Dominant Two-Loop Electroweak Correction to $H \to \gamma\gamma^*$

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[*] F. Fugel, B.A. Kniehl, M. Steinhauser, Nucl. Phys. **B702** (2004) 333.

Contents

1 Introduction

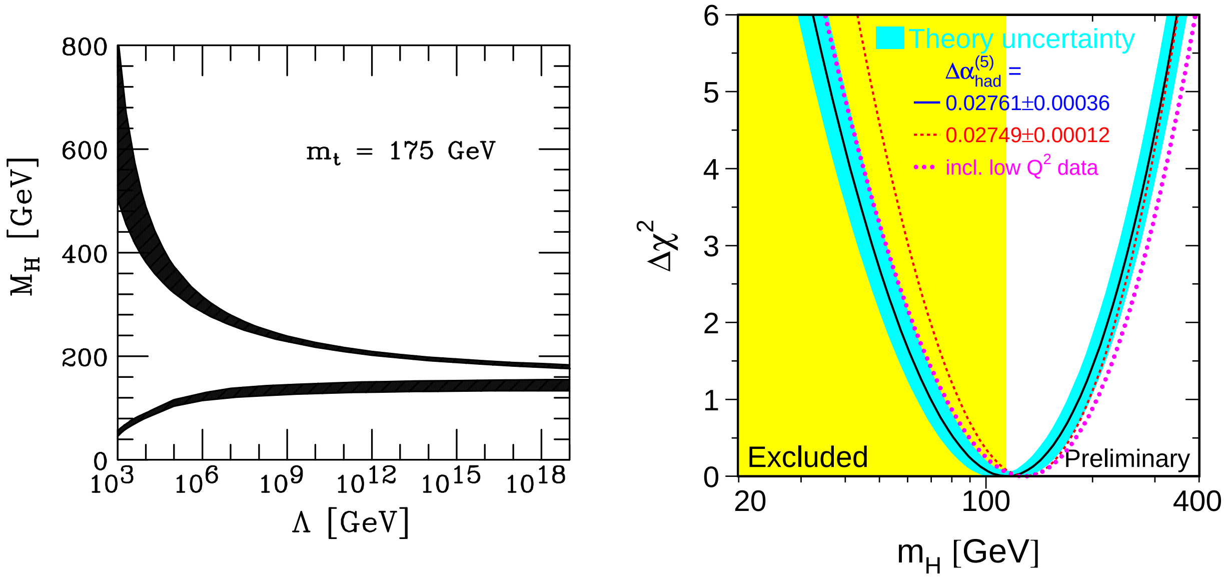

Figure 1: Left: Triviality and vacuum-stability bounds on M_H; right: $\Delta\chi^2 = \chi^2 - \chi^2_{\rm min}$ as a function of M_H.

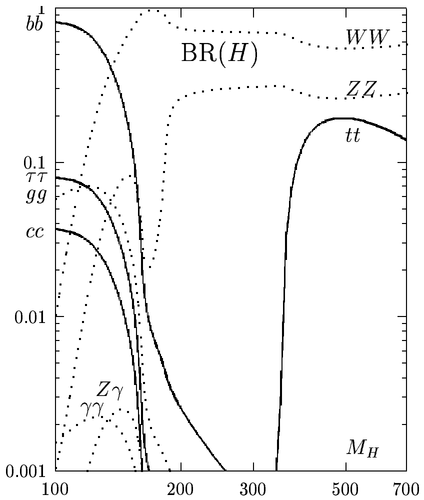

Figure 2: SM Higgs decay branching fractions.

- SM Higgs has intermediate mass: $M_H = 114^{+69}_{-45}$ (EWWG).
- $B(H \to \gamma\gamma) \lesssim 0.3\%$ in this M_H range.
- $H\gamma\gamma$ coupling sensitive to new charged heavy particles.
- At ILC, $H \to \gamma\gamma$ has clear signal.
- At photon collider, $\sigma(\gamma\gamma \to H) \propto \Gamma(H \to \gamma\gamma)$.
- At LHC, $H \to \gamma\gamma$ important discovery mode.

$\rightsquigarrow$ Precise knowledge of $\Gamma(H \to \gamma\gamma)$ required for $M_W < M_H < 2M_W$.

2 One-Loop Result

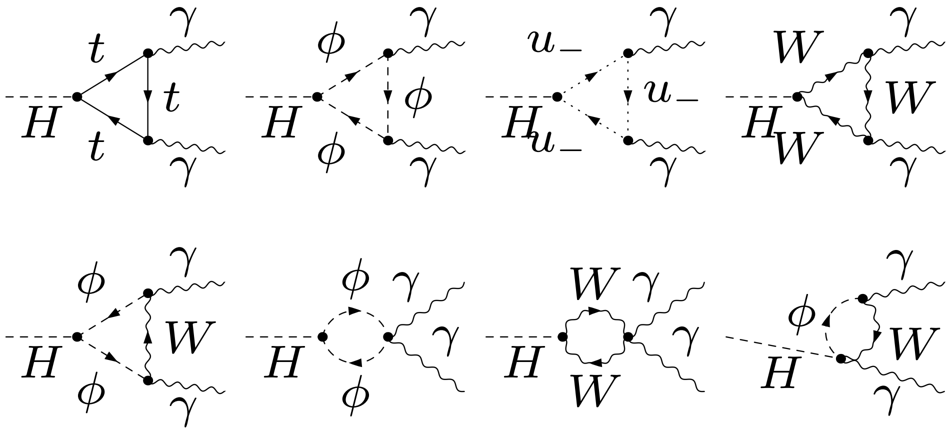

Figure 3: Sample diagrams for $\Gamma(H \to \gamma\gamma)$ at one loop.

Amplitude:

$$\mathcal{T}^{\mu\nu} = (q_1 \cdot q_2\, g^{\mu\nu} - q_1^\nu q_2^\mu)\mathcal{A},$$
$$\mathcal{A} = \mathcal{A}_t^{(0)} + \mathcal{A}_W^{(0)} + \mathcal{A}_{tW}^{(1)} + \cdots$$

Partial decay width:

$$\Gamma(H \to \gamma\gamma) = \frac{M_H^3}{64\pi}|\mathcal{A}|^2.$$

J.R. Ellis, M.K. Gaillard, D.V. Nanopoulos, Nucl. Phys. B 106 (1976) 292;
B.L. Ioffe, V.A. Khoze, Sov. J. Part. Nucl. 9 (1978) 50.

Method

- Put $M_b = 0$ and $V_{tb} = 1$.
- Exploit (formal) hierarchies $M_H^2 = 2q_1{\cdot}q_2 \ll 4M_W^2 \ll M_t^2$.
  $\rightsquigarrow$ Use asymptotic expansion.
- Find leading large-M_t term and its expansion in powers of $\tau_W = (M_H/2M_W)^2$.
- Automatize calculation:
  - `QGRAF` Nogueira: generates diagrams
  - q2e Seidensticker: converts output
  - exp Seidensticker: performs asymptotic expansion and generates relevant subdiagrams according to hard-mass procedure
  - `MATAD` Steinhauser: calculates diagrams
- Use dimensional regularization.
- Use on-mass-shell renormalization scheme.
- Treat tadpoles properly.
- Perform checks:
  - Compute coefficients of $q_1 \cdot q_2\, g^{\mu\nu}$ and $q_1^\nu q_2^\mu$ separately.
  - Work in R_ξ gauge.
  - Verify UV cancellations.
  - Verify cancellation of M_t^4 terms from asymptotic expansion, genuine two-loop tadpoles, and counterterms.
  - Compare with known result for $\Gamma(H \to gg)$ involving only Higgs (H) and Goldstone (χ^0, $\phi^\pm$) bosons.
  - Check convergence properties of τ_W expansions.

t Loops

$$\mathcal{A}_t^{(0)} = \hat{A} N_c Q_t^2 \left\{ \frac{1}{\tau_t} \left[1 + \left(1 - \frac{1}{\tau_t} \right) \arcsin^2 \sqrt{\tau_t} \right] \right\}$$
$$= \hat{A} N_c Q_t^2 \left(\frac{2}{3} + \frac{7}{45}\tau_t + \frac{4}{63}\tau_t^2 + \frac{52}{1575}\tau_t^3 + \frac{1024}{51975}\tau_t^4 + \frac{2432}{189189}\tau_t^5 + \dots \right),$$

where $\hat{\mathcal{A}} = 2^{1/4} G_F^{1/2} (\alpha/\pi)$ and $\tau_t = (M_H/2M_t)^2$.

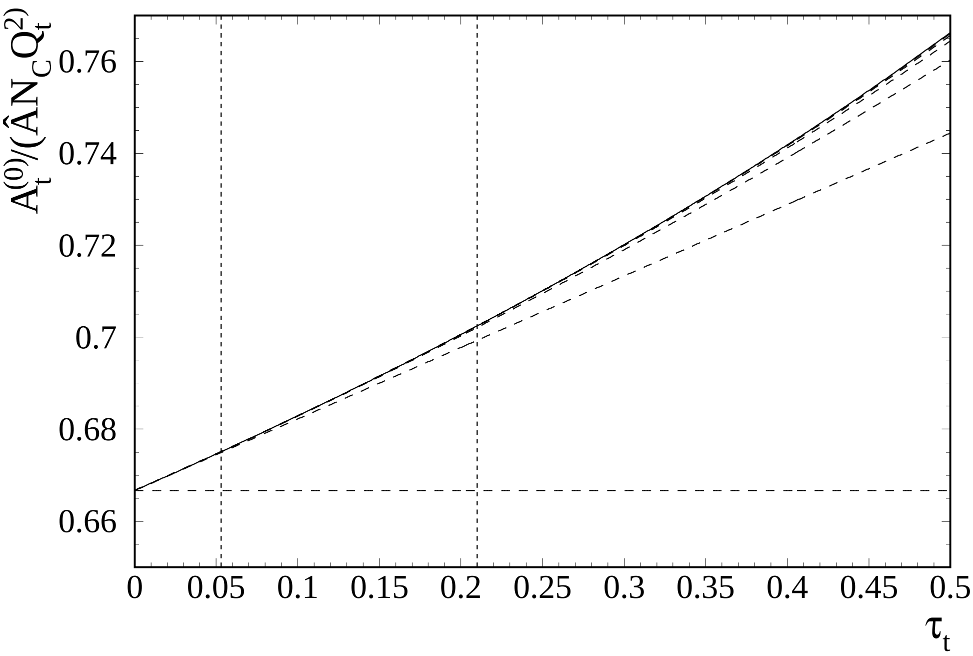

Figure 4: Convergence property of expansion of $\mathcal{A}_t^{(0)}$ in powers of τ_t.

W Loops

$$\mathcal{A}_W^{(0)} = \hat{\mathcal{A}}\left\{-\frac{1}{2}\left[2+\frac{3}{\tau_W}+\frac{3}{\tau_W}\left(2-\frac{1}{\tau_W}\right)\arcsin^2\sqrt{\tau_W}\right]\right\}$$
$$= \hat{\mathcal{A}}\left(-\frac{7}{2}-\frac{11}{15}\tau_W-\frac{38}{105}\tau_W^2-\frac{116}{525}\tau_W^3-\frac{2624}{17325}\tau_W^4 - \frac{640}{5733}\tau_W^5+\ldots\right),$$

where $\tau_W = (M_H/2M_W)^2$.

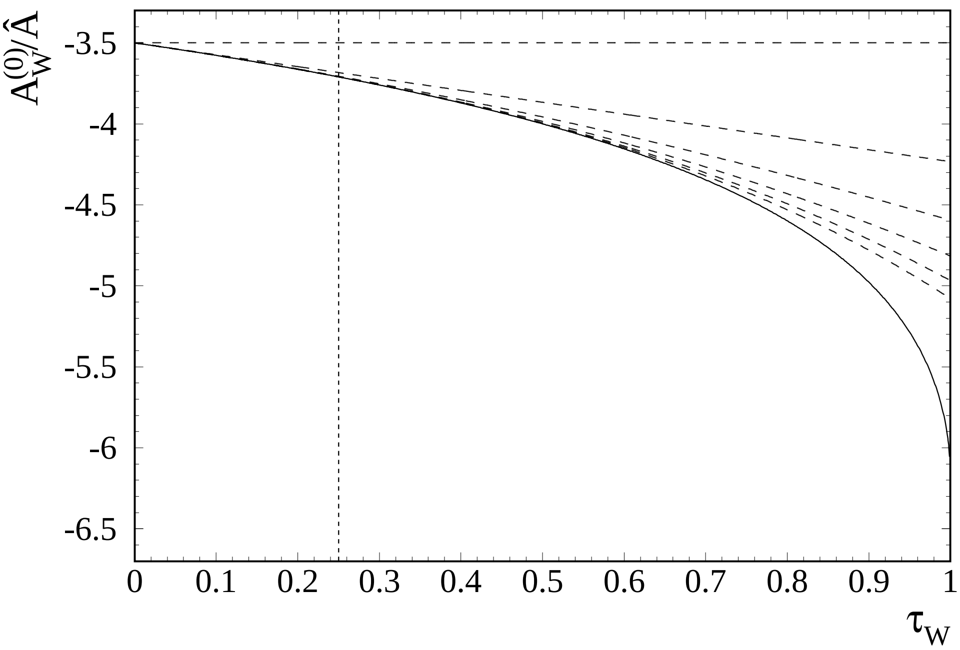

Figure 5: Convergence property of expansion of $\mathcal{A}_W^{(0)}$ in powers of τ_W.

3 $\mathcal{O}(G_F M_t^2)$ Correction

Consider all two-loop electroweak diagrams involving a virtual top quark (1690 in R_ξ gauge).
Two classes:

- t loops with virtual H or χ lines attached to them. $\rightsquigarrow$ Simple Taylor expansion in external momenta.
- t loops with virtual Z lines attached to them. $\rightsquigarrow$ Subleading (below M_t^2).
- Diagrams involving t, b, and W or ϕ. $\rightsquigarrow$ Nontrivial asymptotic expansion; M_t^4 terms occur.

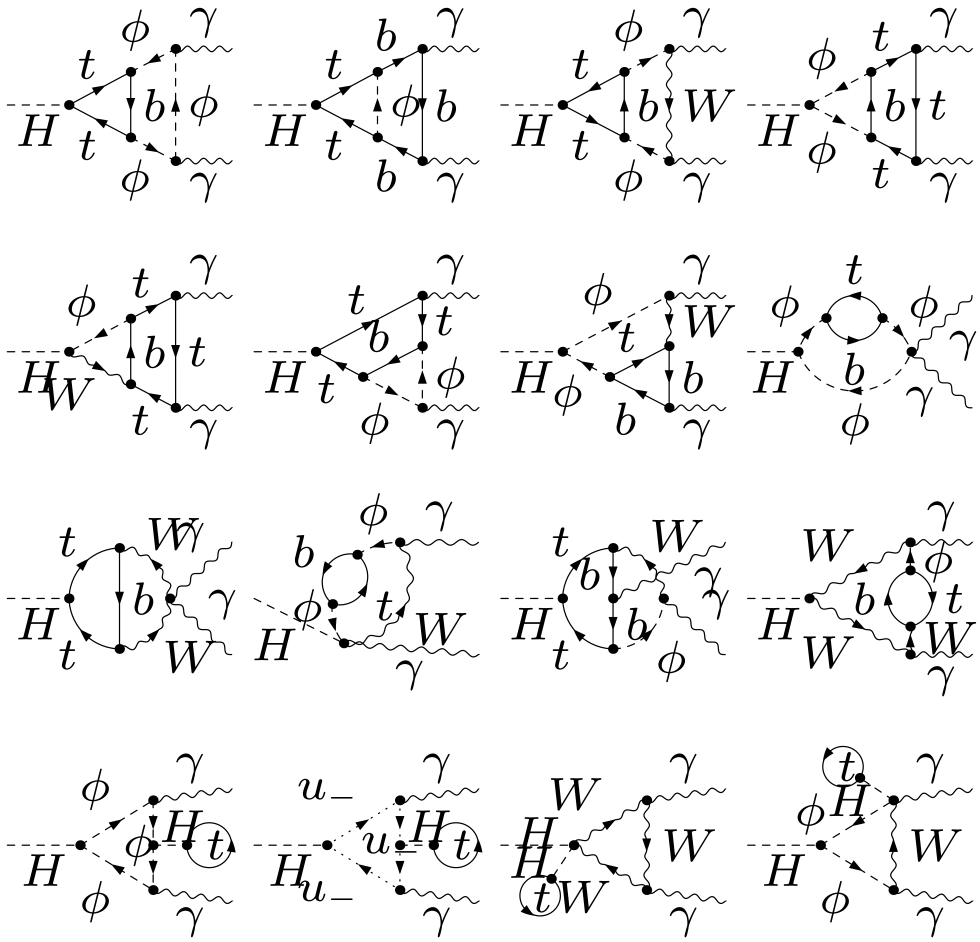

Figure 6: Sample diagrams for $\Gamma(H \to \gamma\gamma)$ at $\mathcal{O}(G_F M_t^2)$.

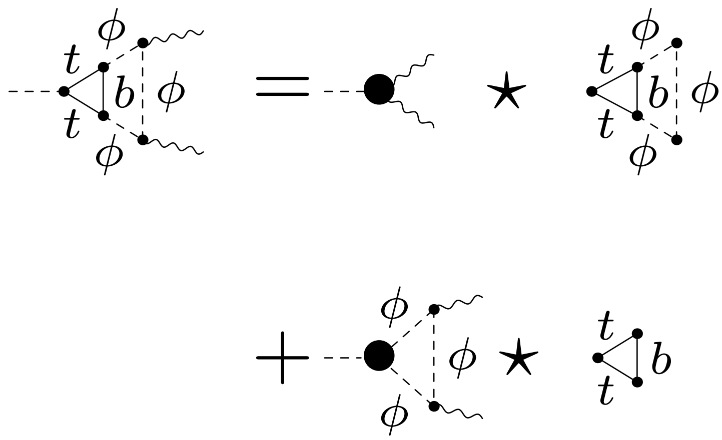

Figure 7: Diagrammatic asymptotic expansion of a diagram that produces M_t^4 terms.

R_ξ gauge for W boson:

$$
\begin{array}{rl}
(i) & M_t^2 \gg M_W^2 = \xi_W M_W^2 \gg M_H^2 \\
(ii) & M_t^2 \gg M_W^2 \gg \xi_W M_W^2 \gg M_H^2 \\
(iii) & M_t^2 \gg \xi_W M_W^2 \gg M_W^2 \gg M_H^2 \\
(iv) & \xi_W M_W^2 \gg M_t^2 \gg M_W^2 \gg M_H^2
\end{array}
$$

Final result ($x_t = G_F M_t^2/(8\pi^2\sqrt{2})$):

$$
\begin{aligned}
\mathcal{A}_{tW}^{(1)} &= \mathcal{A}_u^{(1)} + \mathcal{A}_{H,\chi}^{(1)} + \mathcal{A}_{W,\phi}^{(1)} \\
&= \hat{\mathcal{A}} N_c x_t \left(\frac{367}{108} + \frac{11}{18}\tau_W + \frac{19}{63}\tau_W^2 + \frac{58}{315}\tau_W^3 \right. \\
&\quad \left. + \frac{1312}{10395}\tau_W^4 + \cdots \right)
\end{aligned}
$$

4 Numerical Analysis

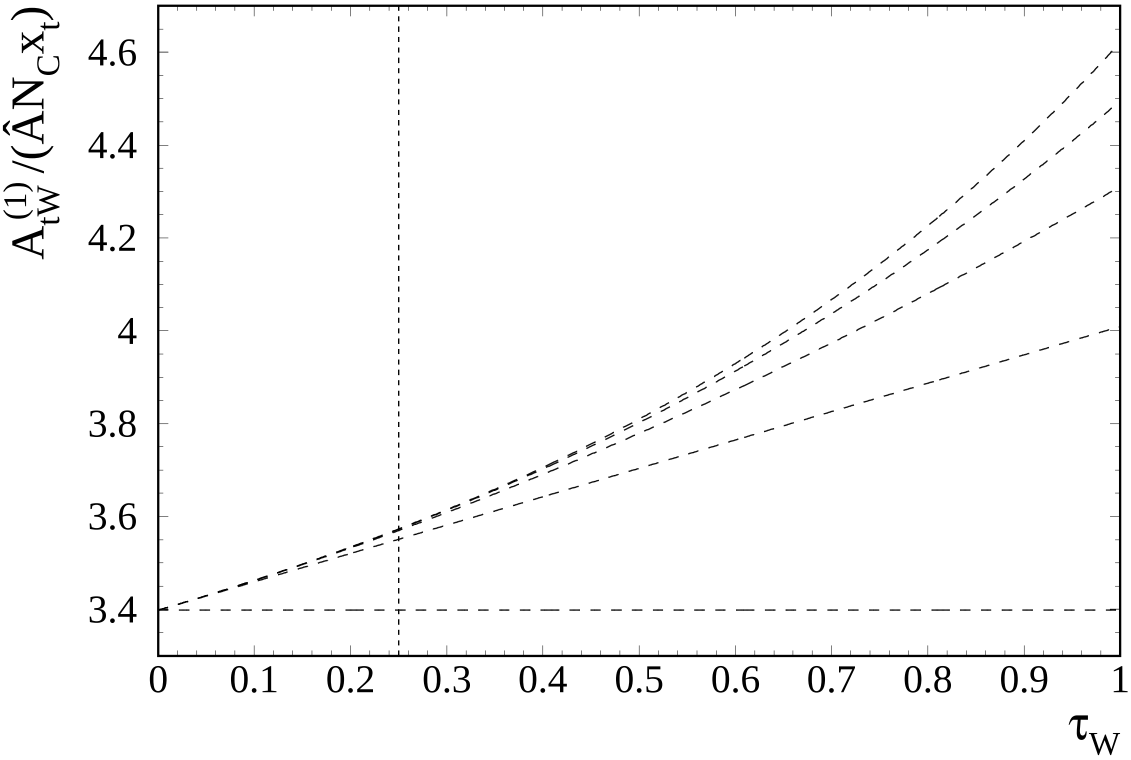

Figure 8: Convergence property of expansion of $\mathcal{A}_{tW}^{(1)}$ in powers of τ_W.

M_H [GeV]	120	140	$2M_W$
$\mathcal{A}_W^{(0)}$	0.4%	1.1%	3.1%
$\mathcal{A}_{tW}^{(1)}$	0.3%	1.0%	2.8%

Table 1: Relative deviation of best approximation from second best one.

Comparison With $\mathcal{O}(\alpha_s)$ Correction

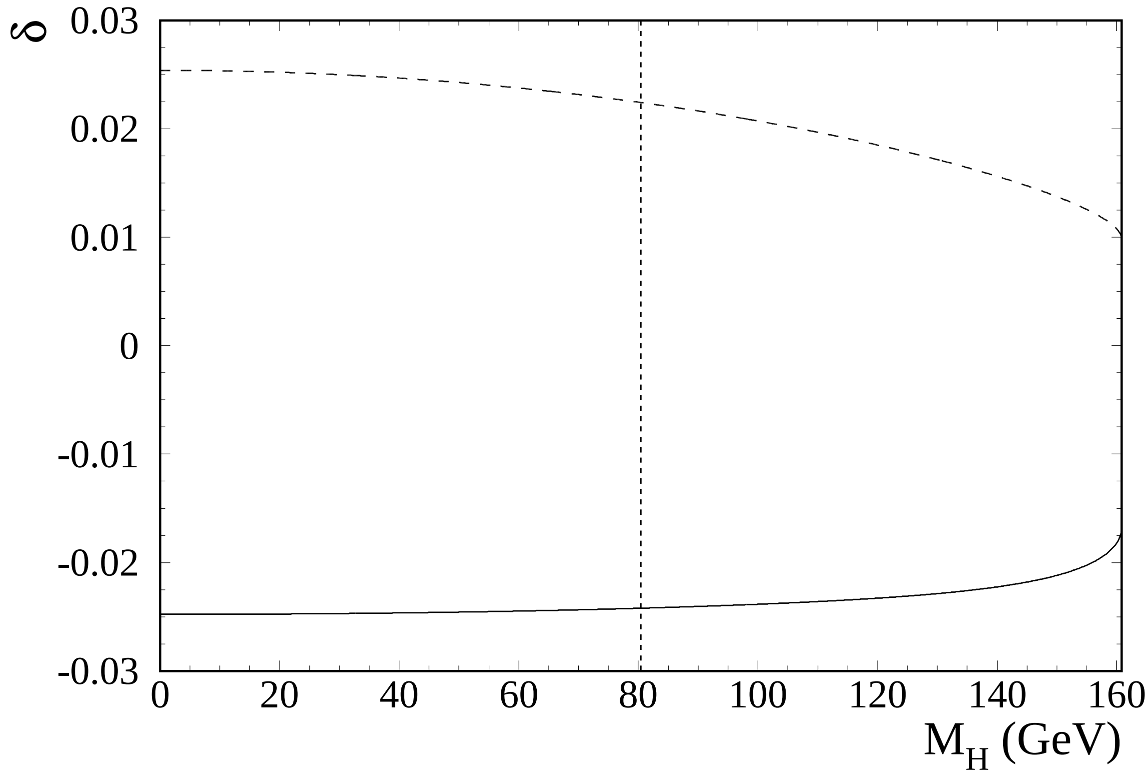

Figure 9: $\mathcal{O}(G_F M_t^2)$ (solid) and $\mathcal{O}(\alpha_s)$ (dashed) corrections to $\Gamma(H \longrightarrow \gamma\gamma)$.

$\mathcal{O}(\alpha_s)$ gluon correction:

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A. Djouadi, M. Spira, J.J. van der Bij, P.M. Zerwas, Phys. Lett. B 257 (1991) 187;

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M. Inoue, R. Najima, T. Oka, J. Saito, Mod. Phys. Lett. A 9 (1994) 1189;
J. Fleischer, O.V. Tarasov, Z. Phys. C 64 (1994) 413;
J. Fleischer, O.V. Tarasov, V.O. Tarasov, Phys. Lett. B 584 (2004) 294.

$\mathcal{O}(n_f G_F M_W^2)$ light-fermion correction: approx. -2% – -1%
U. Aglietti, R. Bonciani, G. Degrassi, A. Vicini, Phys. Lett. **B595** (2004) 432.

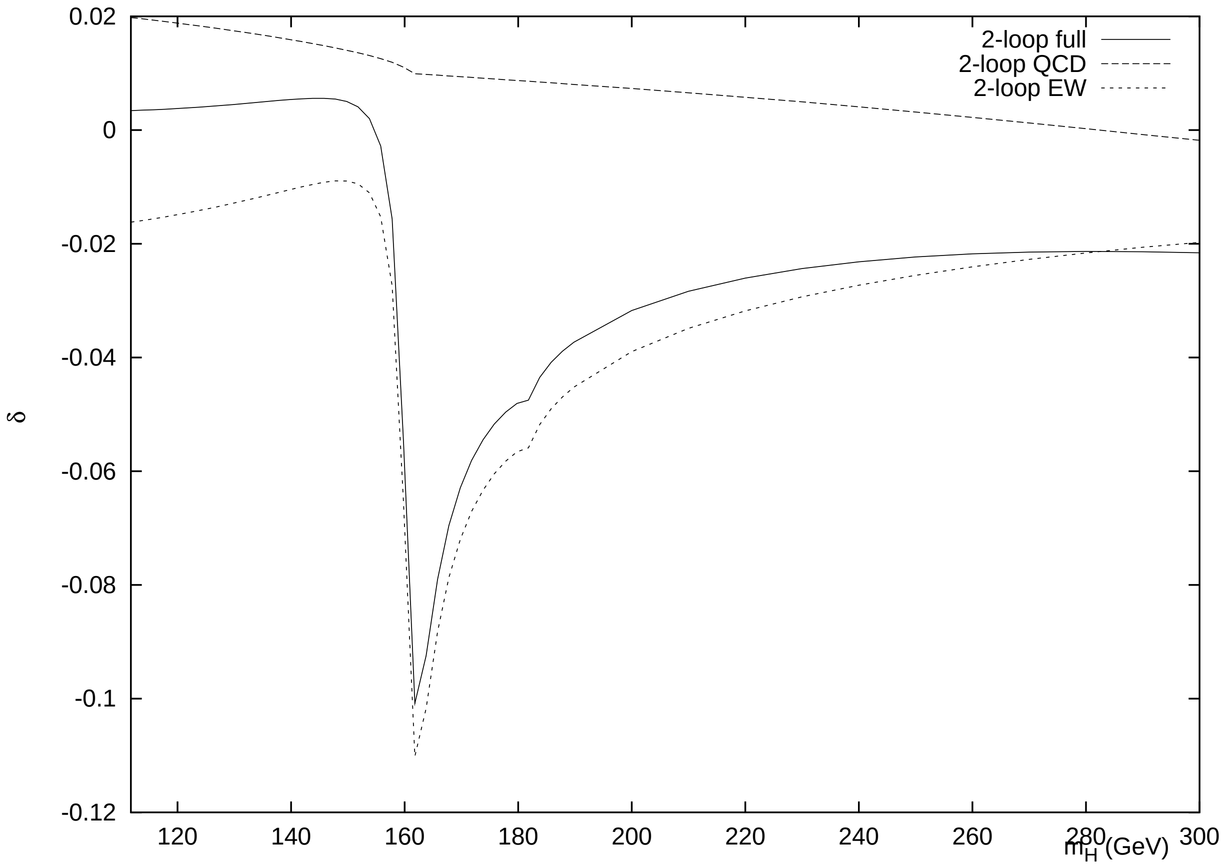

Figure 10: $\mathcal{O}(n_f G_F M_W^2)$ (dotted) and $\mathcal{O}(\alpha_s)$ (dashed) corrections to $\Gamma(H \to \gamma\gamma)$.

5 Conclusions

- Dominant two-loop electroweak $\mathcal{O}(G_F M_t^2)$ correction to $\Gamma(H \to \gamma\gamma)$ for $M_W \lesssim M_H \lesssim 2M_W$ available as expansion in $\tau_W = (M_H/2M_W)^2$ through $\mathcal{O}(\tau_W^4)$.
- Reduction by approx. -2.5% – -2%.
- Positive QCD correction slightly overcompensated.
- Net effect of known corrections -2% – -1%.