Measuring the Beam Energy with Radiative Return Events

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(Master thesis of Arnd Hinze)
Introduction

- The beam energy is needed on the $10^{-4}$ level for mass determinations at the ILC (Higgs, top, SUSY...)
- The beam-energy measurement will mainly come from a magnetic spectrometer
- The absolute calibration of the spectrometer is difficult
- In addition the average beam energy is not necessarily equal to the luminosity weighted centre of mass energy → next slide
- It would thus be useful to have a method to determine the beam energy from annihilation data
- Such a method exists: radiative return events $e^+e^- \rightarrow Z\gamma \rightarrow f\bar{f}\gamma$
- The validity of this method was already proven at LEP
Energy Bias from the Kink-Instability

- Wakefields introduce a correlation $z - E$
- Disruption give a different weight to different parts of the bunch

These effects make the luminosity weighted cms energy different from twice the beam energy.

500 GeV TESLA

<table>
<thead>
<tr>
<th>mean</th>
<th>150 ppm</th>
</tr>
</thead>
<tbody>
<tr>
<td>spread</td>
<td>30 ppm</td>
</tr>
<tr>
<td>max</td>
<td>350 ppm</td>
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</tbody>
</table>
This effect is not visible in the Bhabha acolinearity!

\[ E_1 + E_2 \]

\[ E_1 - E_2 \]

![Graphs showing the comparison between TESLA and TESLA with random ordered E.](image-url)
Basic Idea of the Radiative Return Analysis

- The Z-mass is known with very high precision from LEP
- Assume only one photon is radiated
  \[ \sqrt{s'} \text{ can be calculated from fermion angles only} \]
  \[ \sqrt{s'} \sqrt{s} = \sqrt{\frac{\sin \theta_1 + \sin \theta_2 + \sin(\theta_1 + \theta_2)}{\sin \theta_1 + \sin \theta_2 - \sin(\theta_1 + \theta_2)}} \]
  - \(\gamma\) either along the beampipe or angle is measured
  - This formula assumes that the fermion mass can be neglected
- Assume \(\sqrt{s'} = m_Z \Rightarrow \sqrt{s} = m_Z \sqrt{\frac{\sin \theta_1 + \sin \theta_2 - \sin(\theta_1 + \theta_2)}{\sin \theta_1 + \sin \theta_2 + \sin(\theta_1 + \theta_2)}} \)
First analysis with $e^+e^- \rightarrow \mu^+\mu^-$: 100 fb$^{-1}$ at $\sqrt{s} = 350$ GeV

- $\sigma(\text{rad. ret.}) \sim 0.5$ pb scales approx. with $1/s$
- detector efficiency ($\text{7}\degree$ cut) $\approx 90\%$
- ideal beam with beamstrahlung and $0.2\%$ Gaussian energy spread
Fit method:

- $\chi^2$ fit of data to MC prediction with free normalisation
- MC prediction from linearisation around a default value:
  - two MC samples $MC_{\text{def}}$ at $\sqrt{s}_{\text{def}}$ and $MC_{\Delta}$ at $\sqrt{s}_{\text{def}} + \Delta \sqrt{s}$
  - $N_{\text{pred}}(\sqrt{s}) = \frac{\sqrt{s} - \sqrt{s}_{\text{def}}}{\Delta \sqrt{s}} (MC_{\Delta} - MC_{\text{def}})$
- easy to include all effects into fit
- fit tested to be bias free in region $\sqrt{s}_{\text{def}} \pm \Delta \sqrt{s}$

Cuts:

- $7^\circ < \theta_{1,2} < 183^\circ$ (detector acceptance)
- $m_Z - 5 \text{ GeV} < m(\mu^+\mu^-) < m_Z + 5 \text{ GeV}$
Backgrounds

$\gamma\gamma$ background ($e^+e^- \rightarrow e^+e^-\mu^+\mu^-$)

- very large cross section without cuts
- less than 10% background in fit range after $m(\mu^+\mu^-)$ cut

**Zee:**

- cross section similar to signal
- after cut on visible electrons $\sim$ 25% background remains
- however kinematics similar to signal, so no problem

**WW,ZZ:**

- ZZ already small, WW reduced by $m(\mu^+\mu^-)$ cut
- in the end $\sim$ 1% background
Results

Fit to 100 fb$^{-1}$ including beam effects and background:

\[ \Delta \sqrt{s} = 47 \text{ MeV} \quad \left( \frac{\Delta \sqrt{s}}{\sqrt{s}} = 1.3 \cdot 10^{-4} \right) \]

- without beamstrahlung and energy spread $\sim 10\%$ better
- little effect from background
- slight improvement possible if 2D fit $(\sqrt{s}, \theta)$
Result strongly energy dependent:

- cross section $\propto 1/s$
- resolution deteriorates with $s$
- background rises with $s$
- acceptance worse at larger $s$
- parameterisation:

$$\Delta \sqrt{s} = \left(8.8 + 0.0026\sqrt{s}/\text{GeV} + 0.0032s/\text{GeV}^2\right) \text{ MeV}$$

(However $\Delta \sqrt{s}/\sqrt{s}$ constant if $\mathcal{L} \propto s$)
**Systematics**

- **Background**: no effect for 20-30% background uncertainty
- **Energy spread**: $\Delta \sqrt{s} = 10 \text{ MeV}$ if Gaussian energy spread is replaced by rectangular, no effect if 0.1% instead of 0.2%
- **Beamstrahlung**: method largely cancels errors from beamstrahlung determination
- **Aspect ratio**: LEP error $\Delta \left( \frac{\delta R}{\delta L} \right) = \delta \tan \theta = 5 \cdot 10^{-4}$
  
  $\Rightarrow \Delta \sqrt{s} = 160 \text{ MeV}$

  $\Rightarrow$ need about one order of magnitude better
• **Possible larger statistics:**
  - Bhabha scattering: clean, however diluted by t-channel contribution
  - $e^+ e^- \rightarrow \tau^+ \tau^-$: slight dilution from $\tau$ decay angle, $m(\tau^+ \tau^-)$ against $\gamma \gamma$ does not work because of missing neutrinos
  - $e^+ e^- \rightarrow q\bar{q}$: mass effects significant (e.g. 5 GeV jet mass gives a 2.5 GeV shift in reconstructed $\sqrt{s}$) $\Rightarrow$ large sensitivity to fragmentation and particle losses

• **Global analysis:**
  - beamstrahlung is correlated for the two beams (e.g. from z-position)
  - also the kink instability is correlated for the two beams
  - both effects modify the acolinearity used in the beam spectrum and in the radiative return analysis
  - to understand the interplay of those effects a common analysis taking into account realistic beam effects is needed
Conclusions

- The centre of mass energy can be measured on the $10^{-4}$ level from radiative return events.

- This is a high luminosity analysis, so relative measurements e.g. in a scan are needed from spectrometers.

- The length to radius ratio of the detector needs to be known to better than $10^{-4}$ not to be limited by this effect.

- A global analysis of Bhabha acolinearity for beamstrahlung and radiative return events for the beam energy is needed to understand effects from beam-beam correlations.