

# Generation of Large-Scale Magnetic Fields from Dilaton Inflation in Noncommutative Spacetime

K. Bamba, J. Yokoyama

*Department of Earth and Space Science, Graduate School of Science,  
Osaka University, Toyonaka 560-0043, Japan*

Generation of large-scale magnetic fields is studied in dilaton electromagnetism in noncommutative inflationary cosmology, taking into account the effects of the spacetime uncertainty principle motivated by string theory. We show that it is possible to generate large-scale magnetic fields with sufficient strength to account for the observed fields in galaxies and clusters of galaxies through only adiabatic compression without dynamo amplification mechanism in models of power-law inflation based on spacetime noncommutativity without introducing a huge hierarchy between the dilaton's potential and its coupling to the electromagnetic fields.

## 1. INTRODUCTION

It is well established that magnetic fields with the field strength  $\sim 10^{-6}\text{G}$  and coherent scale  $1 - 10\text{kpc}$  exist in our galaxy and other galaxies (for detailed reviews see [1–5]). There is some evidence that they exist in galaxies at cosmological distances [6]. Furthermore, in recent years magnetic fields in clusters of galaxies have been observed by means of the Faraday rotation measurements (RMs) of polarized electromagnetic radiation passing through an ionized medium [7]. In general, the strength and the coherent scale are estimated as  $10^{-7} - 10^{-6}\text{G}$  and  $10\text{kpc} - 1\text{Mpc}$ , respectively. It is very interesting and mysterious that magnetic fields in clusters of galaxies are as strong as galactic ones and that the coherence scale may be as large as  $\sim\text{Mpc}$ .

Although galactic dynamo mechanisms [8] have been proposed to amplify very weak seed magnetic fields up to  $\sim 10^{-6}\text{G}$ , it is only an amplification mechanism, and so requires initial seed magnetic fields to feed on. Furthermore, the effectiveness of the dynamo amplification mechanism in galaxies at high redshifts or clusters of galaxies is not well established. Hence the origin of the magnetic fields on large scales and/or at high redshift has been an open question.

Proposed generation mechanisms of seed magnetic fields fall into two broad categories. One is astrophysical processes, and the other is cosmological processes in the early Universe, *e.g.*, the first-order cosmological electroweak phase transition (EWPT) [9] or quark-hadron phase transition (QCDPT) [10]. However, it is hardly possible for astrophysical processes to generate magnetic fields on megaparsec scales. Moreover, it is also difficult to make the mechanisms at the cosmological phase transitions operate on these scales today, which are much larger than the horizon scale at the epoch of the field generation (see also [11]).

The most natural origin of such a large-scale magnetic field would be electromagnetic quantum fluctuations generated in the inflationary stage [12] (for a comprehensive introduction to inflation see Refs.

[13, 14]). This is because inflation naturally produces effects on very large scales, larger than Hubble horizon, starting from microphysical processes operating on a causally connected volume. However, there is a serious obstacle on the way of this nice scenario as argued below.

It is well known that quantum fluctuations of massless scalar and tensor fields in the inflationary stage create considerable density inhomogeneities [15] or relic gravitational waves [16, 17]. This is because these fields are not conformally invariant even though they are massless. Since the Friedmann-Robertson-Walker (FRW) metric usually considered is conformally flat, cosmic expansion does not induce particle production if the underlying theory is conformally invariant [18]. The classical electrodynamics is conformally invariant. Hence large-scale electromagnetic fluctuations could not be generated in cosmological background. If the origin of large-scale magnetic fields in clusters of galaxies is electromagnetic quantum fluctuations generated and stretched in the inflationary stage, the conformal invariance must have been broken at that time. Several breaking mechanisms therefore have been proposed [12, 19–22].

Recently we have studied the following model of the dilaton electromagnetism [23]. In addition to the inflaton field  $\phi$  we assumed the existence of the dilaton field  $\Phi$  with a potential  $V[\Phi] = \bar{V} \exp(-\tilde{\lambda}\kappa\Phi)$ , where  $\bar{V}$  is a constant, and introduced the following coupling in the electromagnetic part of the model Lagrangian,

$$\mathcal{L}_{\text{EM}} = -\frac{1}{4}f(\Phi)F_{\mu\nu}F^{\mu\nu}, \quad (1)$$

$$f(\Phi) = \exp(\lambda\kappa\Phi), \quad (2)$$

where  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  is the electromagnetic field-strength tensor,  $\lambda$  and  $\tilde{\lambda}$  are dimensionless constants, and  $\kappa^2 \equiv 8\pi/M_{\text{Pl}}^2$  with  $M_{\text{Pl}} = G^{-1/2} = 1.2 \times 10^{19}\text{GeV}$  being the Planck mass. We use units in which  $k_{\text{B}} = c = \hbar = 1$  and adopt Heaviside-Lorentz units of electromagnetism. Such coupling is reasonable in the light of indications in higher-dimensional

theories including string theory. The coupling was first analyzed by Ratra [19]. In his model, however, the inflaton and the dilaton were identified and only the case the dilaton freezes at the end of inflation was considered. We therefore considered a more realistic situation that the dilaton continues its evolution along with the exponential potential even after reheating but is finally stabilized when it feels other contributions to its potential, say, from gaugino condensation [24] that generates a potential minimum [25, 26]. As it reaches there, the dilaton starts oscillation with mass  $m$  and finally decays into radiation with or without significant entropy production. As a result we have shown magnetic fields with the current strength as large as  $10^{-10}\text{G}$  on cluster scale or even larger scale could be generated, but for this to be the case we had to introduce a huge hierarchy between the coupling constant of the dilaton to the electromagnetic field  $\lambda$  and the coupling one  $\bar{\lambda}$  of the dilaton potential,  $\lambda/\bar{\lambda} \approx 400$ .

Note that the existence of the dilaton under discussion is motivated by higher dimensional theories including string theory. The purpose of the present paper is to argue that if we take another prediction of string theory, namely spacetime uncertainty relation, we can solve the above huge hierarchy problem as well [27]. As emphasized by Yoneya [28], the stringy spacetime uncertainty relation (SSUR) is *not* a modification of the ordinary energy-time uncertainty relation in the framework of quantum mechanics, but simply a *reinterpretation* in terms of strings. Hence the SSUR is likely to be very universal in string theories. It is therefore natural and important to take into account both of the two consequences of string theory, the dilaton and the SSUR, simultaneously to the problem of the generation of primordial magnetic fields in the high energy regime of the early Universe.

## 2. MODEL

### 2.1. U(1) gauge field in exponential inflation

From (1) the equation of motion for the U(1) gauge field  $A_\mu$  in the Coulomb gauge,  $A_0(t, \mathbf{x}) = 0$  and  $\partial_j A^j(t, \mathbf{x}) = 0$ , reads

$$\ddot{A}_i(t, \mathbf{x}) + \left(H + \frac{\dot{f}}{f}\right) \dot{A}_i(t, \mathbf{x}) - \frac{1}{a^2} \partial_j \partial_j A_i(t, \mathbf{x}) = 0, \quad (3)$$

in the spatially flat Robertson-Walker Universe  $ds^2 = -dt^2 + a^2(t) d\mathbf{x}^2$ , where  $H$  is the Hubble parameter and  $a$  is the cosmic scale factor. Through the canonical quantization the expression for  $A_i(t, \mathbf{x})$  is given by

$$A_i(t, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} [\hat{b}(\mathbf{k}) A_i(t, \mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{x}} + \hat{b}^\dagger(\mathbf{k}) A_i^*(t, \mathbf{k}) e^{-i\mathbf{k} \cdot \mathbf{x}}], \quad (4)$$

where  $\hat{b}(\mathbf{k})$  and  $\hat{b}^\dagger(\mathbf{k})$  are the annihilation and creation operators which satisfy

$$\begin{aligned} [\hat{b}(\mathbf{k}), \hat{b}^\dagger(\mathbf{k}')] &= \delta^3(\mathbf{k} - \mathbf{k}'), \\ [\hat{b}(\mathbf{k}), \hat{b}(\mathbf{k}')] &= [\hat{b}^\dagger(\mathbf{k}), \hat{b}^\dagger(\mathbf{k}')] = 0. \end{aligned} \quad (5)$$

Here  $\mathbf{k}$  is comoving wave number, and  $k$  denotes its amplitude  $|\mathbf{k}|$ . It follows from Eq. (3) that the Fourier modes  $A_i(t, k)$  satisfy the equation,

$$\ddot{A}_i(t, k) + \left(H + \frac{\dot{f}}{f}\right) \dot{A}_i(t, k) + \frac{k^2}{a^2} A_i(t, k) = 0, \quad (6)$$

and that the normalization condition for  $A_i(t, k)$  reads

$$\begin{aligned} A_i(t, k) \dot{A}_j^*(t, k) - \dot{A}_j(t, k) A_i^*(t, k) \\ = \frac{i}{f(\Phi)a(t)} \left( \delta_{ij} - \frac{k_i k_j}{k^2} \right). \end{aligned} \quad (7)$$

For convenience in finding the solutions of Eq. (6), we introduce the following approximate form as the expression of  $f$ .

$$f(\Phi) = f[\Phi(t)] = f[\Phi(a(t))] \equiv \bar{f} a^{\beta-1}, \quad (8)$$

where  $\bar{f}$  is a constant and  $\beta$  is a parameter.

In slow-roll exponential inflation models with  $a(t) \propto e^{H_{\text{inf}} t}$ , the model parameter  $\beta$  is given by

$$\beta \approx 1 + \tilde{\lambda}^2 w X, \quad X \equiv \frac{\lambda}{\bar{\lambda}}, \quad (9)$$

$$w \equiv \frac{V[\Phi]}{\rho_\phi}, \quad (10)$$

where  $\rho_\phi \cong \text{const}$  is the energy density of the inflaton  $\phi$  and  $w \ll 1$  because we have assumed that during slow-roll inflation the cosmic energy density is dominated by the inflaton potential and the energy density of the dilaton is negligible. Even if the dilaton was rapidly evolving at the onset, its kinetic energy would soon be dissipated, and it is frozen to a value satisfying  $V''[\Phi] \lesssim H_{\text{inf}}^2$ . Thus  $\beta$  takes a practically constant value. Consequently, the solution of Eq. (6) satisfying Eq. (7) with  $H = H_{\text{inf}}$  is given by

$$A_i(k, a) = \sqrt{\frac{\pi}{4H_{\text{inf}} a f(a)}} H_{\beta/2}^{(1)}\left(\frac{k}{aH_{\text{inf}}}\right) e^{i(\beta+1)\pi/4}, \quad (11)$$

where  $H_\nu^{(n)}$  is an  $\nu$ -th order Hankel function of type  $n$  ( $n = 1, 2$ ), and we have determined the constants of integration in the general solution of Eq. (6) by requiring that the vacuum reduces to the one in Minkowski spacetime at the short-wavelength limit.

The energy density of the large-scale magnetic fields can be evaluated using (11) if we specify cosmic evolution after inflation. Here we adopt the following scenario. After inflation, the inflaton potential is instantaneously converted into radiation and then the Universe is reheated immediately at  $t = t_R$ . On the other hand, even after reheating the dilaton continues its evolution along with the exponential potential  $V[\Phi]$  for a while, but is finally stabilized when it feels other contributions to its potential, say, from gaugino condensation [24] that generates a potential minimum [25, 26]. As it reaches there, the dilaton starts oscillation with mass  $m$  and finally decays into radiation with entropy production, and hence the energy density of magnetic fields is diluted. Then the energy density of the magnetic field on a comoving scale  $L = 2\pi/k$  at the present time is given by

$$\rho_B(L, t_0) = \frac{2^{|\beta|-3}}{\pi^3} \Gamma^2\left(\frac{|\beta|}{2}\right) H_{\text{inf}}^4 \left(\frac{a_R}{a_0}\right)^4 \times \left(\frac{k}{a_R H_{\text{inf}}}\right)^{-|\beta|+5} \exp\left(-\tilde{\lambda}\kappa\Phi_R X\right) (\Delta S)^{-4/3}, \quad (12)$$

$$\Delta S \approx \left(\frac{\bar{V}}{\rho_\phi}\right) \left(\frac{2H_{\text{inf}}}{m}\right)^2 \left(\frac{M_{\text{Pl}}}{m}\right), \quad (13)$$

where  $\Phi_R$  is the dilaton field amplitude at the end of inflation and  $\Delta S$  is the entropy ratio after and before dilaton decay. Here the suffixes ‘R’ and ‘0’ represent the quantities at the end of inflation  $t_R$  and the present time  $t_0$ , respectively. From Eq. (12) we see that the large-scale magnetic fields have a scale-invariant spectrum when  $|\beta| = 5$  [23].

After a number of consistency arguments we have found that the magnetic field could be as large as  $10^{-10}\text{G}$  even with the entropy increase factor  $\Delta S \sim 10^6$ , which is the ratio of the entropy per comoving volume after the dilaton decay to that before decay, provided that the energy scale of inflation is maximal and the spectrum of resultant magnetic field is close to the scale-invariant or the red one, namely,  $\beta \gtrsim 5$  [23]. If the generated magnetic fields are as large as  $10^{-10}\text{G}$ , the observed fields in galaxies and clusters of galaxies could be explained through only adiabatic compression without dynamo amplification mechanism. Incidentally, Caprini, Durrer, and Kahniashvili [29] have recently investigated the effect of gravitational waves induced by a possible helicity-component of a primordial magnetic field on cosmic microwave background (CMB) temperature anisotropies and polarization. According to them, the effect could be sufficiently large to be observable if the spectrum of the primordial magnetic field is close to scale invariant and if its helical component is stronger than  $\sim 10^{-10}\text{G}$ . Hence our scenario may be observationally testable.

On the other hand, the seed field required for the dynamo mechanism,  $B \sim 10^{-22}\text{G}$ , could be accounted

for even when  $\Delta S$  is as large as  $10^{24}$  if model parameters are chosen appropriately to realize nearly scale-invariant spectrum.

The serious problem is that the model parameters should be so chosen that the spectrum of generated magnetic field should not be too blue but close to the scale-invariant or the red one, which is realized only if a huge hierarchy exists between  $\lambda$  and  $\tilde{\lambda}$ , namely,  $X = \lambda/\tilde{\lambda}$  should be extremely larger than unity. For example, if we take  $w = 0.01$  and  $\tilde{\lambda} \sim \mathcal{O}(1)$ , we must have  $X \equiv \lambda/\tilde{\lambda}$  as large as  $\approx 400$  so that the amplitude of the generated large-scale magnetic field could be sufficiently large. This may make it difficult to motivate this type of model in realistic high energy theories.

## 2.2. The case of power-law inflation

If we adopt power-law inflation models instead of exponential inflation with the following exponential inflaton potential,

$$U[\phi] = \bar{U} \exp(-\zeta\kappa\phi), \quad (14)$$

we can relax the constraint on  $X$  to a limited extent. Here  $\bar{U}$  is a constant and  $\zeta$  is a dimensionless constant, the spectral index of curvature perturbation  $n_s$  is given by

$$n_s - 1 = -6\epsilon_U + 2\eta_U = -\zeta^2, \quad (15)$$

with

$$\epsilon_U \equiv \frac{1}{2\kappa^2} \left(\frac{U'}{U}\right)^2, \quad \eta_U \equiv \frac{1}{\kappa^2} \left(\frac{U''}{U}\right), \quad (16)$$

where primes denote derivatives with respect to the inflaton field  $\phi$ . According to the first year Wilkinson Microwave Anisotropy Probe (WMAP) data [30],  $n_s \geq 0.93$ , and hence  $\zeta \leq 0.26$ . In this case the scale factor in the inflationary stage is given by  $a(t) \propto t^p$ , where  $p = 2/\zeta^2 \geq 29$ .

In this background, if power-law inflation lasts for a sufficiently long time, the dilaton will settle to the scaling solution [25] where  $U''[\phi] \approx H^2$  with  $H = p/t$ . Hence the solution of the dilaton in this regime is given by

$$\Phi = \frac{2}{\tilde{\lambda}\kappa} \ln\left(\frac{\sqrt{\bar{V}}\tilde{\lambda}\kappa t}{p}\right). \quad (17)$$

Then we find  $\beta$  is constant given by

$$\beta = \frac{2X}{p} + 1. \quad (18)$$

In this case, the solution of Eq. (6) is given by

$$A_i(k, a) = \sqrt{\frac{p\pi}{4(p-1)Haf(a)}} \times H_{\tilde{\beta}/2}^{(1)}\left(\frac{pk}{(p-1)aH}\right) e^{i(\tilde{\beta}+1)\pi/4}, \quad (19)$$

$$\tilde{\beta} \equiv 1 + \frac{p}{p-1}(\beta - 1) = 1 + \frac{2X}{p-1}. \quad (20)$$

The energy density of the large-scale magnetic fields on a comoving scale  $L = 2\pi/k$  at the present time in the above power-law inflation models,  $\tilde{\rho}_B(L, t_0)$ , is given by

$$\begin{aligned} \tilde{\rho}_B(L, t_0) = & \frac{2^{|\tilde{\beta}|-3}}{\pi^3} \Gamma^2 \left( \frac{|\tilde{\beta}|}{2} \right) \left( \frac{p-1}{p} \right)^{|\tilde{\beta}|-1} H_R^4 \\ & \times \left( \frac{a_R}{a_0} \right)^4 \left( \frac{k}{a_R H_R} \right)^{-|\tilde{\beta}|+5} \\ & \times \exp \left( -\tilde{\lambda} \kappa \Phi_R X \right) \left( \Delta \tilde{S} \right)^{-4/3}, \end{aligned} \quad (21)$$

$$\Delta \tilde{S} \approx \left( \frac{\bar{V}}{\rho_\phi} \right) \left( \frac{2H_R}{m} \right)^2 \left( \frac{M_{\text{Pl}}}{m} \right), \quad (22)$$

where  $H_R$  is the Hubble parameter at the end of inflation. Since  $p \gg 1$  as noted above, we find from Eqs. (12), (13), and (20)–(22) that  $\tilde{\beta} \approx \beta$ , which means  $\tilde{\rho}_B(L, t_0) \approx \rho_B(L, t_0)$  by identifying  $H_R$  with  $H_{\text{inf}}$ . Therefore, if  $\beta \approx 5$ , we find  $X \approx 2p \geq 58$  [23]. Consequently, although some progress has been made to lower the required value of  $X$  by adopting power-law inflation, it is far from sufficient because  $X$  should still be much larger than unity in order that the amplitude of the generated magnetic fields could be sufficiently large at the present time. This is because power-law inflation models are hardly distinguishable from exponential inflation under the constraint imposed by WMAP data as far as the evolution of the dilaton is concerned. Because we cannot expect in realistic high energy theories that a huge hierarchy exists between  $\lambda$  and  $\tilde{\lambda}$ , this is a serious problem of our previous model to be solved.

### 3. EFFECTS OF THE STRINGY SPACETIME UNCERTAINTY RELATION

Next, we consider a possible solution to the above huge hierarchy between  $\lambda$  and  $\tilde{\lambda}$  in our model. In recent years the effect of the stringy spacetime uncertainty relation (SSUR) [28]

$$\Delta t \Delta x_{\text{phys}} \geq L_s^2, \quad (23)$$

on metric perturbations in the early Universe have been investigated [31–33], where  $t$  and  $x_{\text{phys}}$  are the physical spacetime coordinates and  $L_s$  is the string scale. In the presence of the cosmic expansion, long-wavelength perturbations observable today emerged from the string-region in the early Universe, hence string-scale physics might leave an imprint on the primordial spectrum of metric perturbations.

The SSUR is compatible with a homogeneous background, but it leads to changes in the action for the metric fluctuations. Both scalar and tensor metric fluctuations can be described by the action of a free scalar field on the classical expanding background. Brandenberger and Ho have first studied the modified action for the cosmological perturbations in non-commutative spacetime, where they have assumed that matter is dominated by a single scalar field [31]. Spacetime noncommutativity at high energies in the early Universe leads to the following two effects. The first is a coupling between the fluctuation mode and the background which is nonlocal in time. The second is the appearance of a critical time for each mode at which the SSUR is saturated, and which is taken to be the time when the mode is generated in the vacuum state in the absence of cosmological expansion. In a background spacetime with power-law inflation, these two effects lead to a suppression of power for large-wavelength modes, compared to the predictions of power-law inflation in standard general relativity. This is because these modes undergo a shorter period of squeezing than they do in the standard calculations in the commutative geometry, that is, large-scale modes, which correspond to higher energies earlier in inflation, are generated outside the Hubble radius owing to stringy effects, and hence experience less growth than the small-scale modes, which are generated inside the Hubble radius at lower energies, and evolve as in the standard case. There is a critical wavenumber  $k_{\text{crit}}$  such that for  $k < k_{\text{crit}}$  the mode is generated on super-Hubble scales, and hence undergoes less squeezing during the subsequent evolution than it does in commutative spacetime. On the other hand, for  $k > k_{\text{crit}}$  the mode is generated on scales inside the Hubble radius, and since the evolution of the mode after that is not different from that in the case of commutative spacetime, it follows immediately that the spectrum for  $k \gg k_{\text{crit}}$  is the same as that in the classical case. Consequently, the spectrum is blue-tilted for  $k \ll k_{\text{crit}}$  rather than red-tilted as it is in the power-law inflation scenario in commutative spacetime. From now on we call the modes  $k \gg k_{\text{crit}}$  the UV ones and the modes  $k \ll k_{\text{crit}}$  the IR ones, respectively.

The spectrum of cosmic microwave background (CMB) anisotropies predicted by the model in [31] has recently been calculated, and thus the prediction of loss of power for infrared modes has been quantified [32]. In addition, Tsujikawa et al. [32] have performed a likelihood analysis at various angular scales to find the best-fit values to the WMAP data of the cosmological parameters, including the power-law exponent  $p$ , which gives the time dependence on the scale factor, and the critical wavenumber when stringy effects become important. As a result they have shown that high energy stringy effects could account for some loss of power on the largest scales that may be indicated

by recent WMAP data. Moreover, the best-fit value for the power-law exponent  $p$  has been found to be  $p \approx 14$ , which is consistent with the result of [33], in which the likelihood value of  $p$  is derived by using recent WMAP data on only two scales  $k = 0.05 \text{ Mpc}^{-1}$  and  $k = 0.002 \text{ Mpc}^{-1}$ . By using the best-fit values, the string energy scale  $L_s^{-1}$  has been estimated as  $L_s^{-1} \approx 10^{14} \text{ GeV}$ . Furthermore, according to their calculation, even a power exponent as small as  $p \approx 5$  is consistent within the current errors.

If we apply the above consequences of the SSUR in a background spacetime with power-law inflation to our previous model [23], from Eq. (20) and  $p \approx 5$  it is expected that  $X$  could be much smaller than in the case of power-law inflation in commutative geometry which requires  $p \geq 29$ .

One may wonder if electromagnetic quantum fluctuations generated in the inflationary stage are also influenced by spacetime noncommutativity, so that the power for the long-wavelength modes should be suppressed. However, the megaparsec scale, in which we are particularly interested, is smaller than the above critical scale  $2\pi/k_{\text{crit}}$ . In fact, according to Tsujikawa et al. [32], the best-fit value for the crossover scale of the power spectrum of density fluctuation,  $k_*$ , which satisfies  $k_* \gg k_{\text{crit}}$ , has been found to be  $2\pi/k_* \approx 2.7 \times 10^2 \text{ Mpc}$ . Hence the megaparsec scale fluctuations at the present time is the UV modes. It is therefore expected that the SSUR has no significant effect on the megaparsec scale fluctuations at the present time and that the amplitude of the generated magnetic field on 1Mpc scale is the same as that in the case of commutative spacetime.

In fact, to confirm the above expectation, we have considered the modified power spectrum of magnetic fields and the strength of the generated magnetic field on 1Mpc scale at the present time in noncommutative spacetime. As a result, we have confirmed that the SSUR has no significant effect on the megaparsec scale fluctuations at the present time and that the amplitude of the generated magnetic field on 1Mpc scale is the same as that in the case of commutative spacetime [27]. Furthermore, for completeness, we have also considered the modified power spectrum of magnetic fields for the IR modes, which are generated outside the Hubble radius. As a result, we have confirmed that the power for the IR modes, namely, the long-wavelength modes larger than the crossover scale  $\sim 2.7 \times 10^2 \text{ Mpc}$ , tends to be suppressed.

As argued above, the important consequence of the SSUR is that the power-law index,  $p$ , of the power-law inflation could be much smaller than in the case of commutative geometry without conflicting with the nearly scale-invariant spectrum of density fluctuations observed by WMAP. Then from Eq. (20) and, say,  $p \approx 5$ , which is in the allowed range now, we find

$$\tilde{\beta} = 1 + \frac{X}{2}. \quad (24)$$

Hence we can generate magnetic fields as strong as  $B \sim 10^{-10} \text{ G}$  on 1Mpc scale even if the two coupling constants of the dilaton,  $\lambda$  and  $\tilde{\lambda}$ , are of the same order of magnitude,  $X = \lambda/\tilde{\lambda} \approx 8$ . For example, we find  $B(1\text{Mpc}, t_0) = 1.0 \times 10^{-10} \text{ G}$  for  $X = 8.1$ ,  $H_R = 10^8 \text{ GeV}$ , and  $\Delta\tilde{S} = 7.0 \times 10^6$ . Moreover, we find similarly that a sufficient magnitude of magnetic fields on 1Mpc scale at the present time for the galactic dynamo scenario,  $B \sim 10^{-22} \text{ G}$ , could be generated even in the case  $X = \lambda/\tilde{\lambda} \approx 6$ .

## 4. CONCLUSION

In the present paper we have discussed a possible solution to the huge hierarchy between  $\lambda$  and  $\tilde{\lambda}$ , which is required in our previous work [23] in order that the spectrum of generated magnetic field should not be too blue but close to the scale-invariant or the red one, so that the amplitude of the generated magnetic field could be sufficiently large, by taking account of the effects of the SSUR on the primordial power spectrum of metric perturbations in the early Universe. As a result we have found that in power-law inflation models, owing to the consequences of the SSUR on metric perturbations, the resultant magnetic fields could have a nearly scale-invariant spectrum even in the case  $\lambda$  and  $\tilde{\lambda}$  are of the same order of magnitude,  $X = \lambda/\tilde{\lambda} \approx 8$ , so that the amplitude of the generated magnetic field could be as large as  $10^{-10} \text{ G}$  on 1Mpc scale at the present time, which is strong enough to account for the observed fields in galaxies and clusters of galaxies through only adiabatic compression without requiring any dynamo amplification [27]. Since the strength of the magnetic fields on megaparsec scales is expressed by the same formula as in the case of commutative geometry, this result is entirely due to the fact that in the presence of the SSUR the power index of power-law inflation could be much smaller than the case of the commutative geometry in order to reproduce the nearly scale-invariant spectrum of density fluctuation observed by WMAP.

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