HIGH PRECISION OBJECT MONITORING BY THEODOLITES USING GRIDLINE-METHODS

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1. THE IDEA OF GRIDLINE-METHODS

The need for contactless object detection and measurement makes special demands on geodetic measuring methods. The natural structure of the object's surface or temporary targets like projected laser spots can be used to define the necessary targets. The measuring system for gridline-methods is theodolite based, using a pointer- and a video-theodolite working in a master-slave mode. The laser spot, which is projected by the master theodolite, has to be detected by the video-theodolite with an integrated CCD-array. The points representing the object are given by the intersections of the surface and virtual lines (*fig.1*) that are defined in a given co-ordinate frame. The general idea of the measuring method is to trace each of that gridlines until its point of intersection is found.

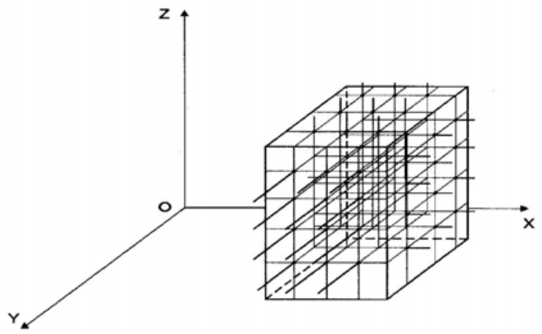

Figure 1: gridlines intersecting an object

The gridline-methods [4, 2] are an answer to the question how to link different measuring epochs if there are no material points on the object which exactly can be found again. The reproducible gridlines are the reference to the initial measurement. Thus deformations or displacements of the object can be detected along these lines. The main field of application could be the survey of barely structured, more or less smooth surfaces.

Fig.2 shows the principle of the search for intersecting points. The points where the gridlines intersect the object are searched in an iterative process. The gridline is given by a point and a vector. On the gridline, there is the intersection point P_0 of the last measurement or the assumed position of the present one. P_0 is targeted by both theodolites, but because of object variations P_0 is not identical with the current intersection point P_1. Therefore the laser spot is projected onto Q. Now, Q is searched and its position is determined. After that an iterative process will shift the projected point to P_1.

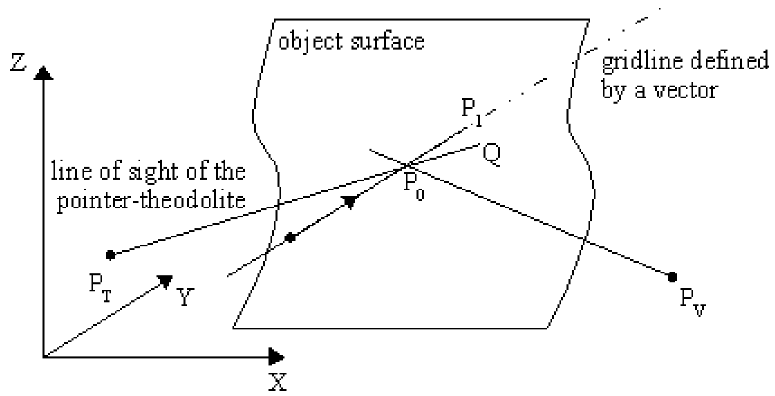

Figure 2: searching the intersecting point P_1

The result is an arbitrarily dense grid of points representing the object's surface. On the assumption that the detected deformations are based on either a deformation or a rigid-body motion the object's change can be reconstructed. Test measurements in the range of industrial applications yielded to point accuracies better than 0.1mm [5].

In the case of deformation, some non-changing reference points are needed, e.g. points on fixed borders. Otherwise it is not possible to distinct clearly deformation and rigid-body motion. Then the object can be modeled by a surface using the known points as mesh points. Each measuring epoch will lead to a slightly modified surface.

The surface is the same at every measuring epoch for rigid-body motions. The shape of the object is known before starting to measure or it is modeled after the first epoch. By defining gridlines with different orientations, e.g. parallel to the axes of co-ordinates, the spatial motion of the object can be captured.

Following the main subject of the explanations are the algorithms to find the intersecting points. A second point of interest is if only straight lines can be used as gridlines or if it is possible to use any spatial curve for monitoring expected motions.

2. SEARCH ALGORITHMS

The gridline-methods were first investigated by [2]. Several algorithms were designed to find the searched point with only a few iteration steps.

Among those algorithms the gridline-method based on the binary search was the most efficient one. The principle of the other algorithms was the optimization of two variables. This led to sophisticated structures but not to an acceleration of the search. Therefore two new algorithms were designed and tested [7] which work with just one variable.

2.1. Measures for controlling the search

For first considerations each gridline *GL* is defined by a given point and a direction vector (*fig.3*): $GL: \quad \vec{x} = \vec{o} + \lambda \cdot \vec{r}$.
On this straight line every point can be determined by the variable λ .

As the searched point has to be situated on the gridline, a more or less arbitrarily chosen distance between the observed point and the gridline is an obviously good criterion to decide whether the intersection point has been found or not. This distance, or distance function resp., must have its zero or minimum there. Finding of a zero of a function is a well known problem and many solutions exist. Finding a minimum is a matter of optimization methods, which is closely related to zero finding if an univariate function is analysed [3].

The distance function f can be chosen in dependence on the parameter λ:

$$f = f(\lambda) \quad .$$

If the laser pointer beams at a certain point $P_i = P_i(\lambda_i)$ on the gridline, the spot will be projected at Q_i on the object. The distance Δl_i between P_i and Q_i along the laser ray can be that function (*fig.3*):

$$f(\lambda_i) = \Delta l_i = |\vec{p}_i| - |\vec{q}_i| \quad .$$

All algorithms suggested and investigated in [7] make use of a search interval. Determined along the gridline in a first step, this interval contains the intersecting point and will be shortened in every iteration step of the search. The interval guarantees the existence of a solution, and concerning the point accuracy it is an interval of uncertainty.

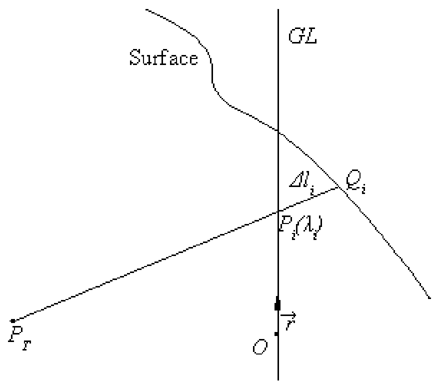

Figure 3: definition of the distance function

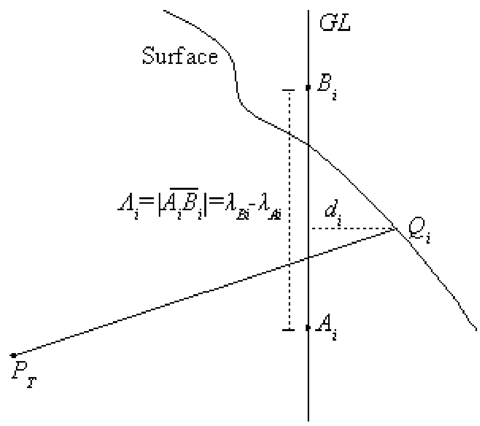

Figure 4: truncation criteria

The iterative search will be truncated when the projected point is located on the gridline within a given tolerance. To keep this tolerance one or two measures are regarded [7], which must be below thresholds. The first one is the (perpendicular) distance d_i between the currently projected point Q_i and the gridline (*fig.4*). Additionally a second one can be taken. This is the length Λ_i of the current interval (*fig.4*).

2.2. Gridline-method based on regula-falsi method

If the truncation criterion is given by the perpendicular distance only, the most efficient algorithm is that one based on the regula-falsi method, a modification of the classical secant method [7].

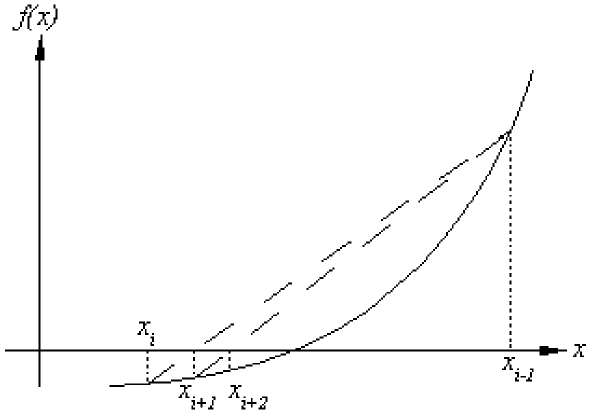

Figure 5: the regula-falsi method

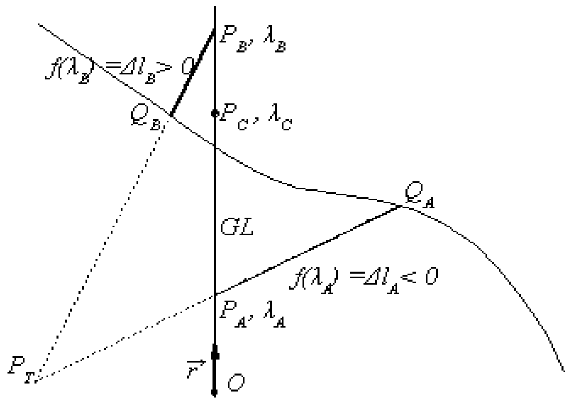

Figure 6: gridline-method using regula-falsi

The classical secant method is a method of linear interpolation. The problem is the possible divergence if the approximating straight line is an extrapolation [3]. In this case both used function values have the same sign. So, the next predicted zero may lie far away from the latest prediction and the real zero. This leads to the slightly modified regula-falsi method (*fig.5*). In this method it is ensured that the function values of the last two approximations x_{i-1} and x_i have different signs because every newly calculated approximation x_{i+1} does not automatically replace x_{i-1} but replaces the older approximation with the same sign in its function value. Therefore every new approximate zero x_{i+1} lies strictly between x_{i-1} and x_i, who define the iteratively reduced interval of uncertainty.

As mentioned before, for the gridline-methods the variable is λ and the corresponding function value is Δl . The next approximate value λ_C (*fig.6*) for the zero is calculated by

$$\lambda_C = \lambda_B - \frac{\lambda_B - \lambda_A}{\Delta l_B - \Delta l_A} \cdot \Delta l_B \qquad .$$

Then, for the next iteration the points P_A and P_B on the gridline (*fig.6*) have to be redefined for spanning the next search interval. The iteration stops if a truncation criterion is fulfilled.

After some simulated tests in MATLAB test measurements were carried out with the real system. Starting with an interval in the range of millimetres up to some centimetres and with a truncation criterion of d<0.5mm the algorithm using the regula-falsi method found the intersecting points after very few search steps, the median of the needed steps in nearly all test cases was one [7].

Defining the truncation criterion by the distance *d* and the interval length Λ means an increase of the necessary iterations. The choice of the kind of the truncation criterion and the magnitudes of the thresholds depend on the particular demands. The main measure of quality and reference to former measuring epochs is the distance between point and gridline. The lower bound is given by the measuring accuracy of the used system, which is about 0.1mm in this case [5].

A possible problem is the demand for a maximum allowed interval length. Since some algorithms, like the regula-falsi method, cannot sufficiently reduce the interval under certain circumstances. The interval must enclose the intersecting point. Therefore its end points have to be different in sign. Thus, it can happen that every newly determined end point of the interval is situated on one side of the zero, i.e. the intersecting point, and the interval will never be shorter than the distance between the other end point and the zero.

During the measurements this phenomenon should happen, but actually it was not of such importance. The demanded interval reduction was achieved in most cases. The reason lies in the inaccuracy of capturing the laser spot. If the spot is already projected in the direct neighbourhood of the intersecting point or on itself, little unavoidable inaccuracies of the measurement and the calculation let the function value fluctuate between positive and negative sign. By this an interval length beneath the measuring accuracy will be achieved, finally.

2.3. Comparison of the results

In addition to the mentioned binary search and regula-falsi method a third algorithm was designed and tested. Without going into details the main idea of this third method is an approximation of the function by a parabola. This parabola method determines the intersecting point by searching the minimum of an unknown true distance function. In every iterative step the next approximate value of the searched point is calculated by three predecessors.

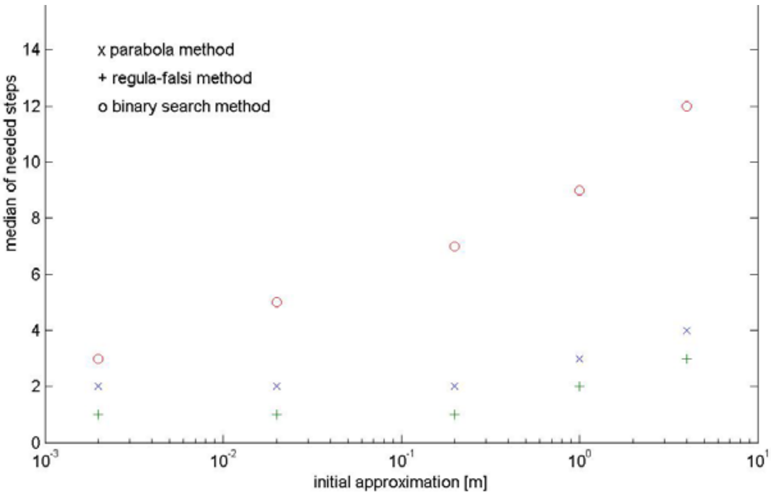

Figure 7: simulated tests, truncation criterion: distance

The typical behaviour of the algorithms can be illustrated by the test case of a planar surface. Both in the simulated (*fig. 7 to 10*) and in the real (*fig. 11, 12*) measurements the surface was given by a plane with a protrusion. For both possible truncation criteria the regula-falsi method is the fastest, i.e. the one with the fewest iterative steps (*fig. 7, 8, 11 and 12*).

The number of unsuccessful searches strongly depends on the chosen kind of truncation criterion and the chosen algorithm itself. The binary search succeeds every time. If the truncation criterion is given only by the distance between point and gridline (*fig. 9*), the regula-falsi method works very reliably, too. If the truncation criterion is given by the distance and the interval length (*fig. 10*), the reliability of the parabola and the regula-falsi method obviously decreases, at least in the simulated measurements. The better reliability in the real measurements are caused by the relatively good initial approximate values (cp. *fig. 11, 12*) and by the accuracy effect already mentioned above (cp. chap. 2.3). In the real measurements, only the parabola method leads to some few unsuccessful searches if additionally an interval length is demanded in the truncation criterion. In all other cases the algorithms succeed.

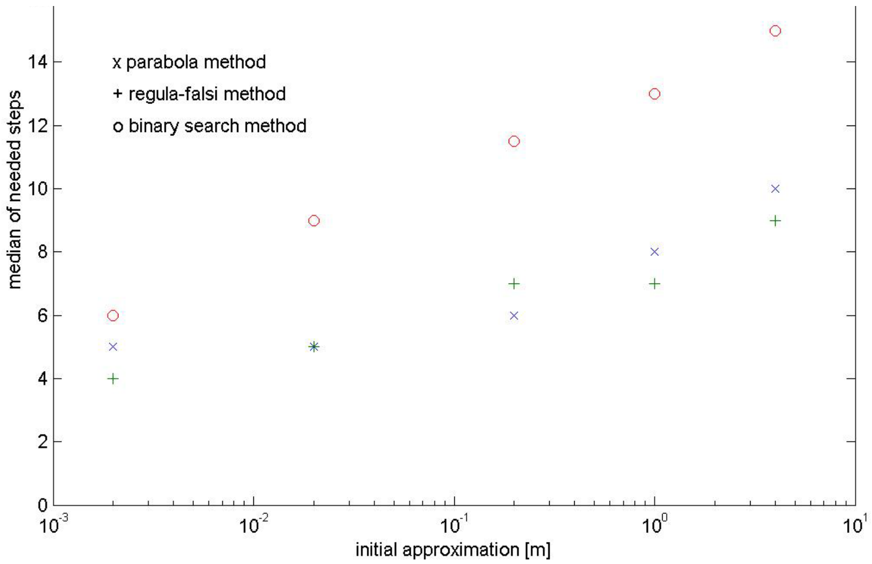

Figure 8: simulated tests, truncation criterion: distance and interval length

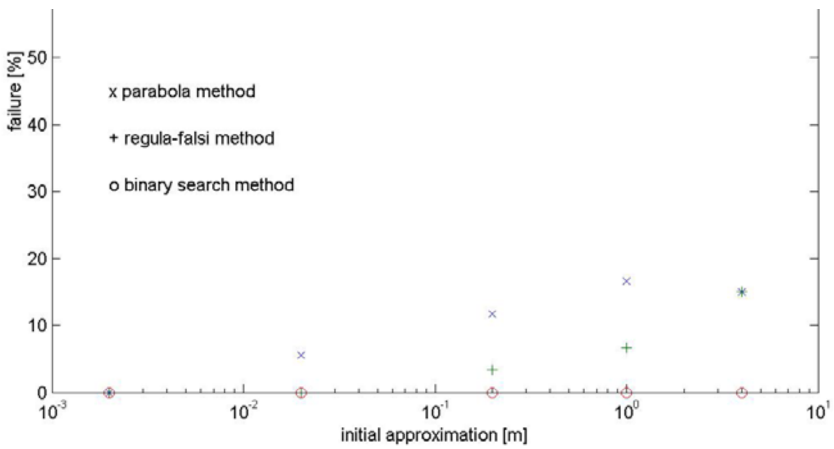

Figure 9: simulated tests, truncation criterion: distance

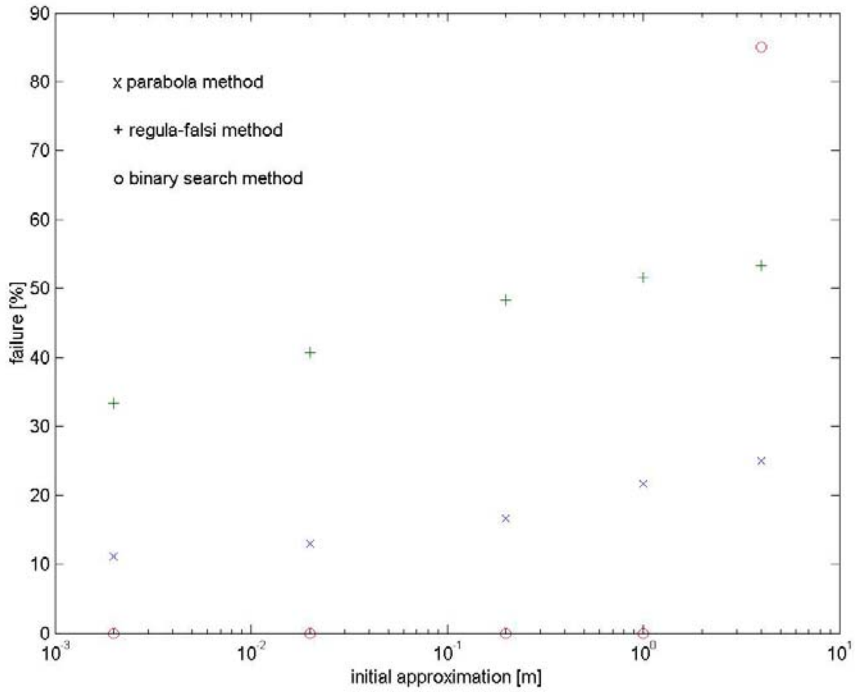

Figure 10: simulated tests, truncation criterion: distance and interval length

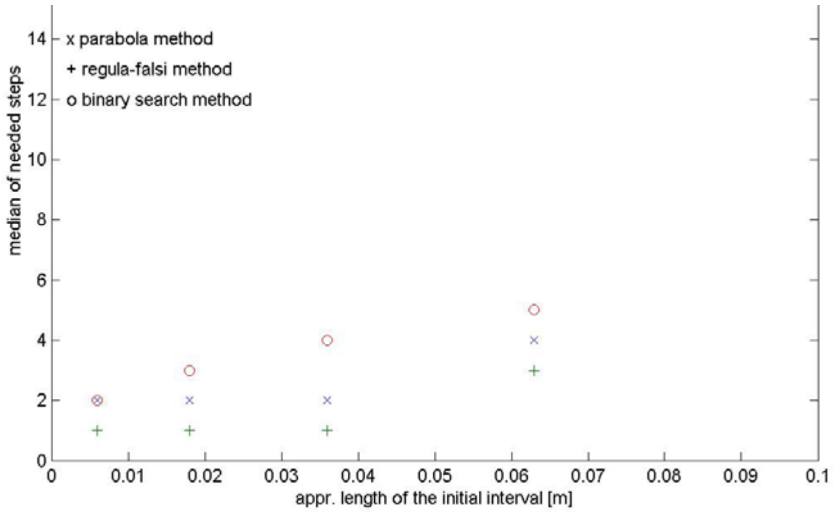

Figure 11: real tests, truncation criterion: distance

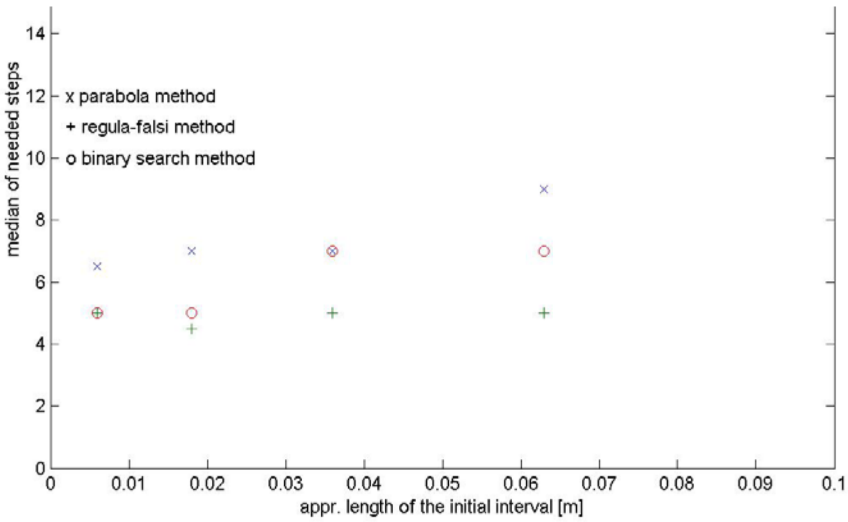

Figure 12: real tests, truncation criterion: distance and interval length

3. SPATIAL CURVES AS GRIDLINES

Straight lines are suitable for monitoring straight object motions, but if the object follows a more general curve it would be of interest to monitor its motion right along this path. If that curves are (approximately) predictable and given in a parametric form $\vec{x} = \vec{x}(t)$, then very similar or even the same algorithms can be used to find the intersecting point of the object and the curve [7]. For this reason a representation by the natural parameter, the arc length *s(t)*, would be the most convenient one. Because of its definition

$$s(t) = s(\vec{x}(t)) = \int_{t_0}^{t} \left| \frac{d\vec{x}}{d\tilde{t}} \right| d\tilde{t}$$

it cannot explicitly be given in every case. But for calculating the interval length used by the truncation criterion a suitable estimate will be sufficient. And for the algorithms themselves any parameter can be used, though it will be less graphic.

A second question is how to define the distance *d* in the truncation criterion, because there is no general formula like for the distance between a point and a straight line. Among some ideas the shortest distance between projected point and curve was chosen [7]. This shortest distance is calculated in an iterative process (*fig.13*) starting with the projected point *Q* and three points on the curve. An optimization method is used to iteratively minimize the distance [1]. As therefore no further measurement is needed the loss of time is entirely negligible.

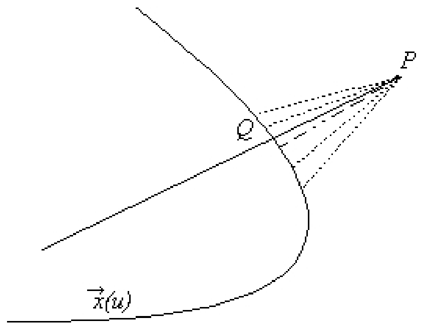

Figure 13: determining the distance between point and curve

As an example a helix was taken as a gridline in simulated measurements. For first investigations it was given as

$$\vec{x}(t) = \begin{pmatrix} a \cdot \cos t \\ a \cdot \sin t \\ b \cdot t \end{pmatrix} \quad \text{with} \quad a = 20 \text{ and } b = 0.5 .$$

The behaviour of the algorithms differs only non significantly form the case of straight lines. If the distance d is the only value in the truncation criterion the regula-falsi method is still most efficient method, which succeeds by very few search steps. For initial approximations in a range up to some centimetres the median values of the needed steps were between one and two. The results are much worse if a maximum interval length Λ is demanded in the truncation criterion. Then there are many unfinished and aborted iterations, as far as the regula-falsi method is concerned. For this case the method based on the binary search is the most promising. It needs some more steps to succeed – the median is about five for the mentioned starting conditions – but it never fails.

Variations of the helix parameters a and b lead to nearly the same results. Even considerable changes in the parameters, e.g. a=5 and b=2, did not markedly alter the results. Therefore, the needed number of iterations to find an intersecting point can be regarded as independent of the curve's curvature and torsion. The similarity to the test case of straight lines as gridlines confirm the view of straight lines as just a special case of spatial curves.

References

[1] Adbey, Dempster, "Introduction to Opimization Methods", Chapman and Hall, London 1974

[2] de Seixas, "3D-Objektrekonstruktion mittels Gitterlinien-Verfahren", doctoral thesis, Vienna University of Technology 2001

[3] Gill, Murray, Wright, "Practical Optimization", chap. 4, AP, London 1981

[4] Kahmen, de Seixas, "Object Recognition with Video-Theodolites and without Targeting the Object", Proceedings, 6th International Workshop on Accelerator Alignment – IWAA 99, Grenoble 1999

[5] Kahmen, Nießner, de Seixas, "On-Line Deformation Measurements with Rotating Computer Controlled CCD Cameras" in: Gruen, Kahmen (Eds.), "Optical 3-D Measurement Techniques V", Vienna 2001

[6] von Webern, "Optimization and Extension of Gridline-Methods to Detect Object Displacements and Deformations" in: Gruen, Kahmen (Eds.), "Optical 3-D Measurement Techniques VI", Vol. II, Zurich 2003

[7] von Webern, "Algorithmen zur Oberflächenerfassung mit Gitterlinien-Verfahren", doctoral thesis in prep., Vienna University of Technology 2004