Baryogenesis and Leptogenesis

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The energy budget of the universe contains two components, dark matter and dark energy, about which we have much to learn. One should not forget, however, that the baryonic component presents its own questions for particle cosmology. In the context of cosmology, baryons would have annihilated with their antiparticles in the early universe, leaving a negligible abundance of baryons, in disagreement with that observed. In this general lecture, delivered at the SLAC 2004 Summer Science Institute, I provide an overview of the central issue and the general principles behind candidate models. I also briefly discuss some popular examples of models that are firmly rooted in particle physics.

1. Introduction

The problem of the baryon asymmetry of the universe is a classic problem of particle cosmology. Particle physics has taught us that matter and antimatter behave essentially identically, and indeed the interactions between matter and antimatter are the focus of successful terrestrial experiments. On the other hand, cosmology teaches us that the early universe was an extremely hot, and hence energetic, environment in which one would expect equal numbers of baryons and antibaryons to be copiously produced. This early state of the universe stands in stark contrast to what we observe in the universe today. Direct observation shows that the universe around us contains no appreciable primordial antimatter. In addition, the theory of primordial nucleosynthesis (for a review see [3]) allows accurate predictions of the cosmological abundances of all the light elements, $^4\text{He}$, $^3\text{He}$, $^7\text{Li}$, $\text{B}$, $\text{D}$, while requiring only that, defining $n_{b(b)}$ to be the number density of (anti)-baryons and $s$ to be the entropy density,

$$2.6 \times 10^{-10} < \eta \equiv \frac{n_{b} - n_{b}}{s} < 6.2 \times 10^{-10},$$

(see, for example, [4]). Very recently this number has been independently determined to be $\eta = 6.1 \times 10^{-10} + 0.3 \times 10^{-10} - 0.2 \times 10^{-10}$ from precise measurements of the relative heights of the first two microwave background (CMB) acoustic peaks by the WMAP satellite [5]. Alternatively we may write the range as

$$0.015(0.011) \lesssim \Omega_B h^2 \lesssim 0.026(0.038),$$

(2)

where $\Omega_B$ is the proportion of the critical energy density in baryons, and $h$ parametrizes the present value of the Hubble parameter via $h = H_0/(100 \text{ Km Mpc}^{-1} \text{ sec}^{-1})$.

The standard cosmological model cannot explain the observed value of $\eta$. To see this, suppose that initially we start with $\eta = 0$. At temperatures $T \lesssim 1 \text{ GeV}$ the equilibrium abundance of nucleons and antinucleons is

$$\frac{n_b}{n_\gamma} \sim \frac{n_\bar{b}}{n_\gamma} \sim \left(\frac{m_p}{T}\right)^{3/2} e^{-\frac{m_p}{T}}.$$  

(3)

As the universe cools, the number of nucleons and antinucleons decreases as long as the annihilation rate $\Gamma_{\text{ann}} \approx n_b \langle \sigma_A v \rangle$ is larger than the expansion rate of the universe $H \approx 1.66 g^{1/2} T^2 / m_p$. The thermally averaged annihilation cross section $\langle \sigma_A v \rangle$ is of the order of $m_p^2$, so at $T \approx 20 \text{ MeV}$, $\Gamma_{\text{ann}} \approx H$, and annihilations freeze out, nucleons and
antinucleons being so rare that they cannot annihilate any longer. Therefore, from (3) we obtain

$$\frac{n_b}{n_\gamma} \approx \frac{n_b}{n_\gamma} \approx 10^{-18},$$

which is much smaller than the value required by nucleosynthesis.

An initial asymmetry may be imposed by hand as an initial condition, but this would violate any naturalness principle. Rather, the goal of this talk was to describe recent progress in understanding scenarios for generating the baryon asymmetry of the universe (BAU) within the context of modern cosmology. Much more complete reviews of this subject, with several different approaches, can be found in [1, 2, 6–10].

In the next section I describe the Sakharov Criteria which must be satisfied by any particle physics theory through which a baryon asymmetry is produced. In section (3) I briefly describe how baryogenesis can proceed in certain Grand Unified Theories and mention how new ideas concerning preheating after inflation have modified this scenario. I also briefly discuss the idea of leptogenesis. In section (4) I review at length one of the most popular models - electroweak baryogenesis - and in section (5) I briefly mention Affleck-Dine (AD) type baryogenesis scenarios. The choices of what topics to cover and how much detail to devote to each were decided by a combination of personal taste and the extent of the direct connection to current and upcoming collider experiments.

Throughout I use a metric with signature +2 and, unless explicitly stated otherwise, I employ units such that \( h = c = k = 1 \).

2. The Sakharov Criteria

As pointed out by Sakharov [11], a small baryon asymmetry \( \eta \) may have been produced in the early universe if three necessary conditions are satisfied: 

\( i ) \) baryon number (\( B \)) violation; 

\( ii ) \) violation of \( C \) (charge conjugation symmetry) and \( CP \) (the composition of parity and \( C \)) and 

\( iii ) \) departure from thermal equilibrium. The first condition should be clear since, starting from a baryon symmetric universe with \( \eta = 0 \), baryon number violation must take place in order to evolve into a universe in which \( \eta \) does not vanish. The second Sakharov criterion is required because, if \( C \) and \( CP \) are exact symmetries, then one can prove that the total rate for any process which produces an excess of baryons is equal to the rate of the complementary process which produces an excess of antibaryons and so no net baryon number can be created. That is to say that the thermal average of the baryon number operator \( B \), which is odd under both \( C \) and \( CP \), is zero unless those discrete symmetries are violated. \( CP \)-violation is present either if there are complex phases in the lagrangian which cannot be reabsorbed by field redefinitions (explicit breaking) or if some Higgs scalar field acquires a VEV which is not real (spontaneous breaking).

Finally, to explain the third criterion, one can calculate the equilibrium average of \( B \)

$$\langle B \rangle_T = \text{Tr} \left( e^{-\beta H} B \right) = \text{Tr} \left[ \left(CPT\right) \left(CPT\right)^{-1} e^{-\beta H} B \right]$$

$$= \text{Tr} \left( e^{-\beta H} \left(CPT\right)^{-1} B \left(CPT\right) \right) = -\text{Tr} \left( e^{-\beta H} B \right),$$

where I have used that the Hamiltonian \( H \) commutes with \( CPT \). Thus \( \langle B \rangle_T = 0 \) in equilibrium and there is no generation of net baryon number.

Of the three Sakharov conditions, baryon number violation and \( C \) and \( CP \)-violation may be investigated only within a given particle physics model, while the third condition – the departure from thermal equilibrium – may be discussed in a more general way, as we shall see. Let us discuss the Sakharov criteria in more detail.

2.1. Baryon Number Violation

2.1.1. \( B \)-violation in Grand Unified Theories

Baryon number violation is very natural in Grand Unified Theories (GUTs) [12], since a general property is that the same representation of the gauge group \( G \) may contain both quarks and leptons, and therefore it is possible for scalar
and gauge bosons to mediate gauge interactions among fermions having different baryon number. However, this alone is not sufficient to conclude that baryon number is automatically violated in GUTs, since in some circumstances it is possible to assign a baryonic charge to the gauge bosons in such a way that at each boson-fermion-fermion vertex the baryon number is conserved. For example, in the particular case of the gauge group SU(5), it turns out that among all the scalar and gauge bosons which couple only to the fermions of the SM, only five of them may give rise to interactions which violate the baryon number.

2.2. B-violation in the Electroweak Theory.

It is well-known that the most general Lagrangian invariant under the SM gauge group and containing only color singlet Higgs fields is automatically invariant under global abelian symmetries which may be identified with the baryonic and leptonic symmetries. These, therefore, are accidental symmetries and as a result it is not possible to violate $B$ and $L$ at tree-level or at any order of perturbation theory. Nevertheless, in many cases the perturbative expansion does not describe all the dynamics of the theory and, indeed, in 1976 't Hooft [13] realized that nonperturbative effects (instantons) may give rise to processes which violate the combination $B + L$, but not the orthogonal combination $B - L$. The probability of these processes occurring today is exponentially suppressed and probably irrelevant. However, in more extreme situations – like the primordial universe at very high temperatures [14–17] – baryon and lepton number violating processes may be fast enough to play a significant role in baryogenesis. Let us have a closer look.

At the quantum level, the baryon and the lepton symmetries are anomalous [18, 19]

$$\partial_\mu j^B_B = \partial_\mu j^L_L = n_f \left( \frac{g^2}{32\pi^2} W^a_{\mu\nu} \tilde{W}^{a\mu\nu} - \frac{g'^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{a\mu\nu} \right),$$

(6)

where $g$ and $g'$ are the gauge couplings of $SU(2)_L$ and $U(1)_Y$, respectively, $n_f$ is the number of families and $\tilde{W}^{a\mu\nu} = (1/2)\epsilon^{\mu\nu\rho\sigma} W_{a \rho\sigma}$ is the dual of the $SU(2)_L$ field strength tensor, with an analogous expression holding for $\tilde{F}$. To understand how the anomaly is closely related to the vacuum structure of the theory, we may compute the change in baryon number from time $t = 0$ to some arbitrary final time $t = t_f$. For transitions between vacua, the average values of the field strengths are zero at the beginning and the end of the evolution. The change in baryon number may be written as

$$\Delta B = \Delta N_{CS} \equiv n_f [N_{CS}(t_f) - N_{CS}(0)].$$

(7)

where the Chern-Simons number is defined to be

$$N_{CS}(t) \equiv \frac{g^2}{32\pi^2} \int d^3x \epsilon^{ijk} \text{Tr} \left( A_i \partial_j A_k + \frac{2}{3} i g A_i A_j A_k \right).$$

(8)

Although the Chern-Simons number is not gauge invariant, the change $\Delta N_{CS}$ is. Thus, changes in Chern-Simons number result in changes in baryon number which are integral multiples of the number of families $n_f$. Gauge transformations $U(x)$ which connects two degenerate vacua of the gauge theory may change the Chern-Simons number by an integer $n$, the winding number. If the system is able to perform a transition from the vacuum $G^{(n)}_{\text{vac}}$ to the closest one $G^{(n+1)}_{\text{vac}}$, the Chern-Simons number is changed by unity and $\Delta B = \Delta L = n_f$. Each transition creates 9 left-handed quarks (3 color states for each generation) and 3 left-handed leptons (one per generation). However, adjacent vacua of the electroweak theory are separated by a ridge of configurations with energies larger than that of the vacuum. The lowest energy point on this ridge is a saddle point solution to the equations of motion with a single negative eigenvalue, and is referred to as the sphaleron [15, 16]. The probability of baryon number nonconserving processes at zero temperature has been computed by 't Hooft [13] and is highly suppressed by a factor $\exp(-4\pi/\alpha_W)$, where $\alpha_W = g^2/4\pi$. This factor may be interpreted as the probability of making a transition from
one classical vacuum to the closest one by tunneling through an energy barrier of height \( \sim 10 \text{ TeV} \) corresponding to the sphaleron. On the other hand, one might think that fast baryon number violating transitions may be obtained in physical situations which involve a large number of fields. Since the sphaleron may be produced by collective and coherent excitations containing \( \sim 1/\alpha_W \) quanta with wavelength of the order of \( 1/M_W \), one expects that at temperatures \( T \gg M_W \), these modes essentially obey statistical mechanics and the transition probability may be computed via classical considerations. The general framework for evaluating the thermal rate of anomalous processes in the electroweak theory was developed in [17]. Analogously to the case of zero temperature and since the transition which violates the baryon number is sustained by the sphaleron configuration, the thermal rate of baryon number violation in the \textit{broken} phase is proportional to \( \exp(-S_3/T) \), where \( S_3 \) is the three-dimensional action computed along the sphaleron configuration,

\[
S_3 = E_{sp}(T) \equiv (M_W(T)/\alpha_W) \mathcal{E}
\]

with the dimensionless parameter \( \mathcal{E} \) lying in the range \( 3.1 < \mathcal{E} < 5.4 \) depending on the Higgs mass. The prefactor of the thermal rate was computed in [20] as

\[
\Gamma_{sp}(T) = \mu \left( \frac{M_W}{\alpha_W T} \right)^3 M_W^4 \exp \left( -\frac{E_{sp}(T)}{T} \right)
\]

where \( \mu \) is a dimensionless constant. Recent approaches to calculating the rate of baryon number violating events in the broken phase have been primarily numerical. The best calculation to date of the broken phase sphaleron rate is undoubtedly that by Moore [21]. This work yields a fully nonperturbative evaluation of the broken phase rate by using a combination of multicanonical weighting and real time techniques.

Although the Boltzmann suppression in (10) appears large, it is to be expected that, when the electroweak symmetry becomes restored at a temperature of around 100 GeV, there will no longer be an exponential suppression factor. Although calculation of the baryon number violating rate in the high temperature \textit{unbroken} phase is extremely difficult, a simple estimate is possible. The only important scale in the symmetric phase is the magnetic screening length given by \( \xi = (\alpha_W T)^{-1} \). Thus, on dimensional grounds, we expect the rate per unit volume of sphaleron events to be

\[
\Gamma_{sp}(T) = \kappa (\alpha_W T)^4
\]

with \( \kappa \) another dimensionless constant. The rate of sphaleron processes can be related to the diffusion constant for Chern-Simons number by a fluctuation-dissipation theorem [22] (for a good description of this see [7]). In almost all numerical calculations of the sphaleron rate, this relationship is used and what is actually evaluated is the diffusion constant.

The simple scaling argument leading to (11) does not capture all of the important dynamics [23–30]. The effective dynamics of soft nonabelian gauge fields at finite temperature has been addressed in [31, 32], where it was found that \( \Gamma_{sp} \sim 5\alpha_W^5 T^4 \ln(1/\alpha_W) \). Further, Lattice simulations with hard-thermal loops included [33] indicate \( \Gamma_{sp} \sim 30\alpha_W^5 T^4 \).

2.2.1. Other Ways of Achieving \( B \)-Violation.

One of the features which distinguishes supersymmetric field theories [34, 35] from ordinary ones is the existence of “flat directions” in field space on which the scalar potential vanishes. At the level of renormalizable terms, such flat directions are generic. Supersymmetry breaking lifts these directions and sets the scale for their potential. From a cosmological perspective, these flat directions can have profound consequences. The parameters which describe the flat directions can be thought of as expectation values of massless chiral fields, the moduli. Many flat directions present in the minimal supersymmetric standard model (MSSM), carry baryon or lepton number, since along them squarks and sleptons have non-zero vacuum expectation values (VEVs). If baryon and lepton number are explicitly broken it is possible to excite a non-zero baryon or lepton number along such directions, as first suggested by Affleck and Dine [131] in a more general context. I’ll explore these ideas more thoroughly in section 5.
2.3. CP-Violation

2.3.1. CP-Violation in Grand Unified Theories.

CP-violation in GUTs arises in loop diagram corrections to baryon number violating bosonic decays. Since it is necessary that the particles in the loop also undergo B-violating decays, the relevant particles are the $X$, $Y$, and $H_3$ bosons in the case of $SU(5)$. In that case, CP-violation is due to the complex phases of the Yukawa couplings $h_U$ and $h_D$ which cannot be reabsorbed by field redefinition. At tree-level these phases do not give any contribution to the baryon asymmetry and at the one-loop level the asymmetry is proportional to $\text{Im} \text{Tr} \left( h_U^* h_U + h_D^* h_D \right) = 0$, where the trace is over generation indices. The problem of too little CP-violation in $SU(5)$ may be solved by further complicating the Higgs sector. For example, adding a Higgs in the 45 representation of $SU(5)$ leads to an adequate baryon asymmetry for a wide range of the parameters [36]. In the case of $SO(10)$, CP-violation may be provided by the complex Yukawa couplings between the right-handed and the left-handed neutrinos and the scalar Higgs.

2.3.2. CP-Violation in the CKM Matrix of the Standard Model.

Since only the left-handed electron is $SU(2)_L$ gauge coupled, $C$ is maximally broken in the SM. Moreover, $CP$ is known not to be an exact symmetry of the weak interactions. This is seen experimentally in the neutral Kaon system through $K_0$, $\bar{K}_0$ mixing. At present there is no accepted theoretical explanation of this. However, it is true that CP-violation is a natural feature of the standard electroweak model. There exists a charged current which, in the weak interaction basis, may be written as $\mathcal{L}_W = (g/\sqrt{2}) \bar{U}_L \gamma^\mu D_L W_\mu + \text{h.c.}$, where $U_L = (u, c, t, \ldots)_L$ and $D_L = (d, s, b, \ldots)_L$. Now, the quark mass matrices may be diagonalized by unitary matrices $V_L^U, V_R^U, V_L^D, V_R^D$ via

$$\text{diag}(m_u, m_c, m_t, \ldots) = V_L^U M^U V_R^U, \quad (12)$$

$$\text{diag}(m_d, m_s, m_b, \ldots) = V_L^D M^D V_R^D. \quad (13)$$

Thus, in the basis of quark mass eigenstates, the charged current may be rewritten as $\mathcal{L}_W = (g/\sqrt{2}) \bar{U}_L' K \gamma^\mu D_L' W_\mu + \text{h.c.}$, where $U_L' \equiv V_L^U U_L$ and $D_L' \equiv V_L^D D_L$. The matrix $K$, defined by $K \equiv V_L^U (V_R^D)^\dagger$, is referred to as the Kobayashi-Maskawa (KM) quark mass mixing matrix. For three generations, as in the SM, there is precisely one independent nonzero phase $\delta$ which signals CP-violation. While this is encouraging for baryogenesis, it turns out that this particular source of CP-violation is not strong enough. The relevant effects are parameterized by a dimensionless constant which is no larger than $10^{-20}$. This appears to be much too small to account for the observed BAU and, thus far, attempts to utilize this source of CP-violation for electroweak baryogenesis have been unsuccessful. In light of this, it is usual to extend the SM in some minimal fashion that increases the amount of CP-violation in the theory while not leading to results that conflict with current experimental data.

2.3.3. CP-Violation in Supersymmetric Models

The most promising and well-motivated framework incorporating CP-violation beyond the SM seems to be supersymmetry [34, 35]. Let us consider the MSSM superpotential

$$W = \mu \hat{H}_1 \hat{H}_2 + h_u \hat{H}_2 \hat{Q} \hat{u}^c + h_d \hat{H}_1 \hat{Q} \hat{d}^c + h_c \hat{H}_1 \hat{L} \hat{e}^c, \quad (14)$$

where we have omitted the generation indices. Here $\hat{H}_1$ and $\hat{H}_2$ represent the two Higgs superfields, $\hat{Q}, \hat{u}$ and $\hat{d}$ are the quark doublet and the up-quark and down-quark singlet superfields respectively and $\hat{L}$ and $\hat{e}^c$ are the left-handed doublet and right-handed lepton singlet superfields.

The lepton Yukawa matrix $h_e^c$ can be always taken real and diagonal while $h_u^c$ and $h_d^c$ contain the KM phase. There are four new sources for explicit CP-violating phases, all arising from parameters that softly break supersymmetry. These are (i) the trilinear couplings

$$\Gamma^u H_2 \hat{Q} \hat{u}^c + \Gamma^d \hat{H}_1 \hat{Q} \hat{d}^c + \Gamma^c H_1 \hat{L} \hat{e}^c + \text{h.c.}, \quad (15)$$

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where we have defined

\[ \Gamma^{(u,d,e)} = A^{(u,d,e)} + h^{(u,d,e)} \]

and the tildes stand for scalar fields, (ii) the bilinear coupling in the Higgs sector \( \mu BH_1 H_2 \), (iii) the gaugino masses \( M \) (one for each gauge group), and (iv) the soft scalar masses \( \tilde{m} \). Two phases may be removed by redefining the phase of the superfield \( \tilde{H}_2 \) in such a way that the phase of \( \mu \) is opposite to that of \( B \). The product \( \mu B \) is therefore real. It is also possible to remove the phase of the gaugino masses \( M \) by an \( R \) symmetry transformation. The latter leaves all the other supersymmetric couplings invariant, and only modifies the trilinear ones, which get multiplied by \( \exp(-\phi_M) \) where \( \phi_M \) is the phase of \( M \). The phases which remain are therefore

\[ \phi_A = \arg(AM) \quad \text{and} \quad \phi_\mu = -\arg(B). \]

These new phases \( \phi_A \) and \( \phi_\mu \) may be crucial for the generation of the baryon asymmetry.

In the MSSM, however, there are other possible sources of \( CP \)-violation. In fact, when supersymmetry breaking occurs, as we know it must, the interactions of the Higgs fields \( H_1 \) and \( H_2 \) with charginos, neutralinos and stops (the superpartners of the charged, neutral gauge bosons, Higgs bosons and tops, respectively) at the one-loop level, lead to a \( CP \)-violating contribution to the scalar potential of the form

\[ V_{CP} = \lambda_1 (H_1 H_2)^2 + \lambda_2 |H_1|^2 H_1 H_2 + \lambda_3 |H_2|^2 H_1 H_2 + \text{h.c.}. \]

These corrections may be quite large at finite temperature [37, 38]. One may write the Higgs fields in unitary gauge as \( H_1 = (\phi_1, 0)^T \) and \( H_2 = (0, \phi_2 e^{i\theta})^T \), where \( \phi_1, \phi_2, \theta \) are real, and \( \theta \) is the \( CP \)-odd phase. The coefficients \( \lambda_{1,2,3} \) determine whether the \( CP \)-odd field \( \theta \) is non zero, signalling the spontaneous violation of \( CP \) in a similar way to what happens in a generic two-Higgs model [39]. If, during the evolution of the early universe, a variation of the phase \( \theta \) is induced, this \( CP \) breakdown may bias the production of baryons through sphaleron processes in electroweak baryogenesis models. The attractive feature of this possibility is that, when the universe cools down, the finite temperature loop corrections to the \( \lambda \)-couplings disappear, and the phase \( \theta \) relaxes to zero. This is turn implies that, unlike for scenarios utilizing \( \phi_A \) and \( \phi_\mu \), one need not worry about present experimental constraints from the physics of \( CP \)-violation. When the constraints from experiment and the strength of the transition are taken into account [40], the relevant region of the parameter space for spontaneous \( CP \)-violation in the MSSM at finite temperature is rather restricted. Nevertheless, in this small region, perturbative estimates need not be reliable, and non-perturbatively the region might be slightly larger (or smaller).

Finally, in scenarios in which the BAU is generated from the coherent production of a scalar condensate along a flat direction of the supersymmetric extension of the SM, \( CP \)-violation is present in the nonrenormalizable superpotentials which lift the flat directions at large field values. I will have more to say about this is section (5).

### 2.4. Departure from Thermal Equilibrium

#### 2.4.1. The Out-of-Equilibrium Decay Scenario

Scenarios in which the third Sakharov condition is satisfied due to the presence of superheavy decaying particles in a rapidly expanding universe, generically fall under the name of out-of-equilibrium decay mechanisms. The underlying idea is fairly simple. If the decay rate \( \Gamma_X \) of the superheavy particles \( X \) at the time they become nonrelativistic (i.e., at the temperature \( T \sim M_X \)) is much smaller than the expansion rate of the universe, then the \( X \) particles cannot decay on the time scale of the expansion and so they remain as abundant as photons for \( T \lesssim M_X \). In other words, at some temperature \( T > M_X \), the superheavy particles \( X \) are so weakly interacting that they cannot catch up with the expansion of the universe and they decouple from the thermal bath while they are still relativistic, so that \( n_X \sim n_\gamma \sim T^3 \) at the time of decoupling. Therefore, at temperature \( T \approx M_X \), they populate the universe with an abundance which is much larger than the equilibrium one. This overabundance is precisely the departure from thermal equilibrium needed to produce a final nonvanishing baryon asymmetry when the heavy states \( X \) undergo
2.4.2. The Electroweak Phase Transition Scenario

At temperatures around the electroweak scale, the expansion rate of the universe in thermal units is small compared to the rate of baryon number violating processes. This means that the equilibrium description of particle phenomena is extremely accurate at electroweak temperatures. Thus, baryogenesis cannot occur at such low scales without the aid of phase transitions and the question of the order of the electroweak phase transition becomes central.

If the EWPT is second order or a continuous crossover, the associated departure from equilibrium is insufficient to lead to relevant baryon number production [17]. This means that for EWBG to succeed, we either need the EWPT to be strongly first order or other methods of destroying thermal equilibrium to be present at the phase transition.

Any thermodynamic quantity that undergoes such a discontinuous change at a phase transition is referred to as an order parameter, denoted by $\varphi$. For a first order transition there is an extremum at $\varphi = 0$ which becomes separated from a second local minimum by an energy barrier. At the critical temperature $T = T_c$ both phases are equally favored energetically and at later times the minimum at $\varphi \neq 0$ becomes the global minimum of the theory. The dynamics of the phase transition in this situation is crucial to most scenarios of electroweak baryogenesis. The essential picture is that around $T_c$ quantum tunneling occurs and nucleation of bubbles of the true vacuum in the sea of false begins. Initially these bubbles are not large enough for their volume energy to overcome the competing surface tension and they shrink and disappear. However, at a particular temperature below $T_c$, bubbles just large enough to grow nucleate. These are termed critical bubbles, and they expand, eventually filling all of space and completing the transition. As the bubble walls pass each point in space, the order parameter changes rapidly, as do the other fields, and this leads to a significant departure from thermal equilibrium. Thus, if the phase transition is strongly enough first order it is possible to satisfy the third Sakharov criterion in this way.

The precise evolution of critical bubbles in the electroweak phase transition is a crucial factor in determining which regimes of electroweak baryogenesis are both possible and efficient enough to produce the BAU. (See [41–49] for possible obstacles to the standard picture) I’ll discuss this, and the issue of the order of the phase transition, in more detail in section 4.

For the bubble wall scenario to be successful, there is a further criterion to be satisfied. As the wall passes a point in space, the Higgs fields evolve rapidly and the Higgs VEV changes from $\langle \phi \rangle = 0$ in the unbroken phase to

$$\langle \phi \rangle = v(T_c)$$  \hspace{1cm} (19)

in the broken phase. Here, $v(T)$ is the value of the order parameter at the symmetry breaking global minimum of the finite temperature effective potential. Now, $CP$-violation and the departure from equilibrium occur while the Higgs field is changing. Afterwards, the point is in the true vacuum, baryogenesis has ended, and baryon number violation is exponentially suppressed. Since baryogenesis is now over, it is imperative that baryon number violation be negligible at this temperature in the broken phase, otherwise any baryonic excess generated will be equilibrated to zero. Such an effect is known as washout of the asymmetry and the criterion for this not to happen may be written as

$$\frac{v(T_c)}{T_c} \gtrsim 1$$  \hspace{1cm} (20)

Although there are a number of nontrivial steps that lead to this simple criterion, (20) is traditionally used to ensure that the baryon asymmetry survives after the wall has passed. It is necessary that this criterion be satisfied for any electroweak baryogenesis scenario to be successful.

2.4.3. Defect-Mediated Baryogenesis

A natural way to depart from equilibrium is provided by the dynamics of topological defects. Topological defects are regions of trapped energy density which can remain after a cosmological phase transition if the topology of the
vacuum of the theory is nontrivial. Typically, cosmological phase transitions occur when a symmetry of a particle physics theory is spontaneously broken. In that case, the cores of the topological defects formed are regions in which the symmetry of the unbroken theory is restored. If, for example, cosmic strings are produced at the GUT phase transition, then the decays of loops of string act as an additional source of superheavy bosons which undergo baryon number violating decays. When defects are produced at the TeV scale, a detailed analysis of the dynamics of a network of these objects shows that a significant baryon to entropy ratio can be generated [50–55] if the electroweak symmetry is restored around such a higher scale ordinary defect [56–58]. Although a recent analysis has shown that $B$-violation is highly inefficient along nonsuperconducting strings [59], there remain viable scenarios involving other ordinary defects, superconducting strings or defects carrying baryon number [60, 61].

3. Grand-Unified Baryogenesis and Leptogenesis

3.1. GUT Baryogenesis

As we have seen, in GUTs baryon number violation is natural since quarks and leptons lie in the same irreducible representation of the gauge group. The implementation of the out-of-equilibrium scenario in this context goes under the name of GUT baryogenesis. The basic assumption is that the superheavy bosons were as abundant as photons at very high temperatures $T \gtrsim M_X$. This assumption is questionable if the heavy $X$ particles are the gauge or Higgs bosons of Grand Unification, because they might have never been in thermal equilibrium in the early universe. Indeed, the temperature of the universe might have been always smaller than $M_{GUT}$ and, correspondingly, the thermally produced $X$ bosons might never have been as abundant as photons, making their role in baryogenesis negligible. All these considerations depend crucially upon the thermal history of the universe and deserve a closer look.

The flatness and the horizon problems of the standard big bang cosmology are elegantly solved if, during the evolution of the early universe, the energy density happened to be dominated by some form of vacuum energy, causing comoving scales grow quasi-exponentially [62–65]. The vacuum energy driving this inflation is generally assumed to be associated with the potential $V(\phi)$ of some scalar field $\phi$, the inflaton, which is initially displaced from the minimum of its potential. As a by-product, quantum fluctuations of the inflaton field may be the seeds for the generation of structure and the fluctuations observed in the cosmic microwave background radiation [65]. Inflation ended when the potential energy associated with the inflaton field became smaller than the kinetic energy of the field. By that time, any pre-inflationary entropy in the universe had been inflated away, and the energy of the universe was entirely in the form of coherent oscillations of the inflaton condensate around the minimum of its potential. Now, we know that somehow this low-entropy cold universe must be transformed into a high-entropy hot universe dominated by radiation. The process by which the energy of the inflaton field is transferred from the inflaton field to radiation has been dubbed reheating. The simplest way to envision this process is if the comoving energy density in the zero mode of the inflaton decays perturbatively into normal particles, which then scatter and thermalize to form a thermal background [66, 67]. It is usually assumed that the decay width of this process is the same as the decay width of a free inflaton field. Of particular interest is a quantity known as the reheat temperature, denoted as $T_r$. This is calculated by assuming an instantaneous conversion of the energy density in the inflaton field into radiation when the decay width of the inflaton energy, $\Gamma_\phi$, is equal to $H$, the expansion rate of the universe. This yields

$$T_r = \sqrt{\frac{\Gamma_\phi m_p}{2}}. \quad (21)$$

There are very good reasons to suspect that GUT baryogenesis does not occur if this is the way reheating happens. First of all, the density and temperature fluctuations observed in the present universe require the inflaton potential to be extremely flat – that is $\Gamma_\phi \ll M_\phi$. This means that the couplings of the inflaton field to the other degrees of freedom cannot be too large, since large couplings would induce large loop corrections to the inflaton potential, spoiling its flatness. As a result, $T_r$ is expected to be smaller than $10^{14}$ GeV by several orders of magnitude. On the other hand, the unification scale is generally assumed to be around $10^{16}$ GeV, and $B$-violating bosons should have
masses comparable to this scale. Furthermore, even the light $B$-violating Higgs bosons are expected to have masses larger than the inflaton mass, and thus it would be kinematically impossible to create them directly in $\phi$ decays, $\phi \to XX$.

One might think that the heavy bosons could be created during the stage of thermalization. Indeed, particles of mass $\sim 10^4$ bigger than the reheating temperature $T_r$ may be created by the thermalized decay products of the inflaton [68–70]. However, there is one more problem associated with GUT baryogenesis in the old theory of reheating, namely the problem of relic gravitinos [71]. If one has to invoke supersymmetry to preserve the flatness of the inflaton potential, it is mandatory to consider the cosmological implications of the gravitino – a spin-(3/2) particle which appears in the extension of global supersymmetry to supergravity. The gravitino is the fermionic superpartner of the graviton and has interaction strength with the observable sector – that is the standard model particles and their superpartners – inversely proportional to the Planck mass. The slow gravitino decay rate leads to a cosmological problem because the decay products of the gravitino destroy $^4$He and $^D$ nuclei by photodissociation, and thus ruin the successful predictions of nucleosynthesis. The requirement that not too many gravitinos are produced after inflation provides a stringent constraint on the reheating temperature, $T_r \lesssim (10^{10} - 10^{11})$ GeV [71]. Therefore, if $T_r \sim M_{\text{GUT}}$, gravitinos would be abundant during nucleosynthesis and destroy the agreement of the theory with observations. However, if the initial state after inflation was free from gravitinos, the reheating temperature is then too low to create superheavy $X$ bosons that eventually decay and produce the baryon asymmetry.

The outlook for GUT baryogenesis has brightened with the realization that reheating may differ significantly from the simple picture described above [72–76]. In the first stage of reheating, called preheating [72], nonlinear quantum effects may lead to extremely effective dissipative dynamics and explosive particle production, even when single particle decay is kinematically forbidden. In this picture, particles can be produced in a regime of broad parametric resonance, and it is possible that a significant fraction of the energy stored in the form of coherent inflaton oscillations at the end of inflation is released after only a dozen or so oscillation periods of the inflaton. What is most relevant for us is that preheating may play an extremely important role in baryogenesis and, in particular, for the Grand Unified generation of a baryonic excess. Indeed, it was shown in [77, 78] that the baryon asymmetry can be produced efficiently just after the preheating era, thus solving many of the problems that GUT baryogenesis had to face in the old picture of reheating. Interestingly, preheating may also play an important role in electroweak baryogenesis [79, 80] in models with very low-scale inflation [81], although I do not have space to discuss this here.

### 3.2. Baryogenesis Via Leptogenesis

Since the linear combination $B - L$ is left unchanged by sphaleron transitions, the baryon asymmetry may be generated from a lepton asymmetry [82]. Indeed, sphaleron transition will reprocess any lepton asymmetry and convert (a fraction of) it into baryon number. This is because $B + L$ must be vanishing and the final baryon asymmetry results to be $B \approx -L$. Once the lepton number is produced, thermal scatterings redistribute the charges. In the high temperature phase of the SM, the asymmetries of baryon number $B$ and of $B-L$ are therefore proportional:

$$B = \left( \frac{8n_f + 4n_H}{22n_f + 13n_H} \right) (B - L),$$

where $n_H$ is the number of Higgs doublets. In the SM as well as in its unified extension based on the group $SU(5)$, $B - L$ is conserved and no asymmetry in $B - L$ can be generated. However, adding right-handed Majorana neutrinos to the SM breaks $B - L$ and the primordial lepton asymmetry may be generated by the out-of-equilibrium decay of heavy right-handed Majorana neutrinos $N_L^c$ (in the supersymmetric version, heavy scalar neutrino decays are also relevant for leptogenesis). This simple extension of the SM can be embedded into GUTs with gauge groups containing $SO(10)$. Heavy right-handed Majorana neutrinos can also explain the smallness of the light neutrino masses via the see-saw mechanism [83–85].
The relevant couplings are those between the right-handed neutrino, the Higgs doublet $\Phi$ and the lepton doublet $\ell_L$

$$\mathcal{L} = \bar{\ell}_L \Phi h \ N^c_L + \frac{1}{2} \mathcal{N}^c_M \ N^c_L + \text{h.c.}$$

(23)

The vacuum expectation value of the Higgs field $\langle \Phi \rangle$ generates neutrino Dirac masses $m_D = h_\nu \langle \Phi \rangle$, which are assumed to be much smaller than the Majorana masses $M$. When the Majorana right-handed neutrinos decay into leptons and Higgs scalars, they violate the lepton number since right-handed neutrino fermionic lines do not have any preferred arrow

$$N^c_L \rightarrow \Phi + \ell,$$

$$N^c_L \rightarrow \Phi + \bar{\ell}.$$  

(24)

The interference between the tree-level decay amplitude and the absorptive part of the one-loop vertex leads to a lepton asymmetry of the right order of magnitude to explain the observed baryon asymmetry and has been discussed extensively in the literature [82, 86–88]. Recently, much attention has been paid to the effects of finite temperature on the $CP$-violation [89], and the contribution to the lepton asymmetry generated by the tree-level graph and the absorptive part of the one-loop self-energy. In particular, it has been observed that $CP$-violation may be considerably enhanced if two heavy right-handed neutrinos are nearly degenerate in mass [90]. It is also important to remember that large lepton number violation at intermediate temperatures may potentially dissipate away the baryon number in combination with the sphaleron transitions. Indeed, $\Delta L = 2$ interactions of the form

$$\frac{m_\nu}{\langle \Phi \rangle^2} \ell_L \ell_L \Phi \Phi + \text{h.c.},$$

(25)

where $m_\nu$ is the mass of the light left-handed neutrino, are generated through the exchange of heavy right-handed neutrinos. The rate of lepton number violation induced by this interaction is therefore $\Gamma_L \sim (m_\nu^2/\langle \Phi \rangle^4) T^3$. The requirement of harmless lepton number violation, $\Gamma_L \lesssim H$ imposes an interesting bound on the neutrino mass

$$m_\nu \lesssim 4 \text{ eV} \left( \frac{T_X}{10^{10} \text{ GeV}} \right)^{-1/2},$$

(26)

where $T_X \equiv \text{Min} \{T_{B-L}, 10^{12} \text{ GeV}\}, T_{B-L}$ is the temperature at which the $B-L$ number production takes place, and $\sim 10^{12} \text{ GeV}$ is the temperature at which sphaleron transitions enter in equilibrium. One can also reverse the argument and study leptogenesis assuming a similar pattern of mixings and masses for leptons and quarks, as suggested by $SO(10)$ unification [91]. This implies that $B - L$ is broken at the unification scale $\sim 10^{16} \text{ GeV}$, if $m_{\nu_e} \sim 3 \times 10^{-3}$ eV as preferred by the MSW explanation of the solar neutrino deficit [92, 93].

4. Electroweak Baryogenesis

Scenarios in which anomalous baryon number violation in the electroweak theory implements the first Sakharov criterion are referred to as electroweak baryogenesis. Typically these scenarios require a strongly first order phase transition, either in the minimal standard model, or in a modest extension. Another possibility is that the out of equilibrium requirement is achieved through TeV scale topological defects, also present in many extensions of the electroweak theory. Finally, a source of $CP$-violation beyond that in the CKM matrix is usually involved.

Historically, the ways in which baryons may be produced as a bubble wall, or phase boundary, sweeps through space, have been separated into two categories.

1. local baryogenesis: baryons are produced when the baryon number violating processes and $CP$-violating processes occur together near the bubble walls.
2. **nonlocal baryogenesis**: particles undergo \(CP\)-violating interactions with the bubble wall and carry an asymmetry in a quantum number other than baryon number into the unbroken phase region away from the wall. Baryons are then produced as baryon number violating processes convert the existing asymmetry into one in baryon number.

In general, both local and nonlocal baryogenesis will occur and the BAU will be the sum of that generated by the two processes. However, if the speed of the wall is greater than the sound speed in the plasma, then local baryogenesis dominates [94]. In other cases, nonlocal baryogenesis is usually more efficient and I will focus on that here.

Nonlocal baryogenesis typically involves the interaction of the bubble wall with the various fermionic species in the unbroken phase. The main picture is that as a result of \(CP\)-violation in the bubble wall, particles with opposite chirality interact differently with the wall, resulting in a net injected chiral flux. This flux thermalizes and diffuses into the unbroken phase where it is converted to baryons. In this section, for definiteness when describing these effects, I assume that the \(CP\)-violation arises because of a two-Higgs doublet structure.

The chiral asymmetry which is converted to an asymmetry in baryon number is carried by both quarks and leptons. However, the Yukawa couplings of the top quark and the \(\tau\)-lepton are larger than those of the other quarks and leptons respectively. Therefore, it is reasonable to expect that the main contribution to the injected asymmetry comes from these particles and to neglect the effects of the other particles.

When considering nonlocal baryogenesis it is convenient to write the equation for the rate of production of baryons in the form [95]

\[
\frac{dn_B}{dt} = -n_f \frac{\Gamma_{sp}(T)}{2T} \sum_i \mu_i ,
\]

where the rate per unit volume for electroweak sphaleron transitions is given by (11). Here, \(n_f\) is again the number of families and \(\mu_i\) is the chemical potential for left handed particles of species \(i\). The crucial question in applying this equation is an accurate evaluation of the chemical potentials that bias baryon number production.

There are typically two distinct calculational regimes that are appropriate for the treatment of nonlocal effects in electroweak baryogenesis. Which regime is appropriate depends on the particular fermionic species under consideration.

**4.0.1 The Thin Wall Regime**

The thin wall regime [95–97] applies if the mean free path \(\ell\) of the fermions being considered is much greater than the thickness \(L_w\) of the wall, i.e. if

\[
\frac{L_w}{\ell} \ll 1 .
\]

In this case we may neglect scattering effects and treat the fermions as free particles in their interactions with the wall.

In this regime, in the rest frame of the bubble wall, particles see a sharp potential barrier and undergo \(CP\)-violating interactions with the wall due to the gradient in the \(CP\) odd Higgs phase. As a consequence of \(CP\)-violation, there will be asymmetric reflection and transmission of particles, thus generating an injected current into the unbroken phase in front of the bubble wall. As a consequence of this injected current, asymmetries in certain quantum numbers will diffuse both behind and in front of the wall due to particle interactions and decays [95–97]. In particular, the asymmetric reflection and transmission of left and right handed particles will lead to a net injected chiral flux from the wall. However, there is a qualitative difference between the diffusion occurring in the interior and exterior of the bubble.

Exterior to the bubble the electroweak symmetry is restored and weak sphaleron transitions are unsuppressed. This means that the chiral asymmetry carried into this region by transport of the injected particles may be converted to an asymmetry in baryon number by sphaleron effects. In contrast, particles injected into the phase of broken symmetry interior to the bubble may diffuse only by baryon number conserving decays since the electroweak sphaleron rate is
exponentially suppressed in this region. Hence, we concentrate only on those particles injected into the unbroken phase.

The net baryon to entropy ratio which results via nonlocal baryogenesis in the case of thin walls has been calculated in several different analyses [96, 97] and [95]. In the following I will give a brief outline of the logic of the calculation, following [95]. The baryon density produced is given by (27) in terms of the chemical potentials \( \mu_i \) for left handed particles. These chemical potentials are a consequence of the asymmetric reflection and transmission off the walls and the resulting chiral particle asymmetry. Baryon number violation is driven by the chemical potentials for left handed leptons or quarks. To be concrete, focus on leptons [95] (for quarks see e.g. [96]). If there is local thermal equilibrium in front of the bubble walls then the chemical potentials \( \mu_i \) of particle species \( i \) are related to their number densities \( n_i \) by

\[
 n_i = \frac{T^2}{12} k_i \mu_i ,
\]

where \( k_i \) is a statistical factor which equals 1 for fermions and 2 for bosons. In deriving this expression, it is important [98] to correctly impose the constraints on quantities which are conserved in the region in front of and on the wall.

Using the above considerations, the chemical potential \( \mu_L \) for left handed leptons can be related to the left handed lepton number densities \( L_L \). These are in turn determined by particle transport. The source term in the diffusion equation is the flux \( J_0 \) resulting from the asymmetric reflection and transmission of left and right handed leptons off the bubble wall.

For simplicity assume a planar wall. If \( |p_z| \) is the momentum of the lepton perpendicular to the wall (in the wall frame), the analytic approximation used in [95] allows the asymmetric reflection coefficients for lepton scattering to be calculated in the range

\[
m_l < |p_z| < m_h \sim \frac{1}{L_w},
\]

where \( m_l \) and \( m_h \) are the lepton and Higgs masses, respectively, and results in

\[
\mathcal{R}_{L\rightarrow R} - \mathcal{R}_{R\rightarrow L} \simeq 2\Delta\theta_{CP} \frac{m_l^3}{m_h |p_z|}.
\]

The corresponding flux of left handed leptons is

\[
J_0 \simeq \frac{v_w m_l^2 m_h \Delta\theta_{CP}}{4\pi^2},
\]

where \( v_w \) is the velocity of the bubble wall. Note that in order for the momentum interval in (30) to be non-vanishing, the condition \( m_l L_w < 1 \) needs to be satisfied.

The injected current from a bubble wall will lead to a “diffusion tail” of particles in front of the moving wall. In the approximation in which the persistence length of the injected current is much larger than the wall thickness we may to a good approximation model it as a delta function source and search for a steady state solution. In addition, assume that the decay time of leptons is much longer than the time it takes for a wall to pass so that we may neglect decays. Then the diffusion equation for a single particle species is easily solved by

\[
L_L(z) = \begin{cases} 
J_0 \frac{\xi_L}{D_L} e^{-\lambda_D z} & z > 0 \\
0 & z < 0 
\end{cases},
\]

with the diffusion root

\[
\lambda_D = \frac{v_w}{D_L},
\]

where \( D_L \) is the diffusion constant for leptons, and \( \xi_L \) that is called the persistence length of the current in front of the bubble wall.
Note that in this approximation the injected current does not generate any perturbation behind the wall. This is true provided $\xi L \gg L_w$ is satisfied. If this inequality is not true, the problem becomes significantly more complex [95].

In the massless approximation the chemical potential $\mu_L$ can be related to $L_L$ by

$$\mu_L = \frac{6}{T^2} L_L$$  \hspace{1cm} (35)

(for details see [95]). Inserting the sphaleron rate and the above results for the chemical potential $\mu$ into (27), and using $1/D_L \approx 8 \alpha_W^2 T$, the final baryon to entropy ratio becomes

$$\frac{n_B}{s} \sim 0.2 \alpha_W^2 (g^*)^{-1} \kappa \Delta \theta_{CP} \frac{1}{v_w} \left(\frac{m_t}{T}\right)^2 m_h \frac{\xi L}{D_L}.$$  \hspace{1cm} (36)

where I have assumed that sphalerons do not equilibrate in the diffusion tail.

Now consider the effects of top quarks scattering off the advancing wall [96, 97]. Several effects tend to decrease the contribution of the top quarks relative to that of tau leptons. Firstly, for typical wall thicknesses the thin wall approximation does not hold for top quarks. This is because top quarks are much more strongly interacting than leptons and so have a much shorter mean free path. An important effect is that the diffusion tail is cut off in front of the wall by strong sphalerons [99, 100]. There is an anomaly in the quark axial vector current in QCD. This leads to chirality non-conserving processes at high temperatures. These processes are relevant for nonlocal baryogenesis since it is the chirality of the injected current that is important in that scenario. In an analogous expression to that for weak sphalerons, we may write the rate per unit volume of chirality violating processes due to strong sphalerons in the unbroken phase as

$$\Gamma_s = \kappa_s (\alpha_s T)^4,$$  \hspace{1cm} (37)

where $\kappa_s$ is a dimensionless constant [101]. Note that the uncertainties in $\kappa_s$ are precisely the same as those in $\kappa$ defined in (11). As such, $\kappa_s$ could easily be proportional to $\alpha_s$, in analogy with (11), perhaps with a logarithmic correction. These chirality-changing processes damp the effect of the injected chiral flux and effectively cut off the diffusion tail in front of the advancing bubble wall. Second, the diffusion length for top quarks is intrinsically smaller than that for tau leptons, thus reducing the volume in which baryogenesis takes place. Although there are also enhancement factors, e.g. the ratio of the squares of the masses $m_t^2/m_\tau^2$, it seems that leptons provide the dominant contribution to nonlocal baryogenesis.

4.0.2. The Thick Wall Regime

Now let’s turn to the thick wall, or adiabatic, regime. This is relevant if the mean free path of the particles being considered is smaller than the width of the wall, $\ell \lesssim L_w$. When the walls are thick, most interactions within the wall will be almost in thermal equilibrium. The equilibrium is not exact because some interactions, in particular baryon number violation, take place on a time scale much slower than the rate of passage of the bubble wall. These slowly-varying quantities are best treated by the method of chemical potentials. The basic idea is to produce asymmetries in some local charges which are (approximately) conserved by the interactions inside the bubble walls, where local departure from thermal equilibrium is attained. These local charges will then diffuse into the unbroken phase where baryon number violation is active thanks to the unsuppressed sphaleron transitions. The latter convert the asymmetries into a baryon asymmetry. Therefore, one has to

i) identify those charges which are approximately conserved in the symmetric phase, so that they can efficiently diffuse in front of the bubble where baryon number violation is fast,

ii) compute the $CP$-violating currents of the plasma induced inside the bubble wall and

iii) solve a set of coupled differential diffusion equations for the local particle densities, including the $CP$-violating sources.

There are a number of possible $CP$-violating sources. To be definite, consider the example where $CP$-violation is due to a $CP$ odd phase $\theta$ in the two-Higgs doublet model [102]. This goes generically under the name of spontaneous baryogenesis. In order to explicitly see how $\theta$ couples to the fermionic sector of the theory (to produce baryons) we
may remove the $\theta$-dependence of the Yukawa couplings arising from the Higgs terms. This is done by performing an anomaly-free hypercharge rotation on the fermions [102], inducing a term in the Lagrangian density of the form $\mathcal{L}_{CP} \propto \partial_\mu \theta J_\mu^Y$, where $J_\mu^Y$ is the fermionic part of the hypercharge current. Therefore, a nonvanishing $\theta$ provides a preferential direction for the production of quarks and leptons; in essence a chemical potential $\mu_B$ (in the rest frame of the thermal bath) for baryon number. Of course, strictly speaking, this is not a chemical potential since it arises dynamically rather than to impose a constraint. For this reason, the quantity $\mu_B$ is sometimes referred to as a "charge potential". A more complete field theoretic approach to the computation of particle currents on a space-time dependent and $CP$-violating Higgs background was provided in [103, 104] where it was shown that fermionic currents arise at one loop, and are proportional to $(h v(T_c)/\pi T)^2 \dot{\theta}$, where $h$ is the appropriate Yukawa coupling. The relevant baryon number produced by spontaneous baryogenesis is then calculated by integrating

$$\frac{d n_B}{d t} = -9 \frac{\Gamma_{np}(T)}{T} \mu_B.$$  (38)

Initially, spontaneous baryogenesis was considered as an example of local baryogenesis. However, it has become clear that diffusion effects can lead to an appreciable enhancement of the baryon asymmetry produced by this mechanism [105, 106].

An alternative way of generating a source for the diffusion equation was suggested by Joyce et al. [107]. The essential idea is that there exists a purely classical chiral force that acts on particles when there is a $CP$ violating field on the bubble wall.

To summarize, the effects of transport in the plasma external to the expanding bubble allow baryon violating transitions in the unbroken phase to transform charge asymmetries produced on the wall into a baryon asymmetry. This means that in the case of nonlocal baryogenesis we do not need to rely on baryon number violating processes occurring in the region where the Higgs fields are changing. The diffusion equation approach to the problem of nonlocal baryogenesis has been very successful. In the thin wall case, it is a valid approximation to assume that the source for the diffusion equation is essentially a $\delta$-function on the wall. This is because one may ignore the effects of particle scattering in this picture. However, in the case of thick walls, significant particle scattering occurs and as a result it is necessary to consider sources for the diffusion equations that extend over the wall.

### 4.1. Electroweak Baryogenesis in the MSSM

As I have mentioned, the most promising and well-motivated framework incorporating $CP$-violation beyond the SM and an enhanced electroweak phase transition seems to be supersymmetry. Electroweak baryogenesis in the framework of the MSSM has attracted much attention in the past years, with particular emphasis on the strength of the phase transition [108–111] and the mechanism of baryon number generation [38, 112–116].

The behavior of the electroweak phase transition in the minimal supersymmetric standard model is dependent on the mass of the lightest Higgs particle, and the mass of the top squark. Recent analytical [117–125] and lattice computations [126–128, 130] have revealed that the phase transition can be sufficiently strongly first order in the presence of a top squark lighter than the top quark. In order to naturally suppress contributions to the $\rho$-parameter, and hence preserve a good agreement with precision electroweak measurements, the top squark should be mainly right handed. This can be achieved if the left handed stop soft supersymmetry breaking mass $m_Q$ is much larger than $M_Z$. For moderate mixing, the lightest stop mass is then approximately given by

$$m_{\tilde{t}}^2 \approx m_{\tilde{t}}^2 + m_{\tilde{t}}^2(\phi) \left( 1 - \frac{|\tilde{A}_t|^2}{m_Q^2} \right),$$  (39)

where $\tilde{A}_t = A_t - \mu^*/\tan \beta$ is the particular combination appearing in the off-diagonal terms of the left-right stop squared mass matrix and $m_{\tilde{U}}$ is the soft supersymmetry breaking mass parameter of the right handed stop.
The preservation of the baryon number asymmetry requires the order parameter \( \langle \phi(T_c) \rangle / T_c \) to be larger than one, see Eq. (20). This quantity is bounded from above

\[
\frac{\langle \phi(T_c) \rangle}{T_c} < \left( \frac{\langle \phi(T_c) \rangle}{T_c} \right)_{\text{SM}} + \frac{2 m_t^3 \left( 1 - \tilde{A}_t^2 / m_Q^2 \right)^{3/2}}{\pi v^2 m_h^2},
\]

where \( m_t = m_t(m_t) \) is the on-shell running top quark mass in the \( \overline{\text{MS}} \) scheme, \( m_h \) is the lightest Higgs boson mass and \( v = \sqrt{v_1^2 + v_2^2} \) is the vev of the Higgs fields. The first term on the right hand side of expression (40) is the Standard Model contribution

\[
\left( \frac{\langle \phi(T_c) \rangle}{T_c} \right)_{\text{SM}} \approx \left( \frac{40}{m_h \text{[GeV]}} \right)^2,
\]

and the second term is the contribution that would be obtained if the right handed stop plasma mass vanished at the critical temperature (see Eq. (42)). The difference between the SM and the MSSM is that light stops may give a large contributions to the effective potential in the MSSM and therefore overcome the Standard Model constraints. The stop contribution strongly depends on the value of \( m_Q^2 \), which must be small in magnitude, and negative, in order to induce a sufficiently strong first order phase transition. Indeed, large stop contributions are always associated with small values of the right handed stop plasma mass

\[
m^\text{eff} = -m_{\tilde{U}}^2 + \Pi_R(T),
\]

where \( m_{\tilde{U}}^2 = -m_{\tilde{U}}^2 \), and

\[
\Pi_R(T) \approx 4 g_5^2 T^2 / 9 + h_t^2 / 6 [2 - \tilde{A}_t / m_Q] T^2
\]

is the finite temperature self-energy contribution to the right-handed squarks. Moreover, the trilinear mass term, \( \tilde{A}_t \), must satisfy \( \tilde{A}_t^2 \ll m_Q^2 \) in order to avoid the suppression of the stop contribution to \( \langle \phi(T_c) \rangle / T_c \). Note that, although large values of \( m_{\tilde{U}} \), of order of the critical temperature, are useful to get a strongly first order phase transition, \( m_{\tilde{U}} \) is bounded from above to avoid the appearance of dangerous charge and color breaking minima [124].

As is clear from (40), in order to obtain values of \( \langle \phi(T_c) \rangle / T_c \) larger than one, the Higgs mass must take small values, close to the present experimental bound. For small mixing and large \( m_A \), the one-loop Higgs mass has a very simple form

\[
m^2 = M_Z^2 \cos^2 2\beta + \frac{3}{4\pi^2} \frac{m_t^4}{v^2} \log \left( \frac{m_t^2 m_{\tilde{t}}^2}{m_t^2} \right) \left[ 1 + \mathcal{O} \left( \frac{\tilde{A}_t^2}{m_Q^2} \right) \right],
\]

where \( m_{\tilde{t}}^2 \approx m_Q^2 + m_t^2 \). Hence, small values of \( \tan \beta \) are preferred. The larger the left handed stop mass, the closer to one \( \tan \beta \) must be. This implies that the left handed stop effects are likely to decouple at the critical temperature, and hence that \( m_Q \) mainly affects the baryon asymmetry through the resulting Higgs mass. A detailed analysis [129], including all dominant two-loop finite temperature corrections to the Higgs effective potential and the non-trivial effects arising from mixing in the stop sector and taking into account the experimental bounds as well as the requirement of avoiding dangerous color breaking minima, concludes that the lightest Higgs should be lighter than about 120 GeV, while the stop mass must be smaller than, or of order of, the top quark mass. This lower bound has been essentially confirmed by lattice simulations [130], providing a motivation for the search for Higgs and stop particles.

As we have seen, the MSSM contains additional sources of \( CP \)-violation besides the CKM matrix phase. These new phases are essential for the generation of the baryon number since large \( CP \)-violating sources may be locally induced by the passage of the bubble wall separating the broken from the unbroken phase during the electroweak phase transition. The new phases appear in the soft supersymmetry breaking parameters associated with the stop mixing angle and the gaugino and neutralino mass matrices. However, large values of the stop mixing angle are strongly restricted in order to preserve a sufficiently strong first order electroweak phase transition. Therefore, an
acceptable baryon asymmetry may only be generated from the stop sector through a delicate balance between the values of the different soft supersymmetry breaking parameters contributing to the stop mixing parameter, and their associated $CP$-violating phases [113]. As a result, the contribution to the final baryon asymmetry from the stop sector turns out to be negligible. On the other hand, charginos and neutralinos may be responsible for the observed baryon asymmetry if a large $CP$-violating source for the higgsino number is induced in the bubble wall. This can happen for sufficiently large values of the phase of the parameter $\mu$ [113, 116].

5. Affleck-Dine Baryogenesis

Now let’s turn to another baryogenesis scenario that has received a lot of attention. This mechanism was introduced by Affleck and Dine (AD) [131] and involves the cosmological evolution of scalar fields carrying baryonic charge. As we shall see, these scenarios are naturally implemented in the context of supersymmetric models. Let us quickly describe this before presenting a detailed analysis.

Consider a colorless, electrically neutral combination of quark and lepton fields. In a supersymmetric theory this object has a scalar superpartner, $\chi$, composed of the corresponding squark $\tilde{q}$ and slepton $\tilde{l}$ fields.

Now, as I mentioned earlier, an important feature of supersymmetric field theories is the existence of “flat directions” in field space, on which the scalar potential vanishes. Consider the case where some component of the field $\chi$ lies along a flat direction. By this I mean that there exist directions in the superpotential along which the relevant components of $\chi$ can be considered as a free massless field. At the level of renormalizable terms, flat directions are generic, but supersymmetry breaking and nonrenormalizable operators lift the flat directions and sets the scale for their potential.

During inflation it is likely that the $\chi$ field is displaced from the position $\langle \chi \rangle = 0$, establishing the initial conditions for the subsequent evolution of the field. An important role is played at this stage by baryon number violating operators in the potential $V(\chi)$, which determine the initial phase of the field. When the Hubble rate becomes of the order of the curvature of the potential $\sim m_{3/2}$, the condensate starts oscillating around its present minimum. At this time, $B$-violating terms in the potential are of comparable importance to the mass term, thereby imparting a substantial baryon number to the condensate. After this time, the baryon number violating operators are negligible so that, when the baryonic charge of $\chi$ is transferred to fermions through decays, the net baryon number of the universe is preserved by the subsequent cosmological evolution.

In this section I will focus on the most recent developments about AD baryogenesis, referring the reader to [10] for more details about the scenario as envisaged originally in [131].

5.1. Supersymmetric Implementations

The most recent implementations of the Affleck-Dine scenario have been in the context of the minimal supersymmetric standard model [132, 133]. In models like the MSSM, with large numbers of fields, flat directions occur because of accidental degeneracies in field space. Although a flat direction is parameterized by a chiral superfield, I shall here focus on just the scalar component. In general, flat directions carry a global $U(1)$ quantum number. I will be interested in those directions in the MSSM that carry $B - L$, since the Affleck-Dine condensate will decay above the weak scale and we must avoid erasure of any asymmetry through rapidly occurring sphaleron processes. Although there are many such directions in the MSSM [133], it is sufficient to treat a single one as typical of the effect. As mentioned above, flat directions can be lifted by supersymmetry breaking and nonrenormalizable effects in the superpotential. An important observation is that inflation, by definition, breaks global supersymmetry because of a nonvanishing cosmological constant (the false vacuum energy density of the inflaton). In supergravity theories, supersymmetry breaking is transmitted by gravitational interactions and the supersymmetry breaking mass squared
is naturally $CH^2$, where $C = O(1)$ [134, 135].

To illustrate this effect, consider a term in the Kähler potential of the form

$$\delta K = C \int d^4 \theta \frac{1}{m_\phi^2} \chi^\dagger \chi \phi \phi^\dagger,$$

(45)

where $\phi$ is the field which dominates the energy density $\rho$ of the universe, that is $\rho \simeq \langle f d^4 \theta \phi \phi^\dagger \rangle$. During inflation, $\phi$ is identified with the inflaton field and $\rho = V(\phi) = 3H^2 m^2$. The term (45) therefore provides an effective $\chi$ mass $m_\chi = -3CH^2$. This example can easily be generalized, and the resulting potential of the AD flat direction during inflation is of the form

$$V(\chi) = H^2 m^2 f(\chi/m_\rho) + H m^3 g(\chi^n/m_\rho^n),$$

(46)

where $f$ and $g$ are some functions, and the second term is the generalized $A$-term coming from nonrenormalizable operators in the superpotential.

Crucial in assessing the possibility of baryogenesis is the sign of the induced mass squared at $\chi = 0$. If the sign of the induced mass squared is negative, a large expectation value for a flat direction may develop, which is set by the balance with the nonrenormalizable terms in the superpotential. In such a case the salient features of the post-inflationary evolution are as follows [133]:

i) Suppose that during inflation the Hubble rate is $H_I \gg m_{3/2}$ and the potential is of the form

$$V(\chi) = (m_{3/2} - |C| H^2) |\chi|^2 + \left[ \frac{\lambda (a H_I + A m_{3/2}) \chi^n}{n M^n} + \text{h.c.} \right] + |\lambda|^2 \frac{|\chi|^{2n-2}}{M^{2n-6}},$$

(47)

where $a$ is a constant of order unity, and $M$ is some large mass scale such as the GUT or the Planck scale. The AD field evolves exponentially to the minimum of the potential, $|\chi| \sim (H_I M^{n-3}/\lambda)^{1/n-2}$. The $A$-term in (47) violates the $U(1)$ carried by $\chi$, and the potential in the angular direction is proportional to $\cos(\theta_a + \theta_\lambda + n\theta)$ where $\chi = |\chi| e^{i\theta}$, and $\theta_a$ and $\theta_\lambda$ are the respective phases of $a$ and $\lambda$. This creates $n$ discrete minima for the phase of $\chi$. During inflation the field quickly settles into one of these minima, establishing the initial phase of the field.

ii) Subsequent to inflation, the minimum of the potential is time dependent since $H$ changes with time and the AD field oscillates near the time dependent minimum $|\chi|(t)$ with decreasing amplitude.

iii) When $H \sim m_{3/2}$ the sign of the mass squared becomes positive and the field begins to oscillate about $\chi = 0$ with frequency $m_{3/2}$ and amplitude $|\langle \chi \rangle|(t \sim m_{3/2}^{-1})$. At this time the $B$-violating $A$-terms are as important as the mass term, and there is no sense in which the baryon number is conserved. When the field begins to oscillate freely, a large fractional baryon number is generated during the initial spiral motion of the field. The important role of $CP$ violation is also dictated by the $A$-terms, since the angular term $\cos(\theta_\lambda + n\theta)$ becomes important at this stage, and a nonzero $\theta$ is generated if $\theta_a \neq \theta_\lambda$. This is of course necessary since the baryon number is given by $n_b = 2 |\chi|^2 \theta$. When the condensate decays, the baryon asymmetry is transferred to fermions.

iv) In the case in which the inflaton decays when $H < m_{3/2}$ (consistently with the requirement that not too many gravitinos are produced at reheating), the resulting baryon to entropy ratio is

$$B \simeq \frac{\rho_\chi}{\rho_\phi} \frac{n_b}{n} \frac{T_r}{m_{3/2}},$$

(48)

where $n_\chi = \rho_\chi/m_{3/2}$, $\rho_\phi \sim m^2_{3/2} m^2_\rho$ is the energy density of the inflaton field when $H \sim m_{3/2}$, and $T_r$ is the reheating temperature. For example $n = 4$ gives [133]

$$B \simeq 10^{-10} \left( \frac{T_r}{10^6 \text{ GeV}} \right) \left( \frac{10^{-3} M_y}{\lambda m^2_\rho} \right).$$

(49)

The MSSM contains many combinations of fields for constructing flat directions carrying a nonvanishing $B - L$ [133]. Particularly appealing directions are the ones which carry $B - L$ and can be lifted at $n = 4$ level. They are the $\chi^2 = LH_2$ directions. The nonrenormalizable operator is then

$$W = \frac{\lambda}{M} (LH_2)^2,$$

(50)
which may be present directly at the Planck scale, or could be generated, as in $SO(10)$ GUTs, by integrating out right-handed neutrino superfields which are heavy standard model singlets.

### 5.2. Initial Conditions for Viable AD Baryogenesis

From the above picture it is clear that a successful AD baryogenesis mechanism is achieved if the effective potential during inflation contains a negative effective mass term and nonrenormalizable terms that lift the flat directions of the potential.

With a minimal Kähler potential only, the effective mass squared $m^2 \sim H_I^2$ during inflation is positive, $\chi = 0$ is stable, and the large expectation values required for baryogenesis do not result. Quantum de Sitter fluctuations do excite the field with $\langle \chi^2 \rangle \sim H_I^2$, but with a correlation length of the order of $H_I^{-1}$. Any resulting baryon asymmetry then averages to zero over the present universe. In addition, the relative magnitude of the $B$ violating $A$-terms in the potential is small for $H_I \ll M$.

A negative sign for the effective mass squared is possible if one either considers general Kähler potentials, or in the case in which inflation is driven by a Fayet-Iliopoulos $D$ term, which preserves the flat directions of global supersymmetry, and in particular keeps the inflaton potential flat, making $D$-term inflation particularly attractive [65]. If the AD field carries a charge of the appropriate sign under the $U(1)$ whose Fayet-Iliopoulos term is responsible for inflation, a nonvanishing vacuum expectation value may develop [136]. Notice that, if the AD field is neutral under the $U(1)$, it may nonetheless acquire a negative mass squared after the end of $D$-term inflation through nonminimal Kähler potentials relating the AD field to the fields involved in the inflationary scenario [137].

Another logical possibility is that the effective mass squared during inflation is positive, but much smaller than $H_I^2$, $0 < C \ll 1$. This happens either in supergravity models that possess a Heisenberg symmetry in which supersymmetry breaking by the inflationary vacuum energy does not lift flat directions at tree level [138], or in models of hybrid inflation based on orbifold constructions, in which a modulus field $T$ is responsible for the large value of the potential during inflation, and a second field $\phi$ with appropriate modular weight is responsible for the roll-over [136]. The correlation length for de Sitter fluctuations in this case is $\ell_c = H_I \exp(3H_I^2/2m^2)$. This is large compared to the horizon size. In fact, the present length corresponding to $\ell_c$ should be larger than the horizon size today. Using $H_I = 10^{13}$ GeV, this gives $(m/H_I)^2 \lesssim 40$, which is easily satisfied. The flat direction is truly flat and the AD baryogenesis may be implemented as originally envisaged [131]. After inflation, these truly flat directions generate a large baryon asymmetry, typically $B = O(1)$. Mechanisms for suppressing this asymmetry to the observed level have been considered in [138]. These include dilution from inflaton or moduli decay, and higher dimensional operators in both GUT models and the MSSM. The observed BAU can easily be generated when one or more of these effects is present.

### 6. Conclusions

The origin of the baryon to entropy ratio of the universe is one of the fundamental initial condition challenges of the standard model of cosmology. Over the past several decades, many mechanisms utilizing quantum field theories in the background of the expanding universe have been proposed as explanations for the observed value of the asymmetry. In this talk I have tried to give a brief overview of GUT scenarios, electroweak baryogenesis, leptogenesis and the Affleck-Dine mechanism. Each of these mechanisms has both attractive and problematic aspects.

While GUT baryogenesis is attractive, since the Sakharov criteria are so naturally satisfied, it is not likely that the physics involved will be directly testable in the foreseeable future. While we may gain indirect evidence of grand unification with a particular gauge group, direct confirmation in colliders seems unrealistic. A second problem with GUT scenarios is the issue of erasure of the asymmetry - unless a GUT mechanism generates an excess $B - L$, any baryonic asymmetry produced will be equilibrated to zero by anomalous electroweak interactions. While this does not invalidate GUT scenarios, it is a constraint. For example, $SU(5)$ will not be suitable for baryogenesis for this reason, while $SO(10)$ may be.
In recent years, perhaps the most widely studied scenario for generating the baryon number of the universe has been electroweak baryogenesis. The physics involved is all testable in principle at realistic colliders and, furthermore, the small extensions of the model involved to make baryogenesis successful can be found in supersymmetry, which is an independently attractive idea. It is worth stressing, however, that electroweak baryogenesis does not rely on supersymmetry.

The testability of electroweak scenarios also leads to tight constraints. At present there exists only a small window of parameter space in extensions of the electroweak theory in which baryogenesis is viable. This is because electroweak baryogenesis to be effective requires a strong enough first order phase transition. This translates into a severe upper bound on the lightest Higgs boson mass: \( m_h < 120 \) GeV, in the case in which the mechanism is implemented in the MSSM. In addition, the stop mass must be smaller than, or of order of, the top quark mass.

If the Higgs and stop are found at the LHC, crucial tests will come from tests of the requisite \( CP \)-violation through \( B \) physics experiments and precision measurements of the Higgs and sfermion sectors at a future linear collider.

Affleck-Dine baryogenesis is a particularly attractive scenario, and much progress has been made in understanding how this mechanism works. As was the case for electroweak baryogenesis, this scenario has found its most promising implementations in supersymmetric models, in which the necessary flat directions are abundant. Particularly attractive is the fact that these moduli, carrying the correct quantum numbers, are present even in the MSSM.

The challenges faced by Affleck-Dine models are combinations of those faced by the GUT and electroweak ideas. In particular, it is necessary that \( B - L \) be violated along the relevant directions (except perhaps in the Q-ball implementations \[139–143\]) and that there exist new physics at scales above the electroweak.

No discussion of baryogenesis is complete these days without mentioning leptogenesis. This idea has found new support in recent years because of the discovery of neutrino masses. If these are due to a heavy right-handed neutrino, then leptogenesis becomes a compelling mechanism.

Whatever the answer, the baryon asymmetry of the universe is clear cosmological evidence for new and interesting physics beyond the standard model of particle physics. With the LHC imminent, a host of impressive \( B \)-physics experiments and a linear collider in the future, the prospects for making progress on this fundamental problem seem good.

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References