Electroweak Corrections to Four-Fermion Production in $\mathrm{e}^+\mathrm{e}^-$ Annihilation

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The recently completed calculation of the full electroweak $\mathcal{O}(\alpha)$ corrections to the charged-current four-fermion production processes $\mathrm{e}^+\mathrm{e}^- \to \nu_\tau\tau^+\mu^-\bar{\nu}_\mu$, $\mathrm{u}\bar{\mathrm{d}}\mu^-\bar{\nu}_\mu$, and $\mathrm{u}\bar{\mathrm{d}}\mathrm{s}\bar{\mathrm{c}}$ is briefly reviewed. The calculation is performed using complex gauge-boson masses, supplemented by complex couplings to restore gauge invariance. The evaluation of the occurring one-loop tensor integrals, which include 5- and 6-point functions, requires new techniques. The effects of the complete $\mathcal{O}(\alpha)$ corrections to the total cross section and to some differential cross sections of physical interest are discussed and compared to predictions based on the double-pole approximation, revealing that the latter approximation is not sufficient to fully exploit the potential of a future linear collider in an analysis of W-boson pairs at high energies.

1. INTRODUCTION

At LEP2, W-pair-mediated four-fermion ($4f$) production was experimentally explored with quite high precision (see Ref. [1] and references therein). The LEP2 measurements had set the scale in accuracy in the theoretical predictions for W-pair-mediated $4f$ production, as reviewed in Refs. [2, 3]. For LEP2 accuracy, it was sufficient to include corrections in the so-called double-pole approximation (DPA), where only the leading term in an expansion about the poles in the two W-boson propagators is taken into account. Different versions of such a DPA have been used in the literature [4, 5, 6, 7, 8]. Although several Monte Carlo programs exist that include universal corrections, only two event generators, YFSWW [5, 6] and RacoonWW [7, 9, 10], include non-universal corrections.

In the DPA approach, the W-pair cross section can be predicted within $\sim 0.5\%$ (0.7%) in the energy range between $180\,\mathrm{GeV}$ $(170\,\mathrm{GeV})$ and $\sim 500\,\mathrm{GeV}$, which was sufficient for the LEP2 accuracy of $\sim 1\%$. In the threshold region ($\sqrt{s} \lesssim 170\,\mathrm{GeV}$), the DPA is not reliable, and the best available prediction results from an improved Born approximation (IBA) based on leading universal corrections only, and thus possesses an intrinsic uncertainty of $\sim 2\%$.

At a future International $\mathrm{e}^+\mathrm{e}^-$ Linear Collider (ILC) [11, 12, 13], the accuracy of the cross-section measurement will be at the per-mille level, and the precision of the W-mass determination is expected to be $\sim 10\,\mathrm{MeV}$ [14] by direct reconstruction and $\sim 7\,\mathrm{MeV}$ from a threshold scan of the total W-pair-production cross section [11, 12]. The theoretical uncertainty (TU) for the direct mass reconstruction at LEP2 is estimated to be of the order of $\sim 5\,\mathrm{MeV}$ [15] to $\lesssim 10\,\mathrm{MeV}$ [16], based on results of YFSWW and RacoonWW; theoretical improvements are, thus, desirable for an ILC. For the cross-section prediction at threshold the TU is $\sim 2\%$, because it is based on an IBA, and thus is definitely insufficient for the planned precision measurement of M_W in a threshold scan. The TU in constraining the anomalous triple-gauge-boson coupling λ_γ was estimated to be ~ 0.005 [17] for the LEP2 analysis. Since a future ILC is more sensitive to anomalous gauge-boson couplings than LEP2 by more than an order of magnitude, a further reduction of the uncertainties resulting from missing radiative corrections is necessary.

Recently we have completed the first $\mathcal{O}(\alpha)$ calculation (improved by higher-order ISR) for the $4f$ final states $\nu_\tau\tau^+\mu^-\bar{\nu}_\mu$, $\mathrm{u}\bar{\mathrm{d}}\mu^-\bar{\nu}_\mu$, and $\mathrm{u}\bar{\mathrm{d}}\mathrm{s}\bar{\mathrm{c}}$, which are relevant for W-pair production. We have presented results on total cross sections in Ref. [19] and on various differential distributions in Ref. [20]. The latter publication also contains technical details of the actual calculation, which is rather complicated.[1] In the following we briefly describe the salient features of the calculation and present a selection of numerical results that are relevant for a future ILC, comprising total cross sections and some phenomenologically interesting distributions.

2. METHOD OF CALCULATION

The actual calculation builds upon the RacoonWW approach [7], where real-photonic corrections are based on full matrix elements and virtual corrections are treated in DPA. Real and virtual corrections are combined either using two-cutoff phase-space slicing or employing the dipole subtraction method [21] for photon radiation. We also include leading-logarithmic ISR beyond $\mathcal{O}(\alpha)$ in the structure-function approach (Ref. [2] and references therein).

2.1. Technical issues

In contrast to the DPA approach, the one-loop calculation of an $\mathrm{e}^+\mathrm{e}^- \to 4f$ process requires the evaluation of 5- and 6-point one-loop tensor integrals. For the generic $f_1\bar{f}_2 f_3\bar{f}_4$ final state, where f_1 and f_3 are different fermions excluding electrons and electron neutrinos and f_2 and f_4 their isospin partners, there are 40 hexagon diagrams, 112 pentagon diagrams, and 227 (220) box diagrams in the conventional 't Hooft–Feynman gauge (background-field gauge). A survey of Feynman diagrams can be found in Ref. [20]. We calculate the 6-point integrals by directly reducing them to six 5-point functions, as described in Refs. [22, 23]. The 5-point integrals are reduced to five 4-point functions following the method of Ref. [24]. Note that this reduction of 5- and 6-point integrals to 4-point integrals does not involve inverse Gram determinants composed of external momenta, which naturally occur in the Passarino–Veltman reduction [25] of tensor to scalar integrals. The latter procedure leads to serious numerical problems when the Gram determinants become small, which happens usually near the boundary of phase space but can also occur within phase space because of the indefinite Minkowski metric.

We use, however, Passarino–Veltman reduction to calculate tensor integrals up to 4-point functions, which involves inverse Gram determinants built from two or three momenta. This, in fact, leads to numerical instabilities in phase-space regions where these Gram determinants become small. For these regions we have worked out two "rescue systems": one makes use of expansions of the tensor coefficients about the limit of vanishing Gram determinants; in the other, alternative method we numerically evaluate a specific tensor coefficient, the integrand of which is logarithmic in Feynman parametrization, and derive the remaining coefficients as well as the scalar integral from it algebraically. This reduction does not involve inverse Gram determinants.

In addition to the evaluation of the one-loop integrals, also the evaluation of the three spinor chains corresponding to the three external fermion–antifermion pairs is non-trivial, because the chains are contracted with each other and/or with four-momenta in many different ways. There are $\mathcal{O}(10^3)$ different chains to calculate, so that an algebraic reduction to a standard form which involves only very few standard chains is desirable. We have worked out algorithms that reduce all occurring spinor chains to $\mathcal{O}(10)$ standard structures without introducing coefficients that lead to numerical problems. These algorithms are described in Ref. [20] in detail.

2.2. Conceptual issues

The description of resonances in (standard) perturbation theory requires a Dyson summation of self-energy insertions in the resonant propagator in order to introduce the imaginary part provided by the finite decay width into the propagator denominator. This procedure in general violates gauge invariance, i.e. destroys Slavnov–Taylor or Ward

[1] Recently the authors of the GRACE/1-LOOP system reported on progress towards a full one-loop calculation for $\mathrm{e}^+\mathrm{e}^- \to \mu^-\bar{\nu}_\mu \mathrm{u}\bar{\mathrm{d}}$ in Ref. [18] so that one can expect that the system will be able to deal with $\mathrm{e}^+\mathrm{e}^- \to 4f$ processes at one loop in the near future.

identities and disturbs the cancellation of gauge-parameter dependences, because different perturbative orders are mixed (see, for instance, Ref. [3] and references therein). The DPA provides a gauge-invariant answer in terms of an expansion about the resonance, but in the full calculation we are after a unified description that is valid both for resonant and non-resonant regions in phase space, without any matching between different treatments for different regions.

For our calculation we have generalized [20] the so-called "complex-mass scheme", which was introduced in Ref. [9] for lowest-order calculations, to the one-loop level. In this approach the W- and Z-boson masses are consistently considered as complex quantities, defined as the locations of the poles in the complex p^2 plane of the corresponding propagators with momentum p. Gauge invariance is preserved if the complex masses are introduced everywhere in the Feynman rules, in particular, in the definition of the weak mixing angle, which is now derived from the ratio of the complex masses. The (algebraic) relations, such as Ward identities, that follow from gauge invariance remain valid, because the gauge-boson masses are modified only by an analytic continuation. Since this continuation already modifies the lowest-order predictions by changing the gauge-boson masses, double-counting of higher-order effects has to be carefully avoided by an appropriate renormalization procedure.

The use of complex gauge-boson masses necessitates the consistent use of these complex masses also in loop integrals. To this end, we have derived all relevant one-loop integrals with complex internal masses. The IR-singular integrals were taken from Ref. [26]. Concerning the non-IR singular cases, we have analytically continued the results of Ref. [27] for the 2-point and 3-point functions, and the relevant results of Ref. [28] for the 4-point functions. We have checked all these results by independent direct calculation of the Feynman-parameter integrals.

2.3. Checks on the calculation

In order to prove the reliability of our results, we have carried out a number of checks, as described in more detail in Ref. [19]. We have checked the structure of the (UV, soft, and collinear) singularities, the matching between virtual and real corrections, and the gauge independence (by performing the calculation in 't Hooft–Feynman gauge and in the background-field gauge [29]). The most convincing check for ourselves is the fact that we worked out the whole calculation in two independent ways, resulting in two independent computer codes the results of which are in good agreement. All algebraic manipulations, including the generation of Feynman diagrams, have been done using independent programs. The amplitudes are generated with FeynArts, using the two independent versions 1 and 3, as described in Refs. [30] and [31], respectively. The algebraic manipulations are performed using two independent in-house programs implemented in Mathematica, one of which builds upon FormCalc [32]. For the calculation of the loop integrals we use the two independent in-house libraries which employ the different calculational methods sketched above for the numerical stabilization.

3. NUMERICAL RESULTS

The precisely defined input for the numerical results presented in the following can be found in Refs. [19, 20].

Figure 1 depicts the total cross section for the energy ranges of LEP2 and of the high-energy phase of a future ILC, focusing on the leptonic final state $\nu_\tau\tau^+\mu^-\bar{\nu}_\mu$. The respective figures for the relative corrections δ to the semileptonic and hadronic final states look almost identical, up to an offset resulting from the QCD corrections. Specifically, the upper plots show the absolute prediction for the cross section including the full $\mathcal{O}(\alpha)$ corrections and improvements from higher-order ISR. The lower plots compare the relative corrections as obtained from the full $\mathcal{O}(\alpha)$ calculation, from an IBA, and from the DPA. The IBA [10] implemented in RacoonWW is based on universal corrections only and includes solely the contributions of the CC03 diagrams. The DPA of RacoonWW comprises also non-universal corrections [7] and goes beyond a pure pole approximation in two respects. The real-photonic corrections are based on the full $\mathrm{e}^+\mathrm{e}^- \to 4f+\gamma$ matrix elements, and the Coulomb singularity is included for off-shell W bosons. Further details can be found in Ref. [7].

A comparison between the DPA and the predictions based on the full $\mathcal{O}(\alpha)$ corrections reveals differences in the relative corrections δ of $\lesssim 0.5\%$ (0.7%) for CM energies ranging from $\sqrt{s} \sim 170\,\mathrm{GeV}$ to $300\,\mathrm{GeV}$ ($500\,\mathrm{GeV}$). This

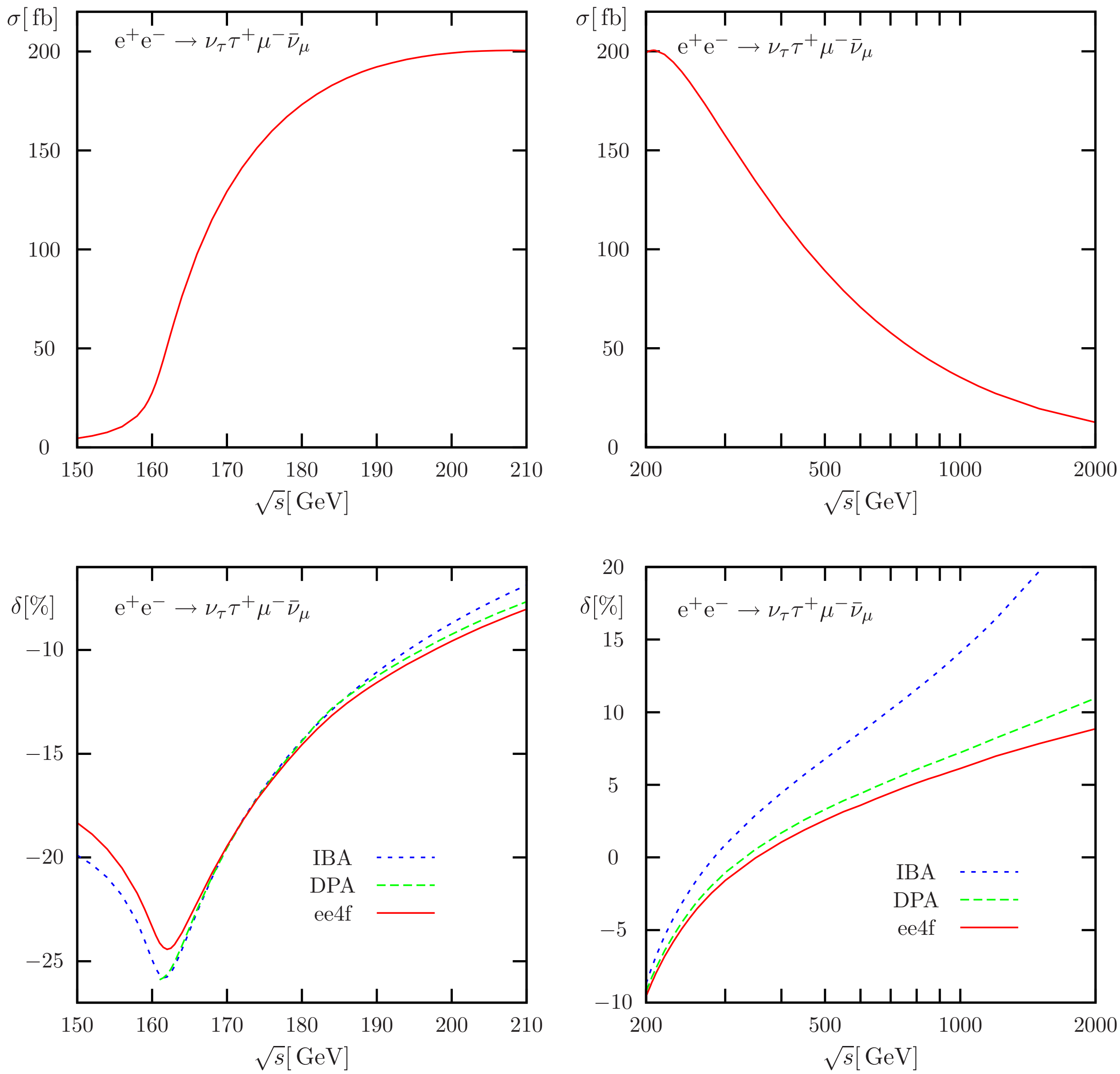

Figure 1: Absolute cross section σ (upper plots) and relative corrections δ (lower plots) to the total cross section without cuts for $e^+e^- \to \nu_\tau \tau^+ \mu^- \bar{\nu}_\mu$ obtained from the IBA, DPA, and the full $\mathcal{O}(\alpha)$ calculation (ee4f). All predictions are improved by higher-order ISR. (Taken from Ref. [19].)

is in agreement with the expected reliability of the DPA, as discussed in Refs. [3, 6, 7]. At higher energies, the deviations increase and reach 1–2% at $\sqrt{s} = 1\text{–}2\,\text{TeV}$. In the threshold region ($\sqrt{s} \lesssim 170\,\text{GeV}$), as expected, the DPA also becomes worse w.r.t. the full one-loop calculation, because the naive error estimate of $(\alpha/\pi) \times (\Gamma_W/M_W)$ times some numerical safety factor of $\mathcal{O}(1\text{–}10)$ for the corrections missing in the DPA has to be replaced by $(\alpha/\pi) \times \Gamma_W/(\sqrt{s} - 2M_W)$ in the threshold region and thus becomes large. In view of that, the DPA is even surprisingly good near threshold. For CM energies below 170 GeV the LEP2 cross section analysis was based on approximations like the shown IBA, which follows the full one-loop corrections even below the threshold at $\sqrt{s} = 2M_W$ within an accuracy of about 2%, as expected in Ref. [10]. More results on total cross sections, including numbers on leptonic, semileptonic, and hadronic final states, can be found in Ref. [19].

The distributions in the invariant mass of the W^+ boson and in the cosine of the W^+ production angle θ_{W^+} are shown in Figure 2 for the process $e^+e^- \to u\bar{d}\mu^-\bar{\nu}_\mu$ at $\sqrt{s} = 500\,\text{GeV}$. Further distributions, also for $\sqrt{s} = 200\,\text{GeV}$,

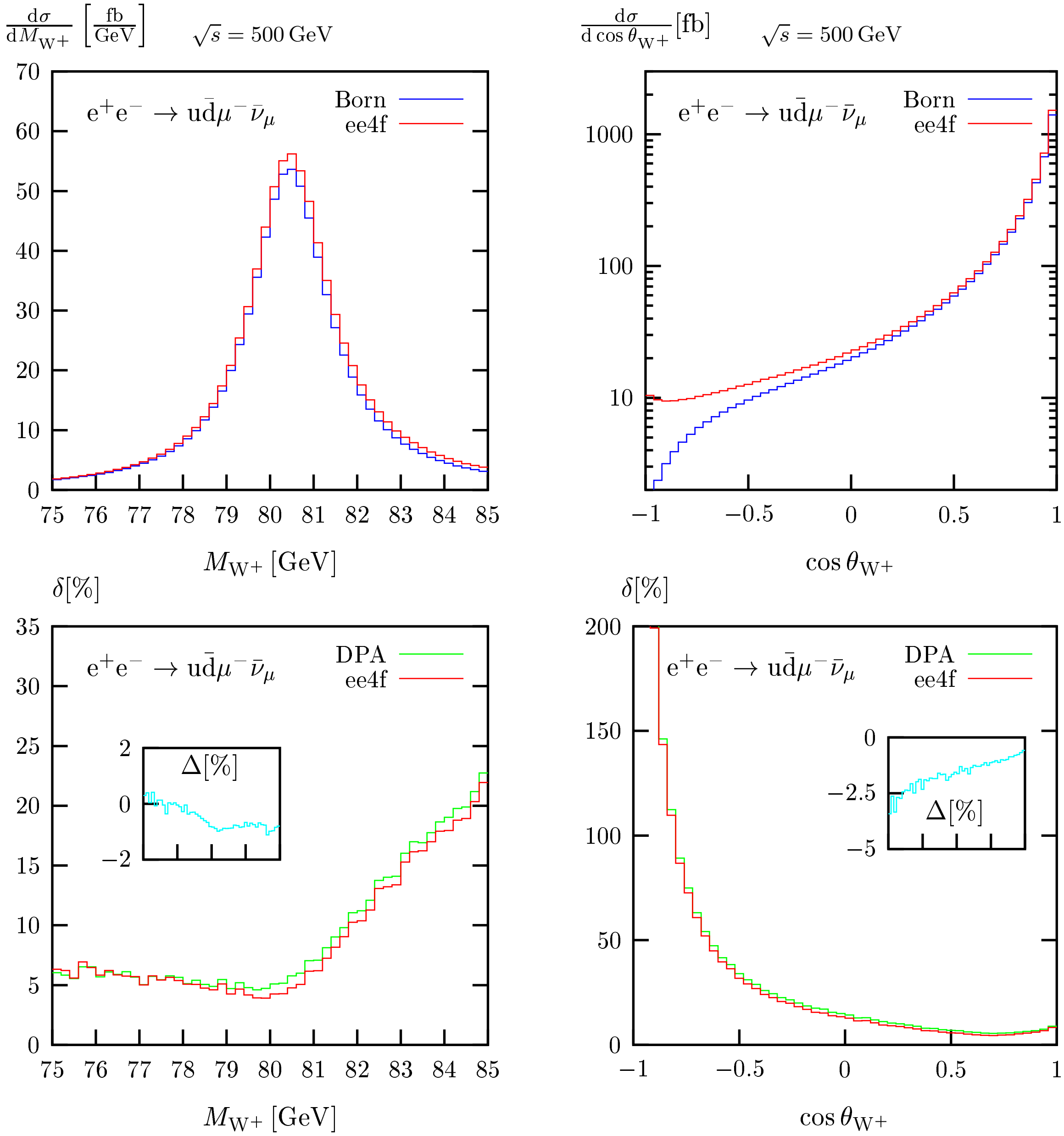

Figure 2: Distributions in the invariant mass of the W^+ boson (l.h.s.) and in the cosine of the W^+ production angle with respect to the e^+ beam (r.h.s.) and the corresponding corrections (lower row) at $\sqrt{s} = 500\,\mathrm{GeV}$ for $\mathrm{e}^+\mathrm{e}^- \to \mathrm{u}\bar{\mathrm{d}}\mu^-\bar{\nu}_\mu$. The inset plot shows the relative difference Δ between the full $\mathcal{O}(\alpha)$ corrections and those in DPA. (Taken from Ref. [20].)

are presented in Ref. [20]. For the invariant-mass distribution the full $\mathcal{O}(\alpha)$ calculation and the DPA agree within $\sim 1\%$ both for LEP2 and ILC energies. For the W-production-angle distribution this is also the case in the LEP2 range (see Fig. 12 of Ref. [20]), but at 500 GeV the difference of the corrections in DPA and the complete $\mathcal{O}(\alpha)$ corrections rises from -1% to about -2.5% with increasing scattering angle. Note that such a distortion of the shape of the angular distribution can be a signal for anomalous triple gauge-boson couplings.

Acknowledgments

This work was supported in part by the Swiss National Science Foundation.

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