Electroweak Baryogenesis
and
the triple Higgs boson coupling

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Reference: PLB606 361 (2005)
Outline

• Introduction
  - Higgs physics/Cosmology interface

• Electroweak baryogenesis
  - Electroweak phase transition in the 2HDM

• Quantum corrections to the hhh coupling constant
  - Collider signals of electroweak baryogenesis?

• Summary
Higgs physics/Cosmology interface

• **Higgs physics at colliders**
  - Discovery of the Higgs boson(s) (@Tevatron, LHC)
  - Measurements of the Higgs couplings with \(\{\) gauge bosons, fermions\(\) \(\mathcal{O}(1)\%\) accuracy (@ILC) ACFA Rep. TESLA TDR
  - Measurements of the Higgs self-couplings (reconstruction of the Higgs potential) \(\mathcal{O}(10 - 20)\%\) accuracy (@ILC) ACFA Higgs WG, Battaglia et al.

• **Cosmology**
  - Baryon Asymmetry of the Universe (BAU) \(n_B/s \sim 10^{-10}\)
  Attempts: GUTs, Affleck-Dine, Leptogenesis, EW baryogenesis, etc...

**Connection with collider physics**

<table>
<thead>
<tr>
<th>Electroweak baryogenesis</th>
<th>based on the Higgs potential at (T \neq 0)</th>
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<tbody>
<tr>
<td>(\downarrow) collider signals??</td>
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| Higgs self-couplings | Higgs potential at \(T = 0\) |

We evaluate the \(hhh\) coupling in the possible region of EW baryogenesis.
Conditions for Baryogenesis

- 3 requirements for generation of the BAU (Sakharov conditions)

1. baryon number violation
2. $C$ and $CP$ violation
3. out of equilibrium

- Baryogenesis in the electroweak theory
  - $B$ violation sphaleron process
  - $C$ violation chiral gauge interaction
  - $CP$ violation KM-phase or other sources in the extension of the SM
  - out of equilibrium 1st order phase transition
Sphaleron process

• A saddle point solution of 4d $SU(2)$ gauge-Higgs system
  
  [Manton, PRD28 ('83)]

\[ \Delta B = N_f \Delta N_{CS} \]

• Transition rate

\[
\Gamma_{\text{sph}}^{(b)} \sim (\alpha_W T)^4 e^{-E_{\text{sph}}/T} \quad \text{(broken phase)}
\]

\[
\Gamma_{\text{sph}}^{(s)} \sim (\alpha_W T)^4 \quad \text{(symmetric phase)}
\]

$B$ violation process is effective at finite temperature, but is suppressed at $T = 0$
Baryogenesis mechanism

- Asymmetry of the charge flow of the particle (due to $CP$ violation)

- Accumulation of the charge in the symmetric phase

- $B$ generation via sphaleron process

- Decoupling of sphaleron process in the broken phase

**Strongly 1st order phase transition**

⇒ Decoupling of the sphaleron process at $T \lesssim T_c$:

$$\Gamma_{sph}^{(b)}/T_c^3 < H(T_c) \quad \Rightarrow \quad \frac{\varphi_c}{T_c} \gtrsim 1$$
In principle, the SM fulfills all the three Sakharov conditions, \textit{BUT}

- Phase transition is \textit{not} 1st order (for $m_h > 114 \text{ GeV}$) out of equilibrium \times
- KM-phase is \textit{too small} to generate sufficient BAU

\[\downarrow\]

\textbf{Extension of the minimal SM Higgs sector}

- Two Higgs Doublet Model (2HDM)
- Minimal Supersymmetric Standard Model (MSSM)
- Next-to-Minimal Supersymmetric Standard Model (NMSSM)
- etc...

In this talk we consider

- 2HDM \textit{simple viable model}
- MSSM
Two Higgs Doublet Model (2HDM)

- Introduce the additional Higgs doublet $\Phi$
- FCNC suppression $\Rightarrow \Phi_1 \rightarrow \Phi_1, \Phi_2 \rightarrow -\Phi_2$ (Type I, II Yukawa int.)

\[ V_{\text{THDM}} = m_1^2 |\Phi_1|^2 + m_2^2 |\Phi_2|^2 - (m_3^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}) \]
\[ + \frac{\lambda_1}{2} |\Phi_1|^4 + \frac{\lambda_2}{2} |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 \]
\[ + \left[ \frac{\lambda_5}{2} (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \right], \quad \Phi_{i=1,2}(x) = \left( \begin{array}{c} \phi_i^+(x) \\ \frac{1}{\sqrt{2}} (v_i + h_i(x) + i a_i(x)) \end{array} \right). \]

- $m_3^2, \lambda_5 \in \mathbb{C}$ (sources of explicit $CP$ violation)

In the MSSM:
\[ \lambda_1 = \lambda_2 = (g_2^2 + g_1^2)/4, \quad \lambda_3 = (g_2^2 - g_1^2)/4, \quad \lambda_4 = g_2^2/2, \quad \lambda_5 = 0 \]

7 independent parameters
- $m_h, m_H, m_A, m_{H^\pm}$: CP-even, CP-odd and charged Higgs boson masses
- $\alpha$: mixing angle between $h$ and $H$, $\tan \beta = v_2/v_1$, ($v = \sqrt{v_1^2 + v_2^2} \sim 246$ GeV)
- $M = \frac{m_3}{\sqrt{\sin \beta \cos \beta}}$ (soft-breaking scale of the $Z_2$ symmetry)
Setup

To avoid complication, we consider [Cline et al PRD54 '96]

\[ m_1 = m_2 \equiv m, \quad \lambda_1 = \lambda_2 = \lambda, \quad \left( \sin(\beta - \alpha) = \tan \beta = 1 \right) \]

- Higgs VEVs: \( \langle \Phi_1 \rangle = \langle \Phi_2 \rangle = \frac{1}{2} \begin{pmatrix} 0 \\ \varphi \end{pmatrix} \)

- Tree-level potential

\[ V_{\text{tree}}(\varphi) = -\frac{\mu^2}{2} \varphi^2 + \frac{\lambda_{\text{eff}}}{4} \varphi^4 \]

\( \mu^2 = m_3^2 - m^2 \), \( \lambda_{\text{eff}} = \frac{1}{4}(\lambda + \lambda_3 + \lambda_4 + \lambda_5) \)

- Field dependent masses of the Higgs bosons

\[ m_h^2(\varphi) = \frac{3}{2} m_h^2(v) \left( \frac{\varphi^2}{v^2} - \frac{1}{3} \right), \]

\[ m_H^2(\varphi) = \left[ m_H^2(v) + \frac{1}{2} m_h^2(v) - M^2 \right] \frac{\varphi^2}{v^2} - \frac{1}{2} m_h^2(v) + M^2, \]

\[ m_A^2(\varphi) = \left[ m_A^2(v) + \frac{1}{2} m_h^2(v) - M^2 \right] \frac{\varphi^2}{v^2} - \frac{1}{2} m_h^2(v) + M^2, \]

\[ m_{H\pm}^2(\varphi) = \left[ m_{H\pm}^2(v) + \frac{1}{2} m_h^2(v) - M^2 \right] \frac{\varphi^2}{v^2} - \frac{1}{2} m_h^2(v) + M^2. \]
1-loop effective potential

• Zero temperature

\[ V_1(\varphi) = n_i \frac{m_i^4(\varphi)}{64\pi^2} \left( \log \frac{m_i^2(\varphi)}{Q^2} - \frac{3}{2} \right) \]

\( (n_W = 6, \ n_Z = 3, \ n_t = -12, \ n_h = n_H = n_A = 1, \ n_{H\pm} = 2) \)

• Finite temperature

\[ V_1(\varphi, T) = \frac{T^4}{2\pi^2} \left[ \sum_{i=\text{bosons}} n_i I_B(a^2) + n_t I_F(a) \right] \]

where \( I_{B,F}(a^2) = \int_0^\infty dx \ x^2 \log \left( 1 + e^{-\sqrt{x^2+a^2}} \right), \quad (a(\varphi) = \frac{m(\varphi)}{T}) \)

▷ High temperature expansion \quad (a^2 \ll 1)

\[ I_B(a^2) = -\frac{\pi^4}{45} + \frac{\pi^2}{12} a^2 - \frac{\pi}{6} (a^2)^{3/2} - \frac{a^4}{32} \left( \log \frac{a^2}{\alpha_B} - \frac{3}{2} \right) + O(a^6), \]

\[ I_F(a^2) = -\frac{7\pi^4}{360} - \frac{\pi^2}{24} a^2 - \frac{a^4}{32} \left( \log \frac{a^2}{\alpha_F} - \frac{3}{2} \right) + O(a^6), \quad \left( \log \alpha_{F(B)} = 2 \log(4)\pi - 2\gamma_E \right) \]

\( \varphi^3 \)-term comes from “boson” loops
Finite temperature Higgs potential

For \( m^2_\Phi(v) \gg M^2, m^2_h(v) \quad m^2_\Phi(\varphi) \simeq m^2_\Phi(v)\frac{\varphi^2}{v^2}, \quad (\Phi = H, A, H^\pm) \)

\[
V_{\text{eff}} \simeq D(T^2 - T_0^2)\varphi^2 - E T \varphi^3 + \frac{\lambda T}{4} \varphi^4
\]

where

\[
E = \frac{1}{12\pi v^3} \left( 6m^3_W + 3m^3_Z + m^3_H + m^3_A + 2m^3_{H^\pm} \right)
\]

additional contributions

At \( T_c \), degenerate minima: \( \varphi_c = 0 \), \( \frac{2ET_c}{\lambda T_c} \)

- The magnitude of \( E \) is relevant for the strongly 1st order phase transition
- We examine the strength of the phase transition without the high temperature expansion.
$T_c$ and $\varphi_c$ vs heavy Higgs boson mass

$m_h = 120$ GeV, $m_\phi = m_H = m_A = m_{H^\pm}$, $\sin(\beta - \alpha) = \tan \beta = 1$
Contours of $\varphi_c/T_c$ in the $m_\Phi-M$ plane

$$\sin^2(\alpha - \beta) = \tan \beta = 1, \ m_h = 120 \text{ GeV, } m_\Phi \equiv m_A = m_H = m_{H^\pm}$$

Contour plot of $\varphi_c/T_c$ in the $m_\Phi-M$ plane

- For $m_\Phi^2 \gg M^2, m_h^2$,
  Strongly 1st order phase transition is possible due to the loop effect of the heavy Higgs bosons ($\varphi^3$-term is effectively large)

- How large is the magnitude of the $\lambda_{hhh}$ coupling at $T = 0$ in such a region?
Quantum corrections to the hhh coupling

[S. Kanemura, S. Kiyoura, Y. Okada, E.S., C.-P. Yuan PL '03]

- **hhh**

\[
\begin{align*}
\lambda_{hhh}^{\text{tree}} &= -\frac{3m_h^2}{v}, \quad \text{(same form as in the SM)} \\
\lambda_{hhh} &\sim -\frac{3m_h^2}{v} \left[ 1 + \frac{c}{12\pi^2 m_h^2 v^2} \left( 1 - \frac{M^2}{m_\Phi^2} \right)^3 \right] \quad (\Phi = H, A, H^\pm)
\end{align*}
\]

\( c = 1 \) for neutral Higgs, \( c = 2 \) for charged Higgs

For \( m_\Phi^2 \gg M^2, m_h^2 \), the loop effect of the heavy Higgs bosons is enhanced by \( m_\Phi^4 \), which does not decouple in the large mass limit. (nondecoupling effect)
Contour plots of $\Delta \lambda_{hhh}/\lambda_{hhh}$ in the $m_\Phi-M$ plane

$\sin^2(\alpha - \beta) = \tan \beta = 1$, $m_h = 120$ GeV, $m_\Phi \equiv m_A = m_H = m_{H^\pm}$

For $m_\Phi^2 \gg M^2, m_h^2$,

- Deviation of the $hhh$ coupling constant from the SM value becomes large.
Contour plots of $\Delta \lambda_{hhh}/\lambda_{hhh}$ and $\varphi_c/T_c$ in the $m_{\Phi}-M$ plane

$\sin^2(\alpha - \beta) = \tan \beta = 1$, $m_h = 120$ GeV, $m_{\Phi} \equiv m_A = m_H = m_{H^\pm}$

[S.Kanemura, Y.Okada, E.S.]

For $m_{\Phi}^2 \gg M^2, m_h^2$,

- Phase transition is strongly 1st order, \textit{AND}
- Deviation of hhh coupling from SM value becomes large. ($\Delta \lambda_{hhh}/\lambda_{hhh} \gtrsim 10\%$)
Contour plots of $\Delta \lambda_{hhh}/\lambda_{hhh}$ and $\varphi_c/T_c$ in the $m_{\Phi}-M$ plane

$$\sin^2(\alpha - \beta) = \tan \beta = 1, \ m_h = 160 \text{ GeV}, \ m_{\Phi} \equiv m_A = m_H = m_{H^\pm}$$

The correlation between $\varphi_c/T_c$ and $\Delta \lambda_{hhh}/\lambda_{hhh}$ is almost same as the lighter $m_h$ case.
Electroweak phase transition in the MSSM

- Light stop scenario [Carena, Quiros, Wagner, PLB380 ('96)]

\[ M_Q^2 \gg M_U^2, m_t^2, \quad m_A^2 \gg m_Z^2 \quad (\sin(\beta - \alpha) \approx 1) \]

\[ m_{\tilde{t}_1}^2(\varphi, \beta) \approx M_U^2 + \mathcal{O}(m_Z^2) + \frac{y_t^2 \sin^2 \beta}{2} \left(1 - \frac{|X_t|^2}{M_Q^2}\right) \varphi^2, \quad (X_t = A_t - \mu \cot \beta) \]

- High temperature expansion

For \( M_U^2 \approx 0, \quad (m_{\tilde{t}_1} \approx m_t) \)

\[ \Delta E_{\tilde{t}_1} \approx \frac{1}{2\pi} \frac{m_t^3}{v^3} \left(1 - \frac{|X_t|^2}{M_Q^2}\right)^{3/2} \]

Stop contribution make the phase transition stronger enough for successful electroweak baryogenesis.

Collider signals \( \Rightarrow \quad m_{\tilde{t}_1} \lesssim m_t, \quad m_h \lesssim 120 \text{ GeV} \)

In this scenario, how large is the magnitude of the \( \lambda_{hhh} \) coupling?
Deviation of the $\lambda_{hhh}$ from the SM value

- Leading contribution of stop loop

$$\frac{\Delta \lambda_{hhh} (\text{MSSM})}{\lambda_{hhh} (\text{SM})} \sim \frac{m_t^4}{2\pi^2 v^2 m_h^2} \left( 1 - \frac{|X_t|^2}{M_Q^2} \right)^3 = \frac{3v^4}{m_t^2 m_h^2} (\Delta E_{\tilde{t}_1})^2.$$

$\varphi_c/T_c = 2E/\lambda T_c > 1$ gives

$$\frac{\Delta \lambda_{hhh} (\text{MSSM})}{\lambda_{hhh} (\text{SM})} \sim 6\%. \quad \text{(for } m_h = 120 \text{ GeV)}$$

In the MSSM, the condition of strongly 1st order phase transition also leads to large quantum corrections to the hhh coupling constant.

- Numerical evaluation without the high temperature expansion

work in progress
Summary

We have studied the collider signature of the successful electroweak baryogenesis.

In the 2HDM

For $m_\Phi^2 \gg M^2, m_h^2$

- Phase transition is strongly 1st order.
- The deviation of the $\lambda_{hhh}$ coupling from the SM prediction becomes large. ($\Delta \lambda_{hhh}/\lambda_{hhh} \gtrsim 10\%$)

**due to the nondecoupling effect of the heavy Higgs bosons**

In the MSSM with light stop scenario

$\Delta \lambda_{hhh}/\lambda_{hhh} \sim \text{several } \%$

**Such deviations can be testable at a future $e^+e^-$ Linear Collider.**
Sensitivity of the $hhh$ coupling at Linear Colliders

\[ hhh \]

\[ e^+ e^- \rightarrow hh\nu\nu \]

\[ e^+ e^- \rightarrow Zhh \]

\[ \lambda_{hhh} \]

Higgs self coupling sensitivity

Higgs mass [GeV]

\[ \delta\lambda_3/\lambda_3 \% \]

\[ Int(L)=1 \text{ ab}^{-1} \]

100% efficiency

[\text{Y.Yasui et al ACFA WG}]
Ring-improved Higgs boson masses

\begin{align*}
m_h^2(\varphi, T) &= \frac{3}{2} m_h^2(v) \frac{\varphi^2}{v^2} - \frac{1}{2} m_h^2(v) + a T^2, \\
m_H^2(\varphi, T) &= \left[ m_H^2(v) + \frac{1}{2} m_h^2(v) - M^2 \right] \frac{\varphi^2}{v^2} - \frac{1}{2} m_h^2(v) + M^2 + a T^2, \\
m_A^2(\varphi, T) &= \left[ m_A^2(v) + \frac{1}{2} m_h^2(v) - M^2 \right] \frac{\varphi^2}{v^2} - \frac{1}{2} m_h^2(v) + M^2 + a T^2, \\
m_{H\pm}^2(\varphi, T) &= \left[ m_{H\pm}^2(v) + \frac{1}{2} m_h^2(v) - M^2 \right] \frac{\varphi^2}{v^2} - \frac{1}{2} m_h^2(v) + M^2 + a T^2, \\
m_{G0}^2(\varphi, T) &= m_{G\pm}^2(\varphi, T) = \frac{1}{2} m_h^2(v) \frac{\varphi^2}{v^2} - \frac{1}{2} m_h^2(v) + a T^2.
\end{align*}

where

\[
a = \frac{1}{12v_0^2} \left[ 6m_W^2(v) + 3m_Z^2(v) + 5m_h^2(v) + m_H^2(v) + m_A^2(v) + 2m_{H\pm}^2(v) - 4M^2 \right].
\]
The magnitude of the self-couplings ($\sin(\beta - \alpha) = \tan \beta = 1$, $m_h = 120$ GeV)

\[ \lambda_1/4\pi = \lambda_2/4\pi = \lambda_3/4\pi \]

\[ \lambda_4/4\pi = \lambda_5/4\pi \]