# Black Hole Mergers Form Population III Binaries 

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#### Abstract

We follow the evolution of first population of massive metal-free binary stars. Due to the low metallicity, the stars are allowed to form with large initial masses and to evolve without significant mass loss. Evolution at zero metallicity, therefore, may lead to the formation of massive remnants. In particular, black holes of intermediate-mass $\left(\sim 100-500 \mathrm{M}_{\odot}\right)$ are expected to have formed in early Universe. Assuming that a fraction of these stars forms in binaries we trace their evolution and investigate a possibility that they form coalescing black hole binaries. We calculate the expected detection rates for such massive binaries in the present day LIGO and VIRGO detectors. We also consider the distribution of chirp masses and discuss the likely data analysis strategies.


## 1. Introduction

Population III stars have been a subject of intense theoretical studies. Zero metallicity stars are stable even when their masses reach few hundred solar masses [1]. The remnants for such massive stars are black holes with the mass close to the initial mass of the star as there is hardly any mass loss, except for the stars with initial masses in the range $140-260 \mathrm{M}_{\odot}$, for which pair instability supernovae occur [2]. The initial mass function for zero metallicity stars has been analyzed by a number of authors [3-5], who found that it is very likely to lean towards the high mass end contrary to the population I stars [6-8]. Recently numerical simulations of rotating zero metallicity clouds have shown that formation of massive zero metallicity binaries takes place for a wide range of parameters [9].

As simple model of evolution of zero metallicity binaries has been constructed by (author?) [10]. They have shown that convalescences of the black hole binaries originating in population III stars should be very prominent in the advanced LIGO detector.

In this paper we apply this model to the properties of the current LIGO and VIRGO detectors. We estimate the expected detection rates and discuss the properties of the observed population of such massive black hole binaries.

## 2. The rates

We assume that the sensitivity of LIGO is given by the equations from [11]. To calculate the VIRGO sensitivity to convalescences of black hole binaries of arbitrary masses we used the VIRGO sensitivity curve (http://www.virgo.infn.it/senscurve/), and we
used the formalism of [11]. We assume that the noise curve can be parameterized as

$$
h_{r m s}(f)= \begin{cases}\infty & f<f_{s} \\ h_{m}\left(\frac{\alpha f}{f_{m}}\right)^{-2} & f_{s} \leq f<\frac{f_{m}}{\alpha} \\ h_{m} & \frac{f_{m}}{\alpha} \leq f \leq \alpha f_{m} \\ h_{m}\left(\frac{f}{\alpha f_{m}}\right)^{3 / 2} & \alpha f_{m}<f\end{cases}
$$

with the following parameters:

$$
\left\{\begin{array}{l}
f_{s}=1.3 \mathrm{~Hz} \\
f_{m}=128 \mathrm{~Hz} \\
\alpha=3.2 \\
h_{m}=5.12 \cdot 10^{-22}
\end{array}\right.
$$

Signal to noise ratio of the merger phase for VIRGO detector has been calculated analytically using the formalism based on [11]. We obtain

$$
S / N=\left\{\begin{array}{c}
\mathcal{F}_{m}\left(\epsilon_{m}, M, z, D\right)^{1 / 2}\left[\frac{4}{3} v^{3} \frac{\kappa^{3}-1}{\kappa^{3}}\right]^{1 / 2} \\
\frac{f_{\text {merge }}}{1+z} \geq \alpha f_{m} \\
\mathcal{F}_{m}\left(\epsilon_{m}, M, z, D\right)^{1 / 2}\left[4 \ln v-\frac{4}{3}\left(\frac{v^{3}-\kappa^{3}}{\kappa^{3}}\right)\right]^{1 / 2} \\
\frac{f_{m}}{\alpha} \leq \frac{f_{\text {merge }}}{1+z}<\alpha f_{m} \leq \frac{f_{q n r}}{1+z} \\
\mathcal{F}_{m}\left(\epsilon_{m}, M, z, D\right)^{1 / 2}[4 \ln \kappa]^{1 / 2} \\
\frac{f_{m}}{\alpha} \leq \frac{f_{\text {merge }}}{1+z}<\frac{f_{q n r}}{1+z} \leq \alpha f_{m} \\
\mathcal{F}_{m}\left(\epsilon_{m}, M, z, D\right)^{1 / 2}\left[1+4 \ln \left(\frac{\kappa \alpha^{2}}{v}\right)-\frac{\alpha^{8}}{v^{4}}\right]^{1 / 2} \\
f_{s} \leq \frac{f_{\text {merge }}}{1+z}<\frac{f_{m}}{\alpha} \leq \frac{f_{q n}}{1+z} \\
\mathcal{F}_{m}\left(\epsilon_{m}, M, z, D\right)^{1 / 2}\left[\alpha^{8} v^{-4}\left(\kappa^{4}-1\right)\right]^{1 / 2} \\
f_{s} \leq \frac{f_{m e r g e}}{1+z}<\frac{f_{q n r}}{1+z} \\
\mathcal{F}_{m}\left(\epsilon_{m}, M, z, D\right)^{1 / 2}\left[\left(\frac{\kappa \alpha^{2}}{v}\right)^{4}-\left(\frac{\alpha f_{s}}{f_{m}}\right)^{4}\right]^{1 / 2} \\
\frac{f_{\text {merge }}}{1+z} \leq f_{s}<\frac{f_{q n r}}{1+z} \leq \frac{f_{m}}{\alpha} \\
\frac{f_{q n r}}{1+z}<f_{s}
\end{array}\right.
$$

Parameters $\mathcal{F}_{m}\left(\epsilon_{m}, M, z, D\right), v$ and $\kappa$ are defined as:

$$
\begin{aligned}
\mathcal{F}_{m}\left(\epsilon_{m}, M, z, D\right) & =\frac{\epsilon_{m} M(1+z)^{2}(4 \mu / M)^{2}}{10 \pi^{2} D(z)^{2} h_{m}^{2} f_{\text {merge }}(\kappa-1)} \\
v & =\frac{(1+z) \alpha f_{m}}{f_{\text {merge }}} \\
\kappa & =f_{\text {qnr }} / f_{\text {merge }}
\end{aligned}
$$

where $f_{q n r}$ is the frequency at which merger phase ends and begins ringdown, $f_{\text {merge }}$ is the transition frequency between inspiral and merger, $\epsilon_{m}$ is the fraction of mass-energy radiated during merger, $D(z)$ is the luminosity distance, and $z$ is the redshift. Following [11] we have assumed that $\epsilon_{m}=0.1$. We have calculated the ringdown signal to noise ratio only numerically, because integration function was too much complex to solve it analytically.

In Figure 1 we present the signal to noise ratio for the LIGO 1 and VIRGO detectors for the merger and ringdown phases of a coalescence of an equal mass black hole binary as a function of its total mass.

We use the numerical model of the evolution of population III binaries to calculate the masses and coalescence times of the binaries. Following (author?) [10] we assume that the binary fraction is $10 \%$, we use the same star formation rate of $R_{\text {sfr }} \simeq$ $1.4 \times 10^{-2} \mathrm{M}_{\odot} \mathrm{Mpc}^{-3} \mathrm{yr}^{-1}$ between redshifts 10 and 30. We first calculate the differential merger rate as a function of redshift $d f_{\text {coal }}(z) / d M$ taking into account the delay between formation and coalescence due to gravitational inspiral. The differential coalescence rate per unit observed mass is

$$
\begin{equation*}
\frac{d R}{d M_{o b s}}=\int_{0}^{z_{M}} \frac{d f_{\mathrm{coal}}(z)}{d M} \frac{1}{1+z} \frac{d V}{d z} d z \tag{1}
\end{equation*}
$$

where $M_{o b s}=M(1+z)$ is the observed (redshifted) total mass, $z_{M}$ is the maximum redshift out to which a binary is observable, and $d V / d z$ is the comoving volume element.

The rates are shown in Figures 2 and 3. At the signal to noise ratio of $(\mathrm{S} / \mathrm{N})=10$ the expected VIRGO rate reaches $\approx 1 \mathrm{yr}^{-1}$. We also present the model where the orbits are tightened by dynamical hardening. In this model we assume that the orbits shrink by a factor of 10 leading to decrease of the coalescence times by a factor of $10^{4}$.

## 3. Discussion

The expected rates - see Figures 2 and 3 are around one per year at the $\mathrm{S} / \mathrm{N}=10$ for the initial configurations of LIGO and VIRGO respectively. While


Figure 1: The signal to noise ratio in the merger (thick lines) and ringdown phases (thin lines) of a coalescence of an equal mass black hole binary from a distance of 1 Gpc observed by LIGO 1 (solid lines) and VIRGO (dashed lines).


Figure 2: The LIGO 1 detection rates of population III black hole binaries in the merger (thin lines) and ringdown phases (thick lines). We also present the evolutionary model with hardening by a factor of 10 (dashed lines).
such number is in principle not large it is still of the same order as the expected coalescence rates for compact object binaries formed in the population I and II evolution, [12-14]. Thus it is equally important to search for the population III binary mergers in the data stream. Figure 4 presents the the distributions of expected total red-shifted masses of the binaries for


Figure 3: The VIRGO detection rates of population III black hole binaries in the merger (thin lines) and ringdown phases (thick lines). We also present the evolutionary model with hardening by a factor of 10 (dashed lines).


Figure 4: Differential rate as a function of redshifted total mass of a black hole binary system for VIRGO. Solid line represents case with no hardening, while dashed - with hardening by a factor of 10 . Here, we performed calculations for $S / N>10$.
two model calculations. depending on the model the distribution may have different shapes, but in general the expected masses stretch from 100 to $600 \mathrm{M}_{\odot}$. A non detection of such mergers shall yield constraints on the formation rate of massive population III stars and their binary fraction.

## Acknowledgments

The authors wish to acknowledge the support of the KBN grant PBZ/KBN/054/P03/2001.

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