30 Years of Lunar Laser Ranging: Implications for Gravity Theory and Cosmology

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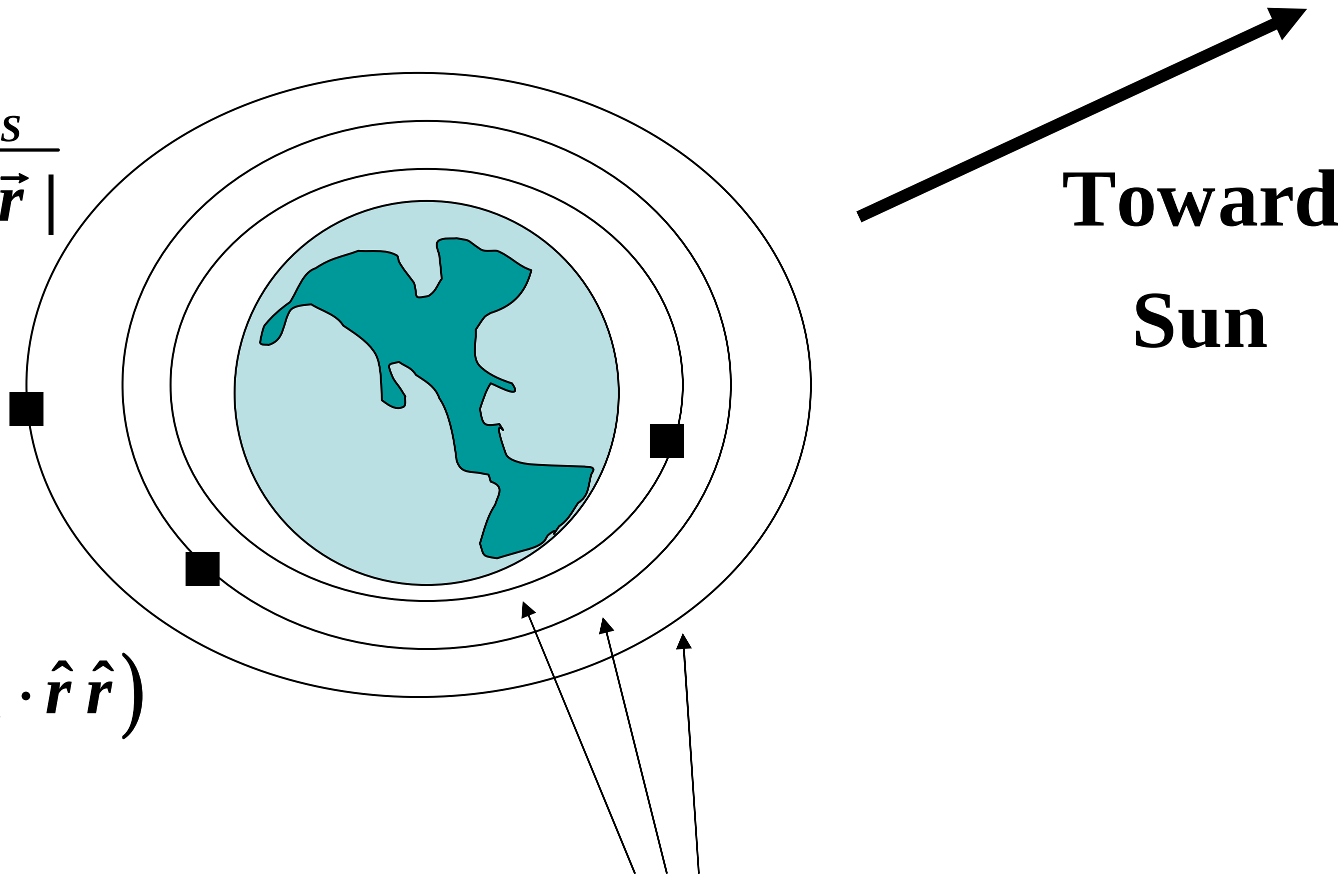
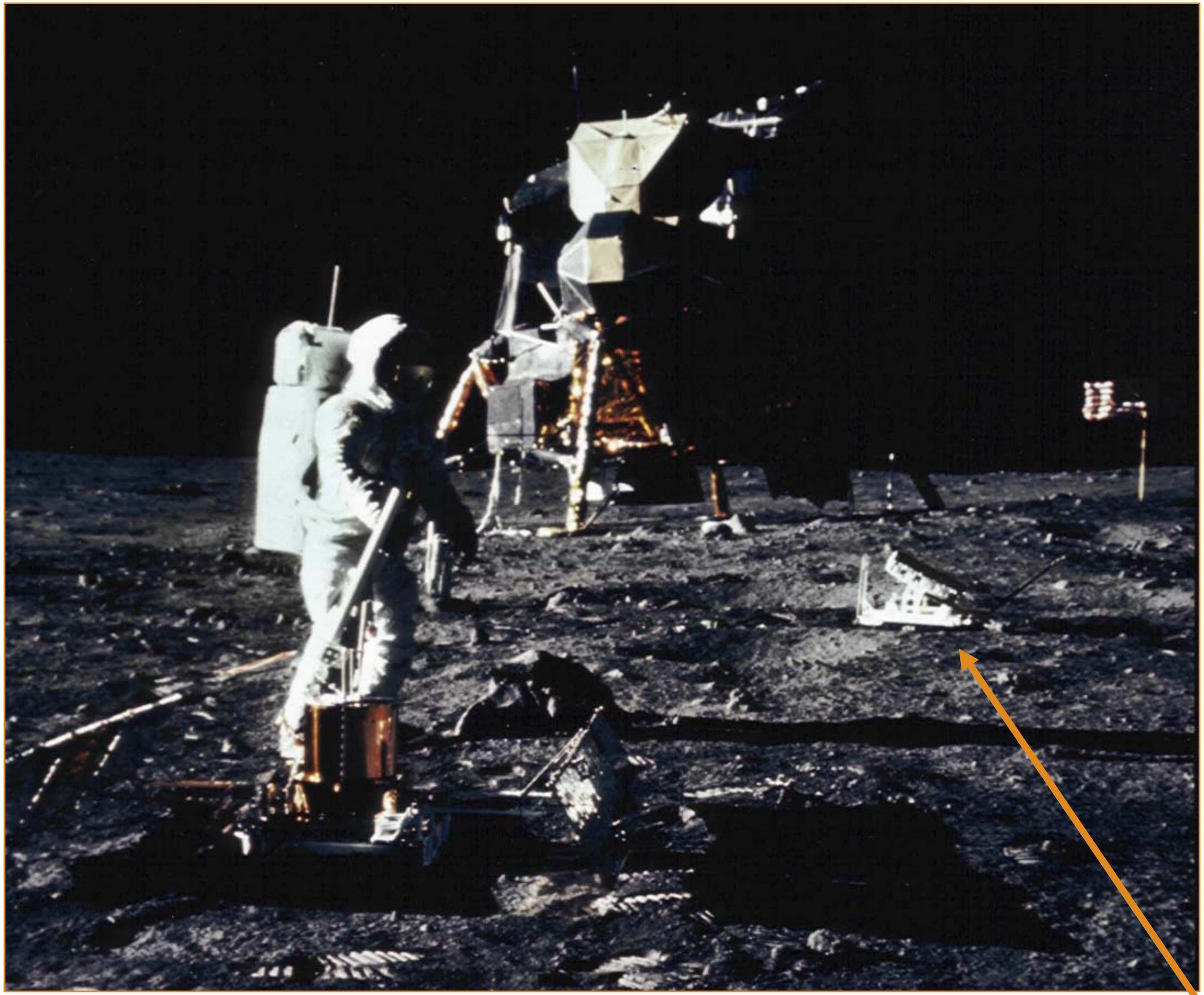
$$\delta\vec{a} \approx \frac{Gm_e}{c^2 r}\vec{\nabla}\frac{GM_S}{|\vec{R}-\vec{r}|}$$



$$\delta\vec{a} = \frac{Gm_e}{2c^2 r}\left(\vec{a}_e + \vec{a}_e \cdot \hat{r}\,\hat{r}\right)$$

Considering lower and lower Earth satellite orbits in order to maximize the strength of General Relativity's non-linear gravitational acceleration, it became clear these non-linear and inertial inductive acceleration fields had to act on the Earth's matter distribution as well.



Passive Laser Reflector Deployed by Apollo Astronaut

Lunar Laser Ranging --- 1969 to the present

Most ranging data from French and Texas Observatories

10 laser pulses per second sent to Moon reflector

Reflected laser photons detected back at Earth every several pulses, and photon round trips timed.

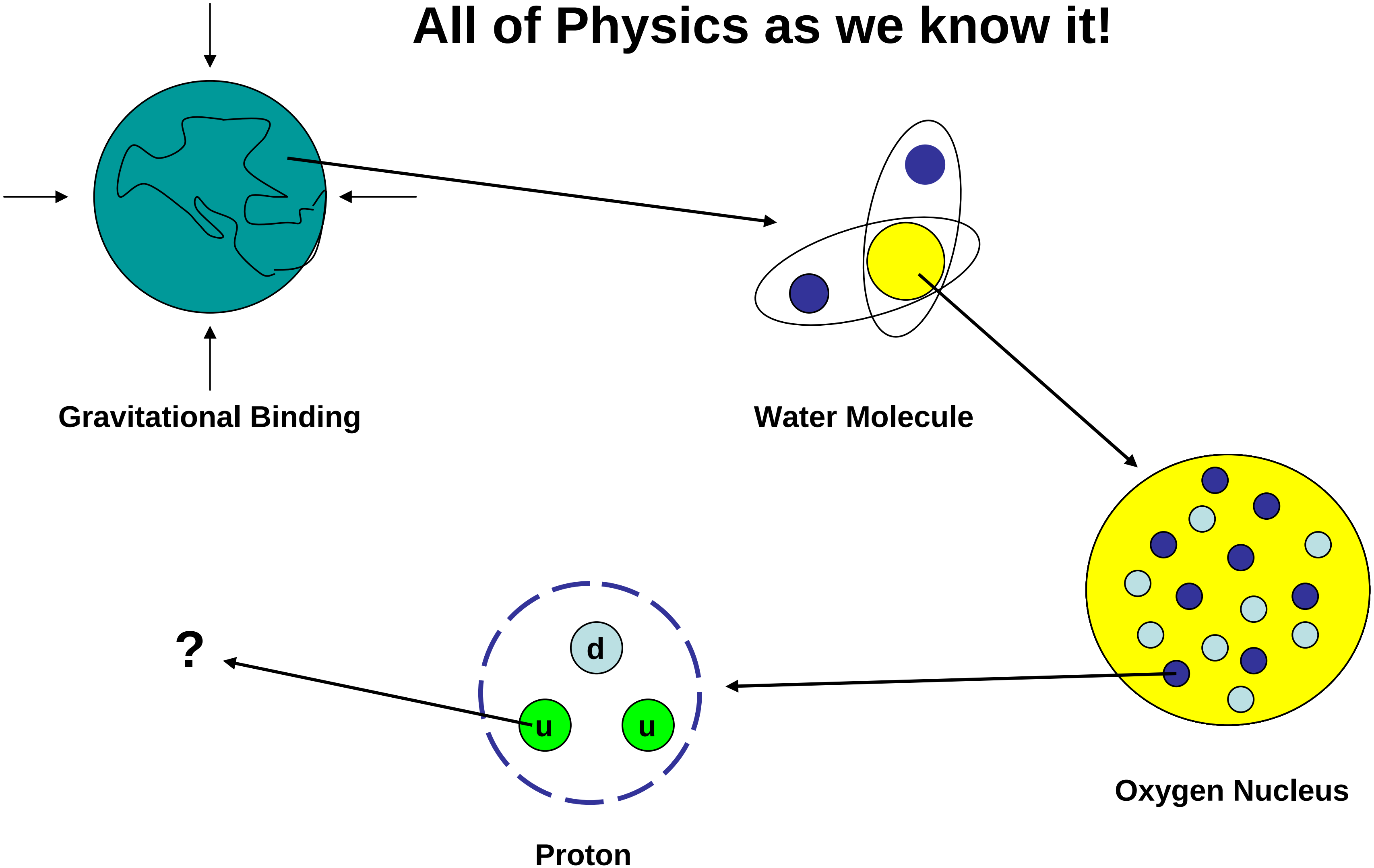
“Range measurement” consists of a number of returns accumuated over tens of minutes, brought to common fiducial time, and averaged.

Over 16,000 range measurements accumulated to date and archived for use by any analysis group.

1 or 2 centimeters = present range accuracy

Most present-day fits of data to theory done by J. Williams and colleagues at Jet Propulsion Lab.

Earth's Mass-Energy consists of nuclear chromodynamic, electromagnetic, weak, kinetic, and gravity contributions. All of Physics as we know it!

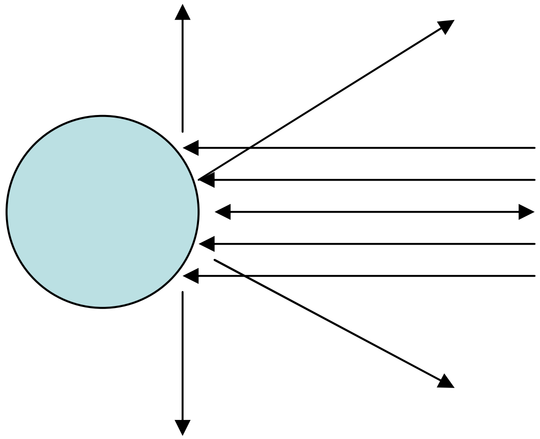


$$\delta \frac{M_{GP}}{M_I} \sim 10^{-13}$$

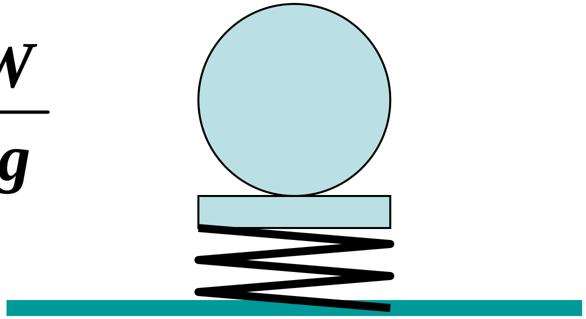
$$\delta \ln GM_{GA} \sim 10^{-10}$$

$$\delta \frac{M_{GA}}{M_{GP}} \sim 10^{-10}$$

$$M_I = \frac{F}{a}$$

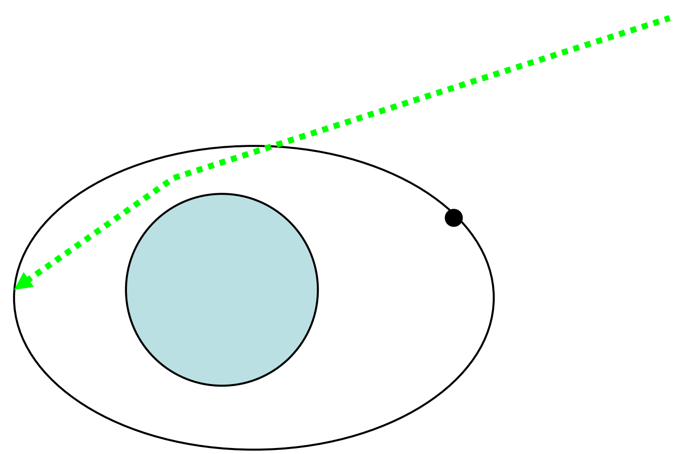


$$M_{GP} = \frac{W}{g}$$

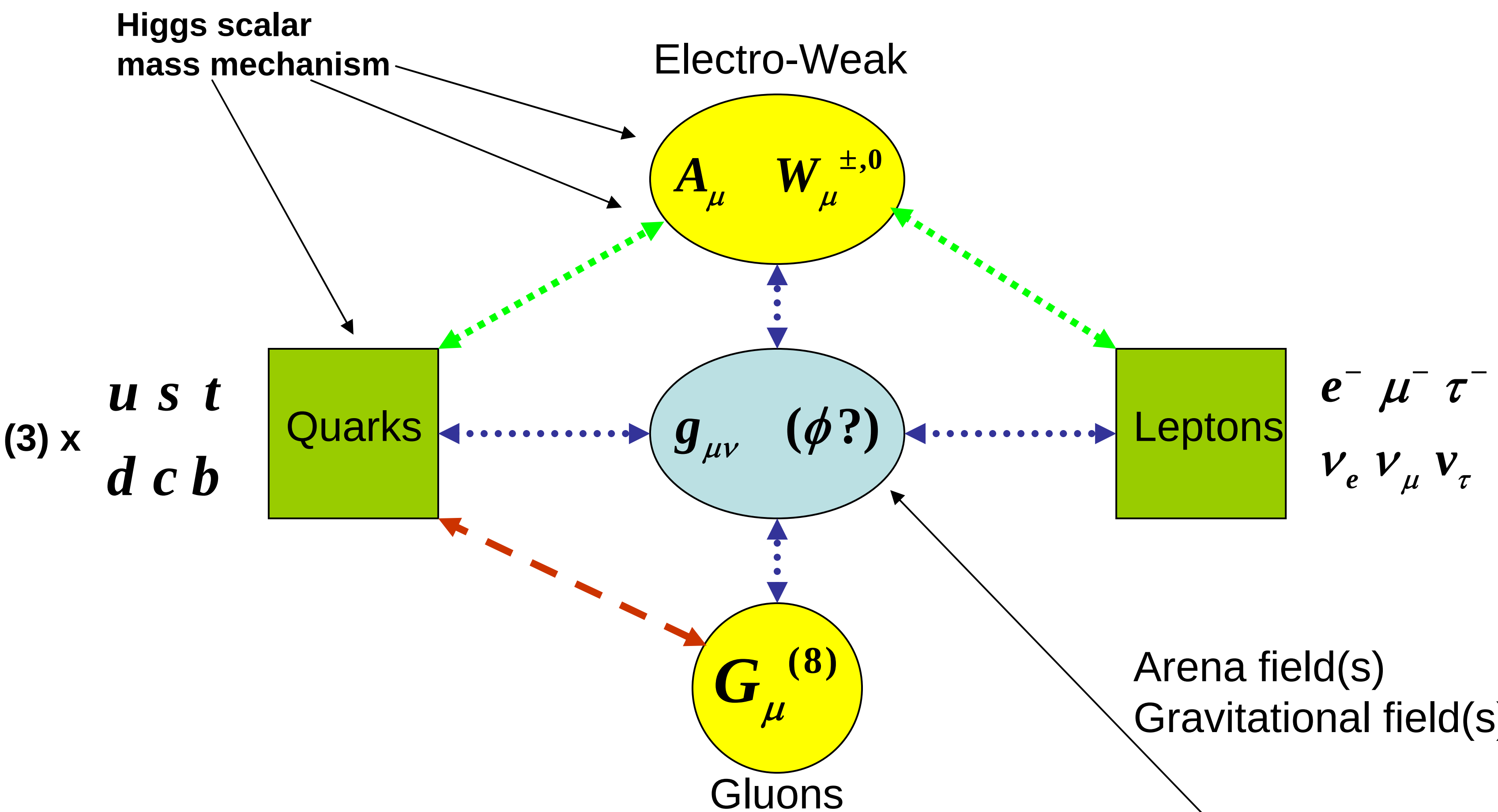


$$\Theta = \frac{4GM_{GA}}{c^2 D}$$

$$\frac{2\pi}{T} = \sqrt{\frac{GM_{GA}}{a^3}}$$



Standard Model plus Gravity

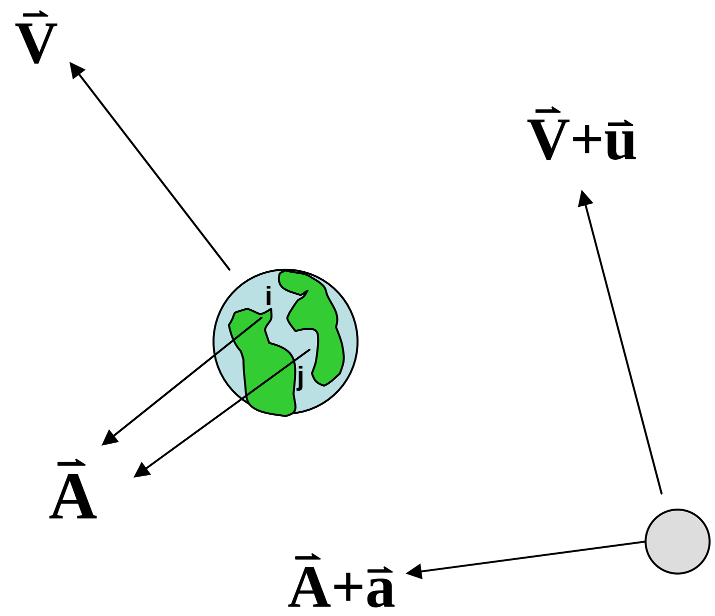


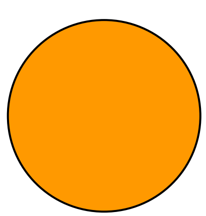
$$\text{"Rigid"} \quad \mathbf{1} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \Rightarrow \begin{pmatrix} g_{00} & \cdots & g_{03} \\ \vdots & \ddots & \vdots \\ g_{30} & \cdots & g_{33} \end{pmatrix} \qquad g_{\mu\nu} = g(\vec{r}, t)_{\mu\nu} \quad \text{"Flexible"}$$

Sun-Earth-Moon System is Comprehensive Relativistic Test Bed

Gravitomagnetism: Earth and Moon exert gravitational forces on each other in proportion to both their motions, $\vec{V}$ and $\vec{V}+\vec{u}$.

Geodetic Precession: Sun's acceleration of Moon differs from Sun's acceleration of Earth because of their different velocities $\vec{V}+\vec{u}$ and $\vec{V}$.





Earth's Gravitational Mass: The non-linear gravity field of (Sun + "i") exerts force on Earth's element "j".

Earth's Inertial Mass: Acceleration of mass element "i" induces an acceleration of Earth's mass element "j".

Post-Newtonian Equation of Motion for N Bodies

$$\vec{a}_i = \left(1 + \frac{dG}{Gdt}(t - t_o)\right)\left(\frac{m_G}{m_I}\right)_i \vec{g}_i \qquad \text{Modified Newtonian}$$

$$\frac{1}{c^2}\left[\begin{array}{ll}
-(2\beta - 1)\sum\limits_{j\neq i}\left(\sum\limits_{k\neq i}\frac{\mu_k}{r_{ik}} + \sum\limits_{k\neq j}\frac{\mu_k}{r_{jk}}\right)\vec{g}_{ij} & \text{Non-linearity} \\
+(2\gamma + 2)\,\vec{v}_i \times \left(\sum\limits_{j\neq i}\vec{v}_j \times \vec{g}_{ij}\right) & \text{Gravitomagnetism} \\
+(\gamma + 1/2)\sum\limits_{j\neq i}\left(v_i{}^2\vec{g}_{ij} - 2\vec{v}_{ij}\cdot\vec{g}_{ij}\,\vec{v}_i\right) & \text{Geodetic Precession} \\
+\sum\limits_{j\neq i}\left(\left[(\gamma + 1)v_j{}^2 - 3(\vec{v}_j \cdot \hat{r}_{ij})^2/2\right]\vec{g}_{ij} - (2\gamma + 1)\vec{v}_j\vec{v}_j\cdot\vec{g}_{ij}\right) & \text{Source Motion} \\
+\sum\limits_{j\neq i}\frac{\mu_j}{2r_{ij}}\vec{a}_j - \frac{1}{2}v_i{}^2\vec{a}_i & \text{Inertial} \\
-(2\gamma + 1)\sum\limits_{j\neq i}\frac{\mu_j}{r_{ij}}\left(\vec{a}_i - \vec{a}_j\right) + \left(\sum\limits_{j\neq i}\frac{\mu_j}{2r_{ij}}\vec{a}_j\cdot\hat{r}_{ij}\,\hat{r}_{ij} - \vec{a}_i\cdot\vec{v}_i\,\vec{v}_i\right) & \text{Misc. Inertial}
\end{array}\right.$$

with $\mu_i = Gm_i$, $\quad \vec{g}_{ij} = \dfrac{Gm_j}{r_{ij}{}^3}\vec{r}_{ji}$, $\quad \vec{g}_i = \sum\limits_{j\neq i}\vec{g}_{ij}$ $\quad$ and $\gamma = 1$, $\beta = 1$ in GR

The Post-Newtonian Lagrangian for N bodies

$$L = -\sum_i m_i \left(c^2 - \frac{u_i^2}{2} - \frac{u_i^4}{8c^2} \right) + \frac{G}{2} \sum_{i \neq j} \frac{m_i m_j}{r_{ij}} \left(1 - \frac{\vec{u}_i \cdot \vec{u}_j + \vec{u}_i \cdot \hat{r}_{ij} \hat{r}_{ij} \cdot \vec{u}_j}{2c^2} \right)$$

$$+ \frac{1 + 2\gamma}{4} \frac{G}{c^2} \sum_{i \neq j} \frac{m_i m_j}{r_{ij}} \left(\vec{u}_i - \vec{u}_j \right)^2 - \frac{\beta^*}{2} \frac{G^2}{c^2} \sum_{i \neq j,k} \frac{m_i m_j m_k}{r_{ij} r_{ik}}$$

$\gamma = \beta^* = 1$ **in General Relativity**

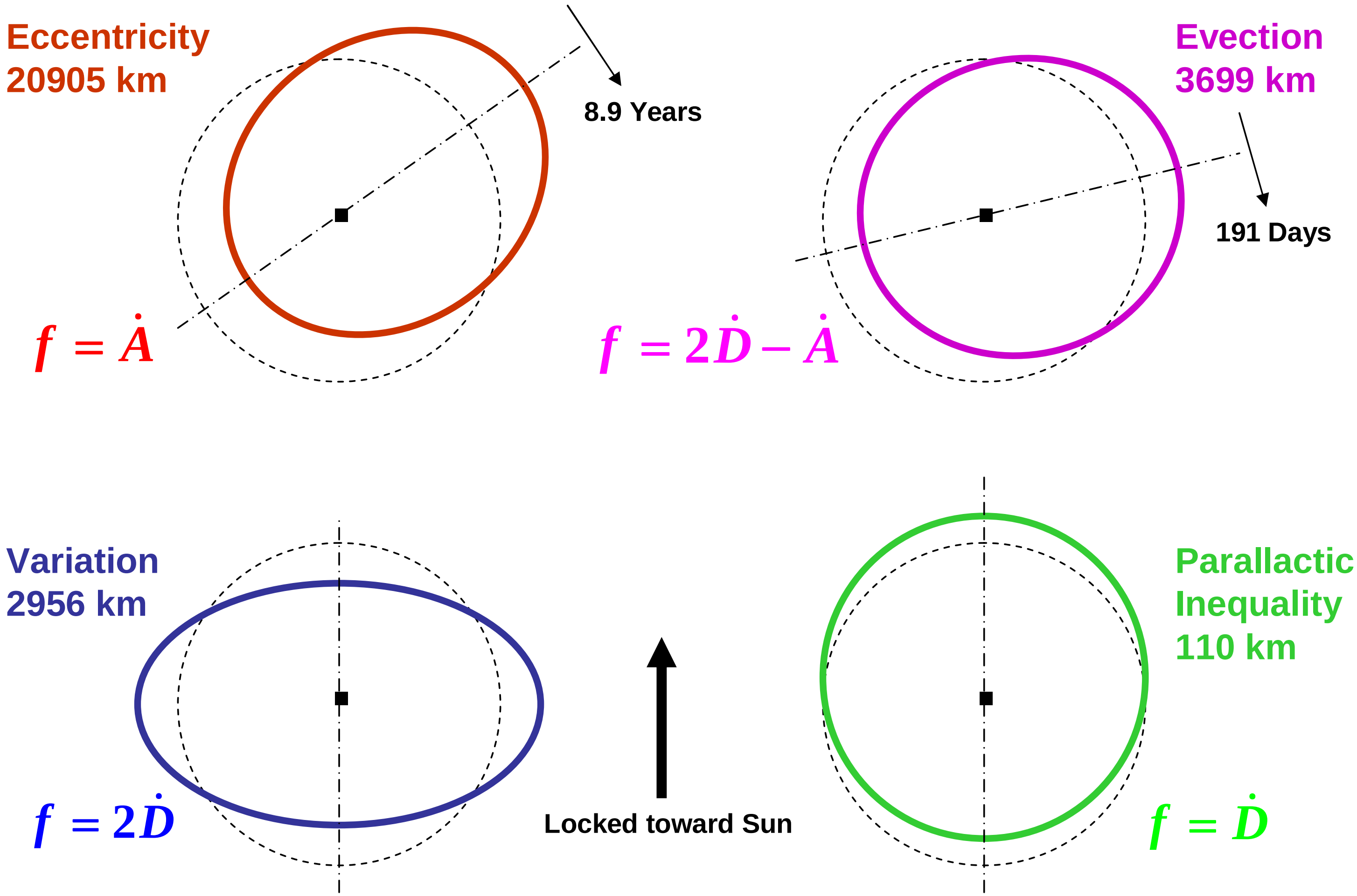
$$\vec{P} = \sum_i \vec{p}_i = \sum_i \frac{\partial L}{\partial \vec{u}_i} \qquad E = \sum_i \vec{p}_i \cdot \vec{u}_i - L \qquad \vec{J} = \sum_i \vec{r}_i \times \vec{p}_i$$

$$\vec{R}_{CE} = \frac{1}{E} \sum_i m_i \vec{r}_i \left(c^2 + \frac{1}{2} u_i^2 - \frac{G}{2} \sum_{j \neq i} \frac{m_j}{r_{ij}} \right) \qquad \vec{V}_{CE} = \frac{c^2 \vec{P}}{E} \qquad \frac{d^2 \vec{R}_{CE}}{dt^2} = 0$$

$$\frac{d\vec{p}_i}{dt} = \frac{\partial L}{\partial \vec{r}_i}$$ **for each body i**

$$\frac{d\vec{P}}{dt} = \frac{dE}{dt} = \frac{d\vec{J}}{dt} = 0$$

Four Key Lunar Orbit Perturbations or “Inequalities”



K. Nordtvedt, LLR, 25 July 2003

$$R(t) = R_o - R_{ecc}\cos(A) - R_{evc}\cos(2D - A) - R_{\mathrm{var}}\cos(2D) - R_{pi}\cos(D) + \dots$$

Anomalistic phase $\quad A = A_o + \dot{A}(t - t_o) + \frac{1}{2}\ddot{A}\,(t - t_o)^2$

Synodic phase $\quad D = D_o + \dot{D}(t - t_o) + \frac{1}{2}\ddot{D}\,(t - t_o)^2$

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1. Whether Sun identically accelerates Earth and Moon determined by R_{pi}.

$$\delta R_{pi} \approx \frac{3}{2}\frac{V_e}{\dot{L}}\Re(\Omega/\omega)\frac{|\vec{a}_e - \vec{a}_m|}{|\vec{g}_s|} \approx 3\,10^{12}\frac{|\vec{a}_e - \vec{a}_m|}{|\vec{g}_s|}\ cm.$$

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2. Whether G varies in time determined by $\ddot{A}$ and $\ddot{D}$.

$$\dot{D} = \dot{L} - \dot{L}_S \qquad \dot{A} = \dot{L}\left(1 - \frac{3}{4}\frac{\Omega^2}{\omega^2} - \frac{225}{32}\frac{\Omega^3}{\omega^3} - \dots\right)$$

$\dot{L}$ **is Moon's sidereal rate** $\qquad \dot{L}_S$ **is Earth's sidereal rate around Sun**

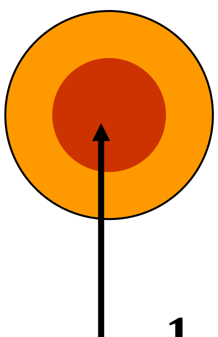
$$\frac{\ddot{L}_S}{\dot{L}_S} = \frac{\ddot{A} - \ddot{D}}{\dot{A} - \dot{D}} + \dots \qquad \text{and} \quad \frac{\dot{G}}{G} = \frac{1}{2}\frac{\ddot{L}_S}{\dot{L}_S} \quad \text{independent of tidal effects}$$

Equivalence Principle Violating Signal (with Feedback)

$$\delta r_{em} \approx \frac{3}{2\dot{L}\dot{L}_S}\left(\frac{1-4\dot{L}_S/\dot{L}+....}{1-7\dot{L}_S/\dot{L}+....}\right)\delta a_{em} \quad \cos D$$

$$\boxed{\delta r_{em} \approx 3\,10^{12}\left|\frac{\delta a_{em}}{g_S}\right| \quad \cos D \quad cm}$$

$$[LLR\ \ fit] \quad \delta r_{em} \approx 3 \pm 4\,mm \quad |\delta a_{em}/g_S| \approx \underline{(-1 \pm 1.4)10^{-13}}$$



$$M_{core} \approx \frac{1}{3}M_e$$

$$[Composition\ dependence] \qquad \boxed{|\delta a_{Fe-Si}| \le 4\,10^{-13}}$$

$$(4\beta - 3 - \gamma) \quad \frac{1}{M_e c^2}\sum_{i,j}\frac{Gm_i m_j}{2r_{ij}} \le 1.3\,10^{-13}$$

$$[Gravity's\ Non-linearity] \qquad \boxed{|\beta - 1| \le\ 0.8\ \ 10^{-5}}$$

If any of the parameters of physical law vary in space-time, there generally will be forces acting on bodies.

$$\vec{F} = - c^2 \frac{\partial M(P)}{\partial \ln P} \vec{\nabla} \ln P \quad \text{for} \quad P = G, \ \frac{e^2}{\hbar c}, \ \ldots.$$

resulting in differential accelerations of bodies

$$\vec{a}_i - \vec{a}_j = -c^2 \frac{\partial \ln M_i / M_j}{\partial \ln P} \vec{\nabla} \ln P$$

From LLR: $\left| c^2 \frac{\vec{\nabla} \ln G}{\vec{g}_{Sun}} \right| \leq 10^{-4}$ $\qquad$ $\left| c^2 \frac{\vec{\nabla} \ln \alpha}{\vec{g}_{Sun}} \right| \leq 10^{-10}$

LLR Results for Time Variation of G

J Williams, S Turyshev, and D Boggs; submitted to Phys. Rev. Lett.

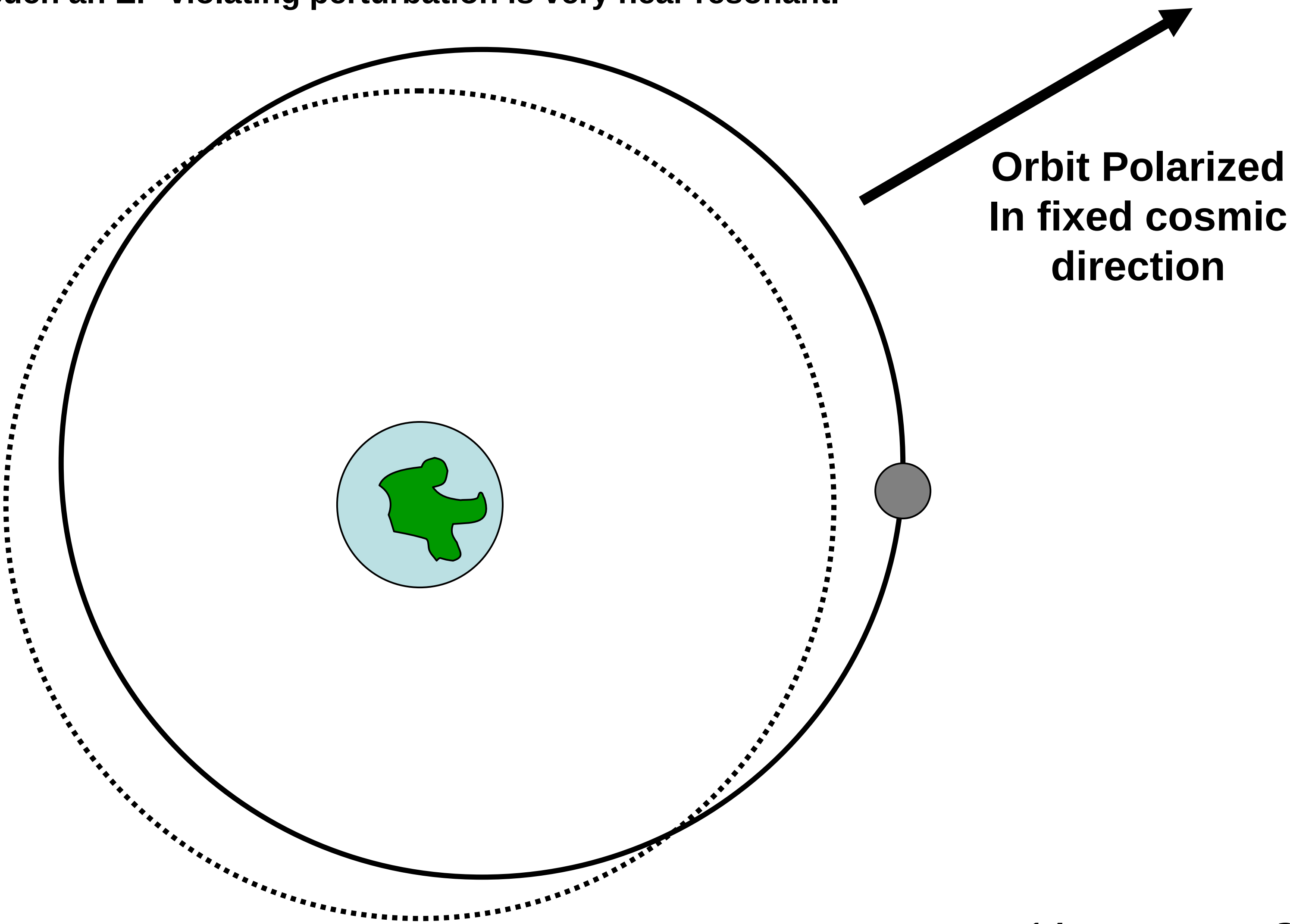
$$\frac{d \ln G}{dt} = (4 \pm 9)\, 10^{-13}\ y^{-1}$$

$$\frac{d \ln G}{H dt} \leq 1.5\, 10^{-2}$$

Unless *dark matter* or *dark energy* act as sources to drive a dG/dt, then the constraint on $|c^2 \vec{\nabla} \ln G / \vec{g}\,|$ from equality of Earth and Moon acceleration toward the Sun is two orders of magnitude more stringent in mapping out relativistic gravity theory.

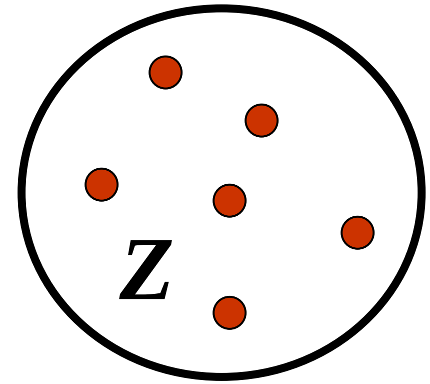
Cosmic or Sidereal EP Violation Test

**Because lunar orbit's perigee precession period is 8.9 years,
such an EP-violating perturbation is very near resonant.**



$$|\vec{a}_e - \vec{a}_m|_{\mathrm{cosmic}} \le 10^{-14} \ \ cm/s^2$$

LLR produces some interesting constraints on space-time variation of the fine structure 'constant'.



$$\delta \vec{a}_i = -\frac{1}{M_i}\frac{\partial M_i(\alpha)}{\partial \ln \alpha} c^2 \frac{\vec{\nabla}\alpha}{\alpha} \qquad \frac{\partial \ln M_i(Z)}{\partial \ln \alpha} \approx 7.5\,10^{-4}\,\frac{Z(Z-1)}{A^{4/3}}$$

$$c^2 \frac{|\vec{\nabla}\ln\alpha|}{|\vec{g}_S|} \leq 10^{-10} \qquad |\vec{\nabla}\ln\alpha|_{Cosmic} \leq 5\,10^{-33}\ cm^{-1}$$

$$\frac{\delta\alpha}{\alpha} \leq 5\,10^{-5} \text{ across Universe} \qquad \frac{\delta\alpha}{\alpha} \leq 1.5\,10^{-10} \text{ across Galaxy}$$

APOLLO

Apache Point Observatory for Lunar Laser-ranging Operations

3.5 Meter Telescope

16 element Avalanche Photodiode Array

Improved Timing Electronics

Millimeter Precision Ranging

Intensive Modeling Improvements

$$\frac{dG}{Gdt} \quad to \quad 10^{-13} \; year^{-1} \qquad |1-\beta| \quad to \quad 10^{-5}$$

$$\left(\frac{M_G}{M_I}\right)_{Earth} - \left(\frac{M_G}{M_I}\right)_{Moon} \quad to \quad 10^{-14}$$