Precision corrections to sfermion masses

LCWS ’05

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Based on:

`hep-ph/0501132` (“TSIL”) with David G. Robertson (Otterbein College)

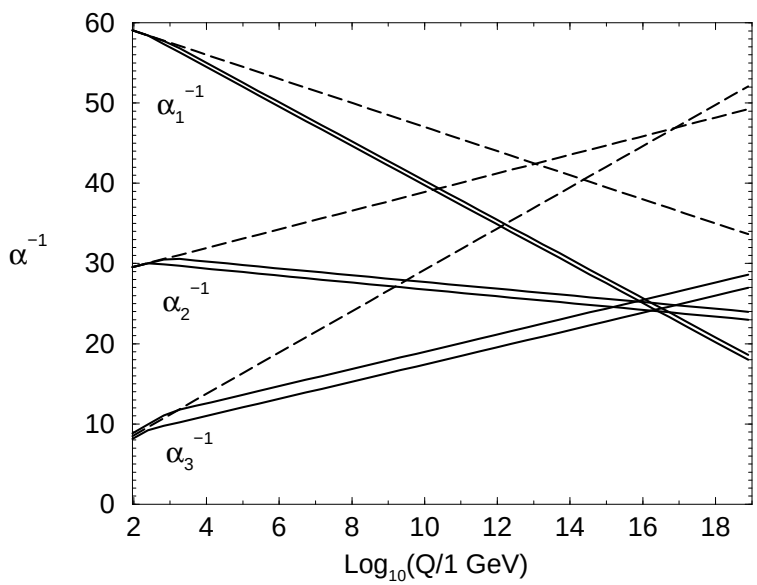
`hep-ph/0502168` and references therein

- Strategy
  - Relating future ILC and LHC results to SUSY models
  - Comments on two-loop calculations for SUSY
- Tactics
  - Differential equations method for multi-loop integrals
  - TSIL (Two-loop Self-energy Integral Library), a computer program for evaluating general two-loop self-energy and vacuum integrals
- Results
  - Two-loop self-energies for scalars in a general renormalizable gauge theory (massless vector approximation)
  - Pole masses of squarks at two-loop order

Masses are key observables in SUSY

Most of what we do not already know about Supersymmetric extensions of the Standard Model involves the soft breaking terms with positive mass dimension.

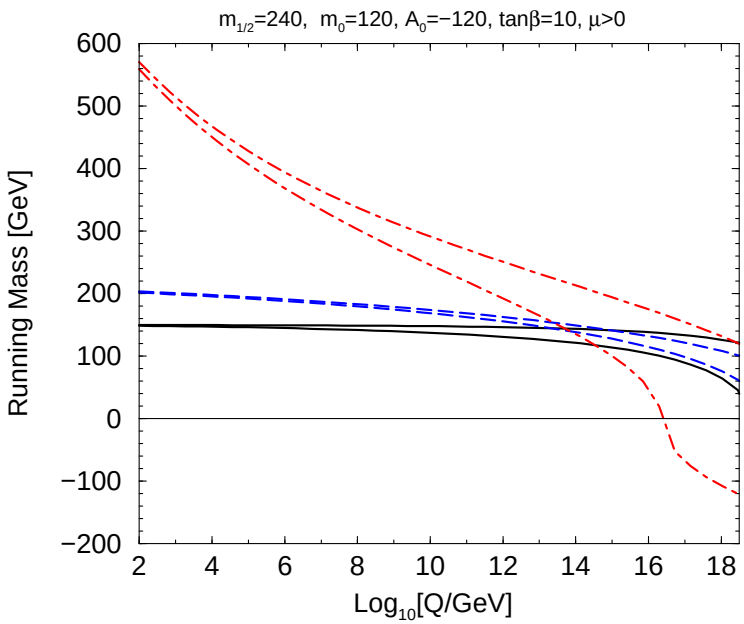
Predictions of specific models (Minimal Supergravity, Gauge Mediation, Anomaly Mediation, Extra-dimensional Mediation, ...) allow/require precise calculations.



The apparent unification of gauge couplings in the MSSM invites us to extrapolate the soft masses up to high scales, to see if they obey some organizing principle.

A case study (Snowmass 2001 P3 working group):

See also Allanach, Blair, Kraml, Martyn, Polesello, Porod, Zerwas, hep-ph/0403133, and earlier references therein, for similar work.



LH squarks, LH sleptons, RH sleptons

Assumptions:

2% uncertainty in $M_{\rm gluino}$

1% uncertainty in $M_{\rm squarks}$

0.5% uncertainty in $M_{\rm sleptons}$

1% uncertainty in α_s

0% theoretical uncertainty (!)

Goal: make the last, unreasonable, assumption as close to reasonable as possible.

TeV-scale SUSY will provide an interesting laboratory for quantum field theory:

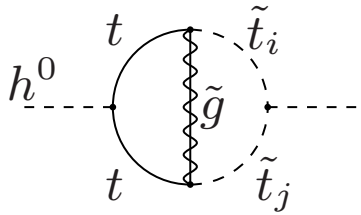
- Fundamental scalar particles
- No new dimensionless couplings in Lagrangian
- QCD coupling is perturbative at the TeV scale, but still strong enough to require multi-loop calculations
  - Corrections to Higgs masses at 1 loop go like y_t^2, at 2 loops go like $y_t^2 g_3^2$.
  - Corrections to gluino mass involve $C g_3^2$ with $C = 3$, rather than $C = 4/3$ for quarks. So, the QCD coupling for the gluino is effectively $9/4$ larger.
  - Corrections to squark, quark masses get large effects from the strongly-coupled, heavy gluino.

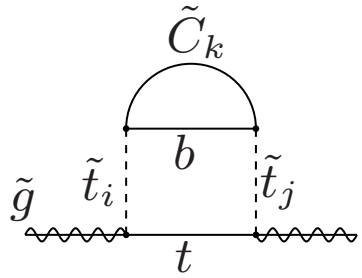
Two-loop corrections to observables will be mandatory if SUSY is correct

Another key feature of the problem: many distinct particles.

- **2-loop diagrams involve many different mass scales simultaneously.**

  For example:





  Large, diverse hierarchies of ratios of squared masses.

- **Method should be generic, reuseable from start to finish.**

  Do calculations for scalars, fermions, vectors in a general field theory. Then apply to Higgs, squarks, sleptons, and quarks, gluino, charginos, neutralinos, etc., or, ???

To calculate physical masses

Evaluate self-energy = sum of 1-particle irreducible Feynman diagrams:

$$\Pi(s) = \Pi^{(1)}(s) + \Pi^{(2)}(s) + \ldots$$

where $s =$ the external momentum invariant.

The complex pole mass

$$s_{\rm pole} = M^2 - i\Gamma M$$

is the solution for complex s of:

$$\begin{aligned} s_{\rm pole} &= m^2_{\rm tree} + \Pi(s_{\rm pole}) \\ &= m^2_{\rm tree} + \Pi^{(1)}(m^2_{\rm tree})\left[1 + \Pi^{(1)\prime}(m^2_{\rm tree})\right] + \Pi^{(2)}(m^2_{\rm tree}) + \ldots \end{aligned}$$

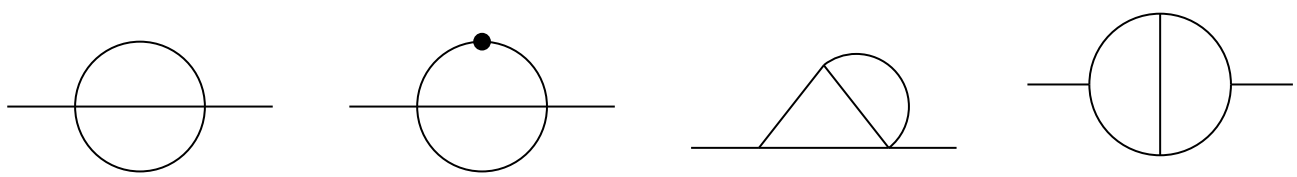
The pole mass is gauge invariant at each order in perturbation theory, can be related to kinematic masses as measured at colliders.

There are a finite number of 2-loop, two-point Feynman diagrams. Why not just do them once, for a general theory, and get it over with?

Method:

- Reduce all self-energies in general theory to a few basis integrals
- Basis integrals contain $\overline{\text{DR}}$ (or $\overline{\text{MS}}$) counter- terms, so finite.
- Numerically evaluate basis integrals quickly and reliably for arbitrary masses.

Tarasov's basis and recurrence relations:



S T U M ("Master integral")

Can always reduce 2-loop self-energies to a linear combination of these, with coefficients rational functions of:

$s = p^2 =$ external momentum invariant

$x, y, z, \ldots$ = internal propagator masses

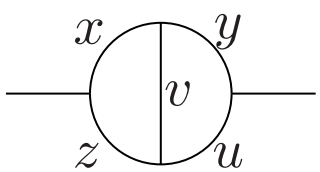
To evaluate basis integrals:

Values at $s = 0$ are known analytically, in terms of logs, polylogs.

$$\frac{\partial}{\partial s}\text{(basis integral)} = \text{(another self-energy integral)}$$
$$= \text{(linear combination of basis integrals)}$$

So, we have a set of coupled, first-order, linear differential equations.

Consider the Master integral $M(x, y, z, u, v)$:



and the basis integrals obtained from it by removing propagators:

$$U(x, z, u, v),\ U(y, u, z, v),\ U(z, x, y, v),\ U(u, y, x, v),$$
$$S(x, u, v),\ T(x, u, v),\ T(u, x, v),\ T(v, x, u),$$
$$S(y, z, v),\ T(y, z, v),\ T(z, y, v),\ T(v, y, z)$$

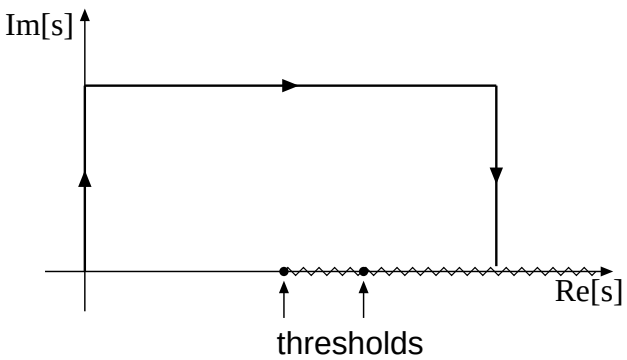
Call these 13 integrals I_n, $(n = 1, \ldots, 13)$.

Differential equations method for basis integrals

$$\frac{d}{ds} I_n = \sum_m K_{nm} I_m + C_n$$

Here K_{nm} are rational functions of s and $x, y, z \ldots$, and C_n are one-loop integrals. These are obtained by using Tarasov's recursion relations.

Solve for basis integrals I_n using Runge-Kutta integration in the complex s-plane, starting from known values at $s = 0$.



Method implemented for S, T, U type integrals by Caffo, Czyz, Laporta, Remiddi.

Dave Robertson and I have extended the method to also work for M:

TSIL = **T**wo-Loop **S**elf-energy **I**ntegral **L**ibrary

D.G. Robertson, SPM, `hep-ph/0501132`

(based in part on SPM, `hep-ph/0307101`)

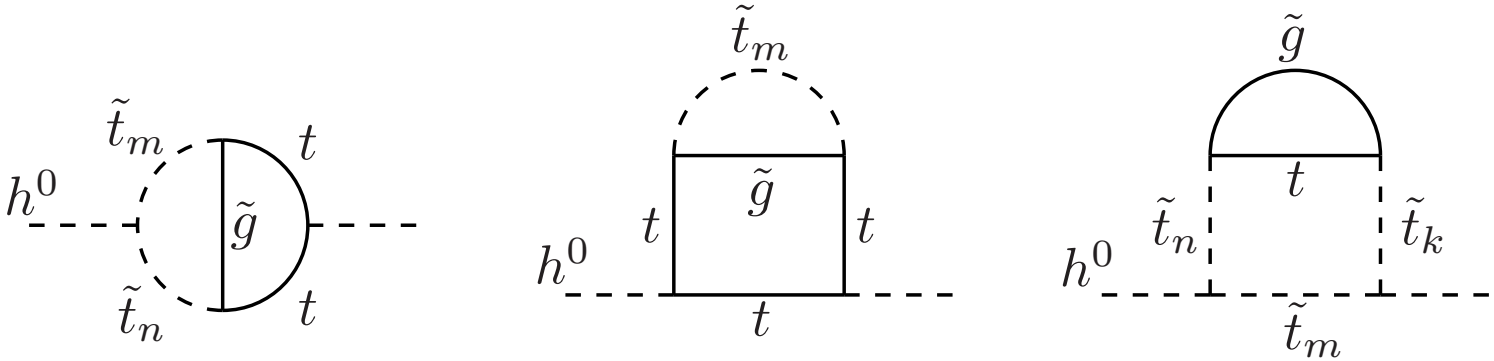
Program written in C, callable from C++, Fortran

Advantages of the method:

- Basis integrals computed for any values of all masses and s.
- All integrals from a given master integral obtained simultaneously in a single numerical computation.
- Checks on the numerical accuracy follow from changing choice of contour.
- Computation times generically $\ll$ 1 second on modern hardware.

In the Hopi culture native to the American southwest, Tsil is the Chili Pepper Kachina. The Kachina are supernatural spirits, represented by masked figurines and impersonated by ceremonial dancers. They communicate between the tribe and their gods, who live in the San Francisco mountains and are never seen directly.

TSIL usage example: Corrections to Higgs pole mass in SUSY



All of the necessary one-loop and two-loop basis integrals for these diagram topologies can be computed in one fell swoop with:

```
for (i=0; i<2; i++) {
 for (j=0; j<i; j++) {
  TSIL_SetParameters (&(result[i][j]),mstop2[i],mt2,mstop2[j],mt2,mgluino2,qq);
  TSIL_Evaluate (&(result[i][j]),s);
 }
}
 ...
 value1 = TSIL_GetFunction(&(result[0][0]), "M");
 value2 = TSIL_GetFunction(&(result[1][0]), "Vzxyv");
```

etc.

Applications to 2-loop scalar self-energies:

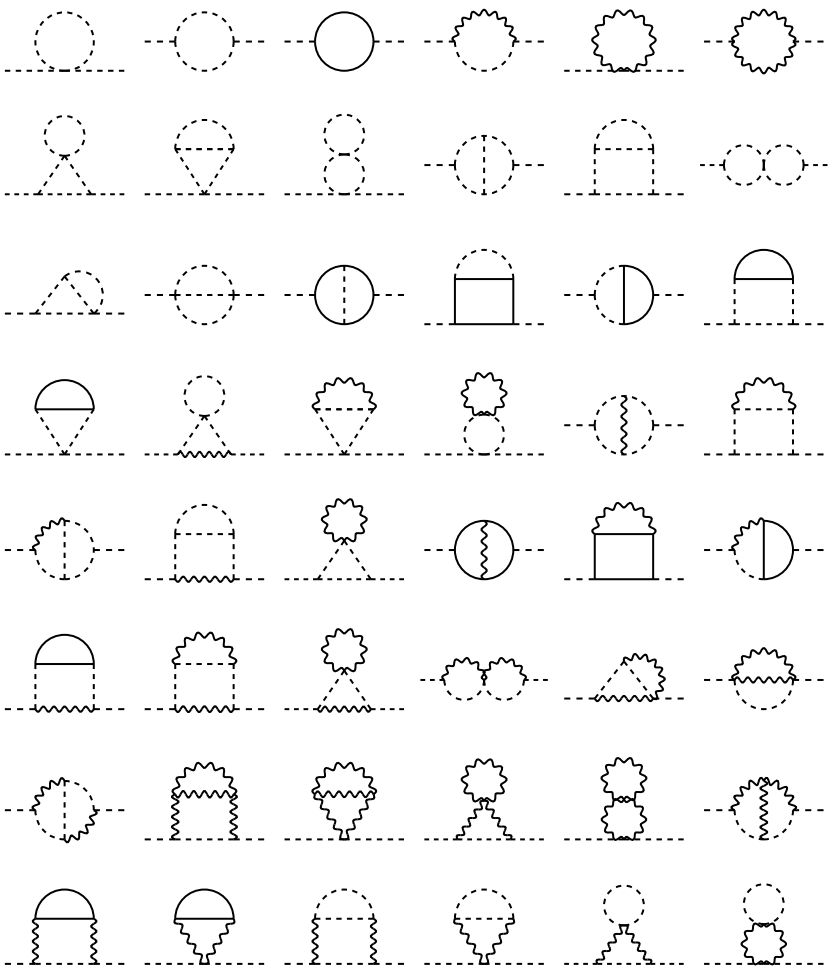
- General renormalizable theory, at leading order in gauge couplings [hep-ph/0312092]
  - Corrections to h^0, H^0, A^0, $H^\pm$ pole masses in the MSSM, including all terms involving SUSYQCD couplings (e.g. $\alpha_s y_t^2$ and $\alpha_s g^2$) , and those that do not vanish as the electroweak gauge couplings are turned off (e.g. y_t^4 and $y_t^2 y_b^2$ and $a_t^2 y_t^2$). [hep-ph/0405022]
- General renormalizable theory with unbroken gauge symmetries [hep-ph/0502168]
  - 2-loop SUSYQCD corrections to squark pole masses (this talk)
  - 2-loop corrections to sfermion pole masses, neglecting only 2-loop effects suppressed by $(m_Z/m_{\text{SUSY}})^4$

The Feynman diagrams

Each diagram is reduced to a linear combination of basis integrals, computed analytically in terms of polylogarithms when possible, otherwise computed numerically using TSIL.

Used $\overline{\mathrm{DR}}$ and $\overline{\mathrm{MS}}$ schemes, general covariant gauge. Vector masses neglected in the 2-loop diagrams with more than one vector propagator.

Various checks made on the calculation . . .



+ fermion mass insertions + ghosts

+ counterterms

Checks on the calculation of scalar pole masses:

- Independent of gauge-fixing parameter

  Individual diagrams depend on ξ; cancels in pole mass

  ($\xi = 0, 1, 3$ in Landau, Feynman, Fried-Yennie gauges)

- Pole mass is renormalization group invariant

  Checked analytically at 2-loop order; numerical check below

- Absence of divergent logs on shell

  Individual diagrams have $\log(1 - p^2/m^2)$, divergent as $p^2 \to m^2$;

  must and do cancel in pole mass

- Independent of unphysical epsilon-scalar masses in SUSY-preserving $\overline{\text{DR}}'$ scheme of I. Jack et al (1993).

SUSYQCD corrections to squark masses in MSSM

Obtained by specializing the general result. Not all diagrams contribute, many terms combine because of (softly broken) supersymmetry.

Example: In the special case of degenerate running masses, $m_{\tilde{Q}} = m_{\tilde{g}} = Q$, the result for the pole mass simplifies:

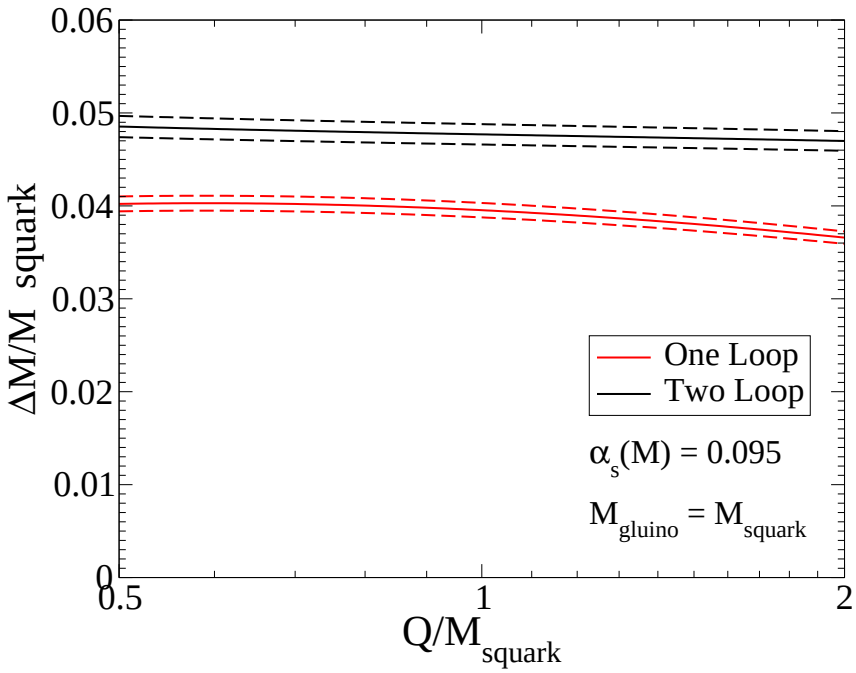
$$M^2_{\tilde{Q}} = m^2_{\tilde{Q}} \left[1 + \frac{\alpha_s}{4\pi}\left(\frac{32}{3}\right) + \left(\frac{\alpha_s}{4\pi}\right)^2 \left\{ \frac{112}{3} + \frac{664\pi^2}{27} + \frac{32\pi^2 \ln 2}{9} - \frac{16\zeta(3)}{3} \right\} \right]$$

$$= m^2_{\tilde{Q}} \left[1 + 0.849\,\alpha_s + 1.89\,\alpha_s^2 \right]$$

There are no large logs here (only one mass scale!), so this illustrates the intrinsic size of typical SUSYQCD 1-loop (~ 4%) and 2-loop (~ 1%) corrections to the squark masses.

Could reasonably guess 3-loop corrections to be of order 0.3% ?

Renormalization scale (Q) dependence of calculated squark pole mass



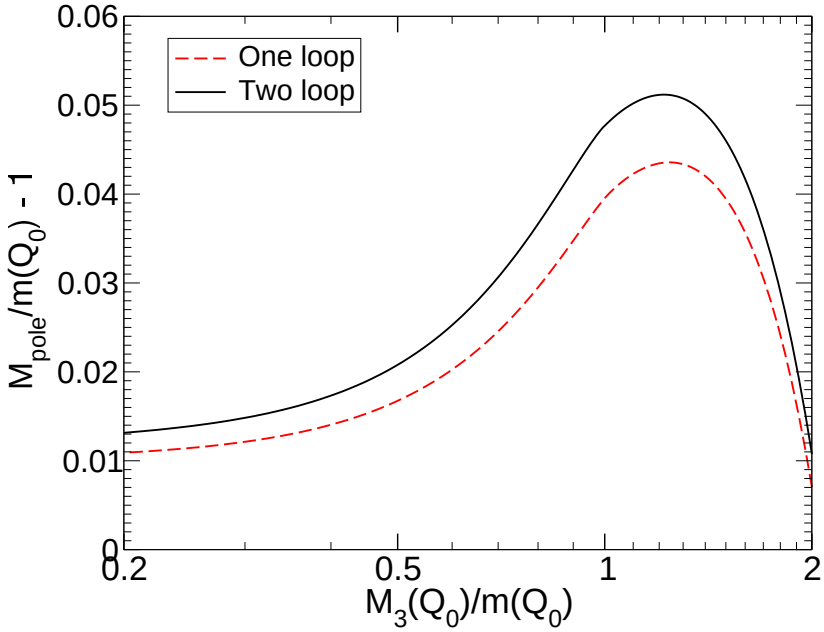
Squark mixing, quark masses, and electroweak effects neglected; all squarks taken degenerate with each other and gluino at tree level.

Dashed lines are $\pm 2\%$ variation of α_s.

Remaining scale dependence (from 3 loops and beyond) is small.

However, 2-loop correction is much larger than 1-loop scale dependence.

Dependence of squark mass correction on the gluino mass



A large part of the squark mass correction is due to the gluino mass.

In realistic models, effects due to variation in squark masses, top and bottom Yukawa effects, electroweak effects are significant, too.

For $M_{\text{gluino}} \gtrsim 2M_{\text{squark}}$, get large negative radiative corrections to squark masses. Resummation of large logarithms may then be necessary.

(This does not usually happen for squarks of the first two families in realistic SUSY-breaking models.)

A non-realistic special case: the supersymmetric limit

Consider N_f flavors of quarks and antiquarks, each degenerate with their squark superpartners, with running $\overline{\text{DR}}$ masses m_i. Then the pole masses are:

$$M_i^2 = m_i^2 \left[1 + \frac{g_3^2}{16\pi^2} C_q \widetilde{\Pi}_i^{(1)} + \frac{g_3^4}{(16\pi^2)^2} C_q \widetilde{\Pi}_i^{(2)} \right]$$

where, with $C_q{=}4/3$, $C_G{=}3$, $I_q{=}1/2$ for $SU(3)_c$:

$$\begin{aligned}
\widetilde{\Pi}_i^{(1)} &= 8 - 4\overline{\ln}(m_i^2) \\
\widetilde{\Pi}_i^{(2)} &= C_q \left[20\zeta(2) - 16\pi^2 \ln 2 + 24\zeta(3) - 28 - 8\overline{\ln}(m_i^2) + 8\overline{\ln}^2(m_i^2) \right] \\
&\quad + C_G \left[66 - 24\zeta(2) + 8\pi^2 \ln 2 - 12\zeta(3) - 36\overline{\ln}(m_i^2) + 6\overline{\ln}^2(m_i^2) \right] \\
&\quad + I_q \sum_{j=1}^{N_f} h(m_j^2/m_i^2),
\end{aligned}$$

with $\overline{\ln}(x) \equiv \ln(x/Q^2)$, and $h(x)$ given in terms of dilogarithms.

This will provide a strong check on a future 2-loop fermion pole mass calculation.

Status for sfermion pole masses

The 2-loop SUSYQCD (α_s^2) corrections to squark masses (including non-degeneracy and mixing effects) are given explicitly in `hep-ph/0502168`.

All other 2-loop sfermion pole masses now given "implicitly" in `hep-ph/0312092`, `hep-ph/0502168`. This means that the results are known for a general theory in terms of Lagrangian running couplings and masses. To evaluate in practice, one must do the tedious, but purely algebraic, exercise of specializing to the MSSM case. (A symbolic manipulation program like Mathematica or FORM can be taught to do this.)

Vector boson masses are included only in the 1-loop diagrams and the 2-loop diagrams with only one vector line. Therefore:

$$\text{(fractional error in 2-loop contribution to pole mass}^2\text{)} \quad \sim \quad N \left(\frac{M_Z}{M_{\text{SUSY}}} \right)^2$$

where N is a dimensionless expansion coefficient. (Experience shows that N is typically less than 1.)

Outlook

- Two-loop calculations for self-energies in the MSSM are necessary, possible
- I favor a Strategy based on:
  - $\overline{\text{DR}}'$ scheme (complementary to on-shell scheme results)
  - Reusable, generic calculations
  - Efficient computations of basis two-loop integrals
- 2-loop SUSYQCD corrections to squarks now known, typically $\lesssim 1\%$
- 2-loop sfermion pole masses are implicitly known
- Some 3-loop calculations (e.g. for h^0, maybe for gluino, squarks) will eventually be necessary to compete with measurement accuracy from a Linear Collider
- Progress continues!