M_W, $\sin^2 \theta_{\text{eff}}$ **and** m_h **in the NMFV MSSM**

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The effects of loop contributions to the electroweak precision observables and m_h in the MSSM with non-minimal flavour violation (NMFV) are analyzed, including the mixing between third and second generation squarks. The mixing-induced shift in M_W can amount to $140\,\text{MeV}$ and to 70×10^{-5} in $\sin^2 \theta_{\text{eff}}$ for extreme values of squarks mixing, allowing to set limits on the NMFV parameters. The corrections for m_h are usually small and can amount up to $\mathcal{O}\,(5\,\text{GeV})$ for large flavour violation.

1. INTRODUCTION

An alternative way, as compared to the direct search for Supersymmetry (SUSY) [1] or Higgs particles, is to probe SUSY via virtual effects of the additional non-standard particles to precision observables. This requires very high precision of the experimental results as well as of the theoretical predictions, such as the prediction for Δr in the M_W–M_Z interdependence and the effective leptonic weak mixing angle, $\sin^2 \theta_{\text{eff}}$.

Radiative corrections to the electroweak precision observables within the Minimal Supersymmetric Standard Model (MSSM) have been discussed at the one-loop and two-loop level in several papers [2]. Moreover, the Higgs sector of the MSSM is considerably affected by loop contributions, thus making m_h yet another sensitive observable. Here we present specifically the impact of non-minimal flavour violation (NMFV) in the MSSM on both the electroweak precision observables and the lightest Higgs-boson mass m_h. Thereby, mixing in the scalar top and charm sectors as well as in the scalar bottom and strange sectors ($\tilde{t}/\tilde{c}$ and $\tilde{b}/\tilde{s}$) is considered. An exhaustive analysis is given in [3].

Mixing between the third and second generation squarks can be numerically significant due to the involved third-generation Yukawa couplings, shown in [4]. On the other hand, there are strong experimental constraints on squark mixing involving the first generation, coming from data on K^0–$\bar{K}^0$ and D^0–$\bar{D}^0$ mixing [5]. Therefore, mixing effects from first-generation squarks are not considered in our analysis. The general up-squark off-diagonal mass matrix in the basis of $(\tilde{c}_L, \tilde{t}_L, \tilde{c}_R, \tilde{t}_R)$ is given by

$$M_{\tilde{u}}^2 \;=\; \begin{pmatrix} M_{\tilde{L}_c}^2 & \Delta_{LL}^t & m_c X_c & \Delta_{LR}^t \\ \Delta_{LL}^t & M_{\tilde{L}_t}^2 & \Delta_{RL}^t & m_t X_t \\ m_c X_c & \Delta_{RL}^t & M_{\tilde{R}_c}^2 & \Delta_{RR}^t \\ \Delta_{LR}^t & m_t X_t & \Delta_{RR}^t & M_{\tilde{R}_t}^2 \end{pmatrix} \tag{1}$$

with

$$\begin{aligned} M_{\tilde{L}_q}^2 &= M_{\tilde{Q}_q}^2 + m_q^2 + \cos 2\beta \; M_Z^2 (T_3^q - Q_q s_{\text{w}}^2) \\ M_{\tilde{R}_q}^2 &= M_{\tilde{U}_q}^2 + m_q^2 + \cos 2\beta \; M_Z^2 Q_q s_{\text{w}}^2 \; (q = t, c) \\ X_q &= A_q - \mu(\tan\beta)^{-2T_3^q} \end{aligned} \tag{2}$$

where m_q, Q_q and T_3^q are the mass, electric charge and weak isospin of the quark q; $M_{\tilde{Q}_q}$, $M_{\tilde{U}_q}$ are the soft SUSY-breaking parameters; A_q are the trilinear Higgs couplings to $\tilde{t}$, $\tilde{c}$; μ is the Higgsino mass parameter and $\tan\beta = v_2/v_1$.

Furthermore, $M_{Z,W}$ are the Z and W boson masses, and $s_W = \sin\theta_W$, $c_W = \cos\theta_W$ the electroweak mixing angle. Similar formula can be generated for the $\tilde{b}/\tilde{s}$ sector by replacing $t \leftrightarrow b, c \leftrightarrow s$.

The numerical discussion of NMFV effects and the illustration of the behaviour of m_h and electroweak observables are performed not for the most general case, but for the simpler and well motivated scenario where only mixing between the left-handed components of $\tilde{t}/\tilde{c}$ and $\tilde{b}/\tilde{s}$ is considered. The only flavour off-diagonal entries in the squark-mass matrices are normalized according to $\Delta_{LL}^{t,b} = \lambda^{t,b} M_{\tilde{Q}_3} M_{\tilde{Q}_2}$, following [4, 5] (the parameters λ^t, λ^b are denoted by $(\delta_{LL}^u)_{23}$, $(\delta_{LL}^d)_{23}$ in the above papers), where $M_{\tilde{Q}_3,\tilde{Q}_2}$ are the soft SUSY-breaking masses for the $SU(2)$ squark doublet in the third and second generation.

In detail, we have $\Delta_{LL}^t = \lambda^t M_{\tilde{L}_t} M_{\tilde{L}_c}$, $\Delta_{LR}^t = \Delta_{RL}^t = \Delta_{RR}^t = 0$, for the entries in the matrix (1) and, correspondingly, $\Delta_{LL}^b = \lambda^b M_{\tilde{L}_b} M_{\tilde{L}_s}$, $\Delta_{LR}^b = \Delta_{RL}^b = \Delta_{RR}^b = 0$ for the down-squark mass matrix. NMFV is thus parametrized in terms of the dimensionless quantities λ^t and λ^b. For the sake of simplicity, we have assumed in the numerical analysis the same flavour mixing parameter in the $\tilde{t} - \tilde{c}$ and $\tilde{b} - \tilde{s}$ sectors, $\lambda^t = \lambda^b = \lambda$. The case of $\lambda^t = \lambda^b = 0$ corresponds to the MSSM with minimal flavour violation (MFV).

In order to diagonalize the two 4×4 squark mass matrices, two 4×4 rotation matrices, $R_{\tilde{u}}$ and $R_{\tilde{d}}$, one for the up-type squarks and the other one for the down-type squarks, are needed. Once the squark mass matrices are diagonalized, one obtains the squark mass eigenvalues and eigenstates that obviously depend on the flavour mixing parameter λ. One important consequence of flavour mixing through the flavour non-diagonal entries in the squark mass matrices is generating large splittings between the squark-mass eigenvalues. For $\tilde{t}$ and $\tilde{b}$ squarks, one of the eigenvalues increases with λ and the other one decreases with λ generating a large mass-splitting. For example, in the $\tilde{b}$-sector, the mass-splitting between $\tilde{b}_1$ and $\tilde{b}_2$ is around $700\,\text{GeV}$, with all the SUSY mass parameters fixed to $1\,\text{TeV}$ and $\tan\beta = 40$ [6]. For more details and a list of Feynman rules for the restructured vertices, i.e. the interaction of one and two Higgs or gauge bosons with two squarks, we refer the reader to [3].

2. $\Delta\rho$ AND ELECTROWEAK PRECISION OBSERVABLES

The loop contribution to the electroweak ρ parameter, $\Delta\rho = \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2}$, with the unrenormalized Z and W boson self-energies at zero momentum, $\Sigma_{Z,W}(0)$, represents the leading universal corrections to the electroweak precision observables induced by mass splitting between partners in isospin doublets [7] and is thus sensitive to the mass-splitting effects induced by non-minimal flavour mixing. Precisely measured observables [8] like the W boson mass, M_W, and the effective leptonic mixing angle, $\sin^2\theta_{\text{eff}}$, are affected by shifts according to

$$\delta M_W \approx \frac{M_W}{2} \frac{c_{\text{w}}^2}{c_{\text{w}}^2 - s_{\text{w}}^2} \Delta\rho, \quad \delta\sin^2\theta_{\text{eff}} \approx -\frac{c_{\text{w}}^2 s_{\text{w}}^2}{c_{\text{w}}^2 - s_{\text{w}}^2} \Delta\rho. \tag{3}$$

Beyond the $\Delta\rho$ approximation, the shifts in M_W and $\sin^2\theta_{\text{eff}}$ originate from the complete squark contributions to the quantity Δr and to other combinations of the various vector boson self energies. However, we have numerically verified that $\Delta\rho$ yields an excellent approximation for the full calculation in the case of NMFV effects [3]. The analytical one-loop result for $\Delta\rho$, resulting from squark-loops based on the general 4×4 mass matrix for both the $\tilde{t}/\tilde{c}$ and the $\tilde{b}/\tilde{s}$ sector, has been implemented into the Fortran code *FeynHiggs2.2* [9].

For the numerical evaluation, the m_h^{max} and the no-mixing scenario [10] have been selected, but with a free scale M_{SUSY}. In the m_h^{max} benchmark scenario the trilinear coupling A_t is not a free parameter, obeying $X_t = 2M_{\text{SUSY}}$ ($X_t = A_t - \mu\cot\beta$). In the no-mixing scenario, A_t is defined by the requirement $X_t = 0$. The results are independent of M_A. The numerical values of the SUSY parameter are $M_{\text{SUSY}} = 1\,\text{TeV}$ and $2\,\text{TeV}$, $\tan\beta = 30$, $\mu = 200\,\text{GeV}$. However, the variation with μ and $\tan\beta$ is very weak, since they do not enter the squark couplings to the vector bosons.

In Fig. 1 (left panel) we show the results for $\Delta\rho$ as function of λ for both, the m_h^{max} and no-mixing scenario for two values of the SUSY mass scale. It is clear that $\Delta\rho$ grows with the λ parameter since the splitting in the squark sector increases. One can also see that the effects are close to zero for $\lambda = 0$ and $M_{\text{SUSY}} = 2\,\text{TeV}$, i.e we recover the expected decoupling for $\lambda = 0$. For large values of M_{SUSY} the contribution increases since the splitting in the squark
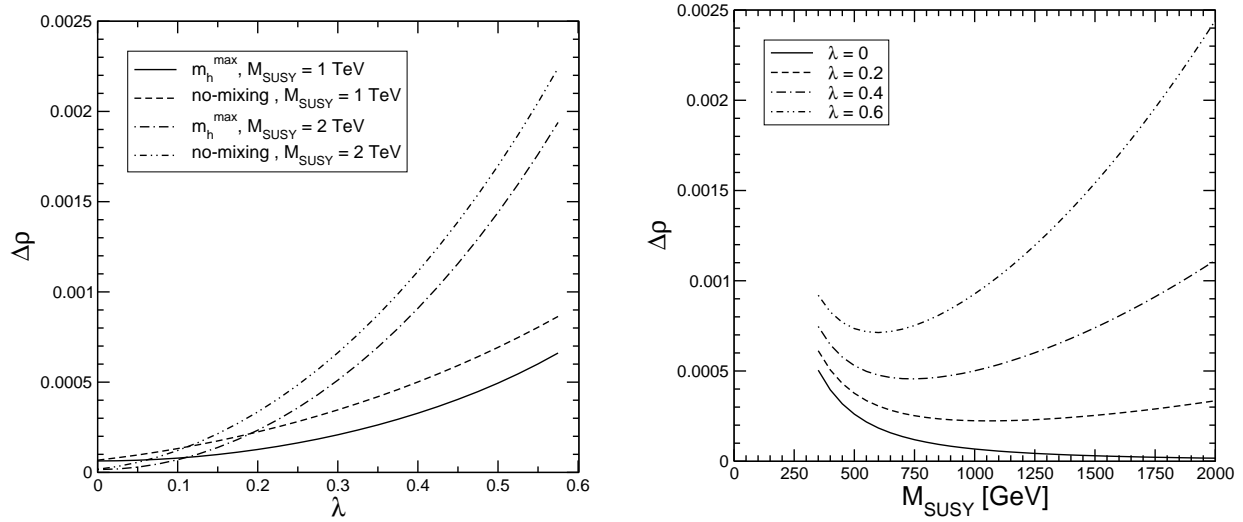
Figure 1: The variation of $\Delta\rho$: with $\lambda = \lambda^t = \lambda^b$ in the $m_h^{\max}$ scenario and no-mixing scenario, for $M_{\mathrm{SUSY}} = 1\,\mathrm{TeV}$ and $2\,\mathrm{TeV}$ (left panel); and with M_{SUSY} in the no-mixing scenario for different values of λ (right panel).

sector increases as well. The dependence on the SUSY mass scale is shown explicitly in Fig. 1 (right panel) in the no-mixing scenario, where the effects on $\Delta\rho$ are largest. The region below $M_{\mathrm{SUSY}} \lesssim 400\,\mathrm{GeV}$ (depending on the scenario) implies too low and hence forbidden values for the squark masses. Correspondingly, for the largest values of λ the excluded region is larger. For $\lambda = 0$ $\Delta\rho^{\tilde{q}}$ decreases, being zero for large M_{SUSY} values as expected. Notice that the experimental bound on $\Delta\rho$, $\Delta\rho \lesssim 2 \times 10^{-3}$, can be saturated.
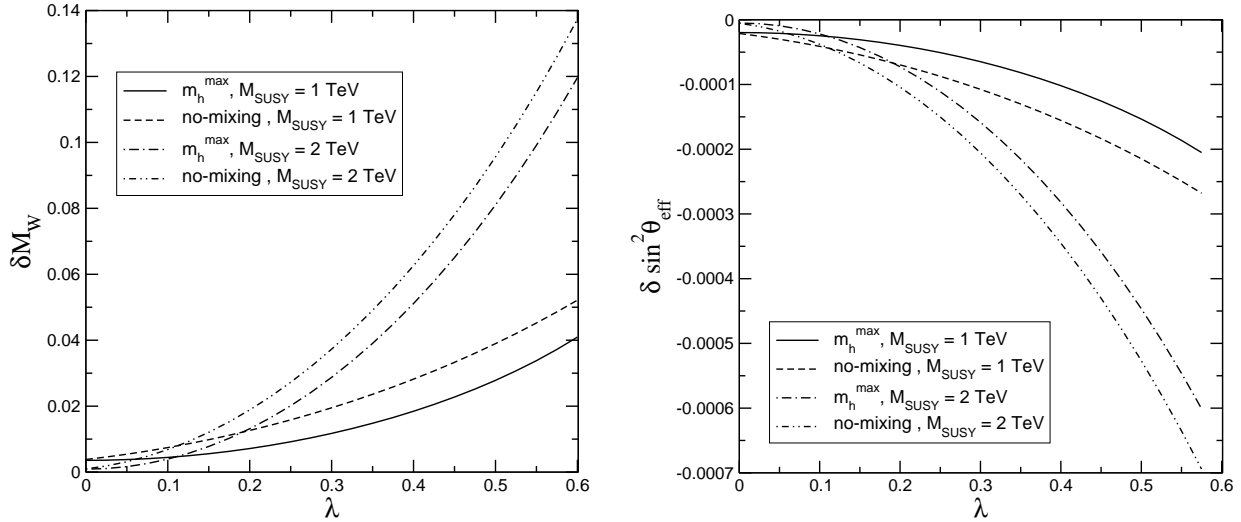

Figure 2: The variation of δM_W and $\delta \sin^2 \theta_{\mathrm{eff}}$ as a function of $\lambda = \lambda^t = \lambda^b$ for the $m_h^{\max}$ and no-mixing scenarios and different choices of M_{SUSY}.

The numerical effects of the NMFV MSSM contributions on the electroweak precision observables, δM_W and $\delta \sin^2 \theta_{\mathrm{eff}}$ as a function of λ are presented in Fig. 2. The $m_h^{\max}$ scenario and no-mixing scenario are included in both plots and we choose two values of M_{SUSY} as before. In the left plot the shifts δM_W are given, which can be as large as $0.14\,\mathrm{GeV}$ for the no-mixing scenario for $M_{\mathrm{SUSY}} = 2\,\mathrm{TeV}$, $\lambda = 0.6$. In the $m_h^{\max}$ scenario δM_W remains smaller, $\delta M_W \lesssim 0.05\,\mathrm{GeV}$, but still sizeable. The correction to $\delta \sin^2 \theta_{\mathrm{eff}}$ is shown in the right plot of Fig. 2. One can see that the shifts $\delta \sin^2 \theta_{\mathrm{eff}}$ can reach values up 7×10^{-4} for $M_{\mathrm{SUSY}} = 2$ TeV and $\lambda = 0.6$ in the no-mixing scenario, being smaller (but still sizeable) for the other scenarios chosen here. These effects have to be compared with the

current experimental uncertainties, $\delta M_W^{\mathrm{exp,today}} = 34\,\mathrm{MeV}$, $\delta \sin^2 \theta_{\mathrm{eff}}^{\mathrm{exp,today}} = 16 \times 10^{-5}$ [8], the expected experimental precision for the LHC, $\Delta M_W = 15 - 20\,\mathrm{MeV}$ [11], and at a future linear collider running on the Z peak and the WW threshold (GigaZ), $\Delta M_W^{\mathrm{exp,future}} = 7\,\mathrm{MeV}$, $\Delta \sin^2 \theta_{\mathrm{eff}}^{\mathrm{exp,future}} = 1.3 \times 10^{-5}$ [12]. Extreme parts of the NMFV MSSM parameter space can be excluded already with todays precision.

3. THE MASS OF THE LIGHTEST HIGGS BOSON

The higher-order corrected masses m_h, m_H of the $\mathcal{CP}$-even neutral Higgs bosons h, H correspond to the poles of the h, H-propagator matrix. The status of the available results for the self-energy contributions to this matrix has been summarized in [13]. Within the MSSM with MFV, the dominant one-loop contributions to these self-energies result from the Yukawa part of the theory (i.e. neglecting the gauge couplings); they are described by loop diagrams involving third-generation quarks and squarks. Within the MSSM with NMFV, the squark loops have to be modified by introducing the generation-mixed squarks. Here we restrict ourselves to the dominant Yukawa contributions resulting from the top and $t/\tilde{t}$ (and $c/\tilde{c}$) sector. Corrections from b and $b/\tilde{b}$ (and $s/\tilde{s}$) could only be important for very large values of $\tan\beta$, $\tan\beta \gtrsim m_t/m_b$, which we do not consider here. The analytical result of the renormalized Higgs boson self-energies, based on the general 4×4 structure of the $\tilde{t}/\tilde{c}$ mass matrix, has then been implemented into the Fortran code *FeynHiggs2.2* [9] that includes all existing higher-order corrections (of the MFV MSSM).

The numerical results for the lightest MSSM Higgs boson mass, m_h, are presented for five benchmark scenarios named "m_h^{max}" (to maximize the lightest Higgs boson mass), "constrained m_h^{max}" (labeled as "$X_t/M_{\mathrm{SUSY}} = -2$"), "no-mixing" (with no mixing in the MFV $\tilde{t}$ sector), "gluophobic Higgs" (with reduced ggh coupling), and "small α_{eff}" scenario (with reduced $hb\bar{b}$ and $h\tau^+\tau^-$ couplings) [10]. For all these benchmark scenarios the soft SUSY-breaking parameters in the three generations of scalar quarks are equal, $M_{\mathrm{SUSY}} = M_{\tilde{Q}_q} = M_{\tilde{U}_q} = M_{\tilde{D}_q}$, as well as all the trilinear couplings, $A_s = A_b = A_c = A_t$.
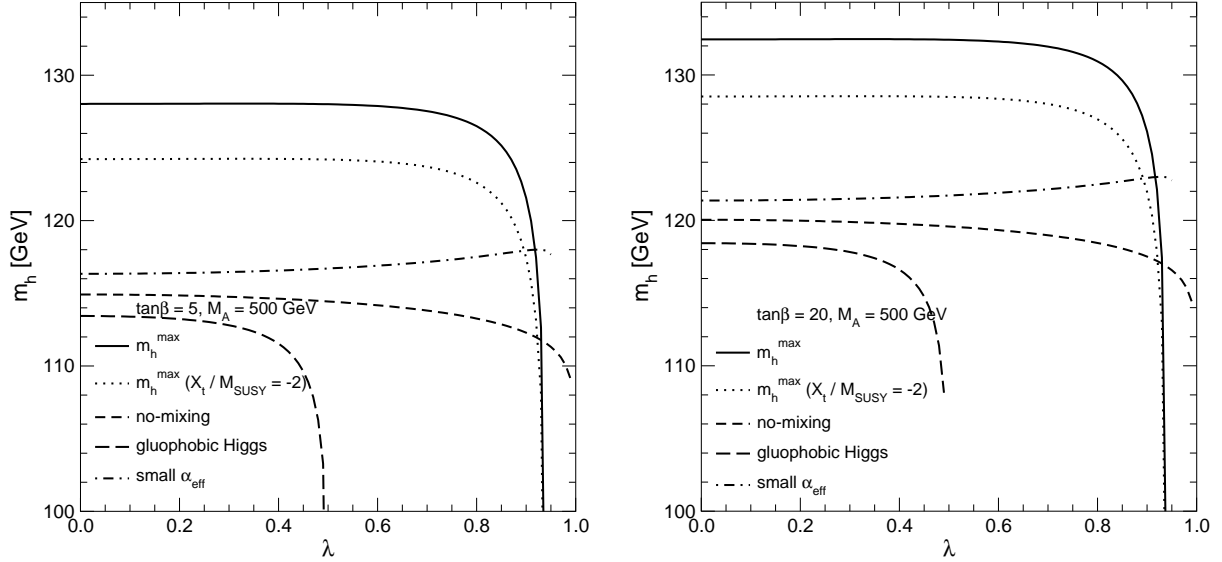

Figure 3: The variation of m_h with $\lambda = \lambda^t$ is shown in five benchmark scenarios [10]. M_A has been fixed to $M_A = 500\,\mathrm{GeV}$, and $\tan\beta$ is set to $\tan\beta = 5$ (left panel) or $\tan\beta = 20$ (right panel).

In Fig. 3 we illustrate the dependence of m_h on $\lambda(= \lambda^t)$ in all five benchmark scenarios. $M_A = 500\,\mathrm{GeV}$ and $\tan\beta$ is set to $\tan\beta = 5$ (left) or $\tan\beta = 20$ (right). All scenarios show a similar behavior. For small to moderate allowed values of λ the variation of m_h is small. Only for large values (around 0.5 in the gluophobic Higgs scenario, and around 0.9 in the other four scenarios) the variation of m_h can be quite strong, up to the $\mathcal{O}(5\,\mathrm{GeV})$. In the gluophobic Higgs scenario unphysical values for the scalar quark masses are reached already for smaller values of λ,

since M_{SUSY} is quite low in this scenario (see [10] for details). Values of λ above 0.5 imply forbidden values for the squark masses in this scenario. In all cases except for the small α_{eff} scenario the lightest Higgs boson mass turns out to be reduced. In the small α_{eff} scenario it can be enhanced by up to $2\,\text{GeV}$. Therefore, the impact of NMFV on m_h is in general rather small, being at present lower than the theoretical uncertainty of m_h, $\delta m_h^{\text{theo}} \approx 3\,\text{GeV}$ [13]. Moreover, independent of low-energy FCNC data on flavour mixing, high values of λ can be constrained by the experimental lower bound on m_h [14].

Results presented in this paper have been later reproduced by using the *FeynArts*, *FormCalc* packages [15], with a model file where the Feynman rules for the general NMFV MSSM (i.e. including the generalized 6×6 squark mixing) have recently been implemented.

4. CONCLUSIONS

We have evaluated the scalar-quark contributions to the lightest MSSM Higgs boson mass, to the ρ-parameter and to the electroweak precision observables M_W and $\sin^2 \theta_{\text{eff}}$, arising from a NMFV mixing between the third and second generation squarks. The analytical results have been obtained for a general 4×4 mixing in the $\tilde{t}/\tilde{c}$ as well as in the $\tilde{b}/\tilde{s}$ sector. They have been included in the Fortran code *FeynHiggs2.2*. The numerical analysis has been performed for a simplified model in which only the left handed squarks receive an additional mixing contribution, parametrized by λ (resp. $(\delta_{LL})_{23}$).

Numerically we compared the effects of NMFV on the mass of the lightest MSSM Higgs boson in five benchmark scenarios. For small and moderate NMFV the effect is small. We have presented the numerical results for the squark contribution to the ρ-parameter. The additional contribution can be of $\mathcal{O}\left(10^{-3}\right)$. Moreover, we have checked that even larger contributions can be obtained if the mixing in the $\tilde{t}/\tilde{c}$ and $\tilde{b}/\tilde{s}$ sector is varied independently. We have also analyzed the NMFV MSSM corrections to the electroweak precision observables M_W and $\sin^2 \theta_{\text{eff}}$, and we conclude that extreme parts of the NMFV MSSM parameter space can be excluded already with todays experimental precision of these observables, and even more for the increasing precision at future colliders. Therefore, scenarios analyzed in the context of B-physics can now be tested, whether they are compatible with electroweak precision observables (where the effects are large) and with Higgs physics (where the effects are small).

Acknowledgments

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