# Electroweak Correction for the Study of Higgs Potential in LC

*LAPTH-Minamitateya Collaboration*

LCWS05

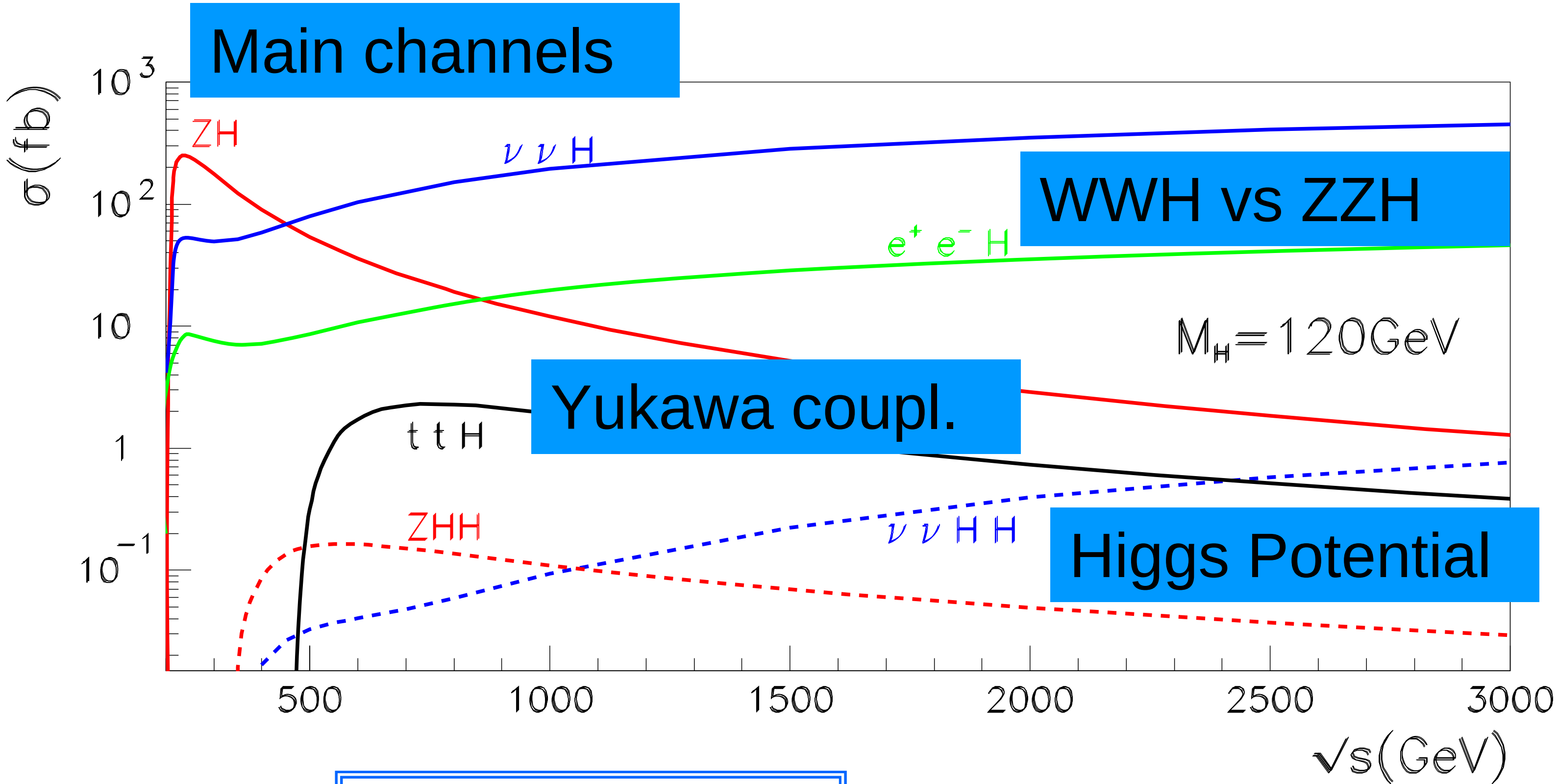
2005.3.19, Stanford U.

presented by K.Kato(Kogakuin U.)

# Introduction

- LC : high-precision experiments
  → Requires the same level theoretical prediction = Needs higher order calculation
- Higgs study : Central target
  → Interesting channels = multi-body final states
- 1-loop correction to $e^+e^- \rightarrow$ 3, 4-bodies : Great Progress since Sept. 2002

# Higgs@LC:tree cross sections




**2→3,4 processes**
**important**

# number of diagrams (GRACE:NLG model)

A.Denner, J.Kublbeck, R.Mertig, M.Bohm, Z.Phys.C56(1992) 261;
B.A.Kniehl, Z.Phys.C55(1992) 605.

| $e^+e^- \to$ | tree | 1-loop |
| --- | --- | --- |
| $ZH$ | 4(1) | 341(119) |

10 years required to develop 2→3 tools

Automated Systems

with(without) e-scalar couplings

*Full 1-loop RC available*

$e^+e^- \to \nu\bar{\nu}H$

GRACE, PLB559(2003)252
Denner et al., NPB660(2003)289

$e^+e^- \to t\bar{t}H$

GRACE, PLB571(2003)163
You et al., PLB571(2003)85
Denner et al., PLB575(2003)290

$e^+e^- \to ZHH$

GRACE, PLB576(2003)152
Zhang et al., PLB578(2004)349

$e^+e^- \to e^+e^-H$

GRACE, PLB600(2004)65

$e^+e^- \to \nu\bar{\nu}\gamma$ $\quad \nu = \nu_\mu, \nu_e$

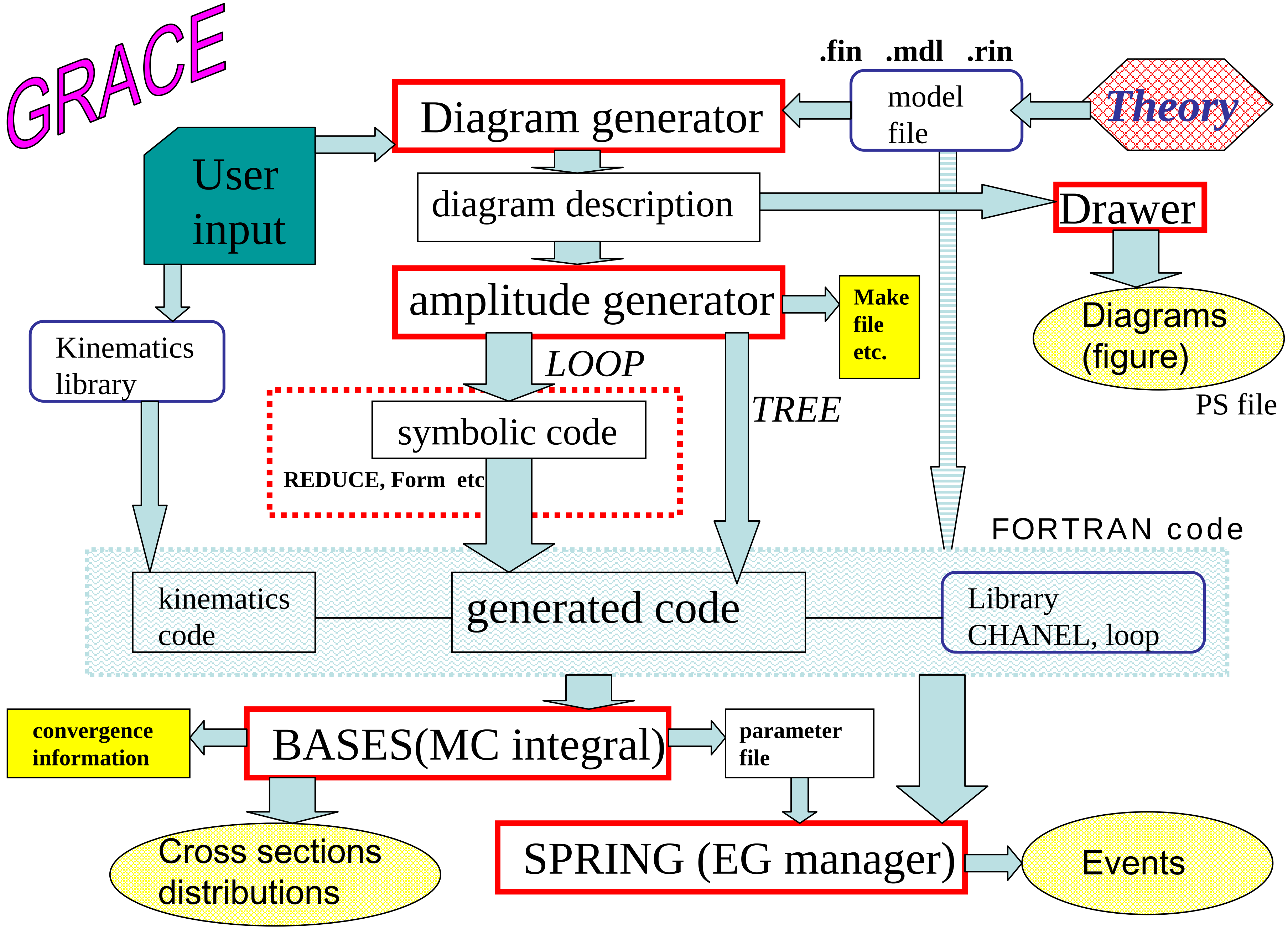
GRACE, NIM A534(2004)334

$e^+e^- \to \nu\bar{\nu}HH$

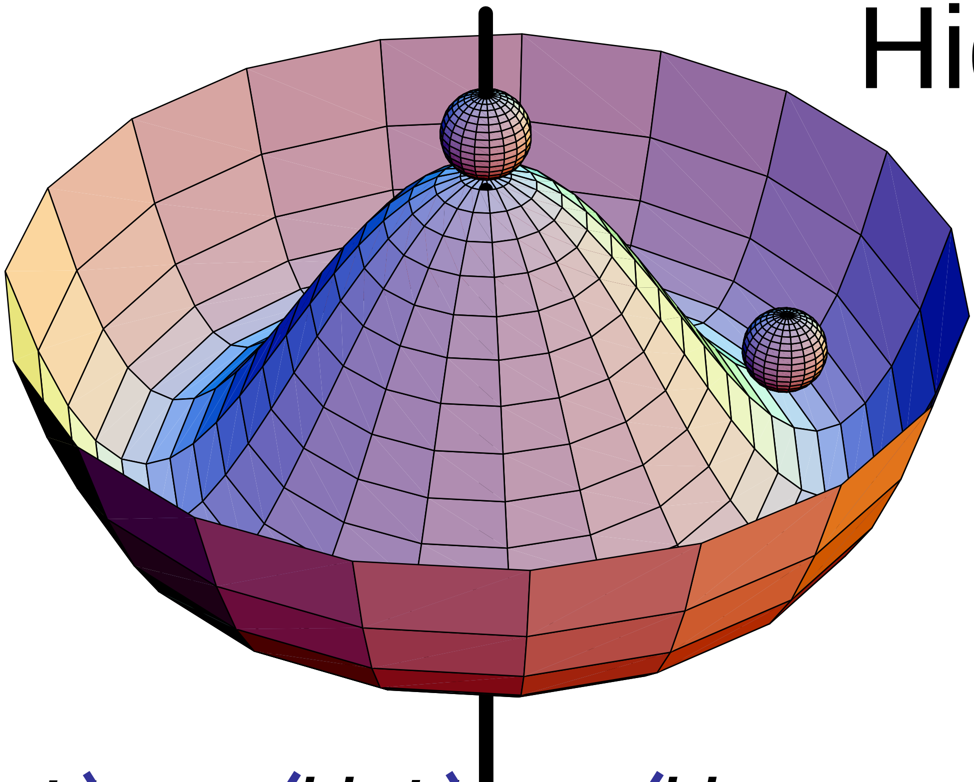
GRACE, Talk by Y.Yasui at Durham(Sep.2004)

$e^+e^- \to \nu_\tau \tau^+ \mu^- \bar{\nu}_\mu,\ u\bar{d}s\bar{c}$

Denner etal., hep-ph/0502063


GRACE
.fin .mdl .rin
Diagram generator
model file
Theory
User input
diagram description
Drawer
amplitude generator
Make file etc.
Diagrams (figure)
PS file
Kinematics library
LOOP
TREE
symbolic code
REDUCE, Form etc
FORTRAN code
kinematics code
generated code
Library CHANEL, loop
convergence information
BASES(MC integral)
parameter file
Cross sections distributions
SPRING (EG manager)
Events

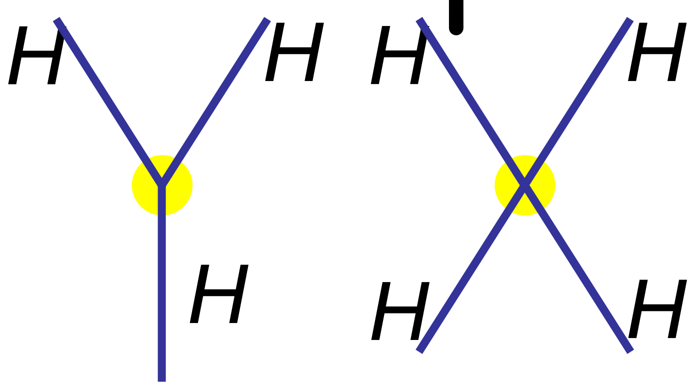
# Higgs Potential



*Something required to provide mass*

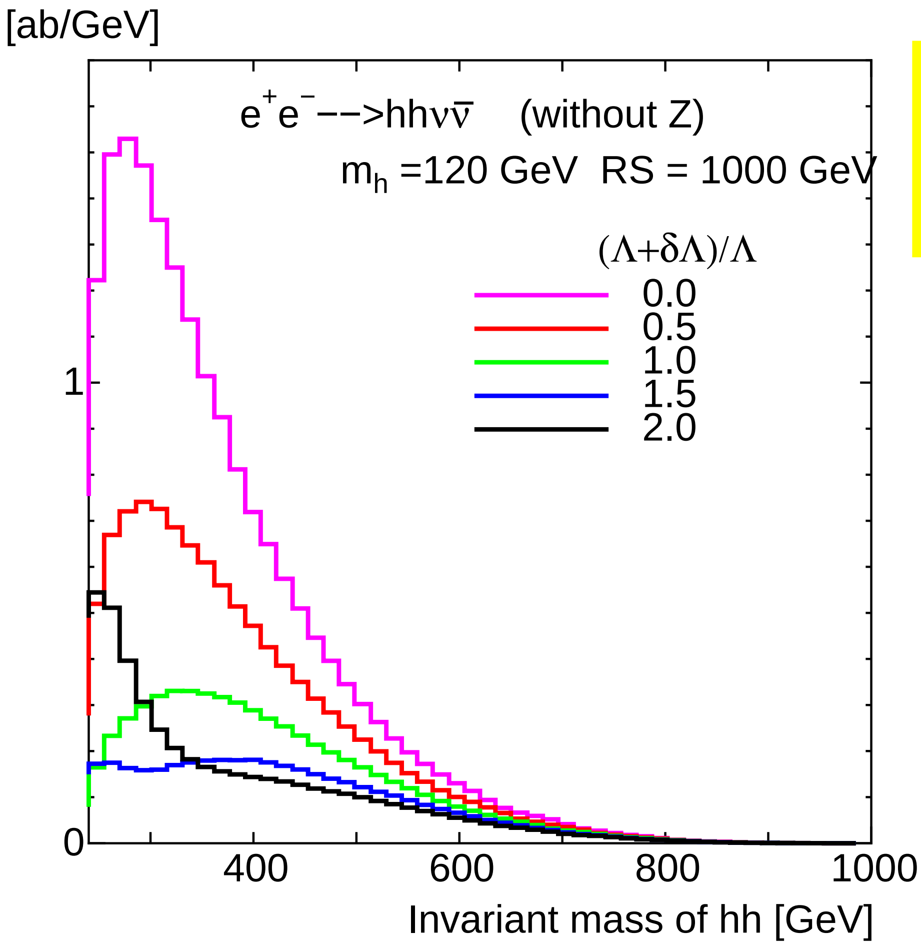
*Key for the structure of standard model*

*Window to New Physics*




Double (triple) Higgs production Process

# $e^+e^- \rightarrow \nu\nu H H$

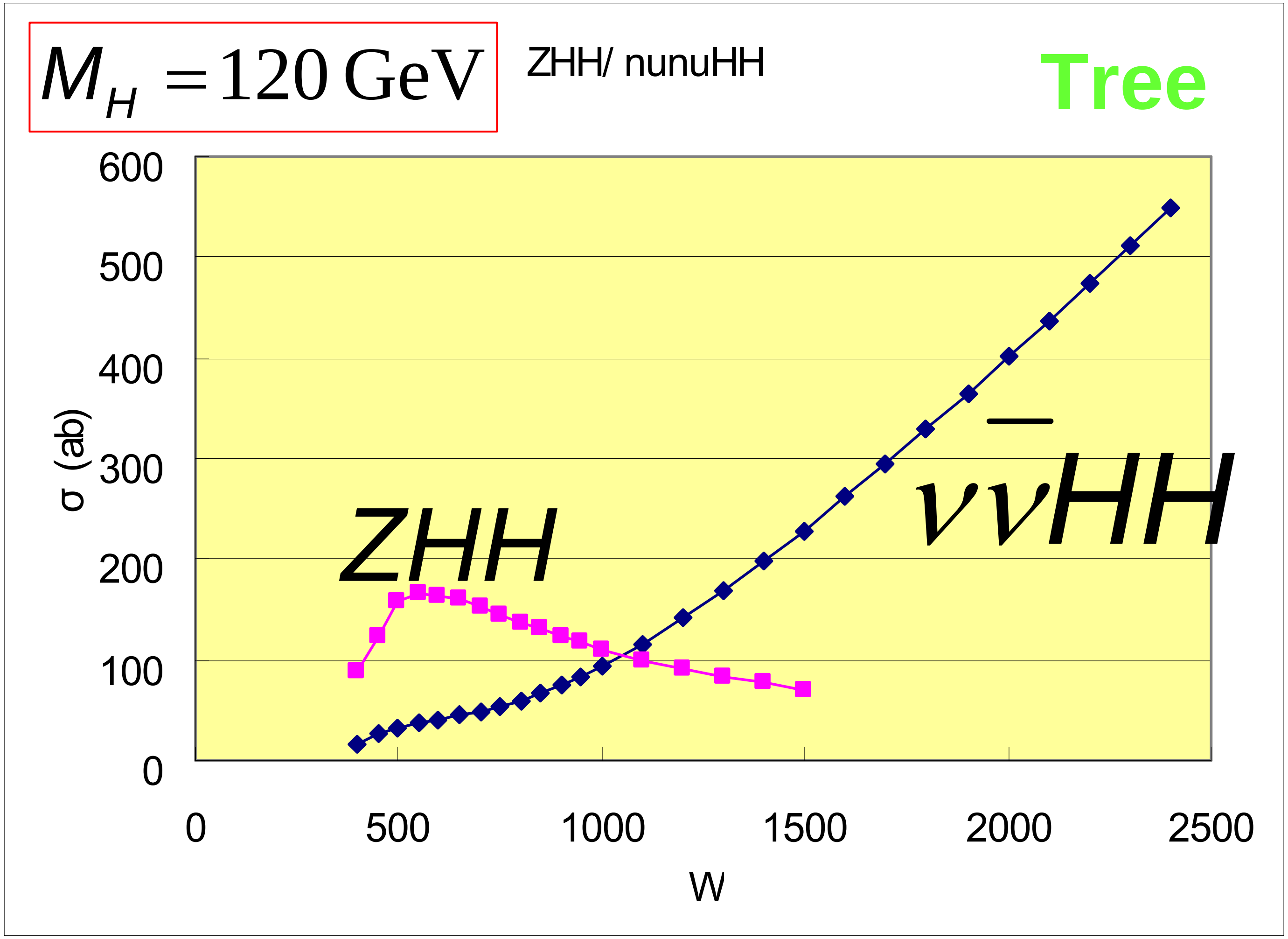



HHH coupling dependence
SM: $\delta\Lambda = 0$

tree

old parameter set

# $\nu\nu$HH vs ZHH



$$e^+e^- \to ZHH$$

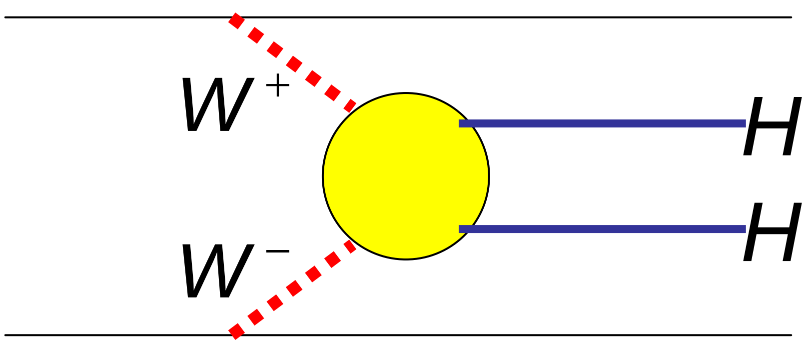
Low W

FULL calculation

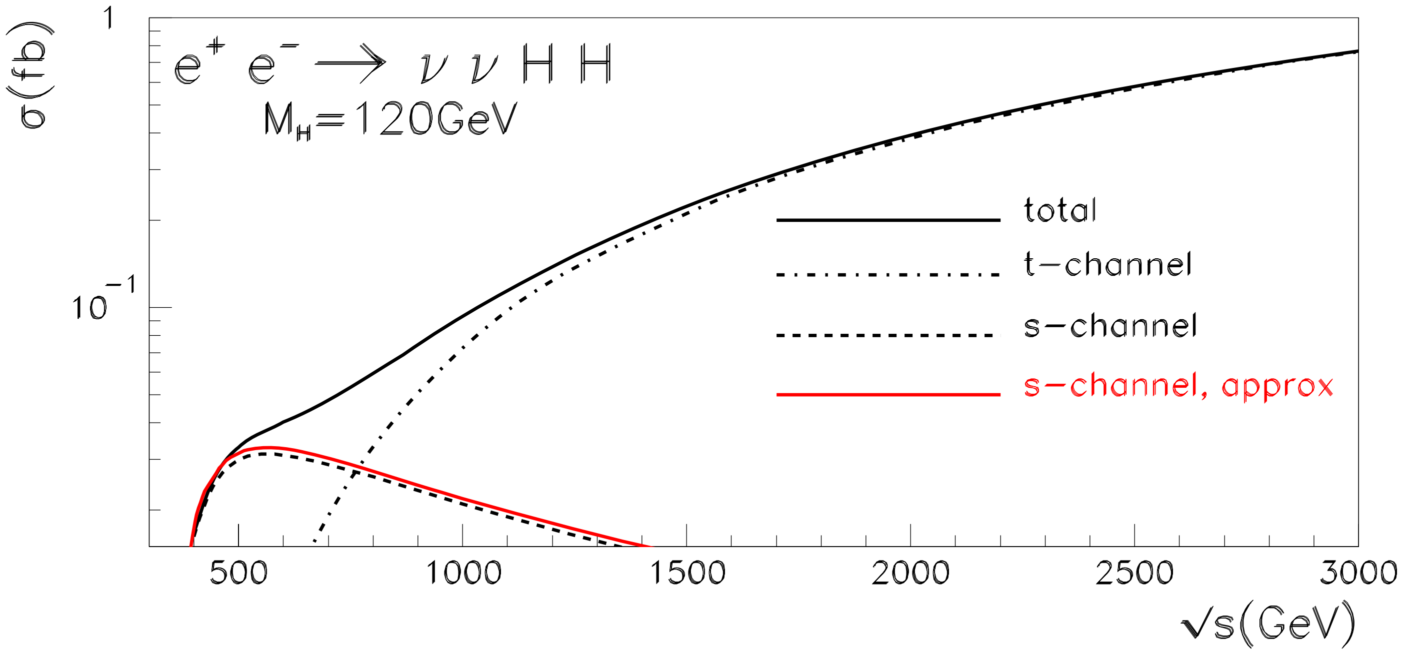
$$e^+e^- \to \nu\bar{\nu}HH$$

High W

dominant mechanism (approximation)

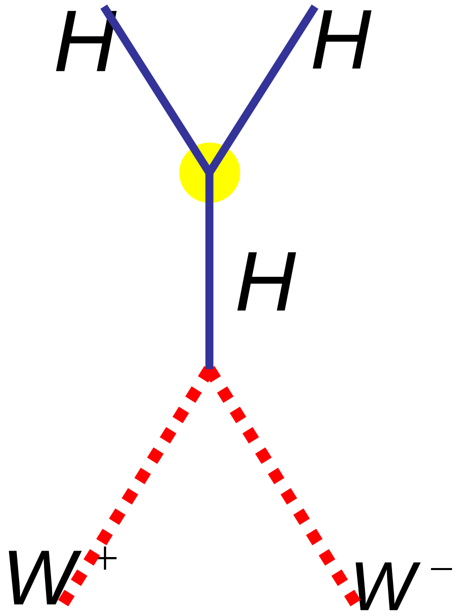


# $e^+e^- \rightarrow \nu\nu H H$

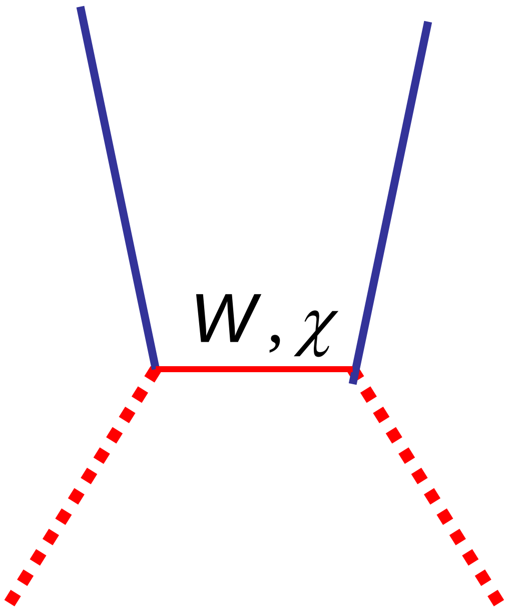


# $W^+W^- \rightarrow HH$

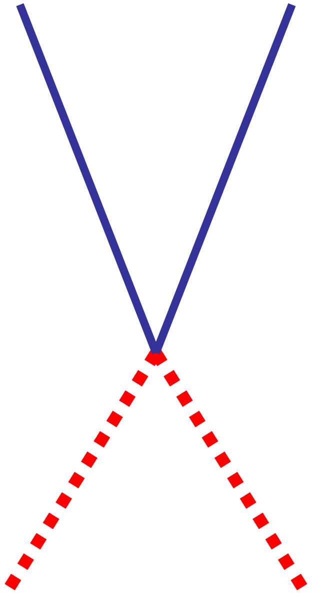
**A**nnihilation




**B**oson Exchange




**C**ontact Interaction



# $W^+W^- \rightarrow HH$

- tree … 6 diagrams
  **A**nnihilation, **B**oson exch., **C**ontact int.
  **B** $\leftarrow\rightarrow$ **A**, **C** … opposite sign
- 1-loop … 827 diagrams
  $\rightarrow$ 13 groups
  **A** : vertex corr .HHH, WWH, AWW, ZWW
  **B** : vertex corr. WWH, W$\chi$H
  **C** : Box

# Check : Cuv independence

| | Cuv=0 | Cuv=100 | Cuv=10000 |
|---|---|---|---|
| $M_1^{(1)}$ | -2.43803300748454E-02 | -2.43803300748515E-02 | -2.43803300745575E-02 |
| $M_2^{(1)}$ | 6.36129553455079E-03 | 6.36129553455044E-03 | 6.36129553460606E-03 |
| $M_3^{(1)}$ | 7.23087719852241E-03 | 7.23087719852186E-03 | 7.23087719846840E-03 |
| $M_4^{(1)}$ | 6.36129553455080E-03 | 6.36129553455055E-03 | 6.36129553462738E-03 |
| $M_5^{(1)}$ | 7.23087719852242E-03 | 7.23087719852180E-03 | 7.23087719845419E-03 |
| $M_6^{(1)}$ | 3.06287858179164E-02 | 3.06287858179168E-02 | 3.06287858180306E-02 |
| $M_7^{(1)}$ | 0.00000000000000E+00 | 0.00000000000000E+00 | 0.00000000000000E+00 |
| $M_8^{(1)}$ | 0.00000000000000E+00 | 0.00000000000000E+00 | 0.00000000000000E+00 |
| $M_9^{(1)}$ | 3.27505572723379E-02 | 3.27505572723362E-02 | 3.27505572717802E-02 |
| $M_{10}^{(1)}$ | 3.00769736052286E-02 | 3.00769736052207E-02 | 3.00769736047325E-02 |
| $M_{11}^{(1)}$ | 3.27505572723379E-02 | 3.27505572723362E-02 | 3.27505572717802E-02 |
| $M_{12}^{(1)}$ | 3.00769736052286E-02 | 3.00769736052209E-02 | 3.00769736047325E-02 |
| $M_{13}^{(1)}$ | -1.87904957771869E-01 | -1.87904957771862E-01 | -1.87904957771593E-01 |
| $M_A^{(1)}$ | 6.24845574307105E-03 | 6.24845574306529E-03 | 6.24845574347307E-03 |
| $M_B^{(1)}$ | 1.52839407221279E-01 | 1.52839407221259E-01 | 1.52839407219181E-01 |
| $M_C^{(1)}$ | -1.87904957771869E-01 | -1.87904957771862E-01 | -1.87904957771593E-01 |
| Total | -2.88170948075182E-02 | -2.88170948075379E-02 | -2.88170948089388E-02 |

# Linear Gauge vs Non-Linear Gauge

| NLG | Cuv=0 | Cuv=100 |
|---|---|---|
| $M_1^{(1)}$ | -5.88235946700174E-02 | -5.88235946700178E-02 |
| $M_2^{(1)}$ | -3.07618745586793E-02 | 3.45210584160196E-01 |
| $M_3^{(1)}$ | -3.28642239568896E-02 | 3.47696096879355E-01 |
| $M_4^{(1)}$ | -3.07618745586793E-02 | 3.45210584160196E-01 |
| $M_5^{(1)}$ | -3.28642239568896E-02 | 3.47696096879354E-01 |
| $M_6^{(1)}$ | 2.36103152312794E-02 | 2.36103152312744E-02 |
| $M_7^{(1)}$ | 0.00000000000000E+00 | -6.93889390390723E-18 |
| $M_8^{(1)}$ | 0.00000000000000E+00 | 0.00000000000000E+00 |
| $M_9^{(1)}$ | 1.84494128486571E-02 | -3.57523045870251E-01 |
| $M_{10}^{(1)}$ | 2.10500413941455E-02 | -3.59510279442103E-01 |
| $M_{11}^{(1)}$ | 1.84494128486572E-02 | -3.57523045870251E-01 |
| $M_{12}^{(1)}$ | 2.10500413941454E-02 | -3.59510279442102E-01 |
| $M_{13}^{(1)}$ | 5.46494731767579E-02 | 5.46494731769278E-02 |
| $M_A^{(1)}$ | -3.52132794387380E-02 | -3.52132794387434E-02 |
| $M_B^{(1)}$ | -4.82532885455325E-02 | -4.82532885456059E-02 |
| $M_C^{(1)}$ | 5.46494731767579E-02 | 5.46494731769278E-02 |
| Total | -2.88170948075125E-02 | -2.88170948074217E-02 |

| LG | Cuv=0 |
|---|---|
| $M_1^{(1)}$ | -2.43803300748454E-02 |
| $M_2^{(1)}$ | 6.36129553455079E-03 |
| $M_3^{(1)}$ | 7.23087719852241E-03 |
| $M_4^{(1)}$ | 6.36129553455080E-03 |
| $M_5^{(1)}$ | 7.23087719852242E-03 |
| $M_6^{(1)}$ | 3.06287858179164E-02 |
| $M_7^{(1)}$ | 0.00000000000000E+00 |
| $M_8^{(1)}$ | 0.00000000000000E+00 |
| $M_9^{(1)}$ | 3.27505572723379E-02 |
| $M_{10}^{(1)}$ | 3.00769736052286E-02 |
| $M_{11}^{(1)}$ | 3.27505572723379E-02 |
| $M_{12}^{(1)}$ | 3.00769736052286E-02 |
| $M_{13}^{(1)}$ | -1.87904957771869E-01 |
| $M_A^{(1)}$ | 6.24845574307105E-03 |
| $M_B^{(1)}$ | 1.52839407221279E-01 |
| $M_C^{(1)}$ | -1.87904957771869E-01 |
| Total | -2.88170948075182E-02 |

# leading $m_t$ formulas

**A**: $C_H + C_W$

**B**: $2C_W$

**C**: $C_4$

$M_W = 80.4163$
$M_Z = 91.1876$
$M_H = 120$
$m_t = 180$
$\alpha = 1/137.0359895$

$$C_H = -\frac{\alpha}{3\pi s_w^2} N_C \frac{m_t^4}{M_W^2 M_H^2}$$

-0.11779616

$$C_W = -\frac{\alpha}{8\pi s_w^2} N_C \frac{m_t^2}{M_W^2} \frac{2s_W^2 + 3}{12 s_w^2}$$

-0.02535196

$$C_4 = -\frac{\alpha}{8\pi s_w^2} N_C \frac{m_t^2}{M_W^2} \frac{8s_W^2 + 3}{6 s_w^2}$$

-0.07033663

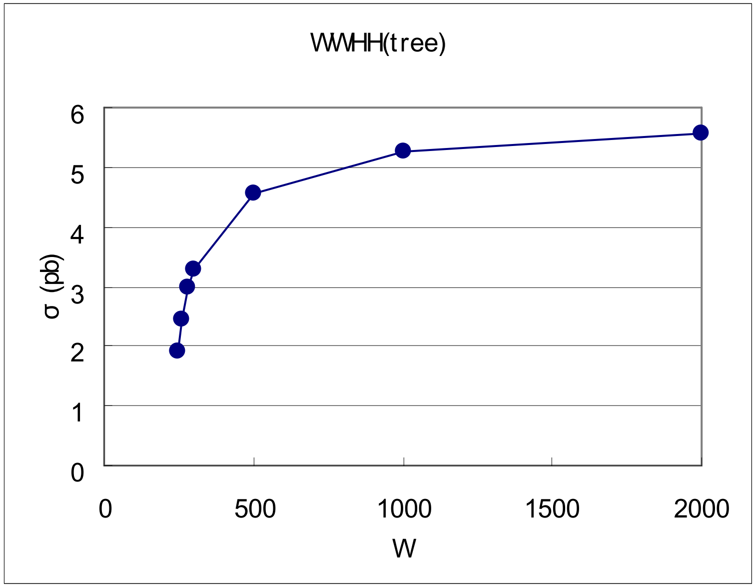
$$m_t = 180 \times 10^{(n-1)} \text{ GeV } (n = 1,2,3,4,5)$$

real world : NOT $m_t$ >> W, $M_H$, $M_Z$,…

| | $m_t = 180$ | $m_t = 180 \times 10^1$ | $m_t = 180 \times 10^2$ | $m_t = 180 \times 10^3$ | $m_t = 180 \times 10^4$ |
|---|---|---|---|---|---|
| $R_1$ | -3.95661868E-02 | -1.16590883E+03 | -1.17784063E+07 | -1.17796040E+11 | -1.17796160E+15 |
| $R_2$ | 1.08836690E-01 | -2.41343274E+00 | -2.53407370E+02 | -2.53518663E+04 | -2.53519683E+06 |
| $R_3$ | 1.09491467E-01 | -2.41224707E+00 | -2.53406171E+02 | -2.53518651E+04 | -2.53519683E+06 |
| $R_4$ | 1.08836690E-01 | -2.41343274E+00 | -2.53407370E+02 | -2.53518663E+04 | -2.53519683E+06 |
| $R_5$ | 1.09491467E-01 | -2.41224707E+00 | -2.53406171E+02 | -2.53518651E+04 | -2.53519683E+06 |
| $R_6$ | 1.39129674E-01 | -2.38545988E+00 | -2.53379444E+02 | -2.53518384E+04 | -2.53519680E+06 |
| $R_9$ | 4.49015626E-02 | -2.47168199E+00 | -2.53465487E+02 | -2.53519244E+04 | -2.53519689E+06 |
| $R_{10}$ | 4.49636773E-02 | -2.47197693E+00 | -2.53465796E+02 | -2.53519247E+04 | -2.53519689E+06 |
| $R_{11}$ | 4.49015626E-02 | -2.47168199E+00 | -2.53465487E+02 | -2.53519244E+04 | -2.53519689E+06 |
| $R_{12}$ | 4.49636773E-02 | -2.47197693E+00 | -2.53465796E+02 | -2.53519247E+04 | -2.53519689E+06 |
| $R_{13}$ | 1.03054258E-01 | -6.88133668E+00 | -7.03232676E+02 | -7.03365171E+04 | -7.03359795E+06 |

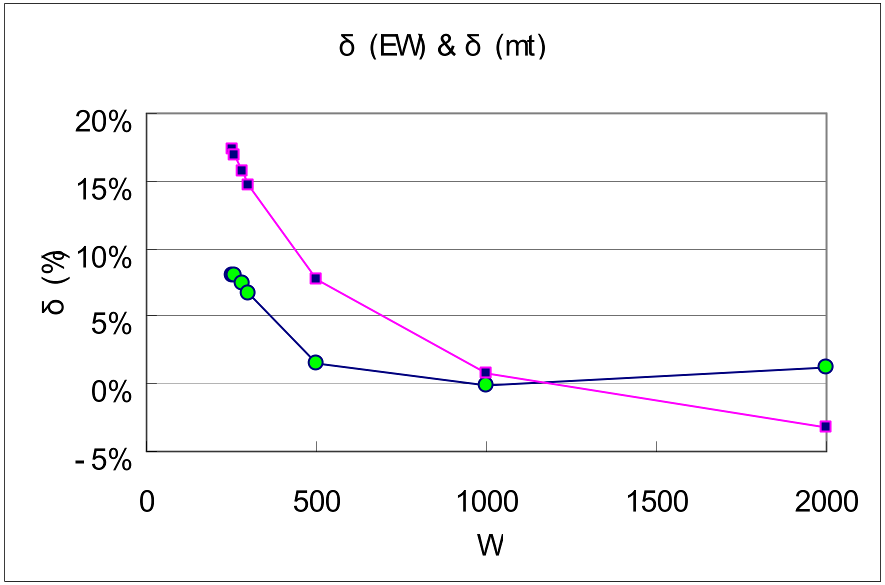
OK

*proof of the formulas and*
*the performance of GRACE system*

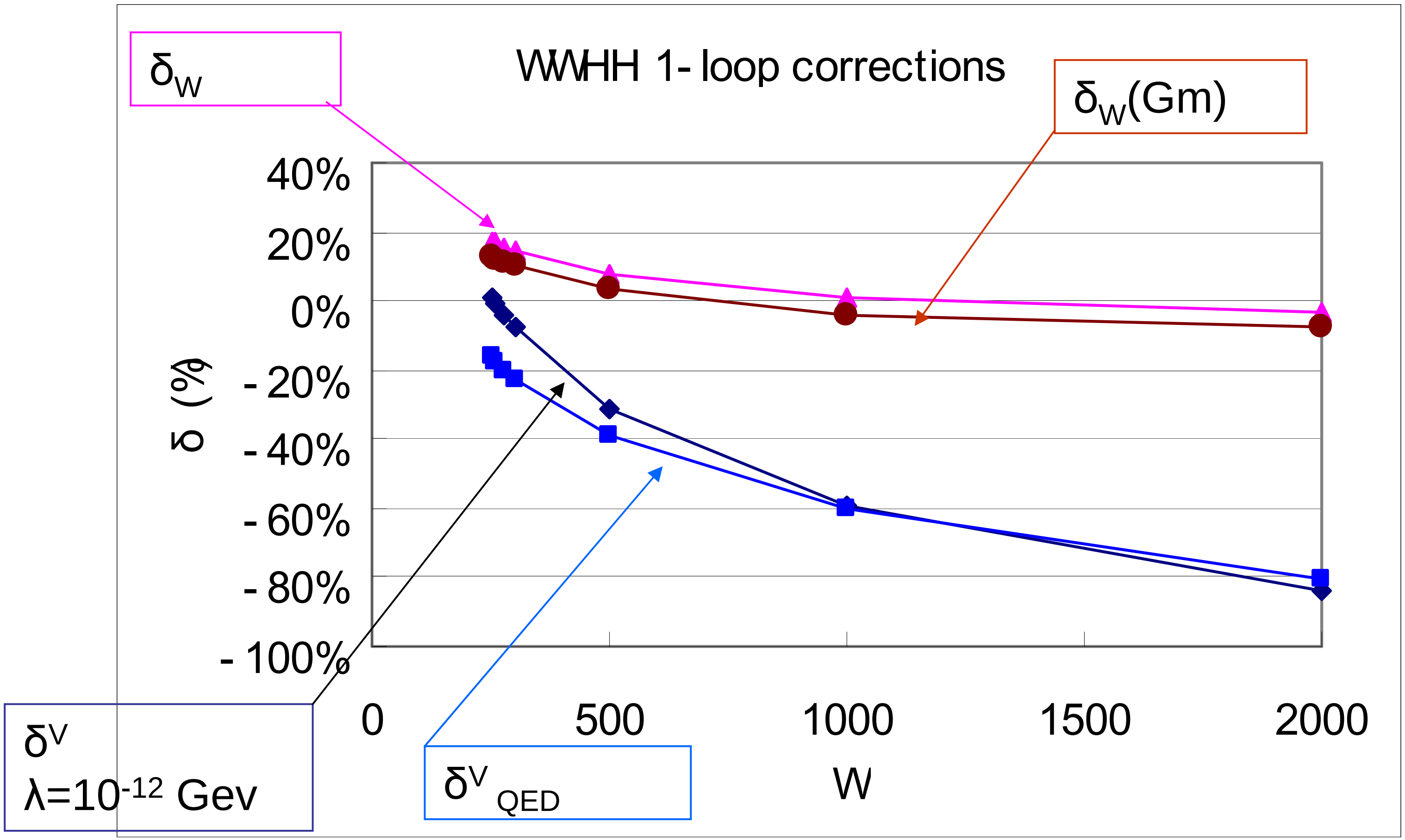



tree+leading mt correction

full EW correction (w/o QED)



# $W^+W^- \rightarrow HH$



# $e^+e^- \rightarrow \nu\nu H H$

- number of diagrams

$\nu_e$ ... tree= 81  loop=19638

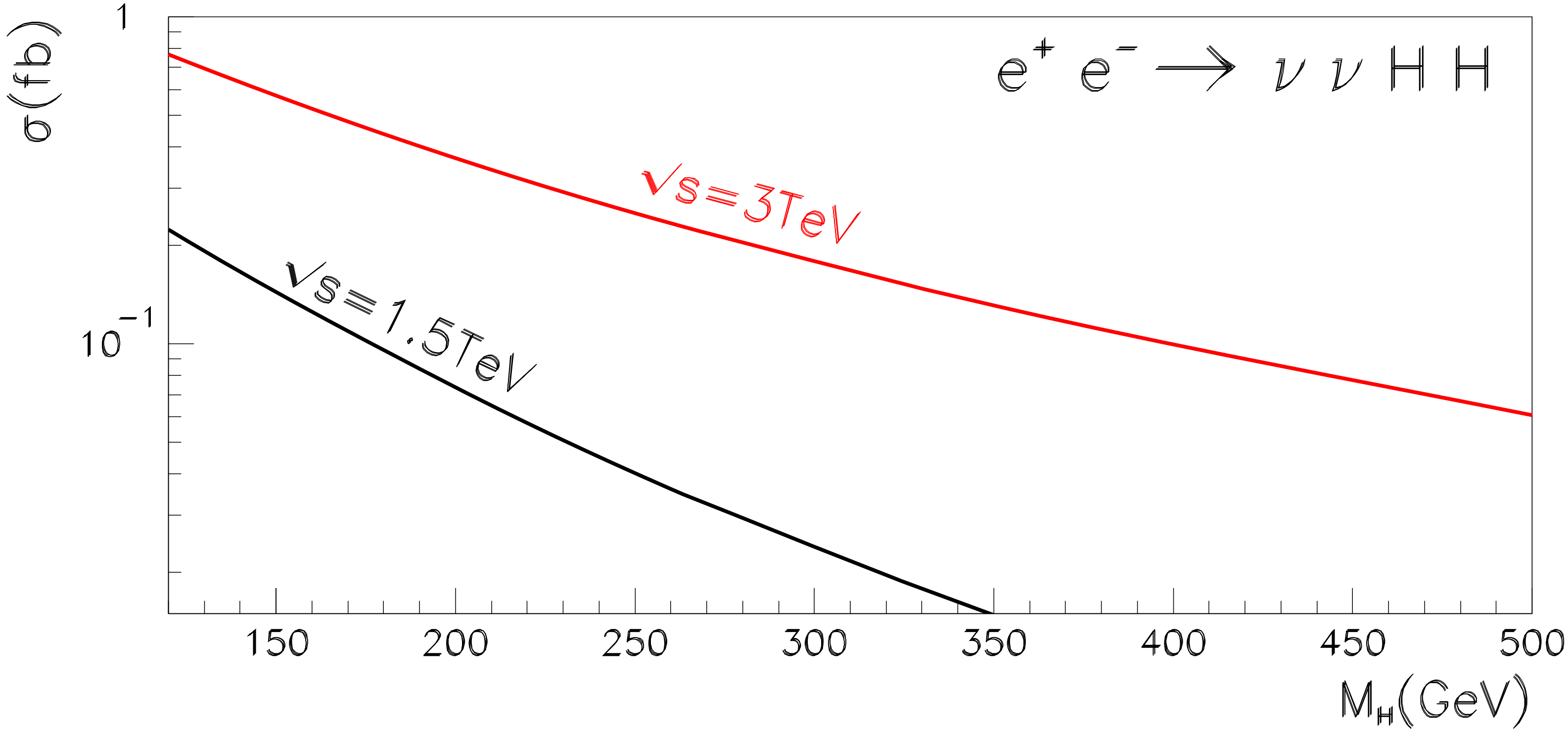
(prod. set  = 12 x 3416)

50M lines of Fortran code

$\nu_{mu}$ ... tree= 27  loop=8292

(prod. set = 6 x 1754)

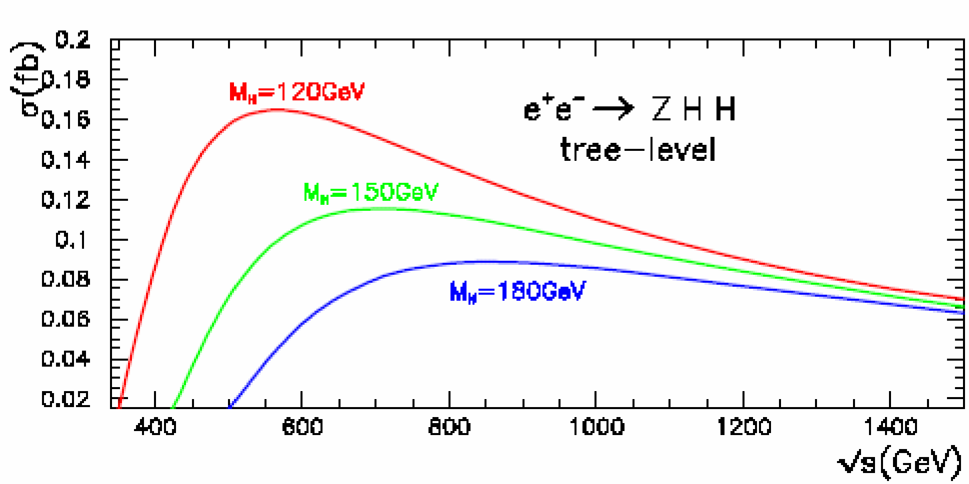
# $e^+e^- \rightarrow \nu\nu H H$

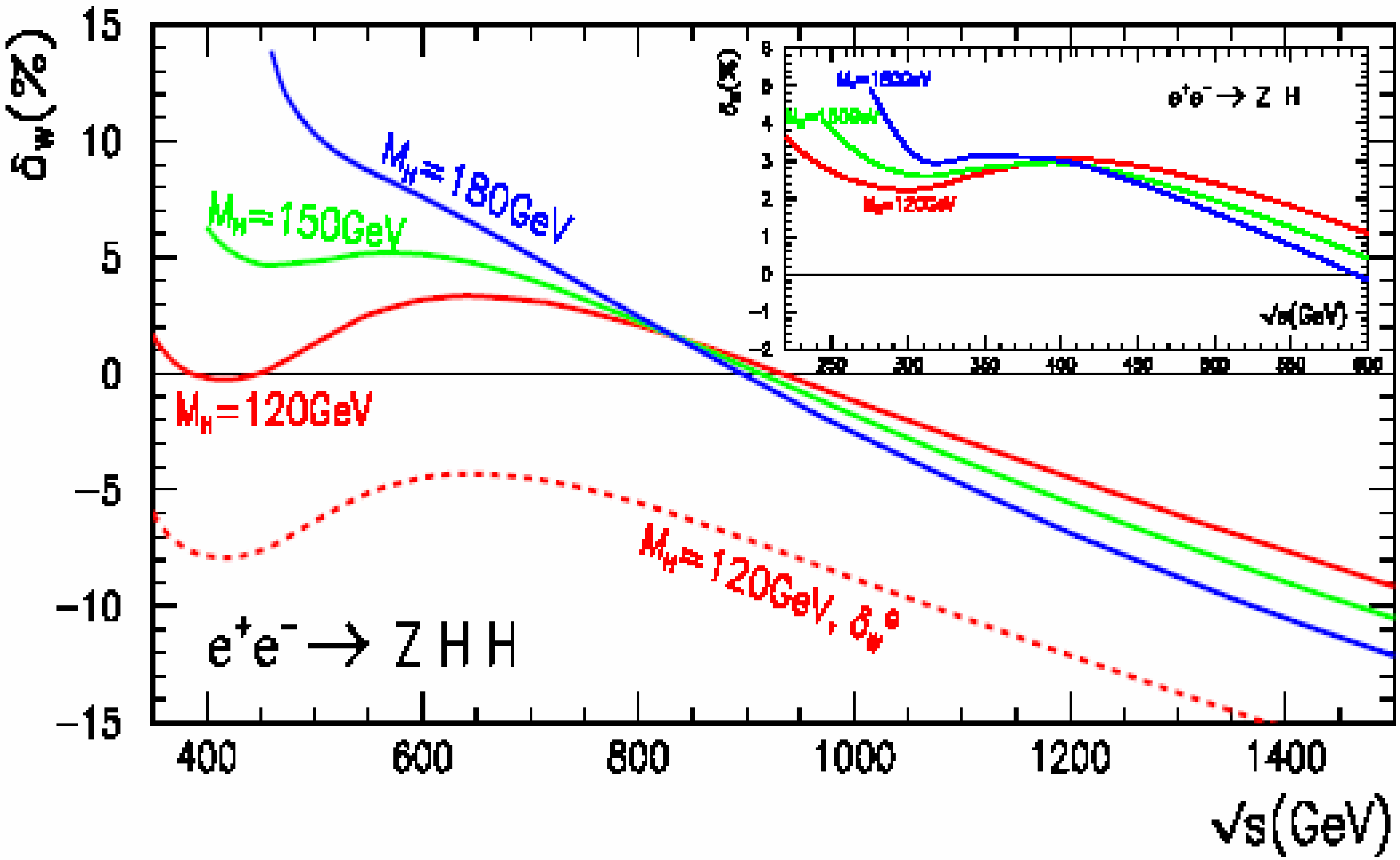


"s-channel"

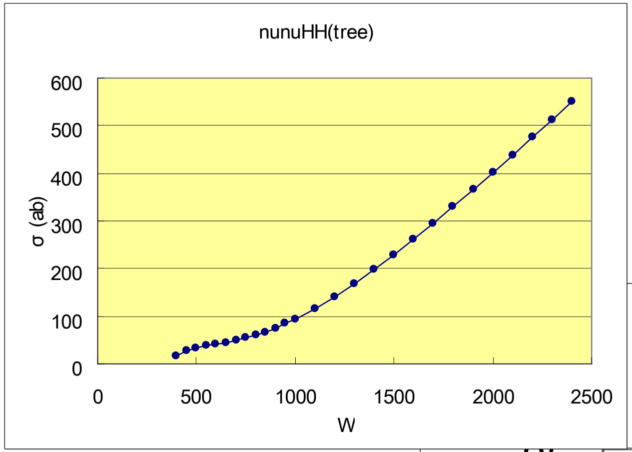
$$e^+e^- \to ZHH$$

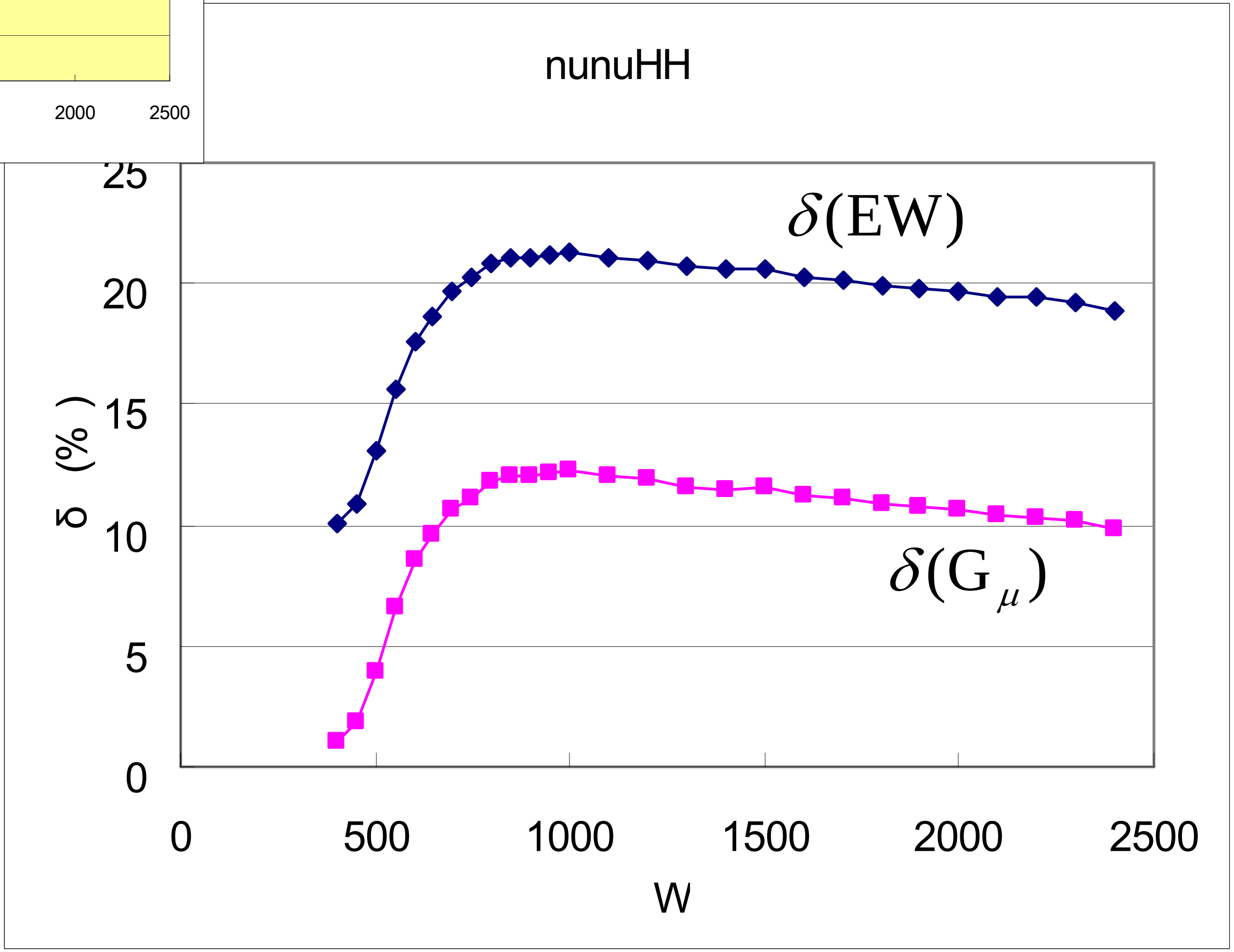
hep-ph/030910
Phys.Lett.B 576(2003)152.



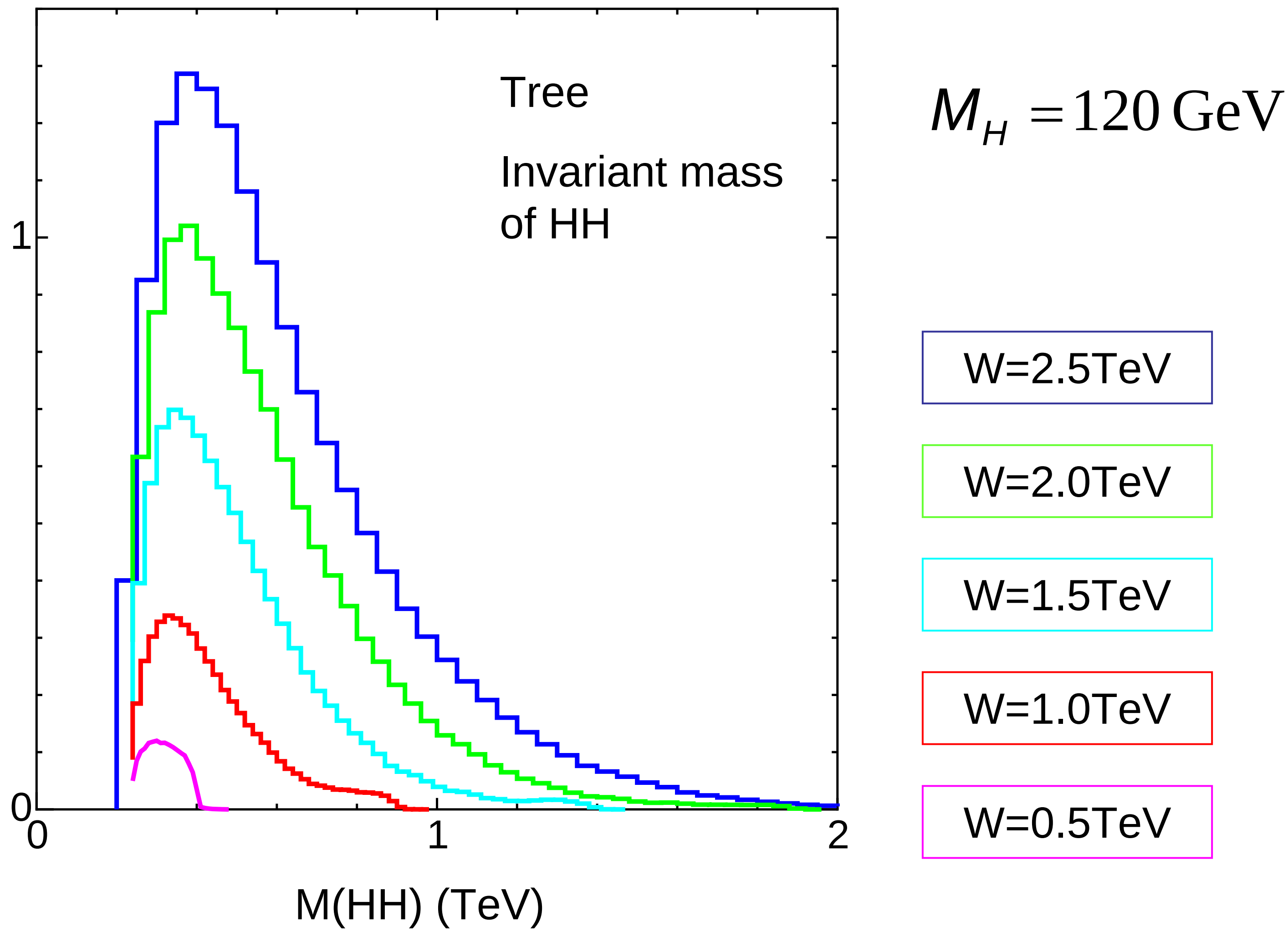
# $e^+e^- \rightarrow \nu\nu H H$




Full calculation
$M_H$=120GeV

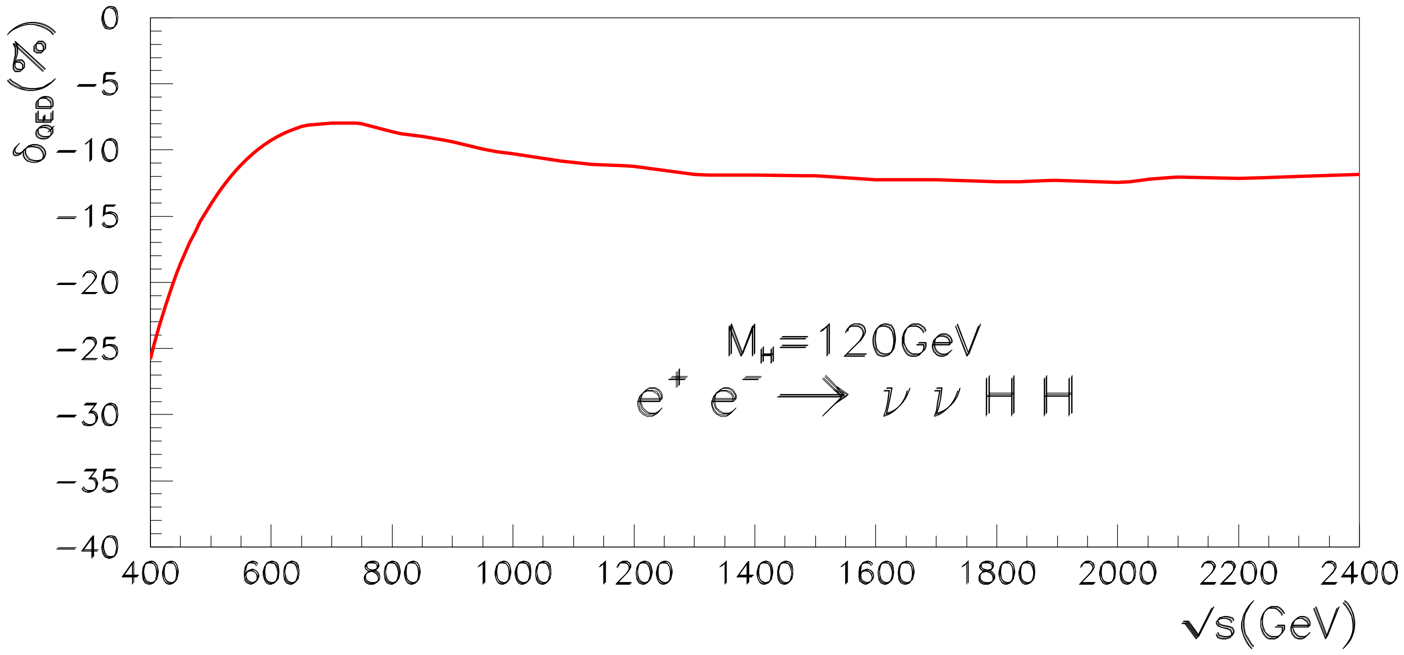
# $e^+e^- \rightarrow \nu\nu H H$



# Summary

- Higgs Potential channel ee→ννHH cross section and its EW correction is calculated. $\sigma = O(100\ ab)$ and greater than ee→ZHH in high-E ILC region ($\sqrt{s} \geqq 1$TeV). δ(Gmu) is ~10% for $\sqrt{s}$ >700GeV. Large diviation from ee→ZHH dominated by s-channel.
- WW→HH is studied to check the t-channel structure of ee→ννHH and has shown validity and limitation of the leading $m_t$ formulas.

# $e^+e^- \rightarrow \nu\nu H H$

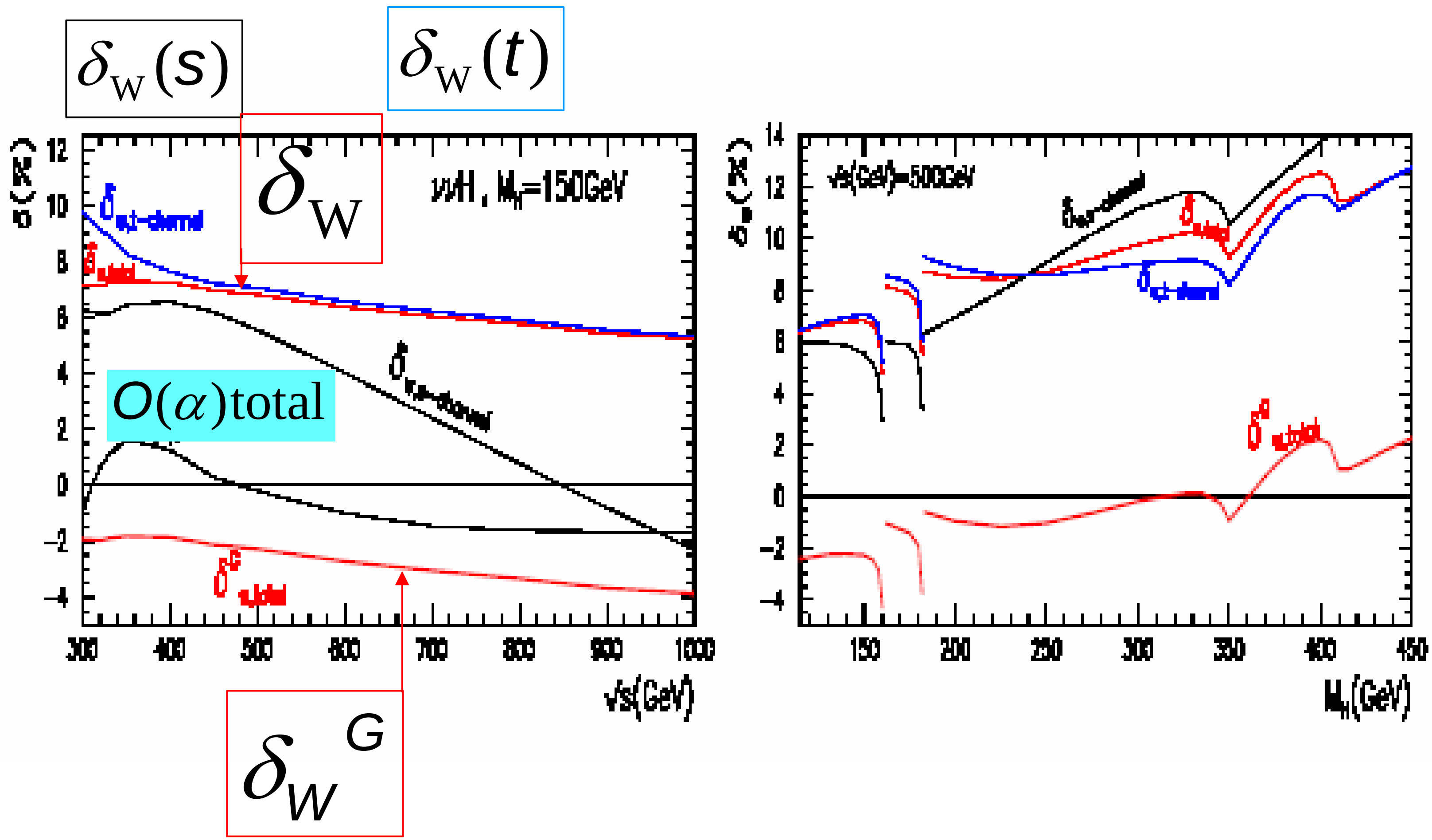


## $e^+e^- \to \nu\bar{\nu}H$

hep-ph/0212261
Phys.Lett.B 559(2003) 252.

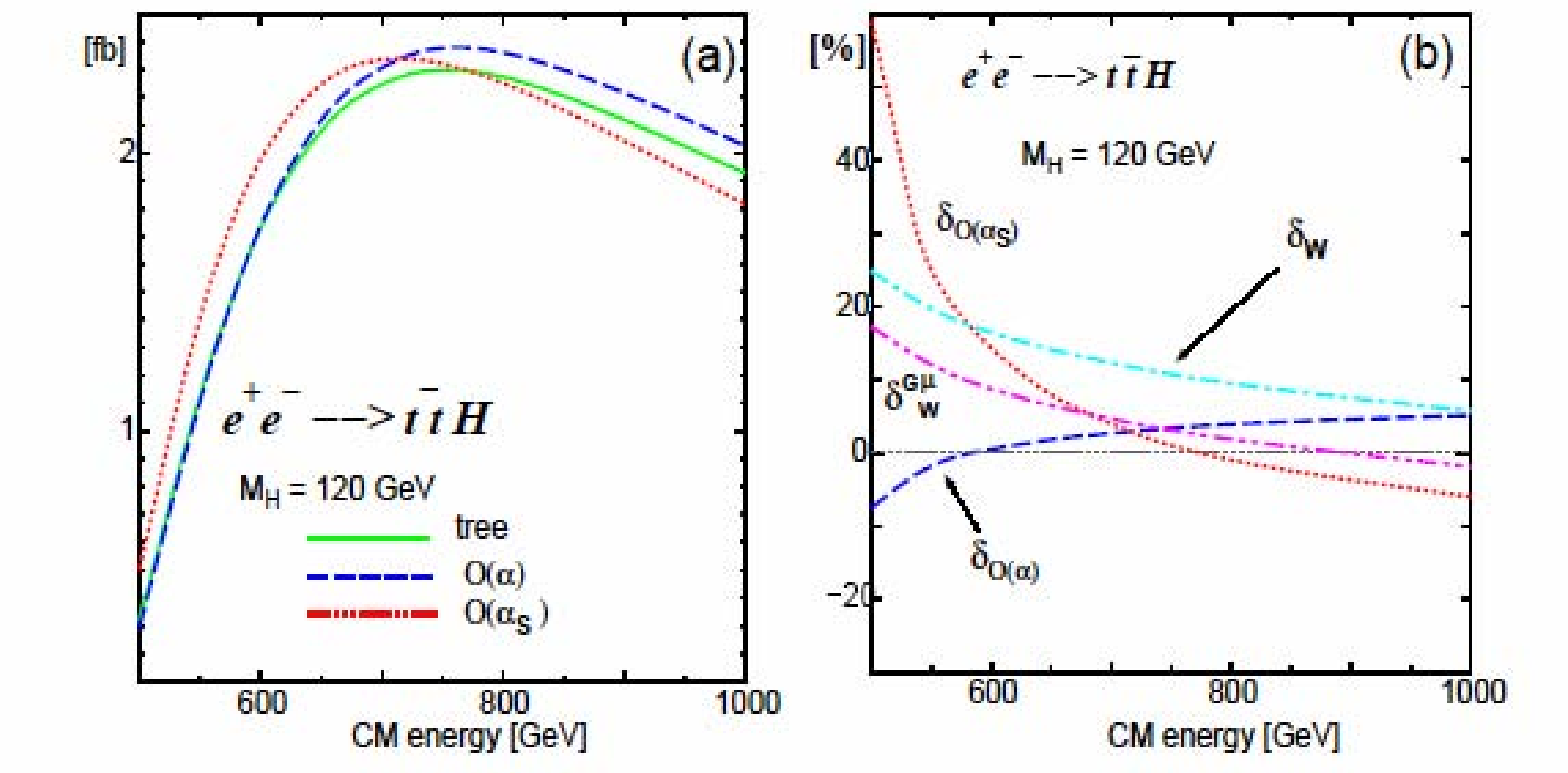
s,t channels show different behavior
genuine weak correction in G-scheme is -2 ~ -4%

$$\delta_{\mathrm{HWW}} = -\frac{5\alpha M_t^2}{16\pi s_W^2 M_W^2} = -1.5\%$$



$$e^+e^- \to t\bar{t}H$$

hep-ph/0307029
Phys.Lett.B 571(2003) 163.



# system components

- Diagram generation for input process
- Amplitude/Matrix element generation
- Kinematics and Integration (efficiency)
- Event generation (efficiency & weight)
- Peripheral tools: rule generator, diagram selection, QED radiation, PDF, loop integral library, multi-process, color flow and interface for hadronization, etc.

# 5-point functions

$N$ rank M

$$I_5 = \sum G_{\mu\nu\cdots\sigma} \int d\ell \, \frac{\ell^\mu \ell^\nu \cdots \ell^\sigma}{D_0 D_1 D_2 D_3 D_4}$$

$$D_0 = \ell^2 + X_0$$

$$D_j = \ell^2 + 2\ell \cdot r_j + X_j$$

$$A_{ij} = r_i \cdot r_j \qquad g^{\mu\nu} = r_i^\mu A_{ij}^{-1} r_j^\nu \qquad \ell^2 = D_0 - X_0$$

$$\ell^\mu = r_i^\mu A_{ij}^{-1}(r_j^\nu \ell) = \tfrac{1}{2} r_i^\mu A_{ij}^{-1}[D_j - D_0 + X_0 - X_j]$$

$$N = \sum_{\alpha=}^{4} E_\alpha(\ell) D_\alpha + F \qquad 1 = \sum_{\alpha=0}^{4} [a_\alpha + b_{\alpha j}(\ell r_j)] D_\alpha$$

scalar 5-pnt

BOX

BOX rank M-1

# δ(QED), δ(EW)

$$\sigma = \sigma_0 (1 + \delta_{QED} + \delta_W)$$

$\delta_W$ non-QED virtual corrections

$$\delta_{QED} = \delta_{QED}^{V} + \delta_{QED}^{soft} + \delta_{QED}^{hard}$$

phase space subtraction $f_{LL}$ =radiator

$$\delta_{QED} = \int \left( d\sigma_0 \delta_{QED}^{v} + d\tilde{\sigma}_0 \otimes f_{LL} \right) + \int_{hard} \left( d\sigma_{1\gamma} - d\tilde{\sigma}_0 \otimes f_{LL} \right)$$

# Non-linear gauge fixing terms

$$L_{GF} = -\frac{1}{\xi_W} F^+ F^- - \frac{1}{2\xi_Z}\left(F^Z\right)^2 - \frac{1}{2\xi}\left(F^A\right)^2$$

$$F^{\pm} = \left(\partial^\mu \mp ie\tilde{\alpha} A^\mu \mp i\frac{ec_W}{s_W}\tilde{\beta} Z^\mu\right) W^{\pm}_\mu \qquad F^A = \partial^\mu A_\mu$$

$$+\xi_W\left(M_W \chi^{\pm} + \frac{e}{2s_W}\tilde{\delta} H \chi^{\pm} \pm i\frac{e}{2s_W}\tilde{\kappa}\chi_3 \chi^{\pm}\right)$$

$$F^Z = \partial^\mu Z_\mu + \xi_Z\left(M_W \chi_3 + \frac{e}{2s_W c_W}\tilde{\varepsilon} H \chi_3\right)$$

# Samples of NLG Feynman rules

$W - W - A$

$$e[g^{\mu\nu}(p_1 - p_2)^\rho + (1+\tilde{\alpha}/\xi_W)(p_3^\nu g^{\mu\rho} - p_3^\mu g^{\nu\rho}) + (1+\tilde{\alpha}/\xi_W)(p_2^\mu g^{\nu\rho} - p_1^\nu g^{\mu\rho})]$$

$W - \chi - A$

$$\mp ieM_W(1-\tilde{\alpha})g^{\mu\nu}$$

modified

---

$\bar{c}^{\mp} - c^A - A - W^{\pm}$

$$-e^2\tilde{\alpha}g^{\mu\nu}$$

$\bar{c}^{\mp} - c^A - \chi^{\pm} - H$

$$\mp ie^2 \frac{1}{2s_W}\tilde{\delta}\xi_W$$

ghost-ghost- vector-vector / ghost-ghost-scalar-scalar

# Non-linear gauge

- Numerator structure is the same as Feynman gauge
  → Loop integral library
- Vertices modified
- general values→ #diagrams

$$g^{\mu\nu} \quad (\text{for } \xi = 1)$$

"old" usage
→reduce #diagram

$\tilde{\alpha} = 1 \Rightarrow$ no AW$\chi$

$$\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\varepsilon}, \tilde{\kappa}$$

Check gauge invariance
→Independence on gauge parameters