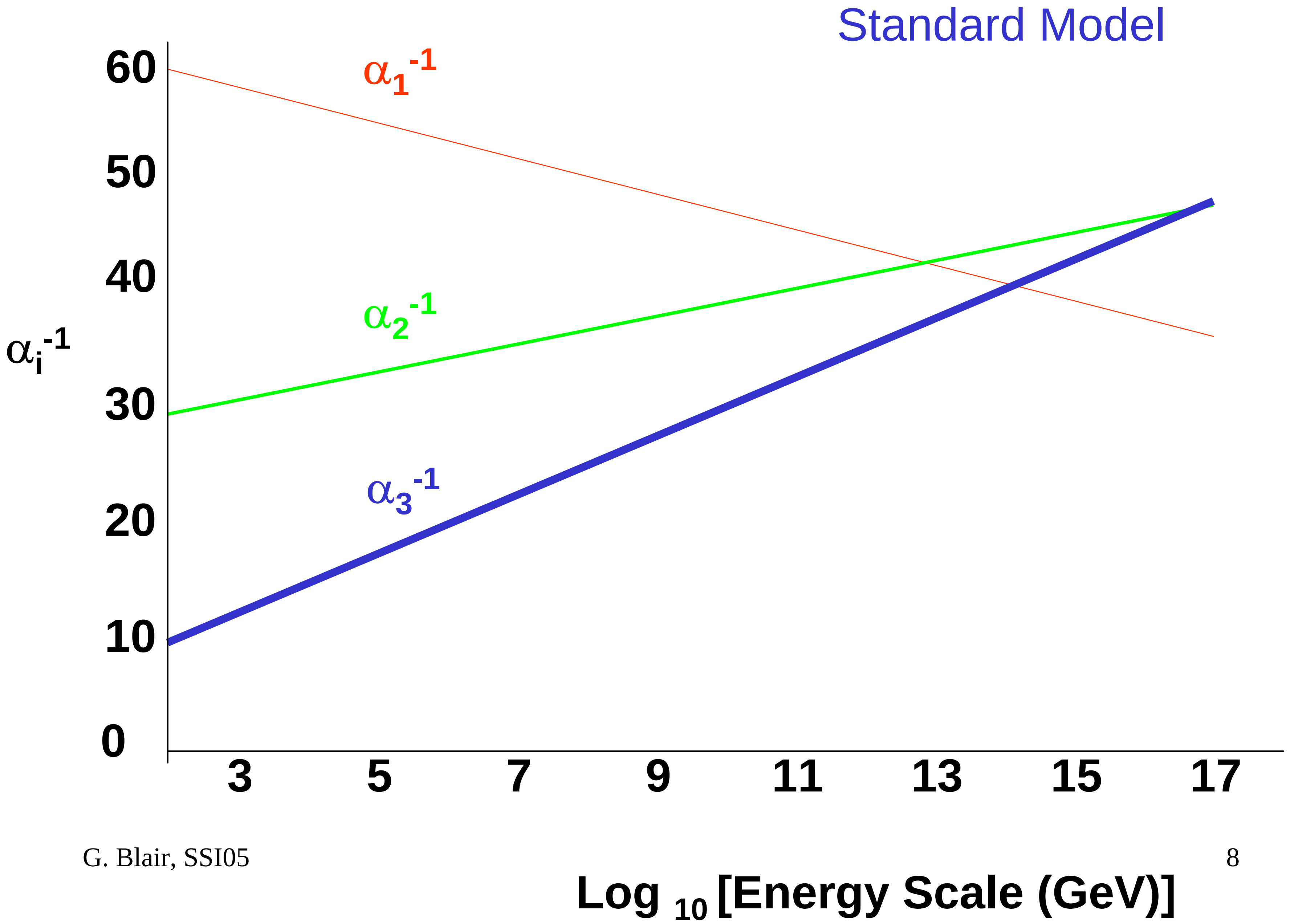
So:

$$\frac{\partial}{\partial \log E}\left(\frac{1}{\alpha_i}\right) = k_i \quad \text{where} \quad \alpha_i \equiv \frac{g_i^2}{(4\pi)^2}$$

Depends on masses and couplings in the ENTIRE theory

So plotting $\frac{1}{\alpha_i}$ against $\log E$ is a straight line, whose slope depends on the matter and couplings of the entire theory

Unification of the Gauge Couplings



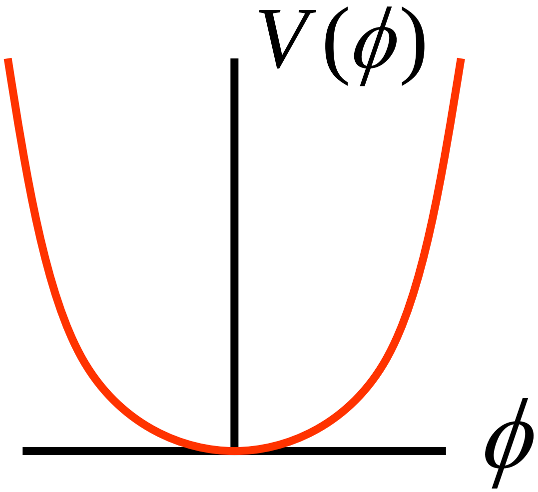
Spontaneous Supersymmetry Breaking

Scalar lagrangian:

$$L = \partial_\mu \phi \; \partial^\mu \phi + \mu^2 \phi^2 + \lambda \phi^4$$

Potential bounded from below:

$\Rightarrow \lambda > 0$



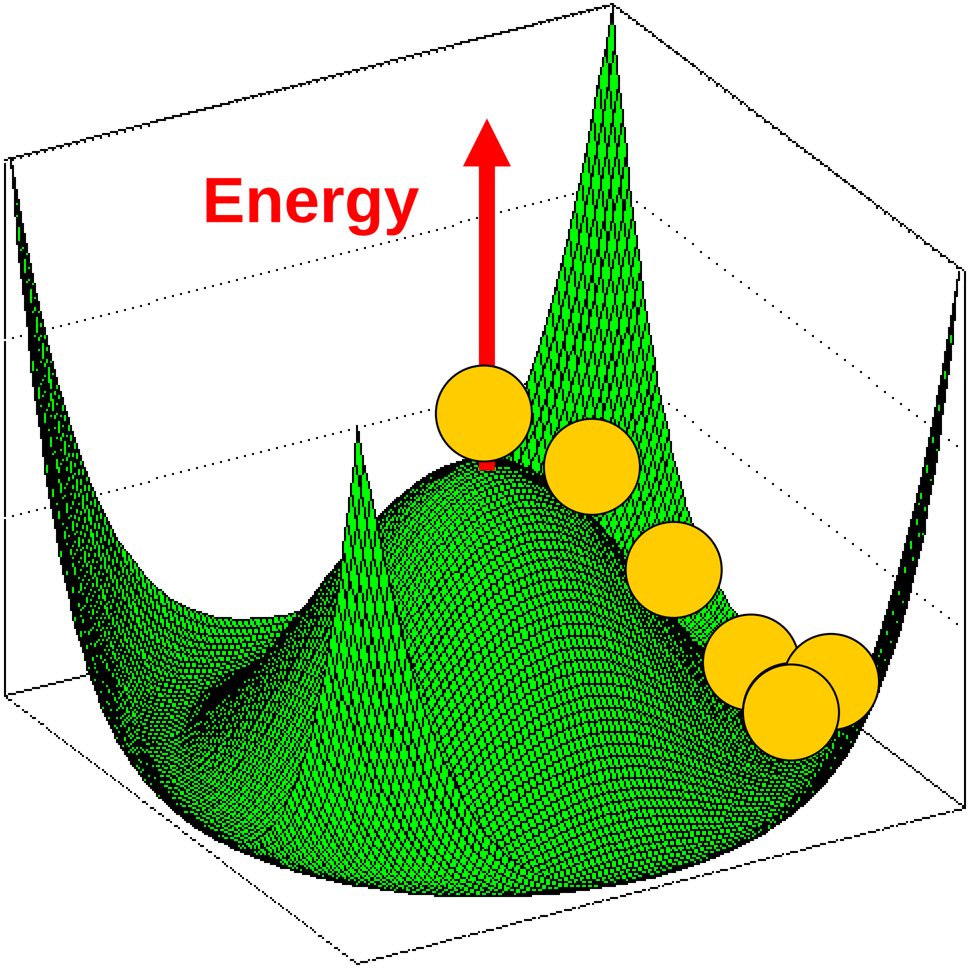
For $\mu^2 > 0$

$$\langle \phi \rangle = 0$$

Symmetric case; μ = free mass

The Higgs Mechanism

The vacuum potential



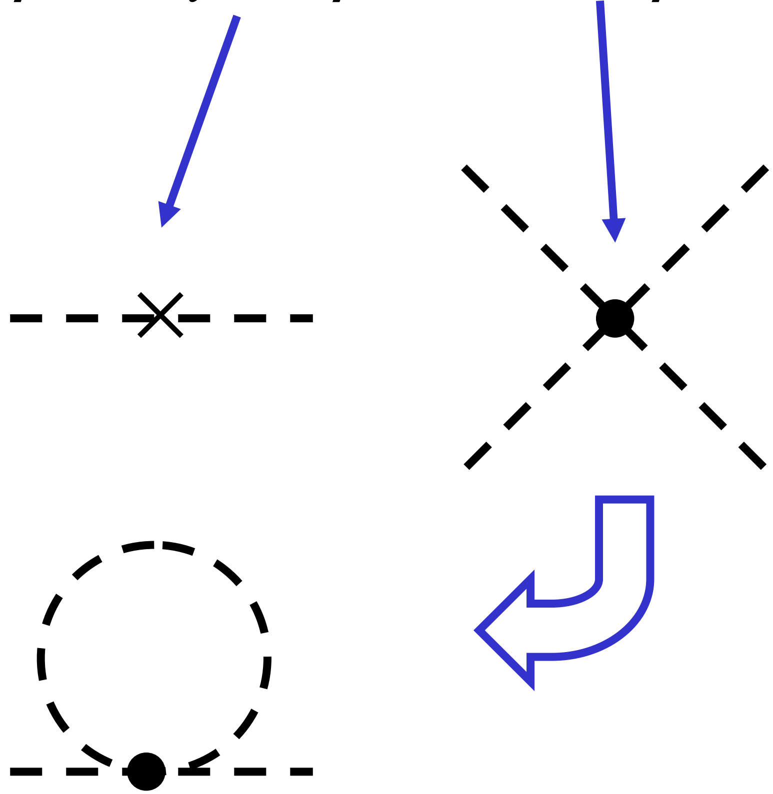
$$\mu^2 < 0$$

Need a Higgs field
Non-zero v.e.v.
m_H not too heavy

Can all be incorporated naturally with SUSY (Supersymmetry)

Problems with scalar mass

$$L = \partial_\mu \phi \ \partial^\mu \phi + \mu^2 \phi^2 + \lambda \phi^4$$



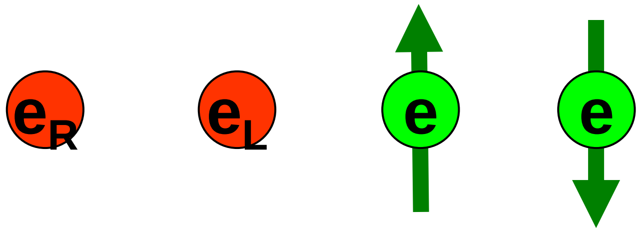
Quadratic divergent
mass generated
by higher order
corrections –
the natural scale of the scalar mass is very large

Either fine tuning or a protective symmetry is required

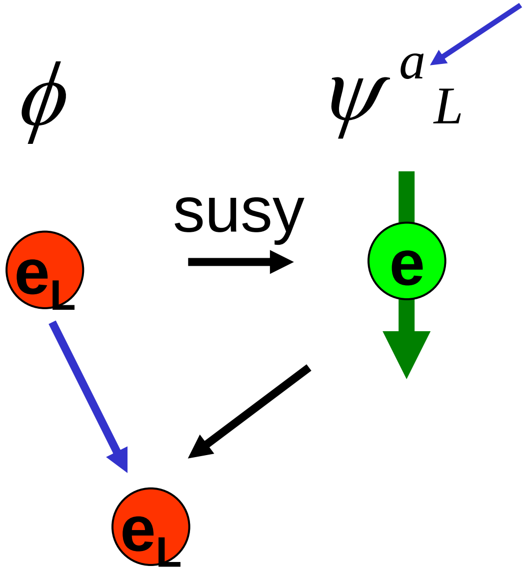
Supersymmetry I

A symmetry relating fermions with bosons

Spin 0 **Spin ½**



Lorentz index



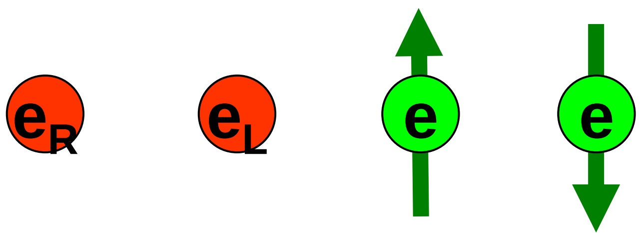
A further doubling of the particle spectrum

Lorentz transform

SUSY involves space-time
⇒Local SUSY involves gravity

Supersymmetry II

Spin 0 **Spin ½**



$$m_f \overline{\psi}_L \psi_R$$

Protected by Chiral symmetry

SUSY – protects scalars indirectly

$$m_s^2 \phi^* \phi$$

Not protected directly

Supersymmetry II

Supersymmetry

To prove existence of SUSY:

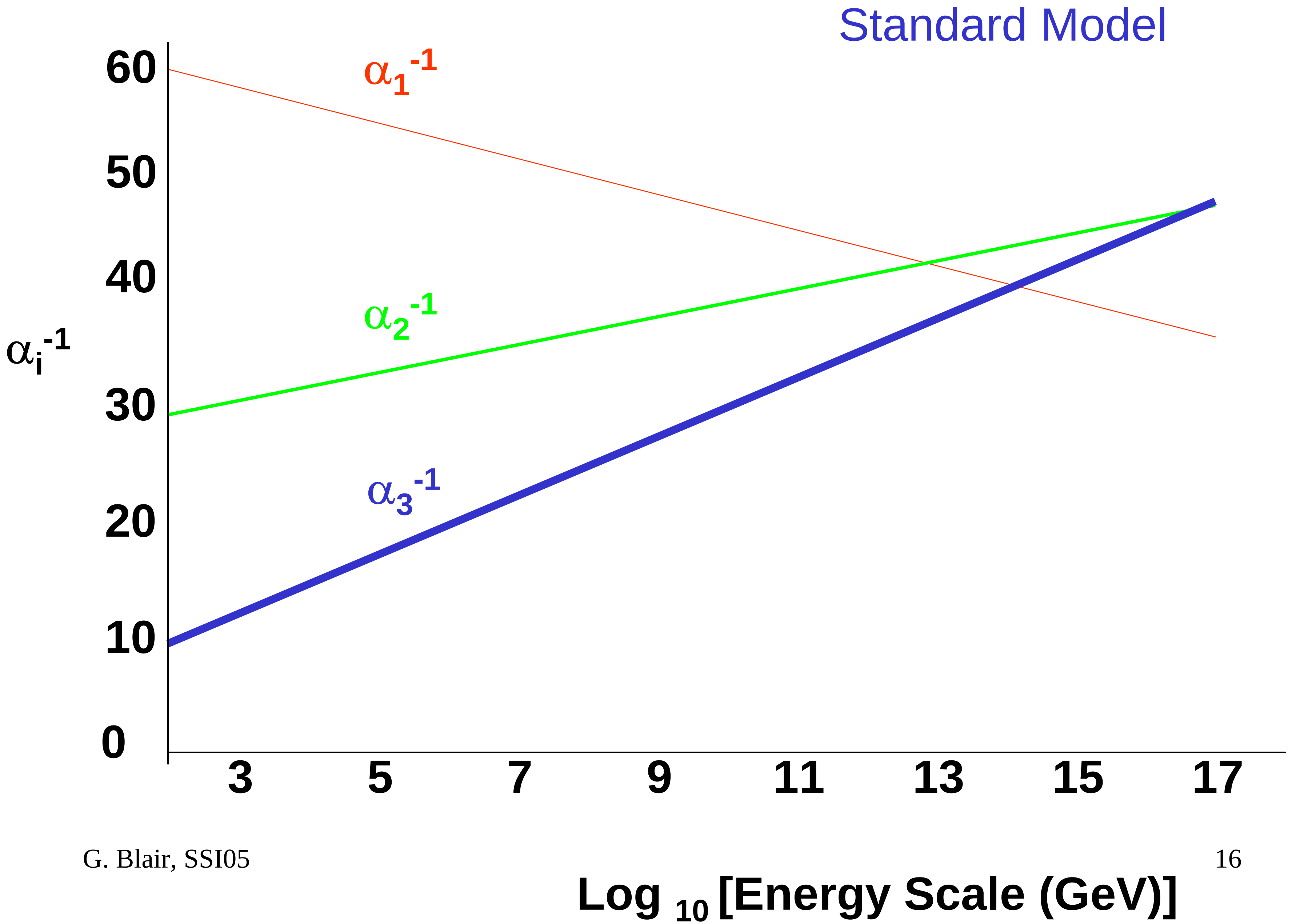
- Need to discover the SUSY partners
- Every SM has a superpartner
- Spins of SM/SUSY partner differ by ½
- Identical gauge quantum numbers
- Identical couplings

Needs accurate measurements of

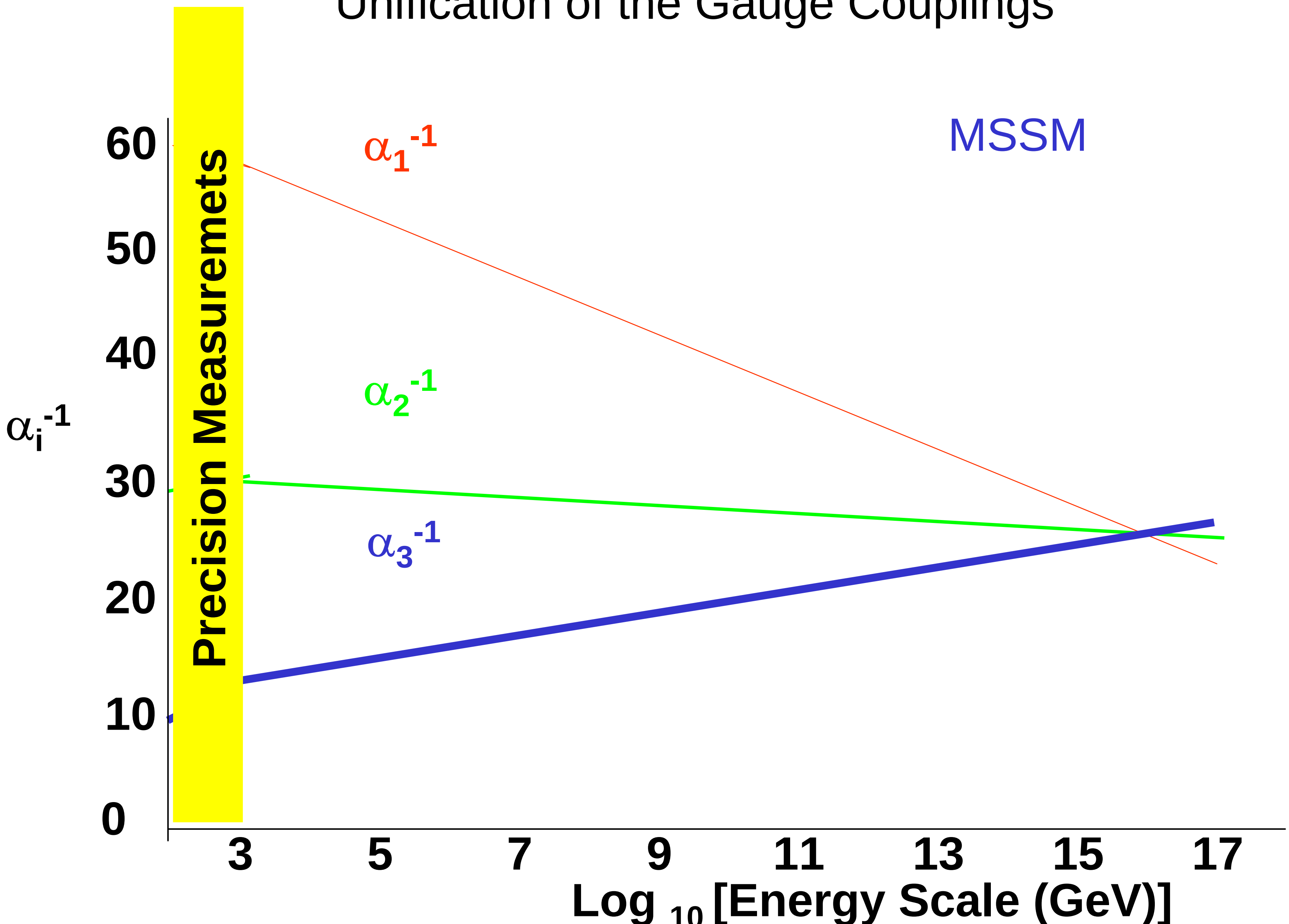
Mass spectra, cross-sections, BRs,

Angular distributions, polarisation

Unification of the Gauge Couplings



Unification of the Gauge Couplings



Supersymmetry Breaking

It is tempting to try to break SUSY by introducing a higgs-like mechanism involving tree-level renormalisable couplings

BUT

$$\sum_{\text{spin } J} \left(-1\right)^{2J} (2J+1) M_J^2 = 0$$

Ferarra-Girardello-Palumbo implies generically that:

tree-level SUSY breaking $\Rightarrow$

$\exists$ Sparticle with mass **less than** its ordinary partner

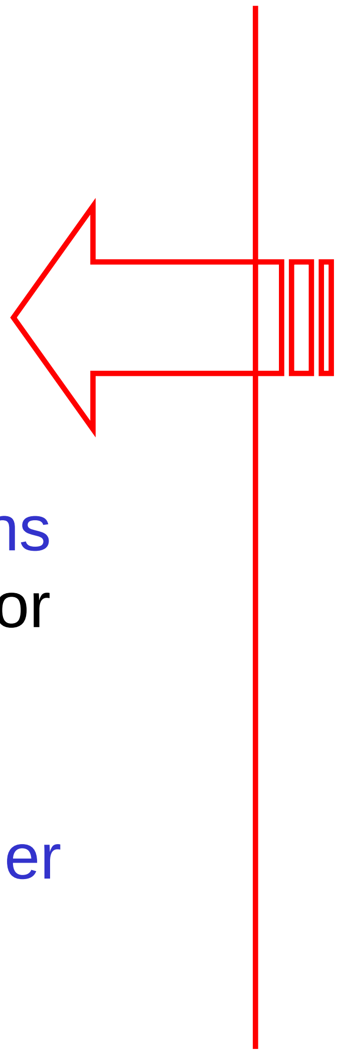
So SUSY breaking must be confined to a hidden sector, with **indirect communication** to the visible one

Supersymmetry – Soft Breaking I

Visible Sector

SUSY broken
softly by
mediating interactions
with the hidden sector

sparticles heavier
than particles



Hidden Sector

SUSY broken
explicitly

Supersymmetry – Soft Breaking II

- Scalar masses are sensitive to radiative corrections
- SUSY breaking requires the introduction of mass terms
- They must **not** lead to quadratically divergent quantum corrections to scalar masses

$$V_{scalar} = \sum_j \left| \frac{\partial W}{\partial \phi_j} \right|^2 + V_D +$$

$$\sum_{i,j} m^2_{i,j} \phi_i^* \phi_j +$$

SUSY Breaking Terms

$$A_U h_U Q_L H_2 U_R + A_D h_D Q_L H_1 D_R + A_E h_E L_L H_1 E_R$$

$$+ B\mu H_1 H_2$$

3×3 matrices in general

|μ| determined by electroweak symmetry breaking

Supersymmetry – Soft Breaking III

Gauginos also acquire masses; parameterised by M_1, M_2, M_3

$$M_{\tilde{\chi}^+} = \begin{pmatrix} M_2 & \sqrt{2} m_W c_\beta \\ \sqrt{2} m_W s_\beta & \mu \end{pmatrix}$$

2 chargino states

$$M_{\tilde{\chi}^0} = \begin{pmatrix} M_1 & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W \\ 0 & M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W \\ -m_Z c_\beta s_W & m_Z c_\beta c_W & 0 & -\mu \\ m_Z s_\beta s_W & -m_Z s_\beta c_W & -\mu & 0 \end{pmatrix}$$

4 neutralino states

Lightest $\Rightarrow$ Dark Matter candidate

$$M_{\tilde{g}} = M_3$$

Supersymmetry – Soft Breaking IV

Sfermion masses are also specified by the soft breaking parameters:

$$M^2_{\tilde{f}} = \begin{pmatrix} m^2_{\tilde{f}_L} & m_f a_f \\ m_f a_f & m^2_{\tilde{f}_R} \end{pmatrix}$$

$$m^2_{\tilde{f}_L} = M^2_{\tilde{F}_L} + (T^3_f - e_f \sin^2\theta_W)\cos 2\beta m_Z^2 + m_f^2$$

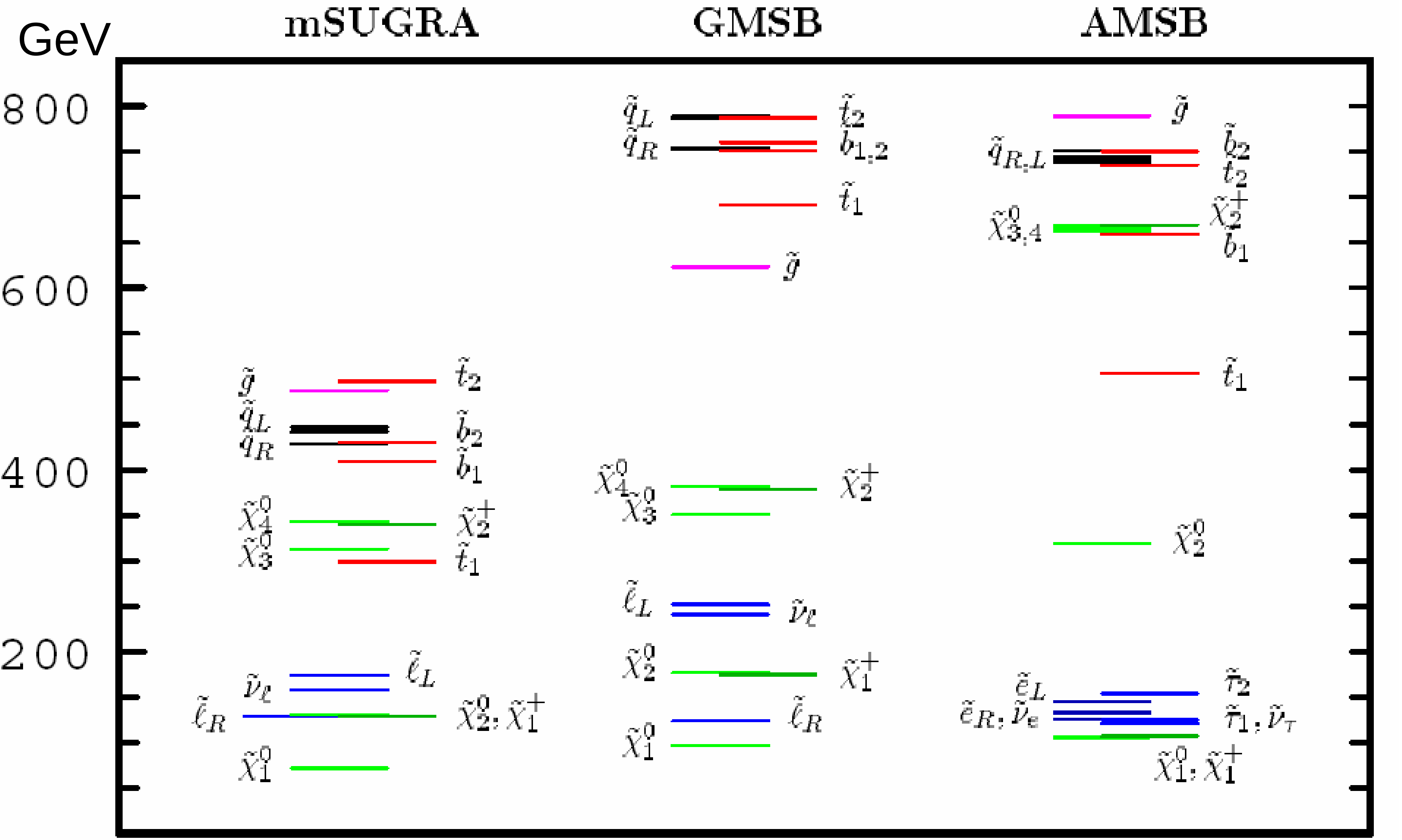
$$m^2_{\tilde{f}_R} = M^2_{\tilde{F}_R} + e_f \sin^2\theta_W \cos 2\beta m_Z^2 + m_f^2$$

$$a_t = A_t - \mu \cot\beta$$

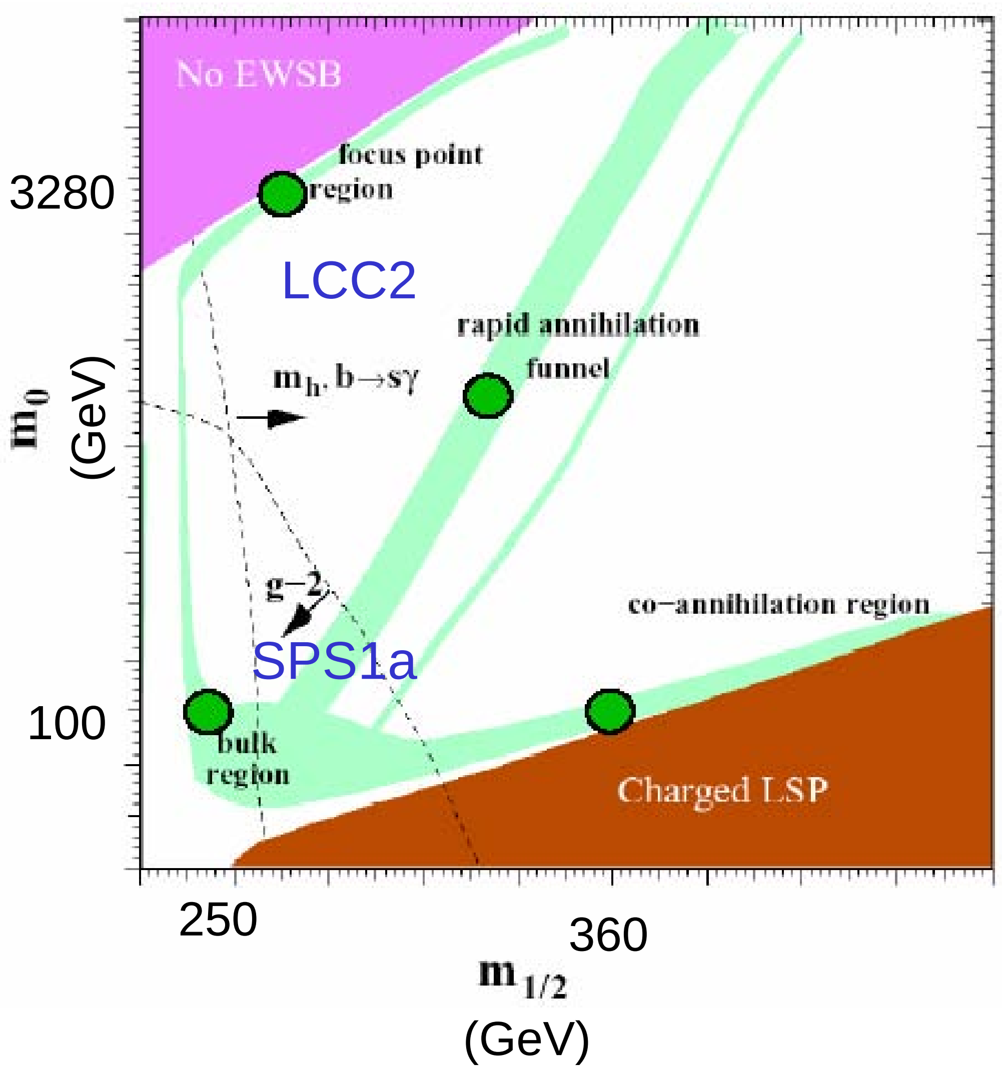
$$a_b = A_b - \mu \tan\beta$$

$$a_\tau = A_\tau - \mu \tan\beta$$

SUSY breaking mechanism leads to characteristic sparticle mass spectra



MSSM Reference Points



Bulk:
slepton exchange

focus point:
WW, ZZ final state

co-annihilation:
exchange of $\tilde{\tau}$ dominant

funnel:
exchange of A (resonant)

Gravity-Mediated SUSY Breaking

In this scenario:

Gravitational interactions generate soft breaking terms
Near the GUT/Planck scale

Gravity is flavour blind $\Rightarrow$ universal values at the GUT scale

M_0	Primarily responsible for scalar masses
$M_{½}$	Primarily responsible for fermion masses
A_0	Tri-linear term
tanβ	ratio of H_1 H_2 vacuum expectation values
sign (μ)	$\lvert \boldsymbol{\mu} \rvert$ determined by requiring electroweak symmetry breaking

SUSY Reference Points

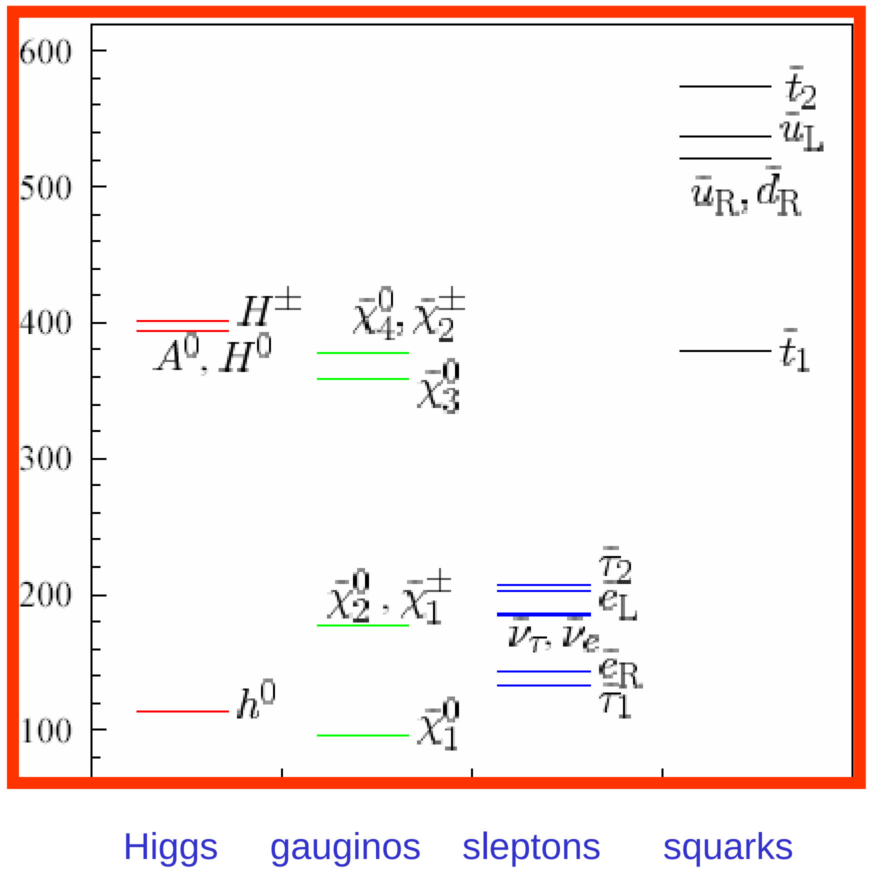
mSUGRA SPS1a:

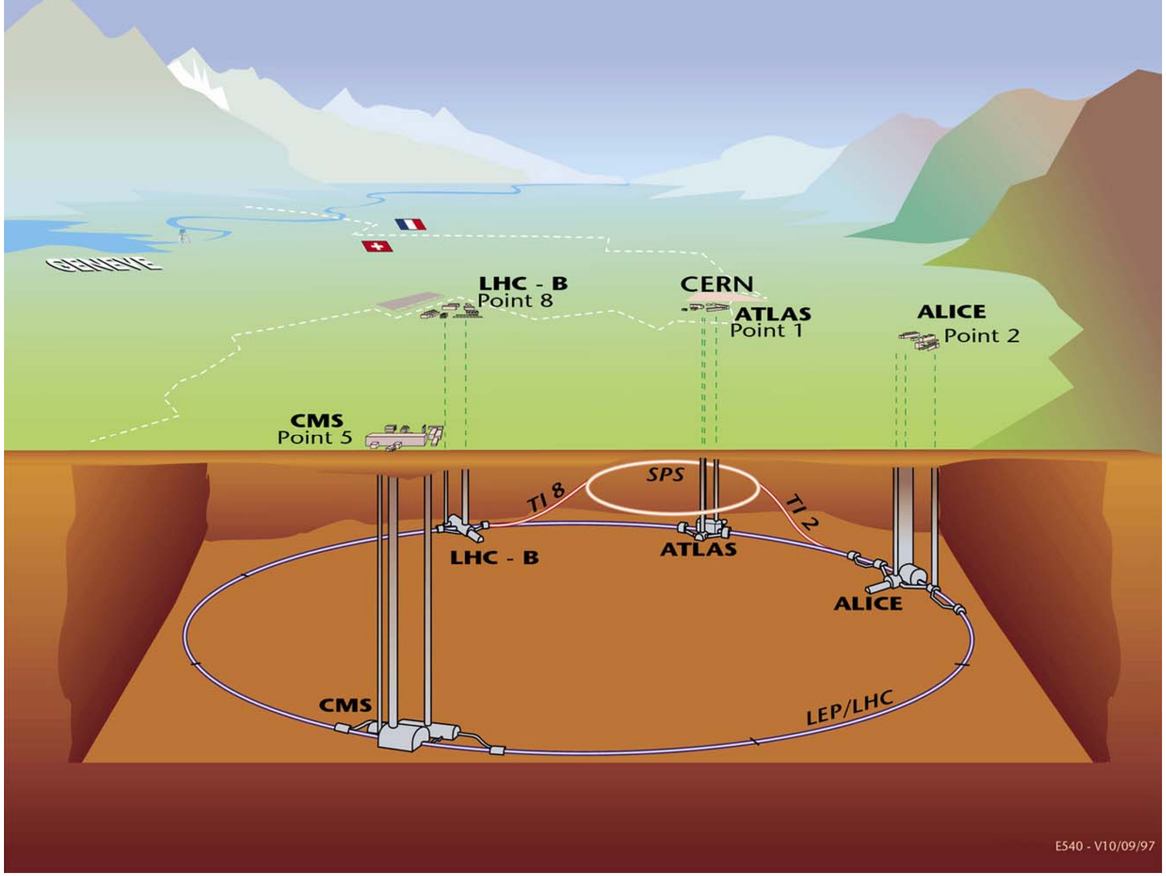
$M_{1/2}$=250 GeV M_0=100 GeV

A0=-100 GeV sign(μ)=+ tanβ=10

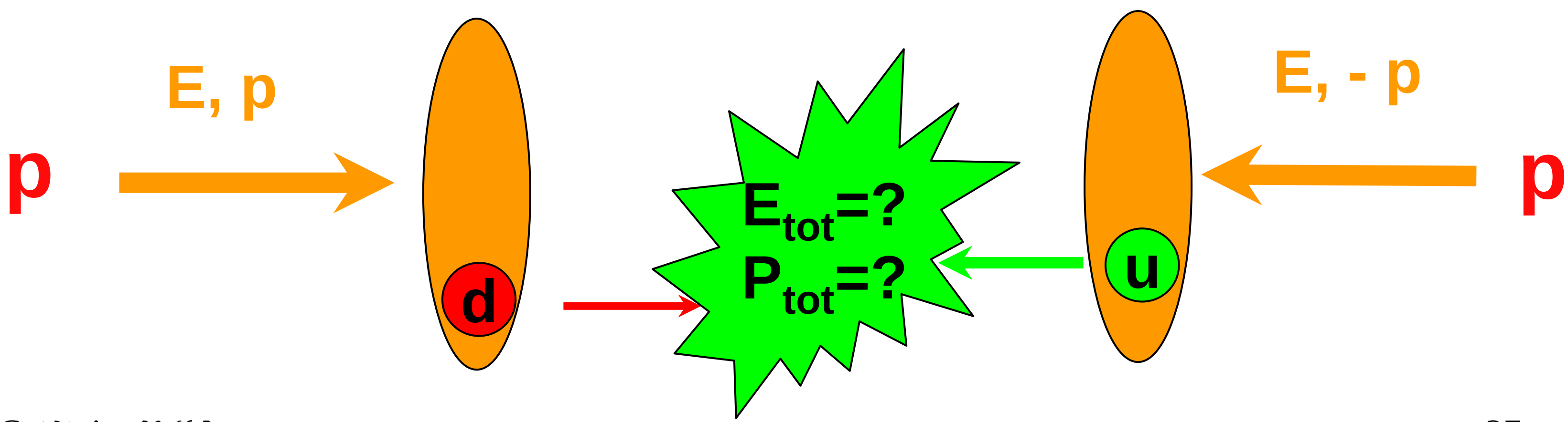
parameters defined at GUT scale

The regularity is not obvious after RGE evolution to TeV scale



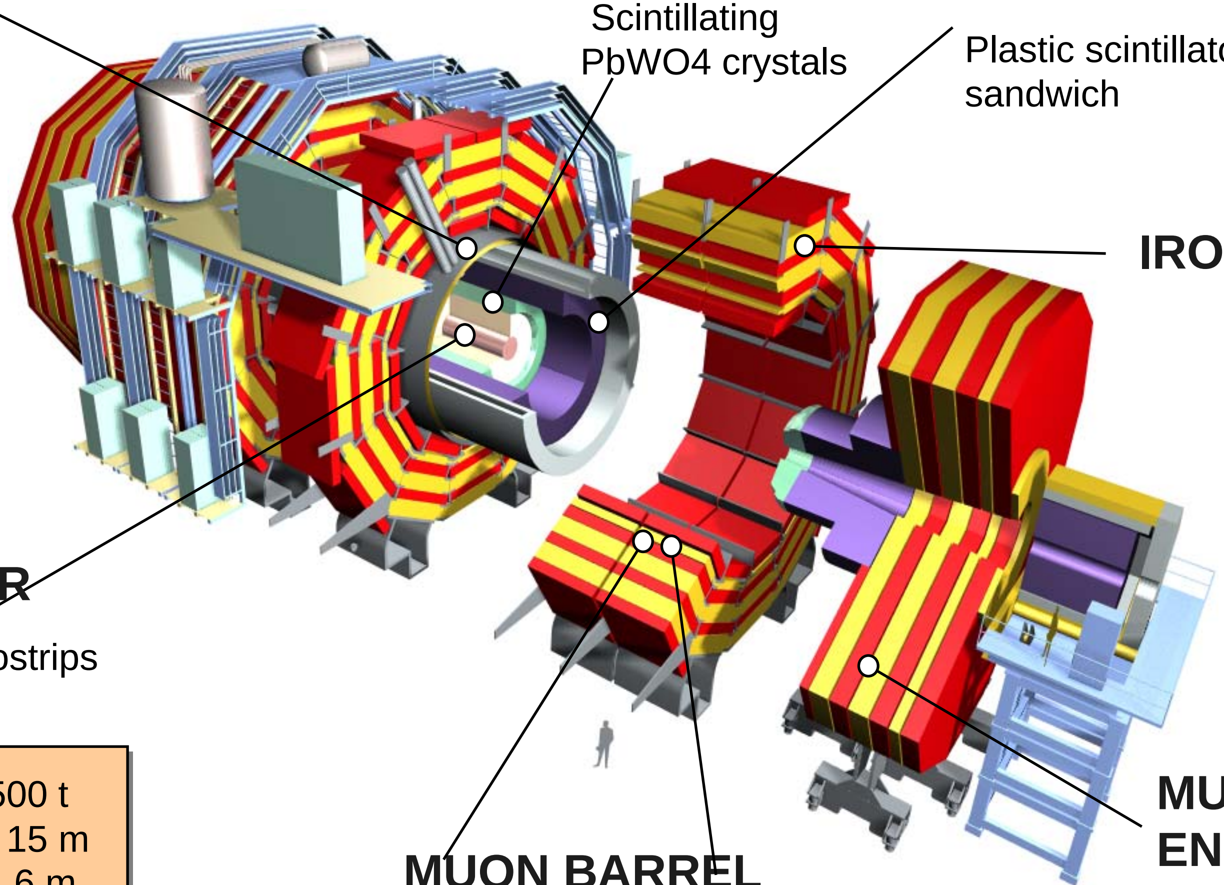


LHC



Broad range of event √s

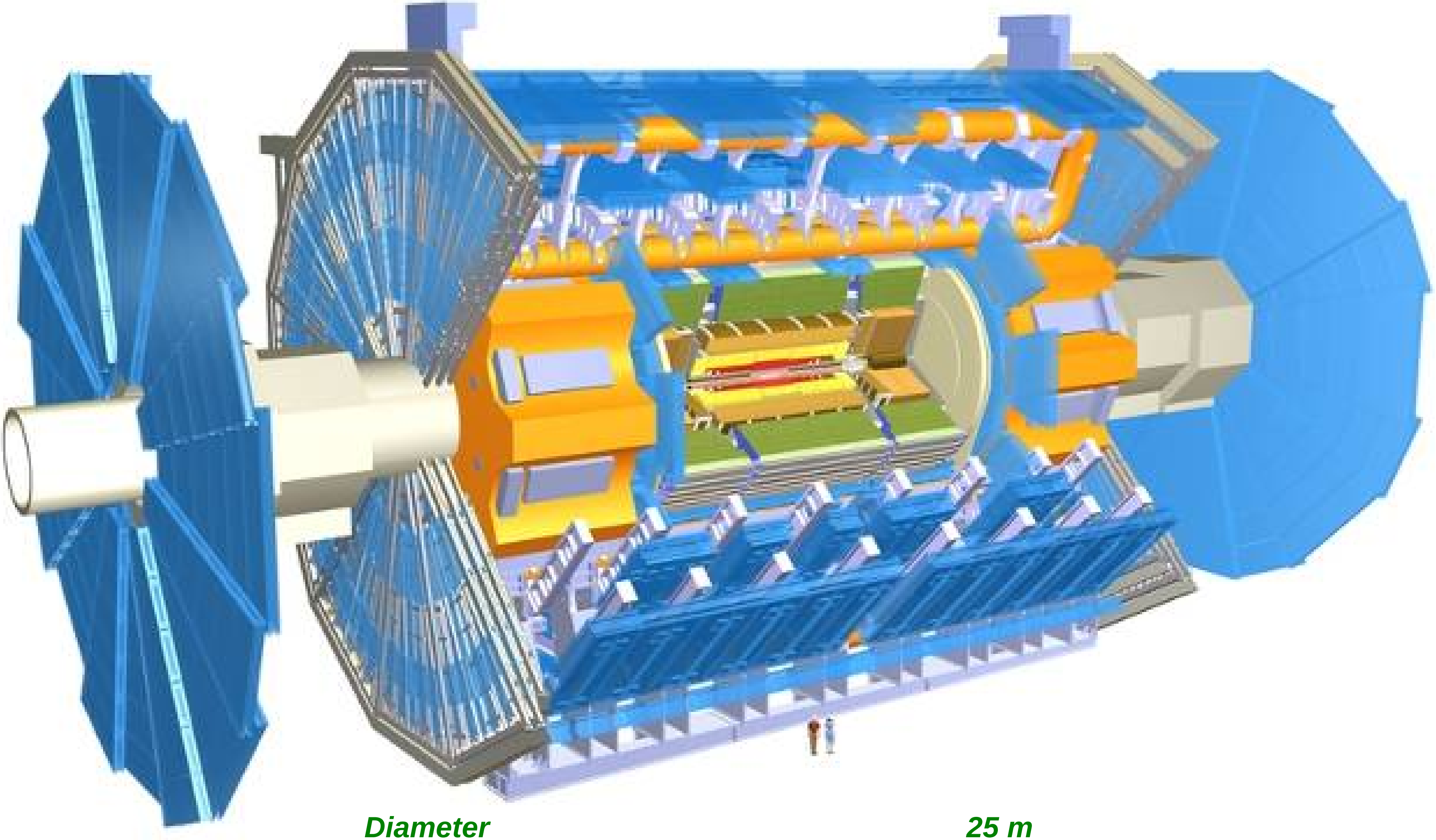
Compact Muon Solenoid



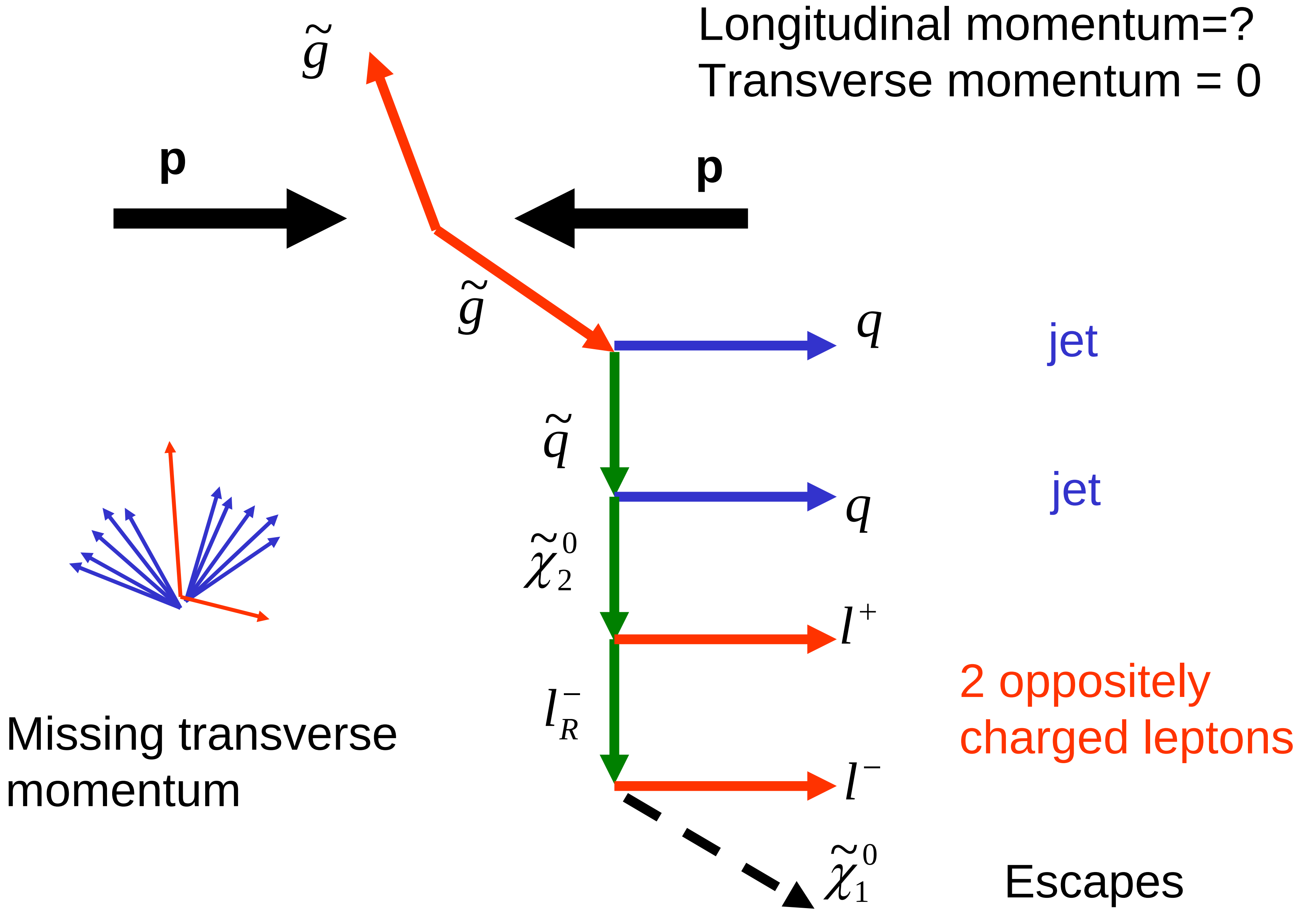
L.Rolandi, LP05

Total weight : 12,500 t
Overall diameter : 15 m
Overall length : 21.6 m
Magnetic field : 4 Tesla

ATLAS

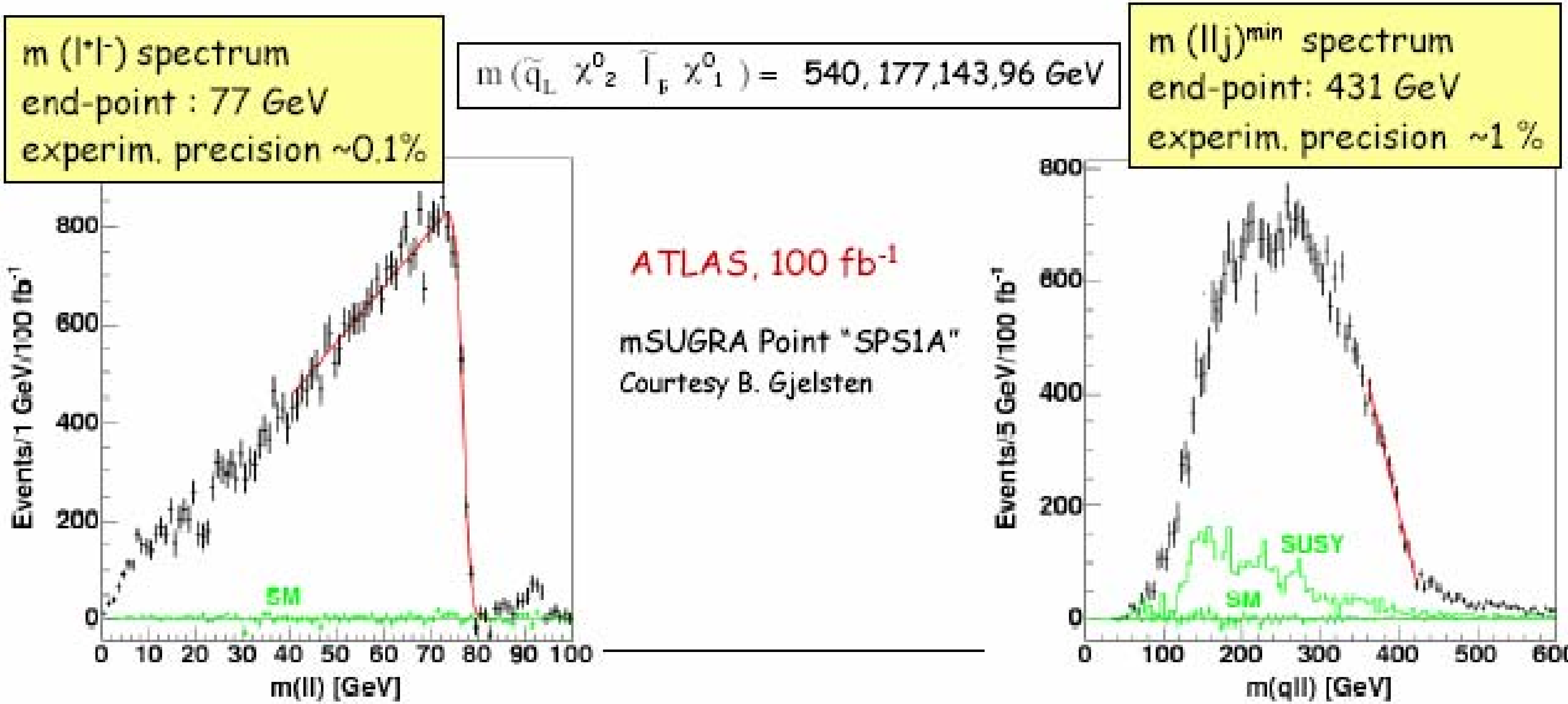


Diameter	*25 m*
Barrel toroid length	*26 m*
End-cap end-wall chamber span	*46 m*
Overall weight	*7000 Tons*



See M. Schmitt’s talk tomorrow for other examples

Simulations for SPS1a at LHC



Mass peaks cannot be directly reconstructed

- Measure invariant masses of endpoint spectra etc.
- Deduce masses in decay chains
- Strong correlations will exist in mass determinations

adapted from talk: M. Carena LP05

SPS1a:

	Mass, ideal	"LHC"
$\tilde{\chi}_1^\pm$	181.5	–
$\tilde{\chi}_2^\pm$	381.2	–
$\tilde{\chi}_1^0$	97.7	8.4
$\tilde{\chi}_2^0$	183.0	8.2
$\tilde{\chi}_3^0$	363.3	–
$\tilde{\chi}_4^0$	381.0	8.9
$\tilde{e}_R$	143.9	8.4
$\tilde{e}_L$	206.6	8.8
$\tilde{\nu}_e$	190.5	–
$\tilde{\mu}_R$	143.8	8.4
$\tilde{\mu}_L$	206.6	8.8
$\tilde{\nu}_\mu$	190.4	–
$\tilde{\tau}_1$	134.5	8.6
$\tilde{\tau}_2$	210.4	–
$\tilde{\nu}_\tau$	189.6	–
$\tilde{u}_R$	547.8	13.6
$\tilde{u}_L$	564.9	13.8

Use to extract soft SUSY breaking parameters

Polesello et al.

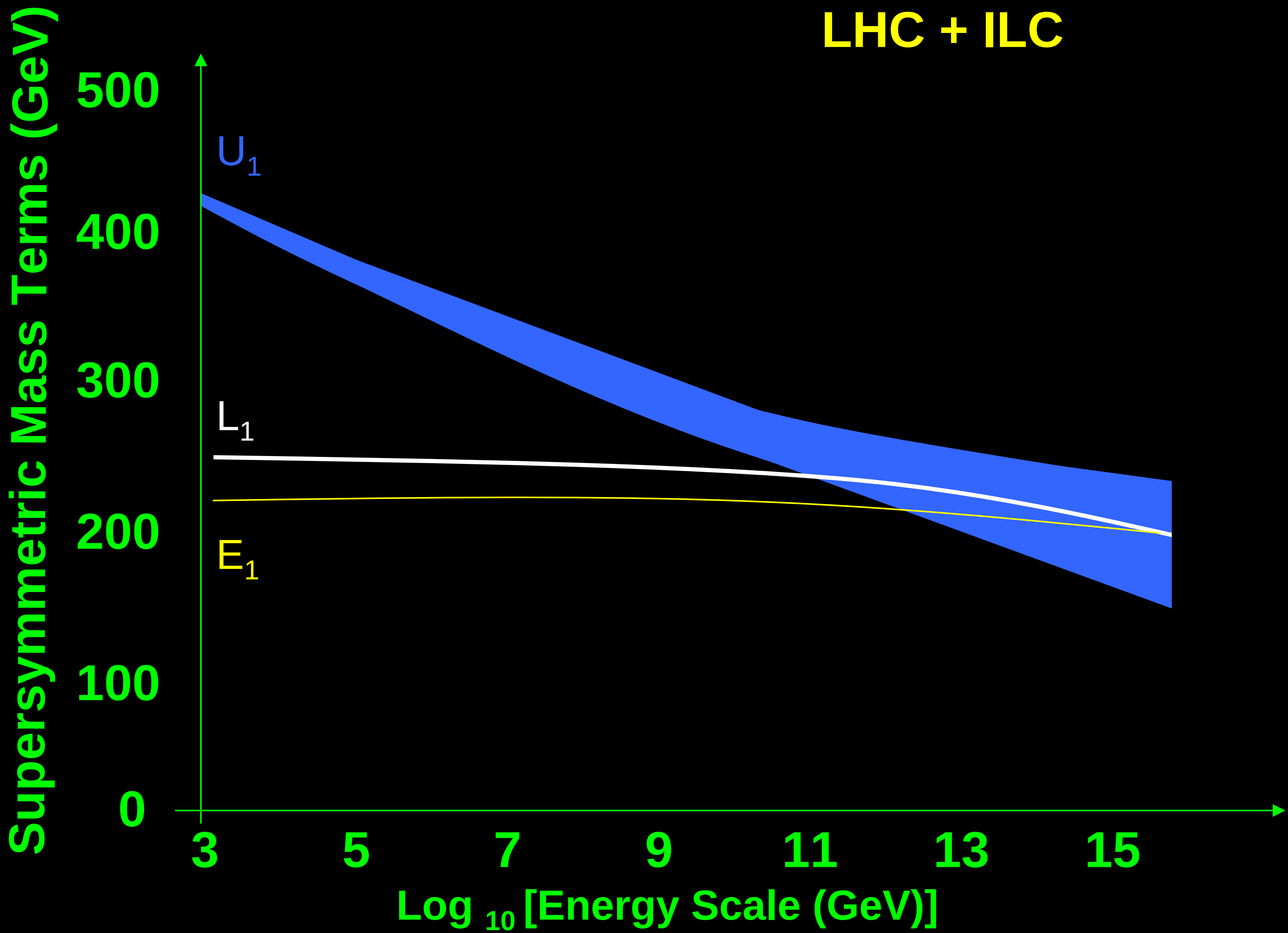
Model-Independent Extrapolation

Renormalisation Group Eqns

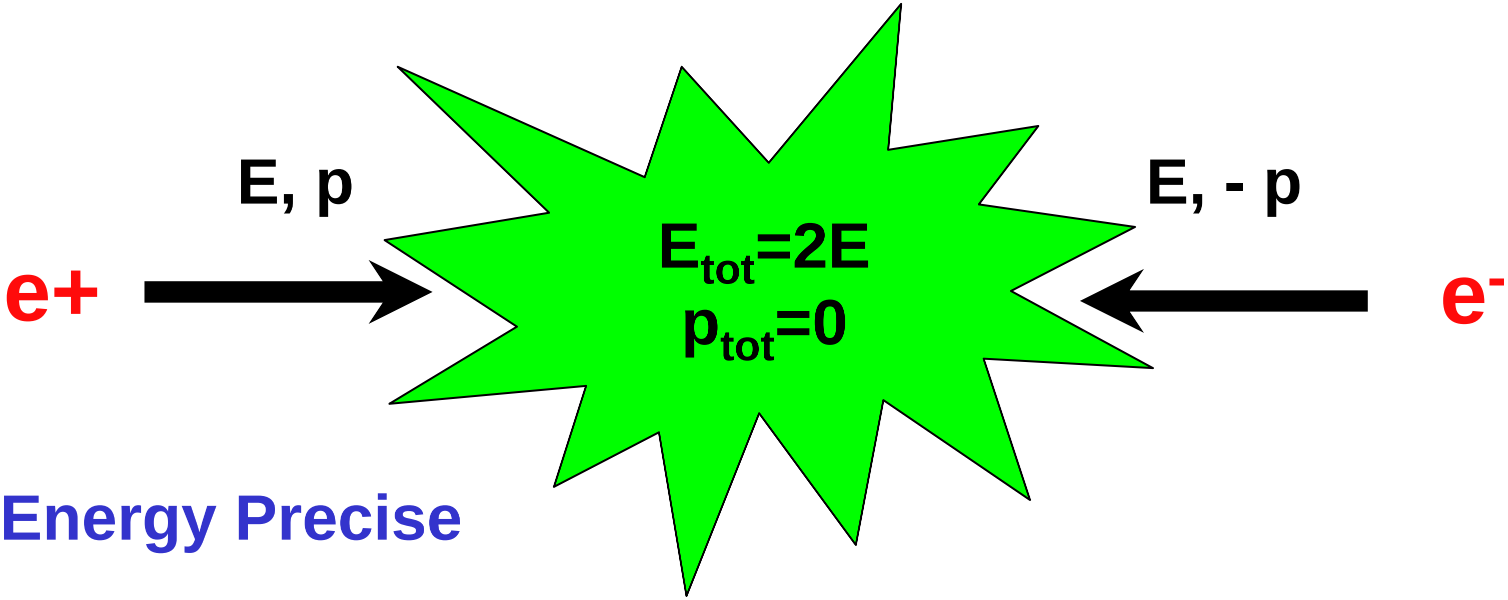
$$\frac{\partial P_i}{\partial E} = f(P_i, m_j, g_k, \ldots)$$

- Measure complete spectrum
- Extract soft SUSY parameters at EW scale
- Input measured masses, couplings into RGEs
- Extrapolate model independently to high scales

The Need for Precision

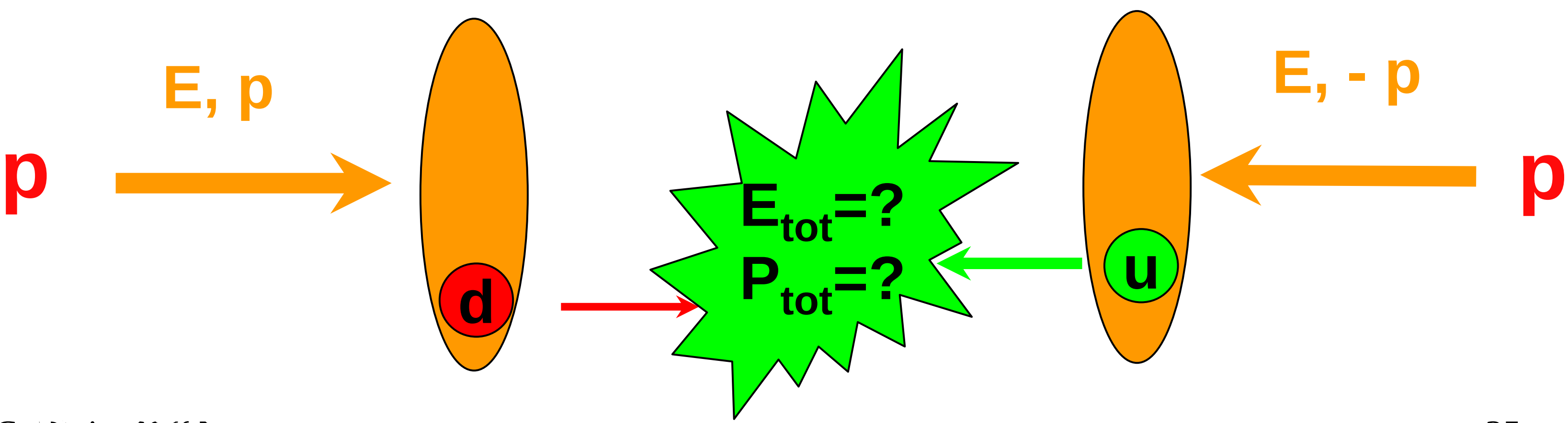


Precision at e^+e^- Colliders

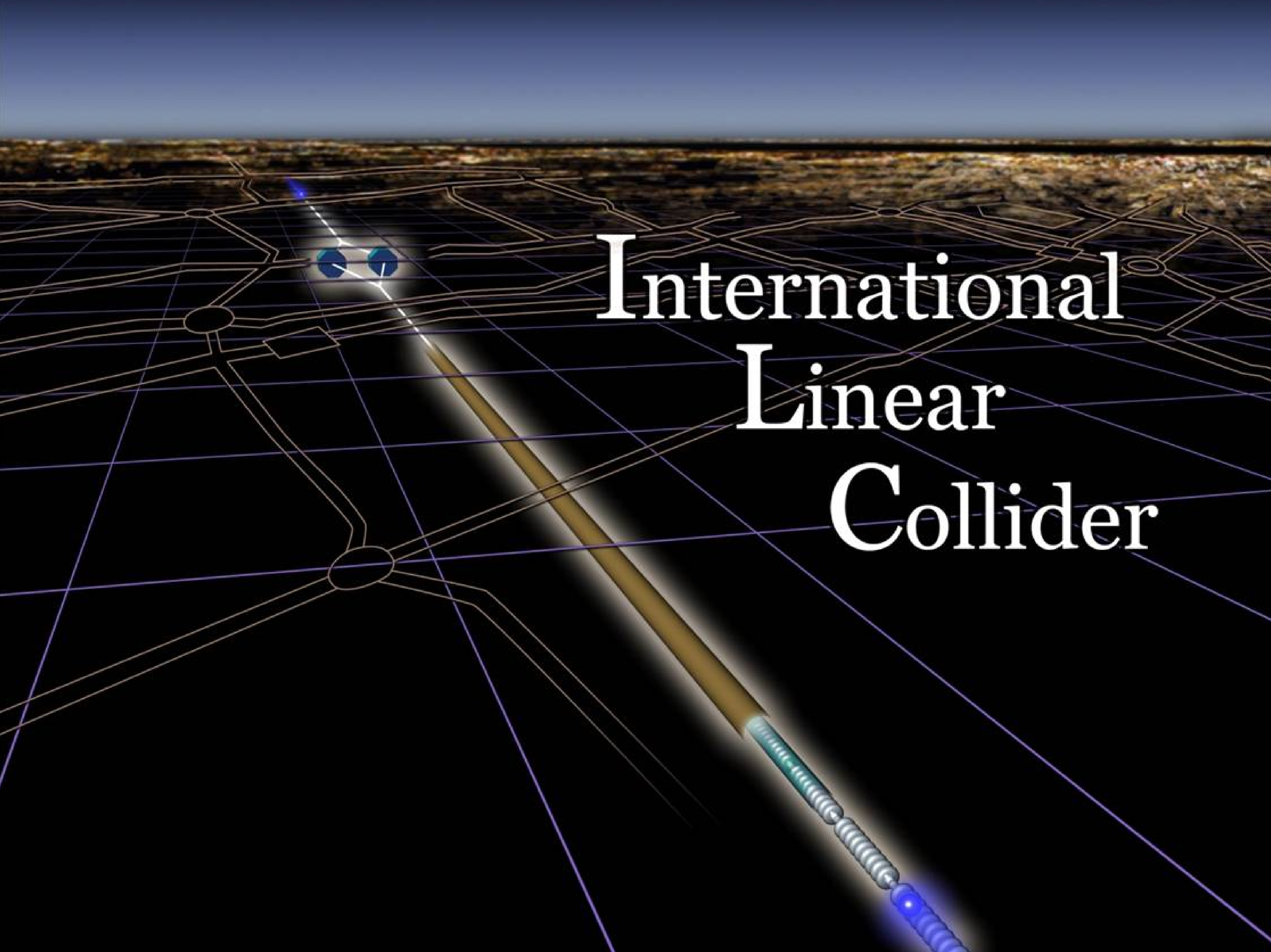


Event Energy Precise

Whereas for proton colliders:



Broad range of event energies


International
Linear
Collider

Initial State

e^-_R → → e^+_L

- W-production suppressed
- s-wave production of charginos ~ $\beta \Rightarrow$ sharp threshold
- Specific polarisations for specific couplings (eg SUSY)

http://www.ippp.dur.ac.uk/~gudrid/power/

e^-_R → ← e^-_R

- s-wave production of selectrons ~ $\beta \Rightarrow$ sharp threshold

γ_R → ← γ_R

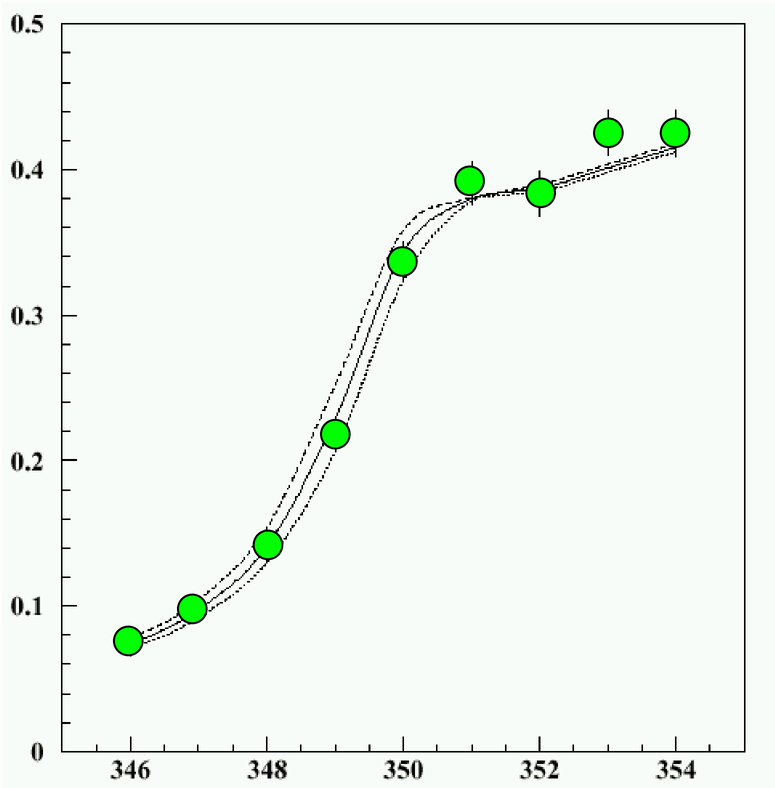
- Direct production of higgs

Threshold scans – eg: Top Mass

Precision measurement of fundamental particle properties

The top quark is the heaviest: most sensitive to new physics

Cross section (pb)



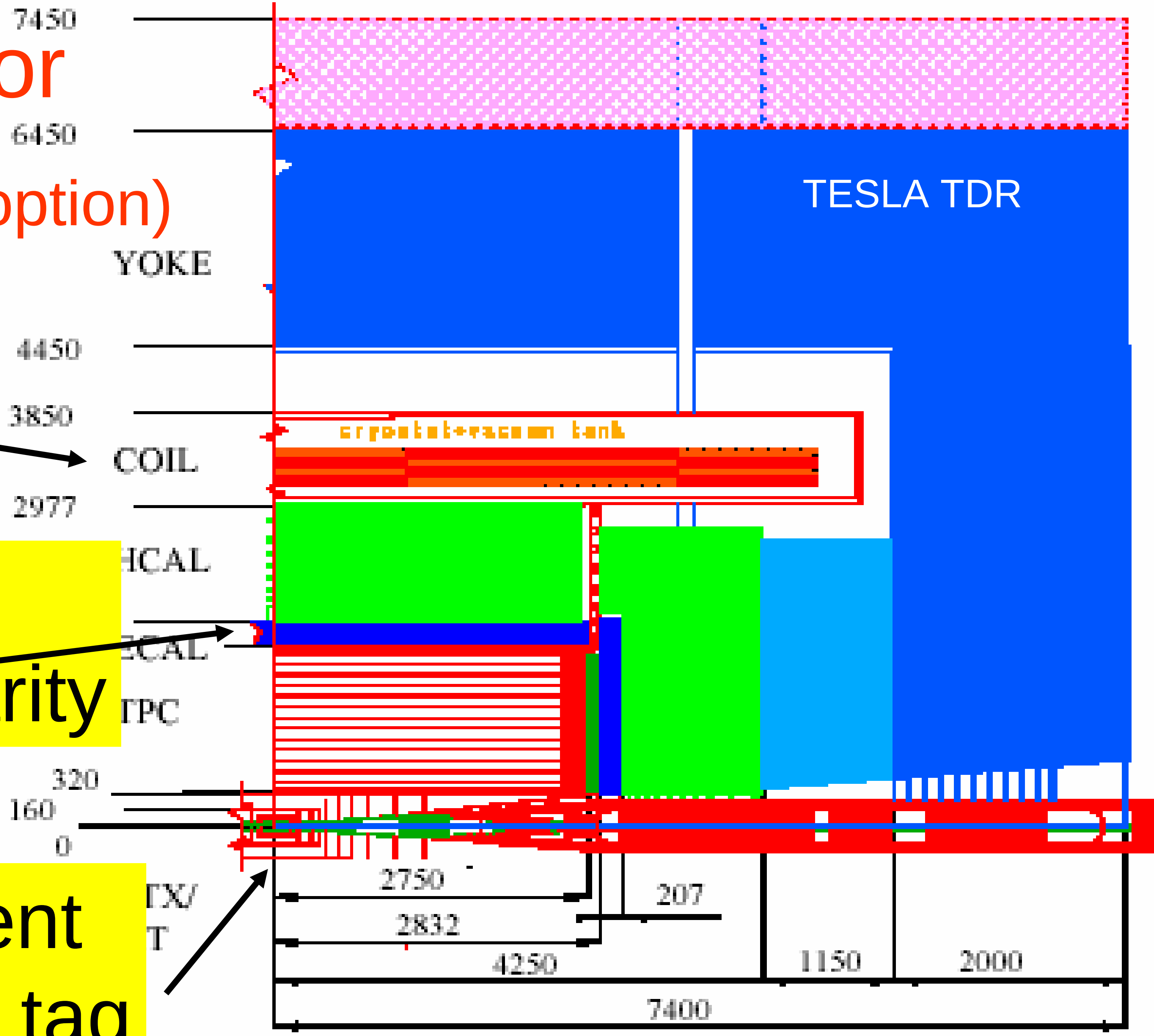
E_{tot}(GeV)

M_{top}=175 GeV
100 fb^{-1} per point

Statistical Precision ~0.05 GeV ⇒0.02%

Detector

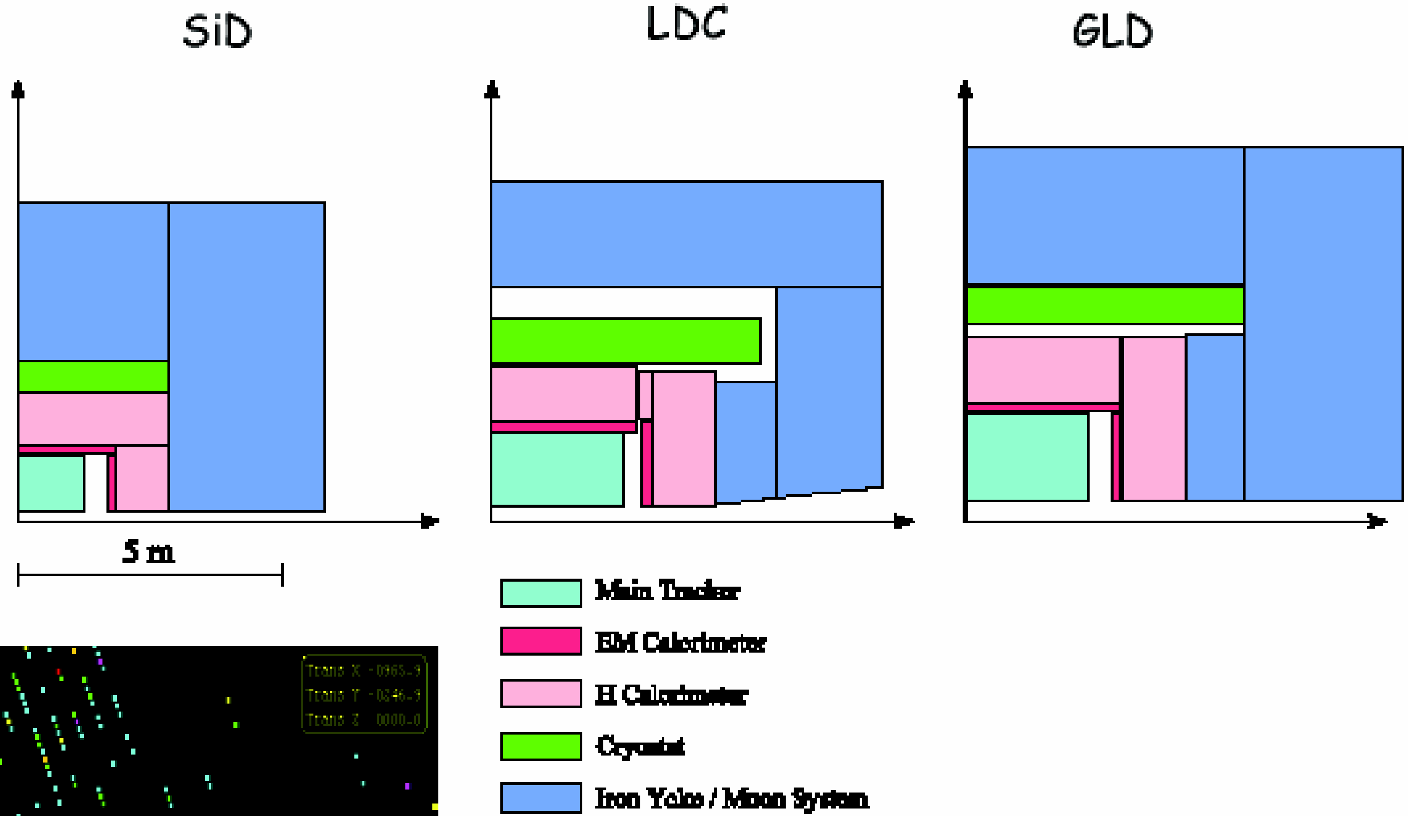
('LDC' option)

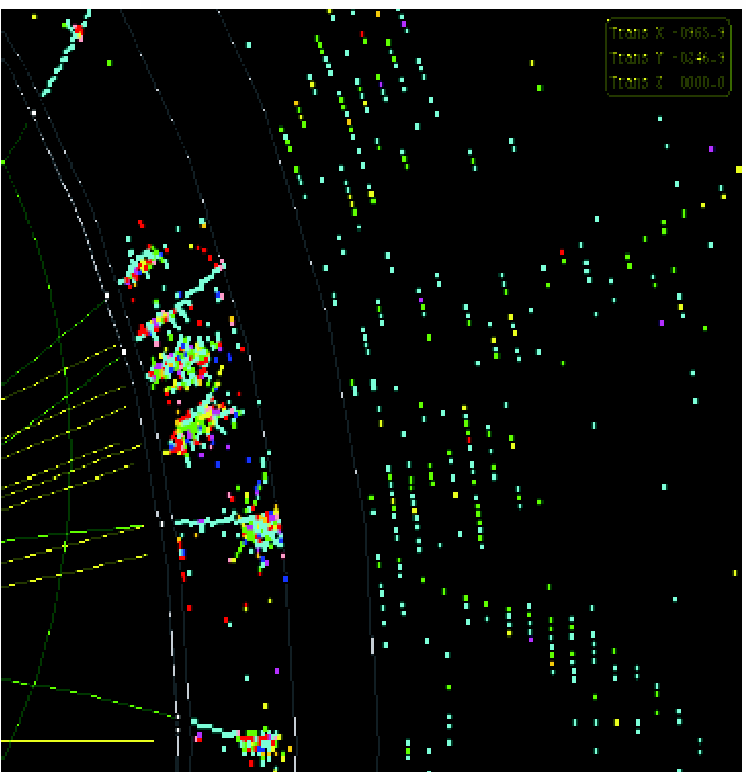


4 T

v. high granularity

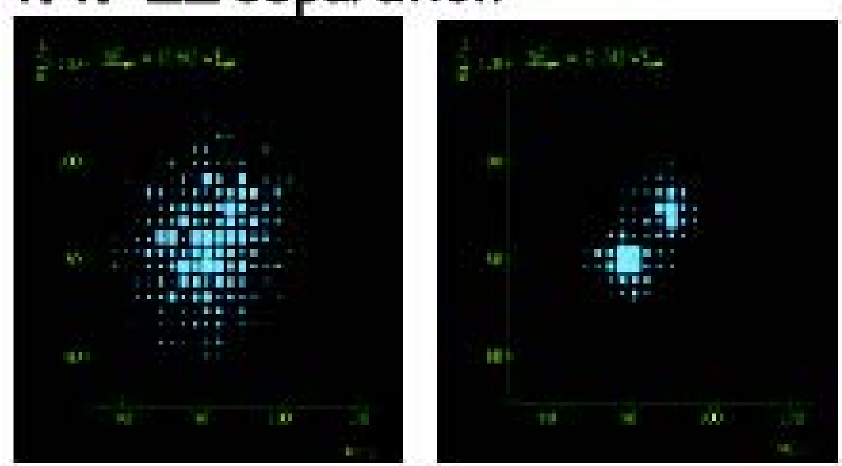
Excellent flavour tag





Energy Flow

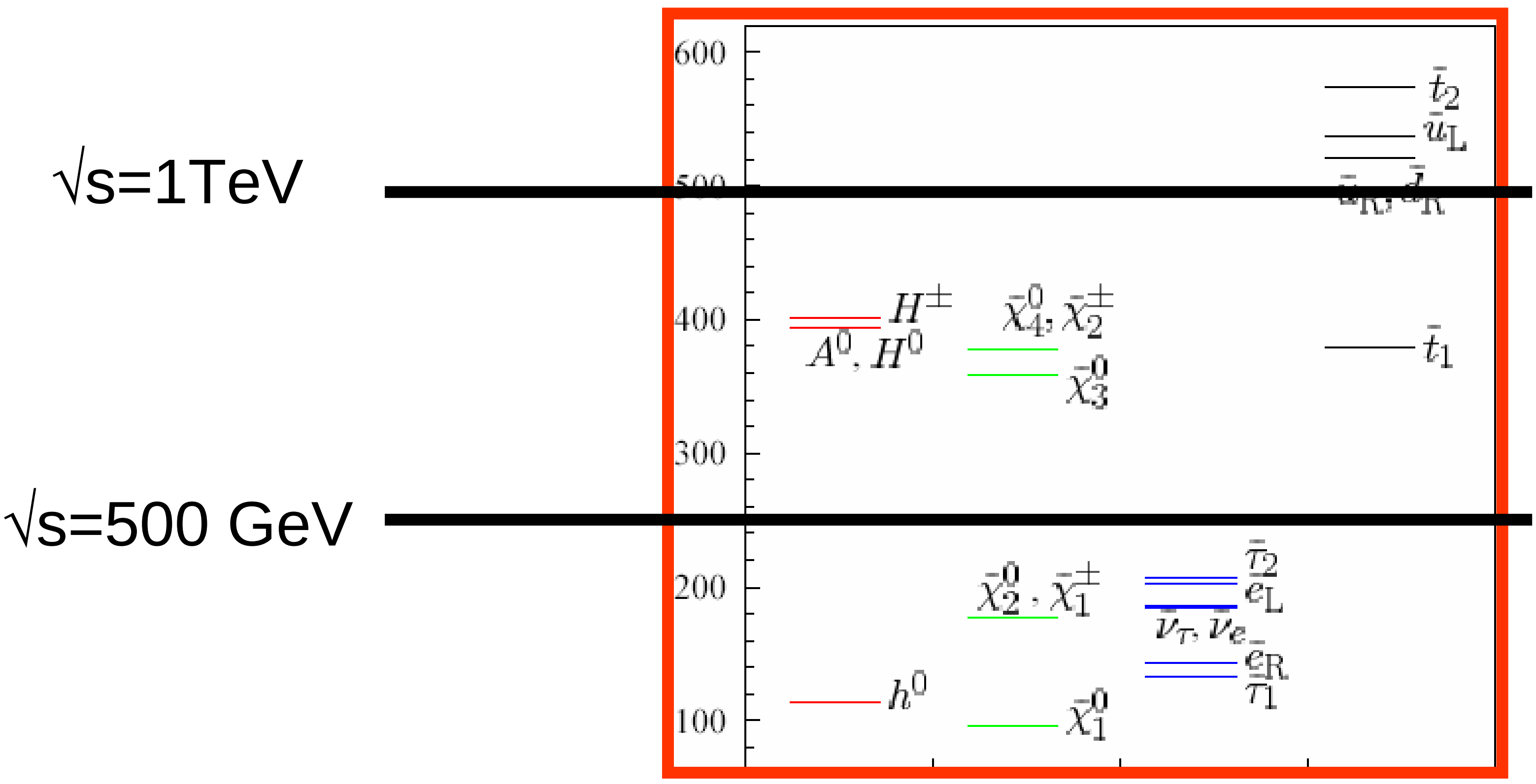
WW-ZZ separation



mSUGRA SPS1a:

$M_{1/2}$=250 GeV M_0=100 GeV

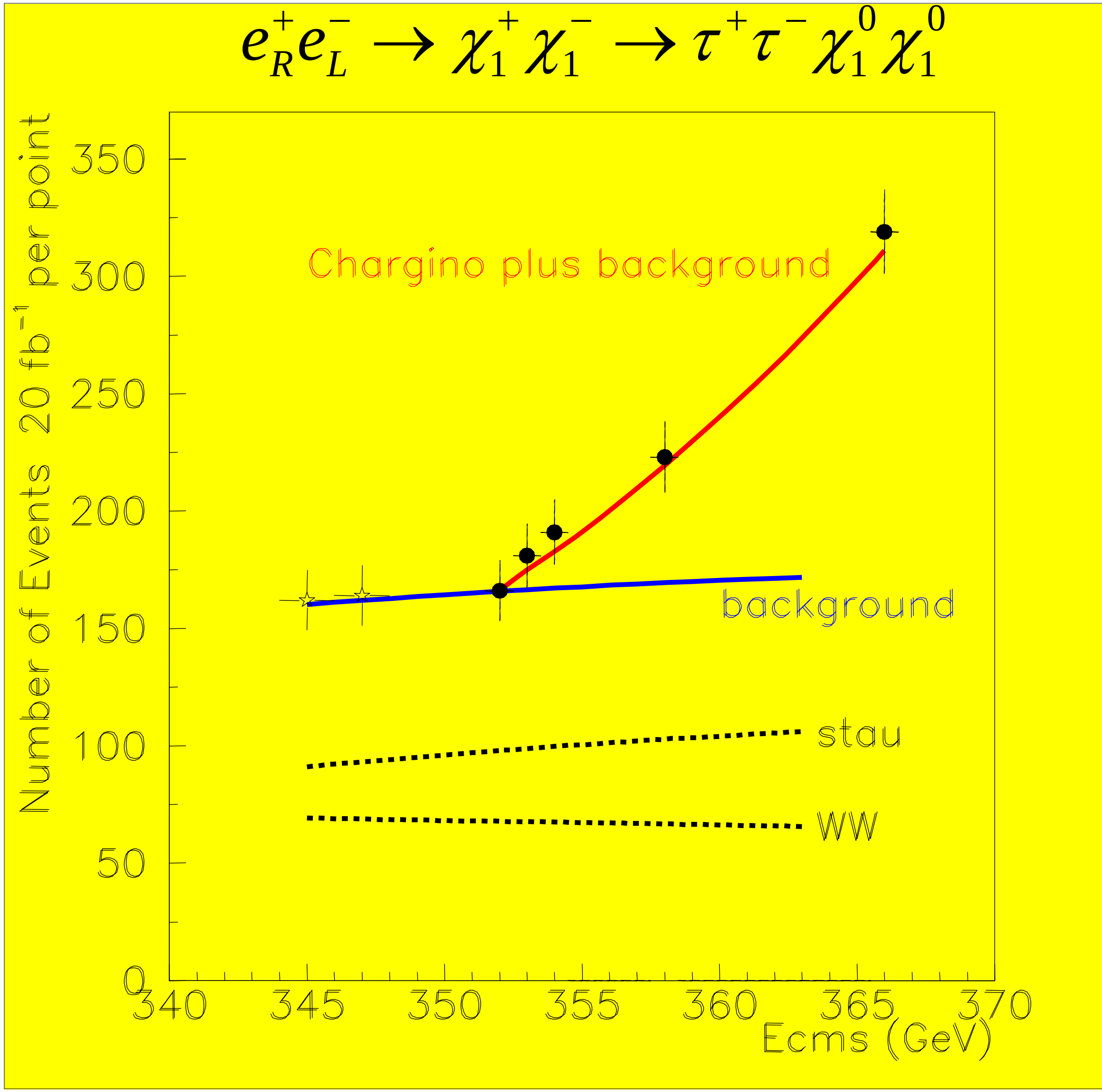
A0=-100 GeV sign(μ)=+ tanβ=10



Mass Measurements

Threshold scans
chargino ~ β
slepton ~ β^3

100 fb^{-1}

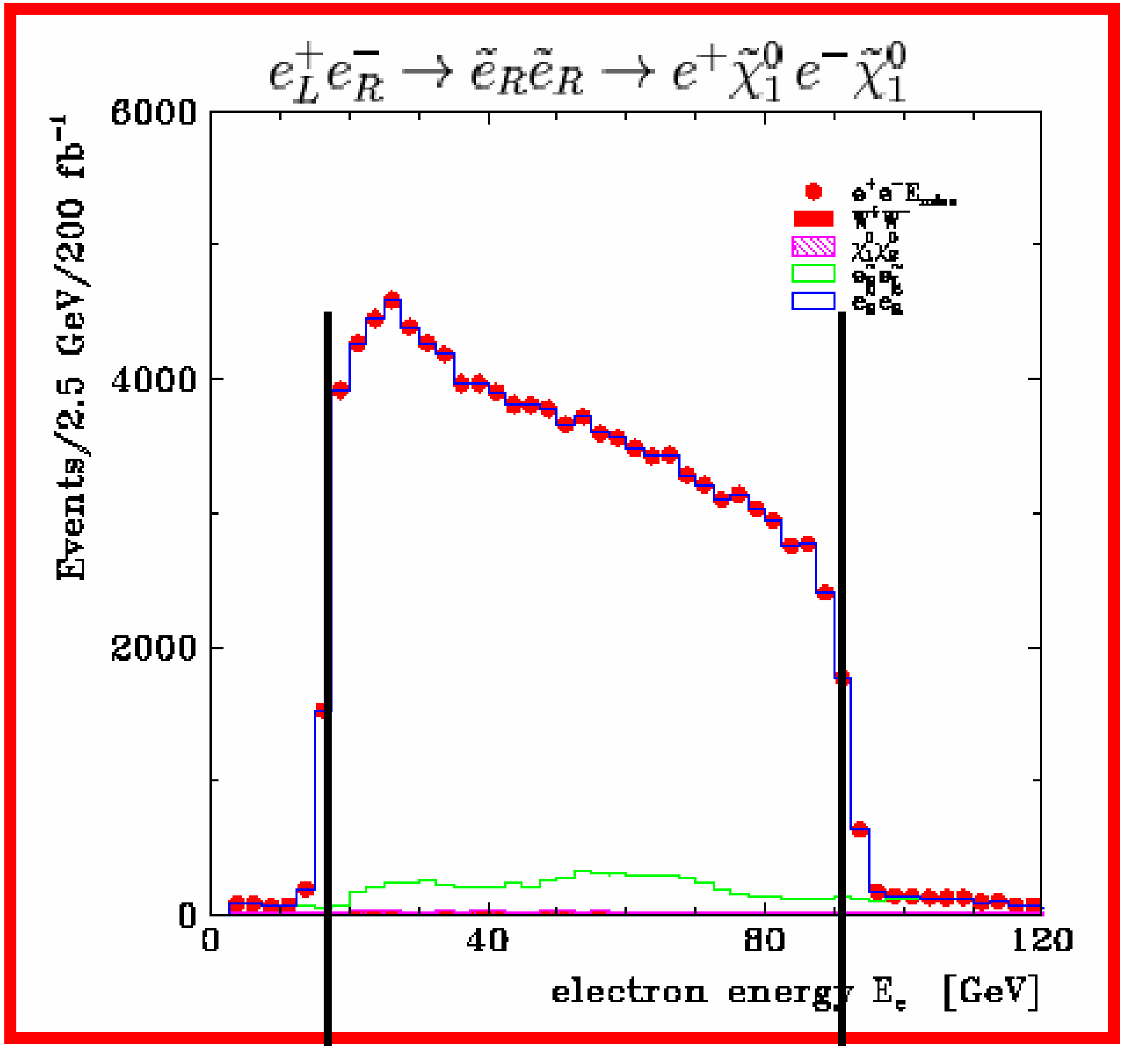


$$m_{\chi^{\pm}} = 181.5 \pm 0.55$$

Endpoint Measurements

$\sqrt{s}$=400 GeV
L=200 fb^{-1}

$\Rightarrow$ **Both** sparticle masses



Absolute + accurate measurement of mass of $\tilde{\chi}^0_1$

NB: $\sqrt{s}$ known

Martyn

Accurate measurement of mass of $\tilde{\chi}_1^0$ improves many LHC measurements

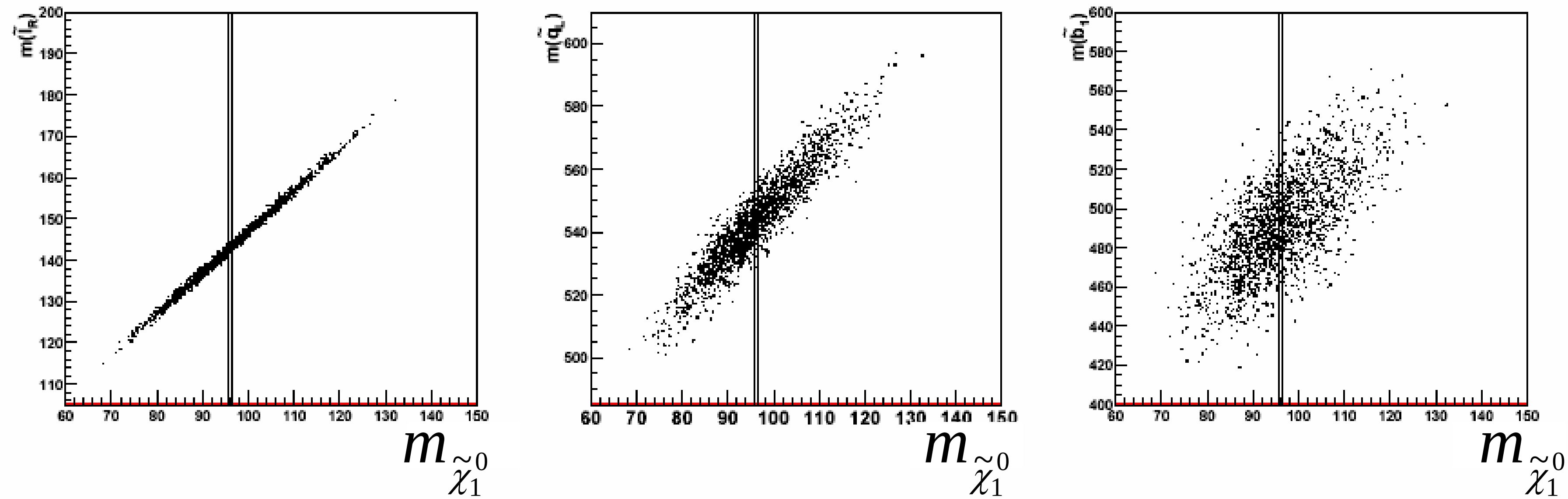
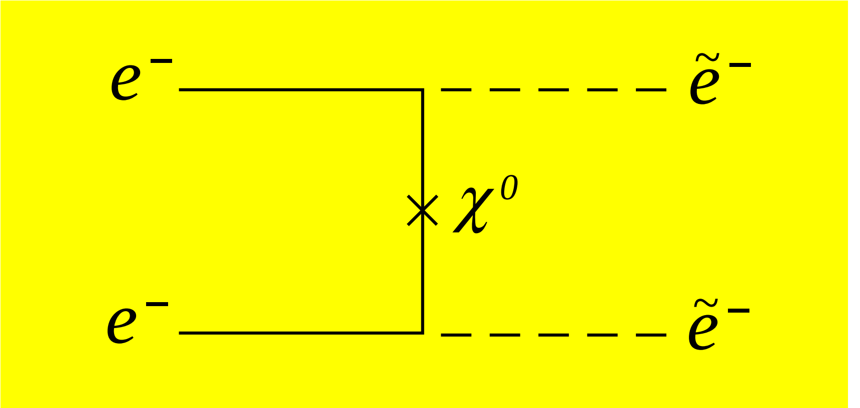


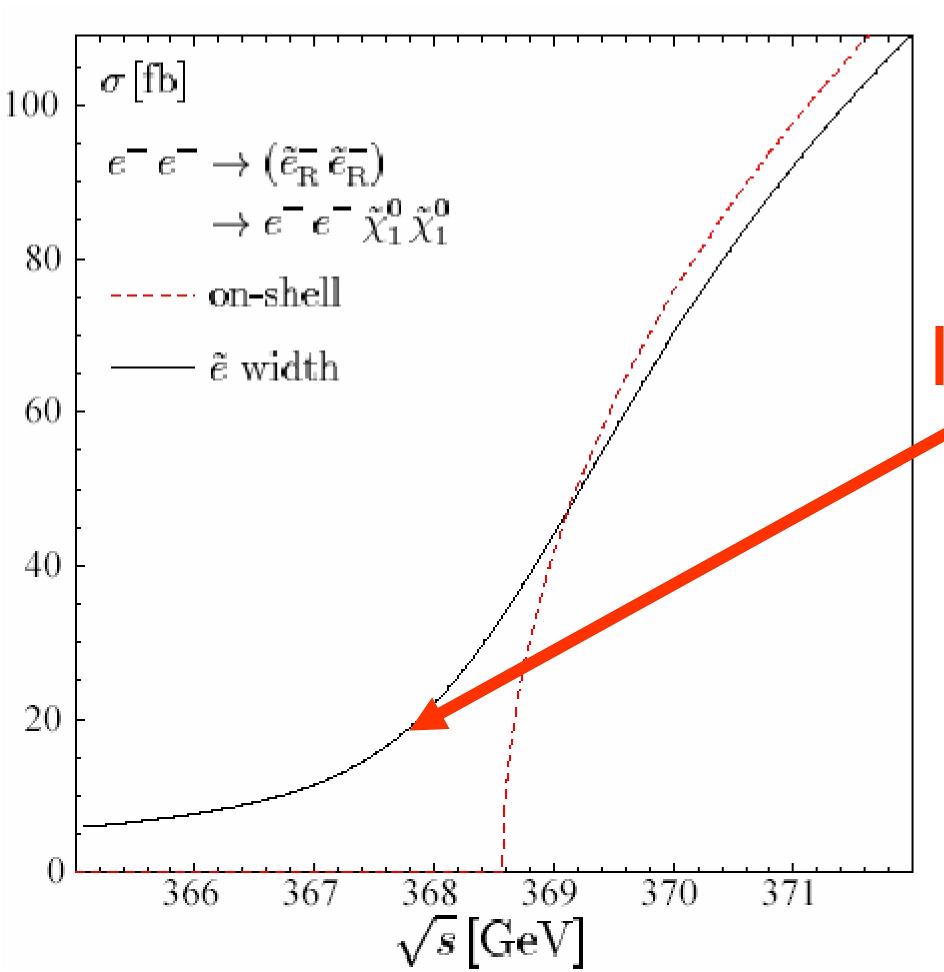
Figure 5.33: Mass correlation plots. Dots: LHC alone. Vertical bands: Fixing $m_{\tilde{\chi}_1^0}$ to within $\pm 2\sigma$ with LC input ($\sigma = 0.2\%$).

"Physics Interplay of the LHC and the ILC"

Weiglein et al. hep-ph/0410364

e^-e^- running





Including width effects

Δm~50 MeV for 4 fb^{-1}

Feng, Peskin
Freitas, Miller, Zerwas

SPS1a:

LC Threshold

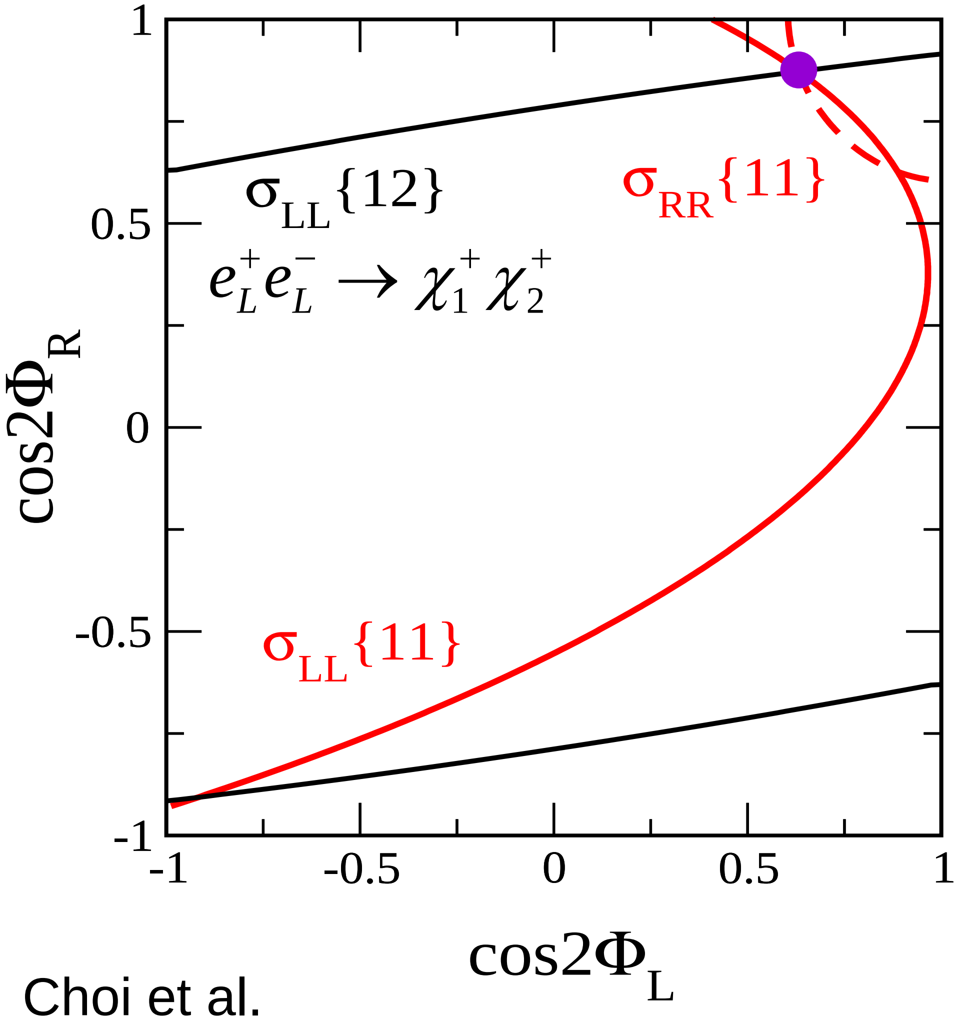
LC Endpoint

e^-e^- Threshold

LHC+LC

	Mass, ideal	"LHC"	"LC"	"LHC+LC"
$\tilde{\chi}_1^\pm$	181.5	–	0.55	0.55
$\tilde{\chi}_2^\pm$	381.2	–	3.0	3.0
$\tilde{\chi}_1^0$	97.7	8.4	0.05	0.05
$\tilde{\chi}_2^0$	183.0	8.2	1.2	0.08
$\tilde{\chi}_3^0$	363.3	–		
$\tilde{\chi}_4^0$	381.0	8.9		2.0
$\tilde{e}_R$	143.9	8.4	0.05	0.05
$\tilde{e}_L$	206.6	8.8	0.2	0.2
$\tilde{\nu}_e$	190.5	–	0.7	0.7
$\tilde{\mu}_R$	143.8	8.4	0.2	0.2
$\tilde{\mu}_L$	206.6	8.8	0.5	0.5
$\tilde{\nu}_\mu$	190.4	–		
$\tilde{\tau}_1$	134.5	8.6	0.3	0.3
$\tilde{\tau}_2$	210.4	–	1.1	1.1
$\tilde{\nu}_\tau$	189.6	–	–	–
$\tilde{u}_R$	547.8	13.6	–	7.4
$\tilde{u}_L$	564.9	13.8	–	3.3

Martyn, Polesello
Porod, Zerwas, GB



Choi et al.

$$\begin{pmatrix} M_2 & \sqrt{2} m_W \cos\beta \\ \sqrt{2} m_W \sin\beta & \mu \end{pmatrix}$$

$$\cos 2\phi_L = 0.645 \pm 0.020$$
$$\cos 2\phi_R = 0.844 \pm 0.005$$

For mSugra:
M_0=100 GeV $M_{1/2}$=200 GeV
tanβ=3 sign(μ)=+

Using 2× 500 fb-1
at √s=800 GeV

Luminosity Budget

Beams	Energy	Pol.	$\int \mathcal{L}dt$	$[\int \mathcal{L}dt]_{equiv}$	Comments
e^+e^-	500	L/R	335	335	Sit at top energy for sparticle masses
e^+e^-	M_Z	L/R	10	45	Calibrate with Z's
e^+e^-	270	L/R	100	185	Scan $\tilde{\chi}_1^0\,\tilde{\chi}_2^0$ threshold (L pol.) Scan $\tilde{\tau}_1\,\tilde{\tau}_1$ threshold (R pol.)
e^+e^-	285	R	50	85	Scan $\tilde{\mu}_R^+\,\tilde{\mu}_R^-$ threshold
e^+e^-	350	L/R	40	60	Scan $t\bar{t}$ threshold Scan $\tilde{e}_R\,\tilde{e}_L$ threshold (L & R pol.) Scan $\tilde{\chi}_1^+\,\tilde{\chi}_1^-$ threshold (L pol.)
e^+e^-	410	L	60	75	Scan $\tilde{\tau}_2\,\tilde{\tau}_2$ threshold Scan $\tilde{\mu}_L^+\,\tilde{\mu}_L^-$ threshold
e^+e^-	580	L/R	90	120	Sit above $\tilde{\chi}_1^\pm\,\tilde{\chi}_2^\mp$ threshold for $\tilde{\chi}_2^\pm$ mass
e^-e^-	285	RR	10	95	Scan with e^-e^- collisions for $\tilde{e}_R$ mass

- Several running modes required.
- Input will already exist from LHC

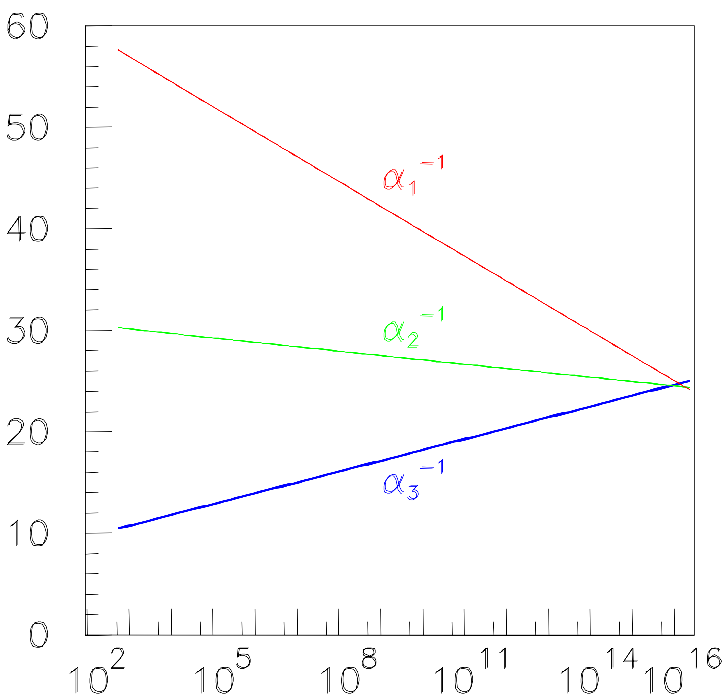
Grannis et al.

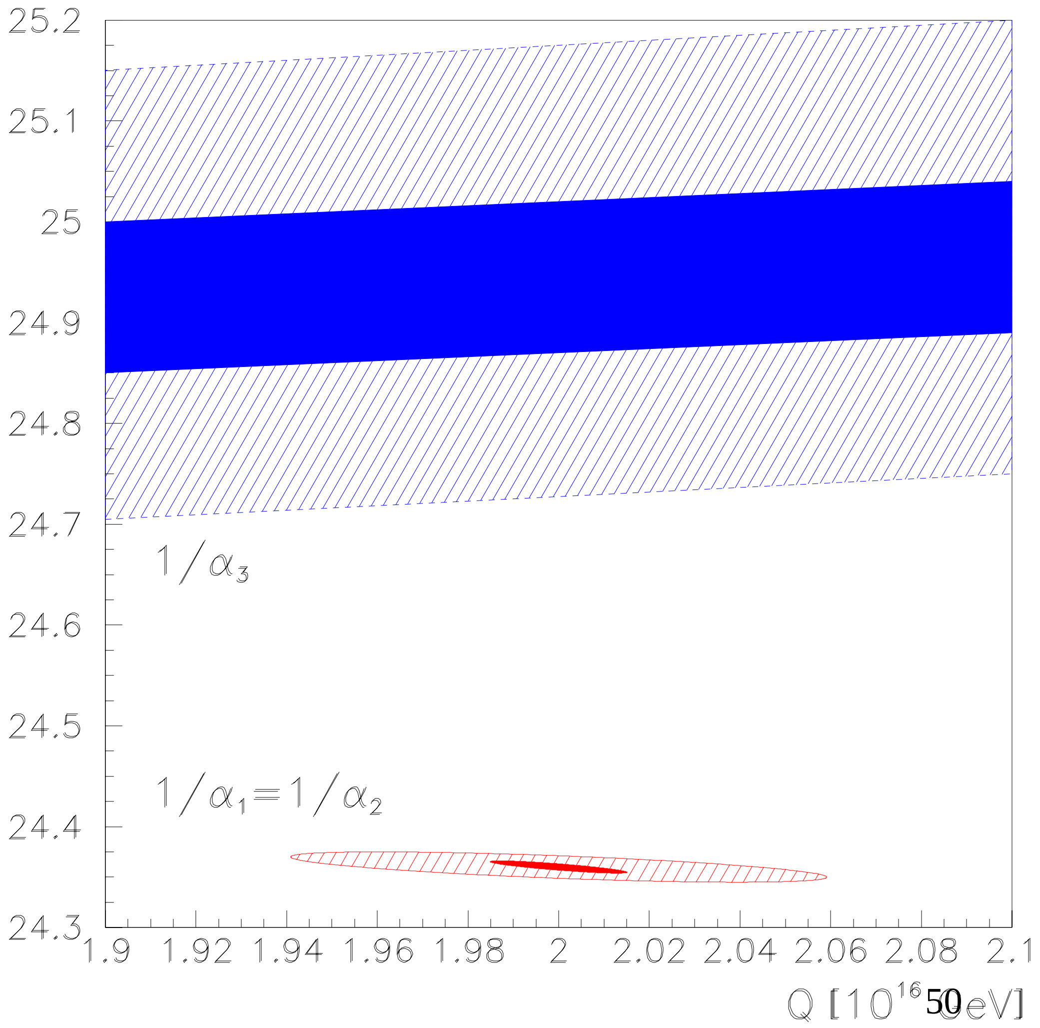
GigaZ

- The LC can also provide high luminosity running at the Z-pole and at W-threshold
- Approximately 100 fb^{-1} per year
- Needs specific linac bypass design

	LEP/SLC/Tev [19]	ILC
$\sin^2\theta^{\ell}_{\rm eff}$	0.23146 ± 0.00017	± 0.000013
lineshape observables:		
M_Z	91.1875 ± 0.0021 GeV	± 0.0021 GeV
$\alpha_s(M_Z^2)$	0.1183 ± 0.0027	± 0.0009
$\Delta\rho_\ell$	$(0.55 \pm 0.10) \cdot 10^{-2}$	$\pm 0.05 \cdot 10^{-2}$
N_ν	2.984 ± 0.008	± 0.004
heavy flavours:		
$\mathcal{A}_{\rm b}$	0.898 ± 0.015	± 0.001
$R^0_{\rm b}$	0.21653 ± 0.00069	± 0.00014
M_W	80.436 ± 0.036 GeV	± 0.006 GeV

Unification of Gauge Couplings

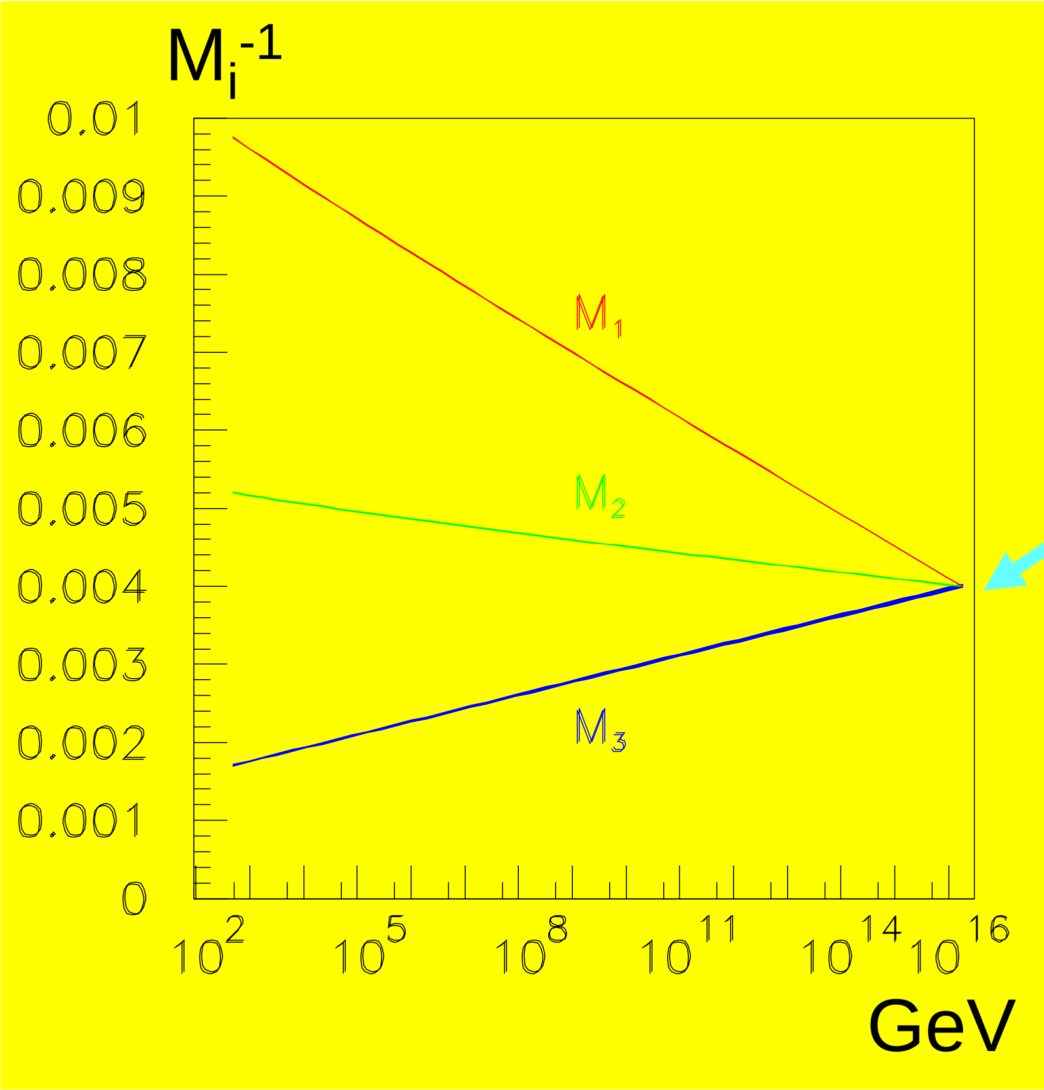


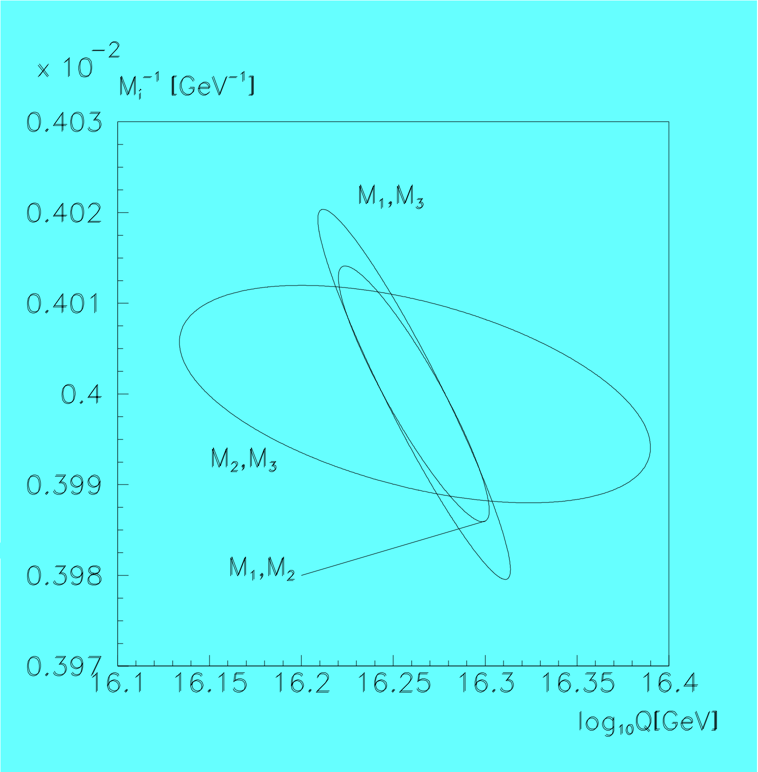


Improved measurement of GUT scale

Heavy Threshold effects eg colour triplet higgs …

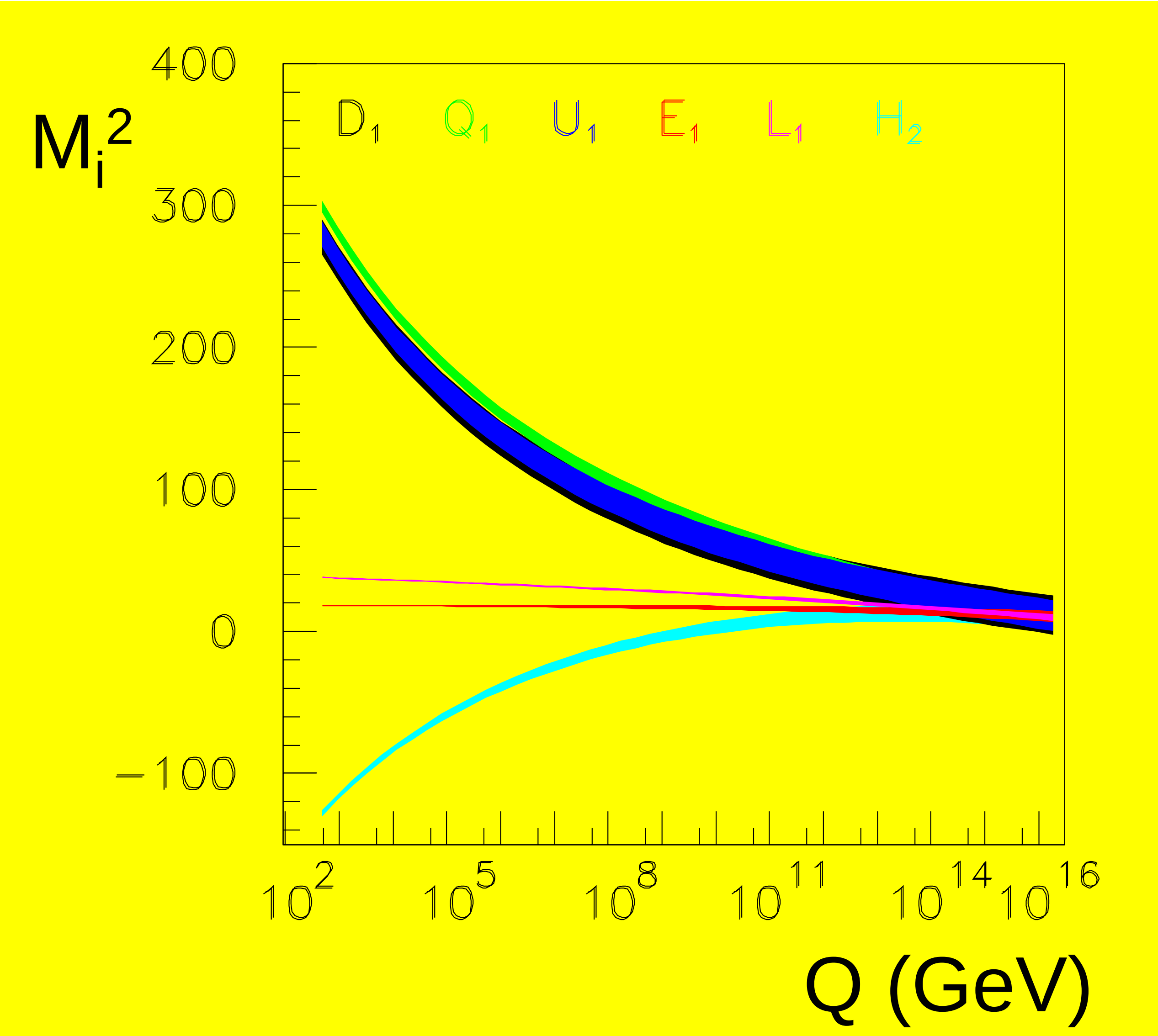
Extrapolation: gaugino





Porod, Zerwas, GB

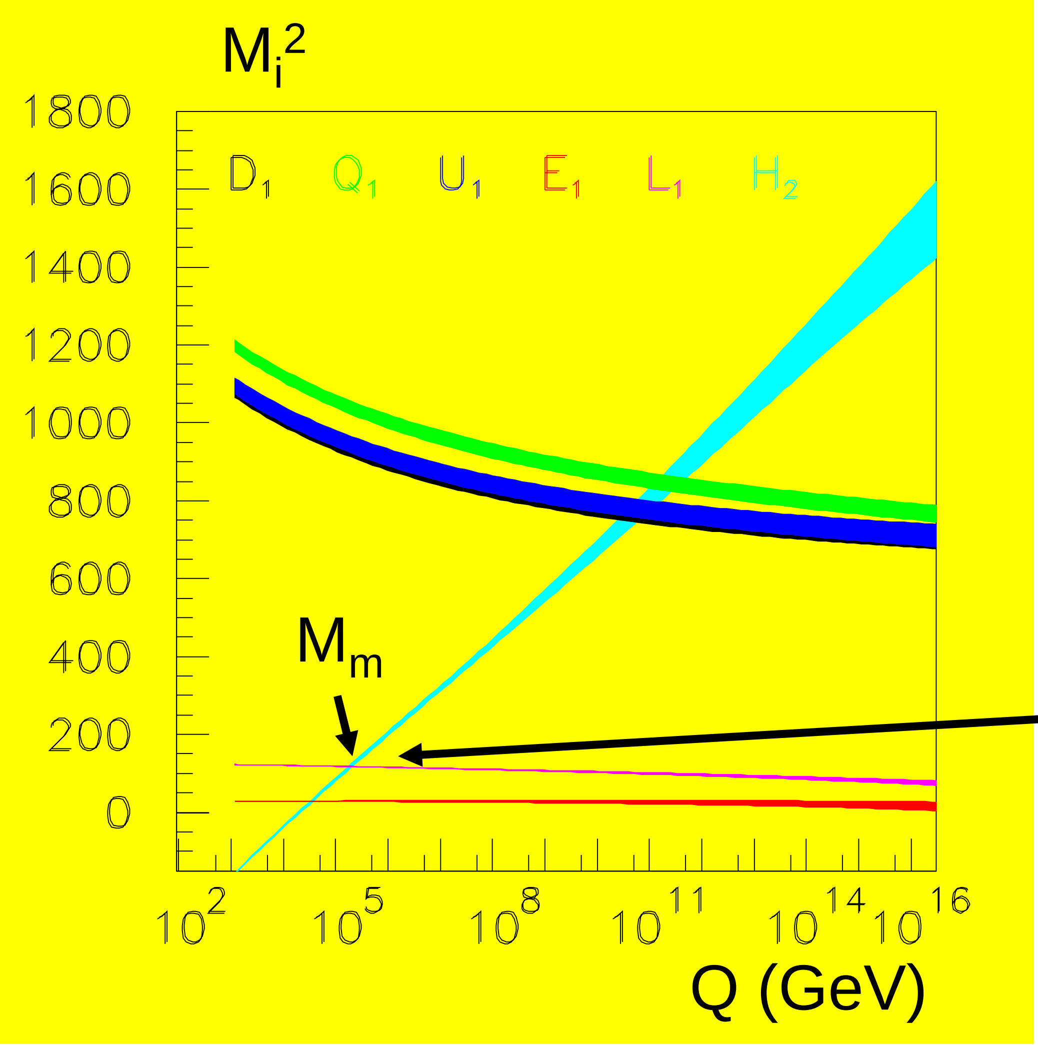
Extrapolations mass terms

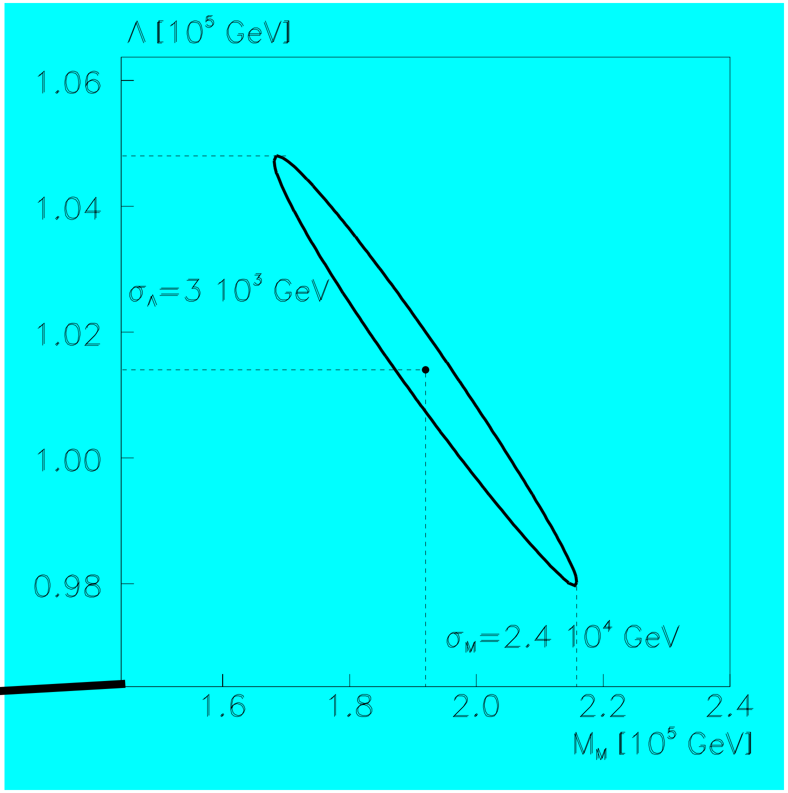


mSUGRA
structure
reconstructed

Fine structure?

Gauge Mediated SB





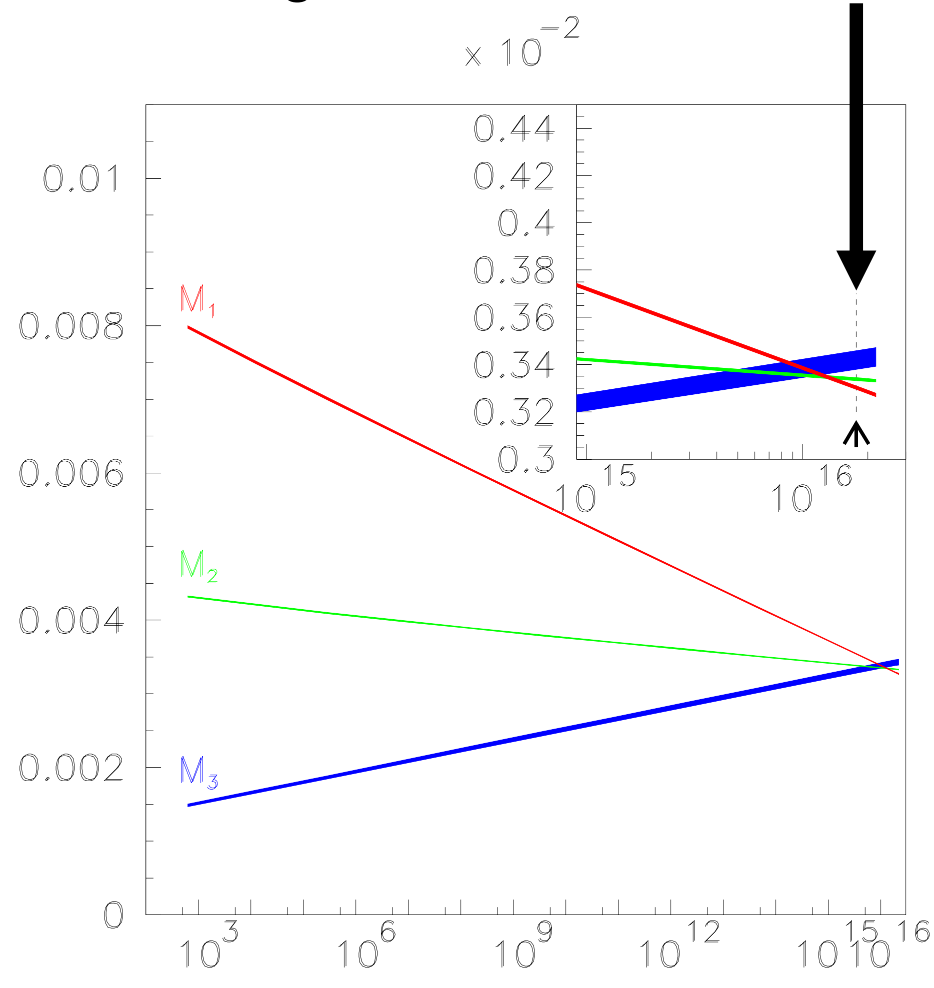
GMSB reconstructed
Messenger Scale measured

M_m=200 TeV, Λ=100 TeV, N_5=1, tanβ=15 sign(μ)=+

String Effective Theory

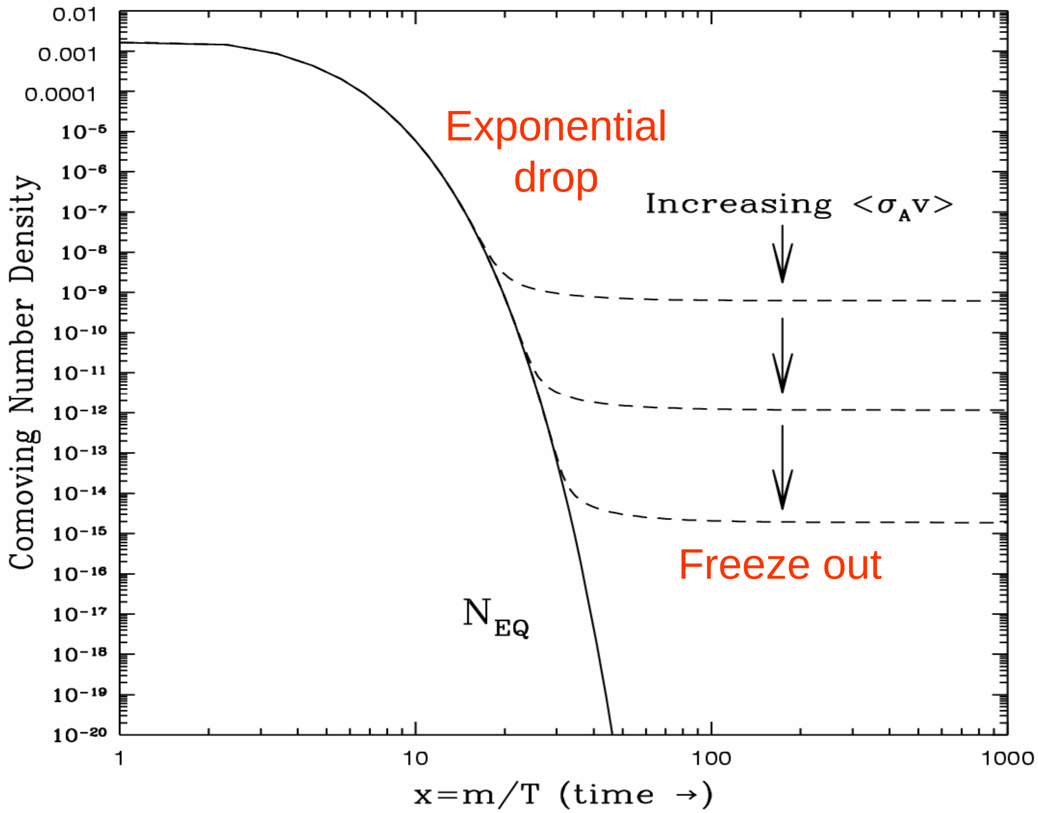
Parameter	Ideal	Reconstructed
$m_{3/2}$	180	179.9 ± 0.4
<S>	2	1.998 ± 0.006
<T>	14	14.6 ± 0.2
$Sin^2\theta$	0.9	0.899 ± 0.002
G_s^2	0.5	0.501 ± 0.002
δ_{GS}	0	0.1 ± 0.4
n_L	-3	-2.94 ± 0.04
n_E	-1	-1.00 ± 0.05
n_Q	0	0.02 ± 0.02
n_U	-2	-2.01 ± 0.02
n_D	1	0.80 ± 0.04
n_{H1}	-1	-0.96 ± 0.06
n_{H2}	-1	-1.00 ± 0.02
Tanβ	10	10.00 ± 0.13

breaking of universality
from string threshold corrections

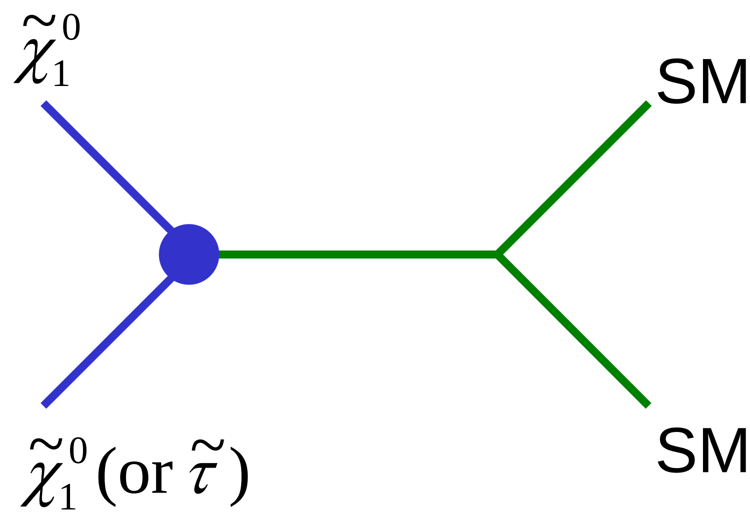


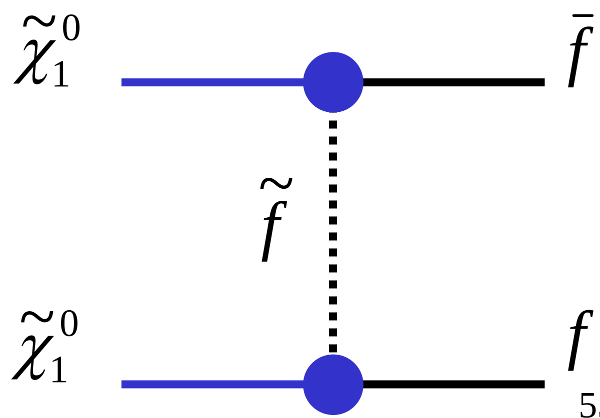
reproduction of modular
weights as integers at % level

Abundance of WIMPs



Annihilation cross section σ





- Annihilation rate depends on couplings and masses
- Both must be known accurately

LCC2:

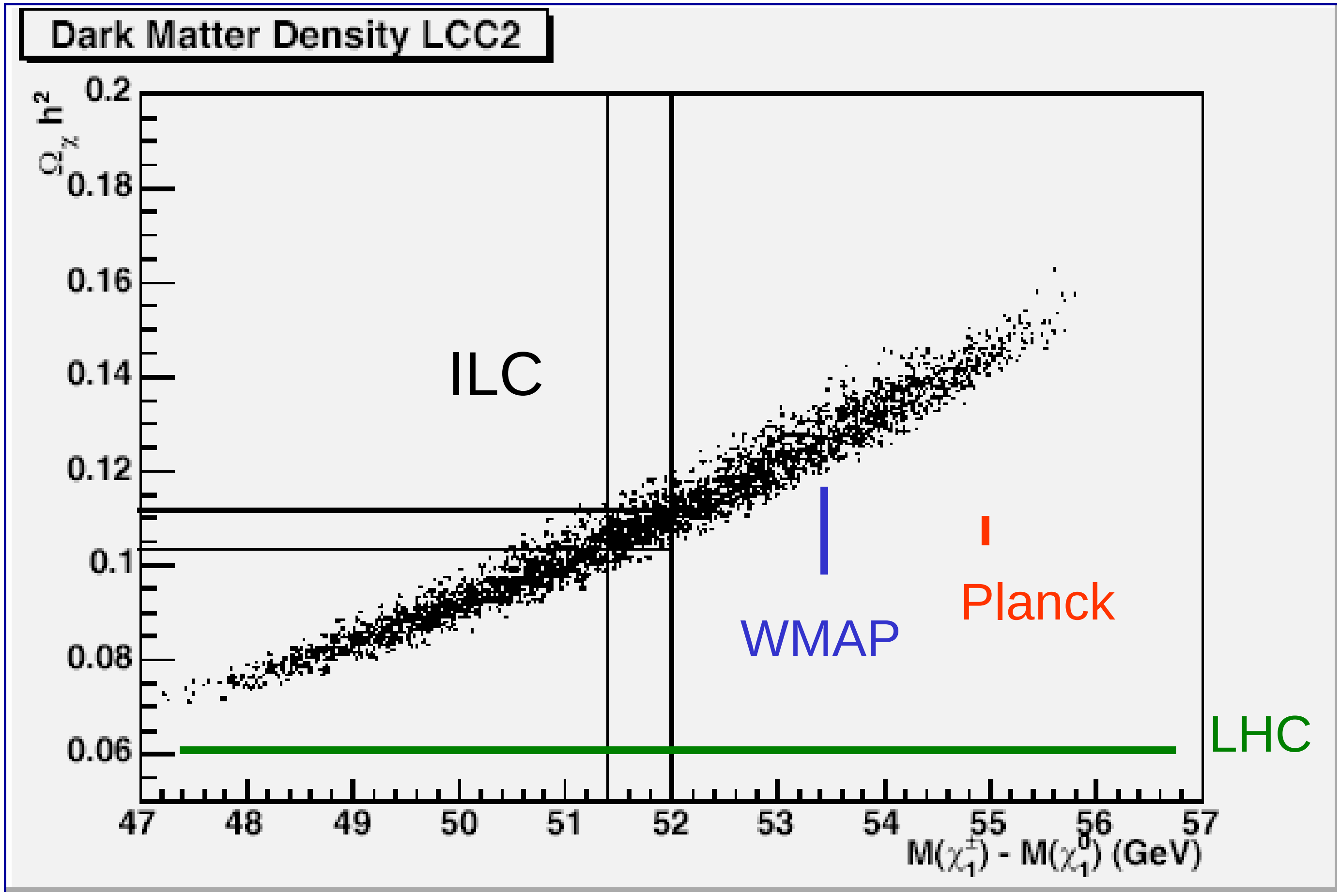
$M_{1/2}$=300 GeV

A0=0 GeV

Relic Abuncance

M_0=3280 GeV

tanβ=10

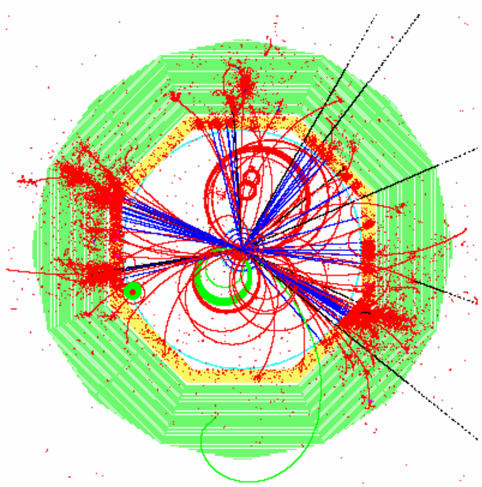


Battaglia, Feng LCWS05

Summary

$\tilde{\chi}_1^0$ $\tilde{\chi}_1^0$ $\tilde{\chi}_1^0$ $\tilde{\chi}_1^0$ $\tilde{\chi}_1^0$

$\tilde{\chi}_1^0$ $\tilde{\chi}_1^0$ $\tilde{\chi}_1^0$



G. Blair, SSI05

$$m_\nu \approx \frac{m_D^2}{M_{\text{Majorana}}}$$