Measuring the Beam Energy with Radiative Return Events

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This paper studies the possibility to measure the centre of mass energy using $e^+e^- \rightarrow Z\gamma \rightarrow \mu^+\mu^-\gamma$ events at the ILC. With $\mathcal{L} = 100\ \text{fb}^{-1}$ at $\sqrt{s} = 350\,\text{GeV}$ a relative error of around 10^{-4} is possible. The potentially largest systematic uncertainty comes from the knowledge of the aspect ratio of the detector.

1. INTRODUCTION

The beam energy at the ILC is needed to a precision of around 10^{-4} for accurate mass determinations of the top quark, the Higgs boson and in supersymmetry [1]. This measurement will mainly be done with a magnetic spectrometer [2]. However the absolute calibration of such a spectrometer is difficult and in addition the luminosity weighted centre of mass energy is not necessarily identical to twice the beam energy. It is thus very useful to have a method to measure the luminosity weighted centre of mass energy directly from annihilation data. Fortunately this is possible using radiative return events to the Z using the fact that the Z mass is known to very high precision. The method was already pioneered at LEP2, where it was however limited by the available statistics [3, 4, 5].

2. ENERGY BIAS FROM THE KINK-INSTABILITY

Wakefields in the main accelerator introduce a correlation between the z position of an electron in the bunch and its energy. Due to the disruption of the bunch in the interaction not all parts of the bunch contribute with the same weight to the luminosity. The combination of both effects introduces a bias in the luminosity weighted centre of mass energy. A detailed study can be found in [6].

For the TESLA design the effect is on average 150 ppm with a spread of 30 ppm and a maximum of 350 ppm which is on the edge of being relevant. Figure 1 shows the centre of mass energy and the energy difference of colliding particles for TESLA. The histogram shows the real simulated distribution while the points show the artificial case where the energies have been ordered randomly. The bias can be seen from the shift of the mean of the two distributions in the centre of mass energy. The two distributions agree well in the energy difference. This means that the bias cannot be measured using the Bhabha acolinearity, which is proposed to measure the beam energy spectrum due to beamstrahlung [7]. If one does not want to rely completely on beam simulations methods using annihilation data are thus the only way to control such effects.

3. THE RADIATIVE RETURN METHOD

The process $e^+e^- \rightarrow Z\gamma \rightarrow \mu^+\mu^-\gamma$ is well suited for the reconstruction of of the centre of mass energy since the Z mass is well known from LEP [8] and thus the γ energy depends only on the centre of mass energy, $\sqrt{s}$. If one assumes that exactly one photon is radiated and that the energy of the two beams is the same, the mass of the $\mu^+\mu^-$ system, $\sqrt{s'}$, can be reconstructed only from the angles of the particles neglecting all energy measurements:

$$\frac{\sqrt{s'}}{\sqrt{s}} = \sqrt{\frac{\sin\theta_1 + \sin\theta_2 + \sin(\theta_1 + \theta_2)}{\sin\theta_1 + \sin\theta_2 - \sin(\theta_1 + \theta_2)}} \quad (1)$$

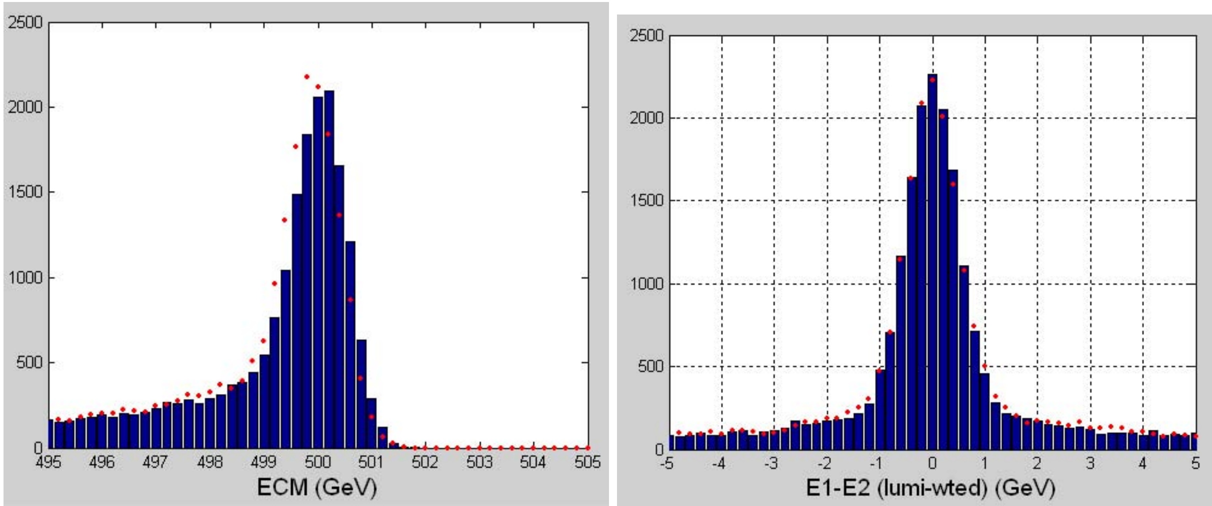

Figure 1: Centre of mass energy (left) and energy difference (right) of colliding electron-positron pairs in the TESLA design. The histogram shows the real simulation while for the dots the particle energies are ordered randomly in the bunch.

where $\theta_{1,2}$ are the angles between the two muons and the photon. In most cases the photon is lost in the beampipe. In this case the photon direction can be replaced by the z-axis signed by the negative $\mu^+\mu^-$ momentum vector. In addition it is assumed that the fermion mass can be neglected. Setting $\sqrt{s'} = m_Z$ one gets

$$\sqrt{s} = m_Z \sqrt{\frac{\sin\theta_1 + \sin\theta_2 - \sin(\theta_1+\theta_2)}{\sin\theta_1 + \sin\theta_2 + \sin(\theta_1+\theta_2)}}. \tag{2}$$

Equation 2 thus allows in principle to reconstruct the beam energy without measuring energies with the detector. Only angles, which can be measured with better precision and less systematic uncertainties are used. In reality it is possible that more than one photon is radiated or that one or both beams have lost energy due to beamstrahlung. These effects can easily be accounted for in the fit, however they have to be known accurately. Figure 2 a) shows the true $\mu^+\mu^-$ invariant mass and the one reconstructed according to equation 1 for $\sqrt{s} = 350\,\mathrm{GeV}$. Multiple radiation and beamstrahlung is responsible for the shift of the Z-peak towards higher energy. The reconstructed centre of mass energy is shown in figure 2 b).

The cross section $\sigma(\mathrm{e^+e^-} \to \mathrm{Z}\gamma \to \mu^+\mu^-\gamma)$ is about 0.5 pb for $\sqrt{s} = 350\,\mathrm{GeV}$ and scales approximately like $1/s$. The detector accepts charged particles above $\theta = 7^\circ$ [9] which results in an efficiency of about 90%. For the simulation an ideal beam with a Gaussian energy spread of 0.2% and the CIRCE parameterisation of the beamstrahlung [10] has been used [11].

4. Background

Potential backgrounds are given by all events that have exactly two muons in the detector. These are

- two photon events and $\mathrm{e^+e^-} \to Z\mathrm{e^+e^-}$ events where the electrons are lost below $\theta = 7^\circ$;
- $\mathrm{e^+e^-} \to \mathrm{ZZ}$ events where one Z decays into muons and the other into neutrinos;
- $\mathrm{e^+e^-} \to \mathrm{W^+W^-}$ events where both W-bosons decay into a muon and a neutrino.

If there is no resonant Z-boson in the event the background can be rejected efficiently by a cut around the reconstructed $\mu^+\mu^-$ mass, where the cut has to be sufficiently loose not to reintroduce a dependence on the energy calibration. For the analysis presented here a cut $m_Z - 5\,\mathrm{GeV} < m(\mu^+\mu^-) < m_Z + 5\,\mathrm{GeV}$ has been applied. Events with neutrinos

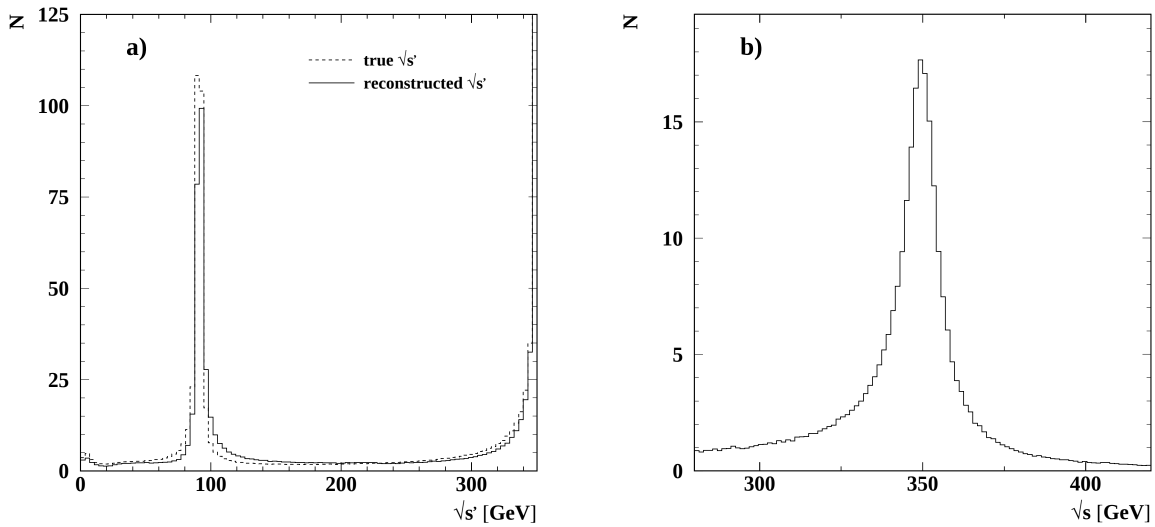

Figure 2: True and reconstructed $\sqrt{s'}$ (a) and reconstructed $\sqrt{s}$ for $e^+e^- \to Z\gamma \to \mu^+\mu^-\gamma$ at $\sqrt{s} = 350\,\mathrm{GeV}$

(WW, ZZ) can in principle be rejected further by a cut on the tranverse momentum ballance, however this has been found to be not necessary. The final sample contains a background of around 10% from two-photon events, 25% from Zee events and about 1% from WW and ZZ. The Zee background is rather large, but this does nor pose any problem. The typical topology for these events is one electron of very high energy while the momentum of the other one is very low. These events thus have a sensitivity to the beam energy very similar to the signal events.

5. Fit Method

To fit the beam energy from the reconstructed centre of mass energy a Monte Carlo linearising around a default value [7] has been used. In this method it is assumed that the differential cross section at a given reconstructed centre of mass energy is a linear function of the true centre of mass energy in a range around the nominal value larger than the expected error:

$$\begin{aligned} \sigma(\sqrt{s}, \sqrt{s}_{\mathrm{rec}}) &= \sigma(\sqrt{s}_0, \sqrt{s}_{\mathrm{rec}}) + A(\sqrt{s}_0, \sqrt{s}_{\mathrm{rec}}) \left(\sqrt{s} - \sqrt{s}_0\right) \\ A(\sqrt{s}_0, \sqrt{s}_{\mathrm{rec}}) &= \frac{\sigma(\sqrt{s}_1, \sqrt{s}_{\mathrm{rec}}) - \sigma(\sqrt{s}_0, \sqrt{s}_{\mathrm{rec}})}{\sqrt{s}_1 - \sqrt{s}_0}. \end{aligned}$$

$\sigma(\sqrt{s}_i, \sqrt{s}_{\mathrm{rec}})$, $i = 0, 1$ is calculated with the simulation including all effects like background, detector resolution, beamstrahlung etc. Apart from the linearity assumption the fit is bias free per construction and this assumption can be tested with the simulation to be valid.

In the fit the data a binned in $\sqrt{s}_{\mathrm{rec}}$ and a χ^2 is built as a function of $\sqrt{s}$, summing over the bins in $\sqrt{s}_{\mathrm{rec}}$. Not to be dependent on the luminosity measurement the total normalisation was treated as a second free parameter in the fit.

6. Results

Monte Carlo data corresponding to an integrated luminosity of $\mathcal{L} = 100\ \mathrm{fb}^{-1}$ at $\sqrt{s} = 350\,\mathrm{GeV}$ have been fitted with the method described above. Including background, beamstrahlung and energy spread an error of $\Delta\sqrt{s} = 47\,\mathrm{MeV}$ or $\frac{\Delta\sqrt{s}}{\sqrt{s}} = 1.3 \cdot 10^{-4}$ has been achieved. If beamstrahlung and energy spread are omitted the error is about 10% smaller. The influence of the background is negligible.

It has been shown that this error can be improved by a factor two to four if the muon momenta are included in the fit [12]. For this improvement a momentum resolution with a constant term of around $2 \cdot 10^{-5}/\,\mathrm{GeV}$ and a multiple scattering term of around 10^{-3} is needed. Furthermore it is assumed that the systematic uncertainty on the momentum resolution can be described by a single scale factor which is included as a free parameter in the fit.

As shown in figure 3 the error depends strongly on the centre of mass energy. For constant luminosity the error can be parametrised as

$$\Delta\sqrt{s} = \left(8.8 + 0.0026\sqrt{s}/\,\mathrm{GeV} + 0.0032 s/\,\mathrm{GeV}^2\right)\ \mathrm{MeV}.$$

It should, however, be noted that the relative error is almost constant if the luminosity increases proportional to s.

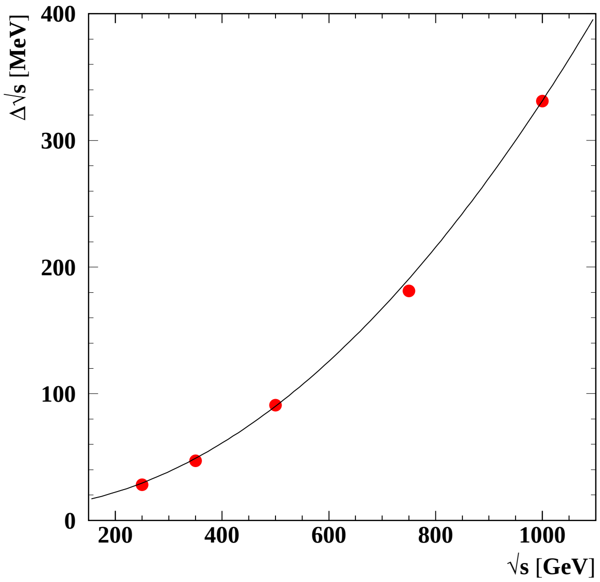

Figure 3: Energy dependence of $\Delta\sqrt{s}$ for $\mathcal{L} = 100\ \mathrm{fb}^{-1}$.

Several sources of systematic uncertainty have been studied. The background has no effect if an uncertainly of less than 20-30% on the amount of background is assumed. If instead of a Gaussian energy spread a rectangular shape is assumed the reconstructed centre of mass energy changes by 10 MeV. There is no change if the width is changed from 0.2% to 0.1%.

If the parameters describing the beamstrahlung in Circe are varied by values as suggested in [7] a shift of the beam energy up to 40 MeV has been found. This shift is, however, strongly anticorrelated with the shift of the mean beam energy due to the parameter change, so that the uncertainty on the average beam energy is very small.

The by far largest error may come from an uncertainty in the polar angle measurement of the detector. At LEP is was assumed that the ratio of the detector radius and detector length is $\Delta\left(\frac{R}{L}\right) = \Delta \tan\theta = 5 \cdot 10^{-4}$. If the same uncertainty hold for the ILC detector the uncertainty on the reconstructed centre of mass energy would be $\Delta\sqrt{s} = 160\,\mathrm{MeV}$. The aspect ratio of the detector thus needs to be known an order of magnitude more accurate than at LEP to make the beam energy measurement with radiative return events useful.

6.1. Future Work

It would be useful to increase the statistics of the radiative return measurement. Bhabha scattering ($\mathrm{e^+e^-} \to \mathrm{e^+e^-}$) is in principle clean, however the signal is diluted by the t-channel contribution. However with a cut on the production angle in the centre of mass system a useful measurement should still be possible. The resolution for $\tau^+\tau^-$ events will be somewhat diluted because of the kink of the charged particles in the τ decay. The main problem, however, is that due to the missing neutrinos the cut on the $\tau^+\tau^-$ invariant mass to reject two-photon background is not very effective.

In principle there is a much larger statistics using $Z \to q\bar{q}$ events. As already said, equation 2 assumes, however, that the mass of the final state particle is negligible. A 5 GeV jet mass results in a shift of 2.5 GeV in $\sqrt{s'}_{\mathrm{rec}}$. It is thus very improbable that fragmentation can be understood well enough to make these events useful.

To get a final estimate of the radiative return method a global analysis will be needed. Beamstrahlung and the kink instability are correlated between the two beams. These correlations influence the Bhabha acolinearity to measure the beamstrahlung and the reconstructed $\sqrt{s}$ from the radiative return analysis simultaneously. A common analysis using both methods is thus needed to see how these effects modify the reconstructed centre of mass energy.

7. Conclusions

The centre of mass energy can be measured on the 10^{-4} level from radiative return events using only the measured angles of the final state muons. This is, however, a high luminosity analysis. The statistics is not sufficient to measure $\sqrt{s}$ for example point by point in a mass scan. These relative measurements still have to be done using spectrometers.

The potentially largest systematic uncertainty comes from the aspect ratio of the detector. Great care has to be taken in the detector design to make sure that this quantity is understood on the 10^{-4} level.

To draw final conclusions on this method a global analysis of the acolinearity of Bhabha events for beamstrahlung and of the radiative return events for the beam energy is needed to understand the effects from beam-beam correlations.

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