The Origin of Mass in QCD*

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In this talk I discuss the origin of mass in quantum chromodynamics in the context of the classical and quantum symmetries of the theory.

1. Introduction

"As we know,
There are known knowns.
There are things we know we know.
We also know
There are known unknowns.
That is to say
We know there are some things
We do not know.
But there are also unknown unknowns,
The ones we don't know
We don't know."
– *Donald Rumsfeld, U.S. Secretary of Defense, Feb. 12, 2002*

Using the classification suggested by Donald Rumsfeld, the subject of the majority of this conference is the "known unknowns" – those questions which can usefully be framed in the context of the standard model of particle physics (and cosmology), but whose answers remain elusive. The unknown, unknowns are the subject of philosophy.

In contrast the subject of this talk, quantum chromodynamics or QCD, is a "known known." Why should we spend time studying this topic at this conference? I hope to convince you in the course of this lecture that there are at least three reasons to do so:

1. The elucidation of the strong force is one of the great intellectual triumphs of quantum field theory [1].

2. QCD is the only experimentally studied strongly-interacting quantum field theory and, as such, illustrates many subtle issues in field theory (many of which are the subject of this lecture).

3. QCD is a paradigm for the sort of strongly-interacting field theories which may be involved in the solution of the "known unknowns" discussed in the rest of this conference.

After reviewing the basics of QCD, the bulk of the lecture will discuss the origin of mass in QCD in terms of the classical symmetries of the QCD Lagrangian and their quantum analogs, and I will conclude with some applications of the properties of QCD (or QCD-like theories) to other issues in particle physics.

1.1. What is Mass?

There are many overlapping definitions of mass, arising from

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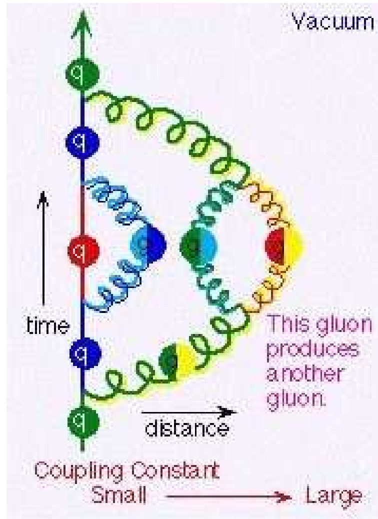

Figure 1: Contribution to "self-energy" of the quark, illustrating the coupling of gluons to themselves.

- Newton's Second Law: $\vec{F} = m\vec{a}$.
- The Relativistic Dispersion Relation: $E^2 = p^2 + m^2$.
- Newton's Principle of Equivalence: $m_{grav} = m$.
- Einstein's Principle of Equivalence: $G_{\mu\nu} \propto T_{\mu\nu}$.

Each of these definitions is subtle and interesting, and subject to a range of important experimental tests and theoretical limitations. In this talk, we will be interested in how the theory of the strong interations – quantum chromodynamics – affects the masses of the physical particles as inferred by *any* of these definitions. Even more interesting: each of the definitions above involve kinematic tests on individual particles – but the strong constituents of matter, quarks and gluons, are confined! What, precisely, do we mean by the mass of these particles?

1.2. What is QCD?

Quantum chromodynamics is the $SU(3)$ Yang-Mills theory of interacting quarks and gluons, and may be summarized by the Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^{(a)}_{\mu\nu} F^{(a)\mu\nu} + i \sum_q \bar{\psi}_{qi} \left[\gamma^\mu (D_\mu)^i_j - m_q \delta^i_j \right] \psi^j_q \ , \tag{1}$$

where the ψ^j_q are the quark fields (labeled by flavor q and color j) transforming in the fundamental (3) representation of the color $SU(3)$ gauge group and m_q is the "Lagrangian" mass of quark q. D_μ is the covariant derivative

$$(D_\mu)^i_j = \delta^i_j \partial_\mu + i g_s \sum_a \frac{(\lambda^a)^i_j}{2} A^a_\mu \ , \tag{2}$$

where λ^a $(a = 1, \ldots, 8)$ are the $SU(3)$ Gell-Mann matrices, the A^a_μ are the gluon fields, and g_s is the QCD (strong) coupling constant. Finally, $F^{(a)}_{\mu\nu}$ is the gluon field-strength tensor

$$F^{(a)}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_s f_{abc} A^b_\mu A^c_\nu \ , \tag{3}$$

where the constants f_{abc} are the $SU(3)$ structure constants. A summary of the quarks and their quantum numbers is given in fig. 2, and a summary of our knowledge of the Lagrangian quark masses is given in fig. 3.

Property \ Quark	d	u	s	c	b	t
Q – electric charge	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$
I – isospin	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
I_z – isospin z-component	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	0	0
S – strangeness	0	0	-1	0	0	0
C – charm	0	0	0	$+1$	0	0
B – bottomness	0	0	0	0	-1	0
T – topness	0	0	0	0	0	$+1$

Figure 2: The Quarks.

The distinguishing feature of non-abelian gauge theories like QCD is that the gauge-bosons, the gluons, carry charge and couple to themselves. The relevant couplings arise from the non-linear terms in the field-strength (eqn. 3), and are illustrated in fig. 1. It is this property of QCD which gives rise to the numerous non-trivial features of the theory.

1.3. The Quark Model

Explaining the Lagrangian quark masses of fig. 3 is the content of the "flavor problem" – the subject of many talks at this summer school. The Lagrangian masses of the light quarks, a few to 10 MeV for the up and down quarks, and around 100 MeV for the strange quark, should be contrasted with what we might expect from the quark model – illustrated in fig. 4. Naively, looking at the baryons or heavy mesons, we expect the quark masses to be of order a third the proton mass or about 300 MeV. While this would seem reasonable for the baryons or heavy mesons, the quark model doesn't explain why the pions are so light! So, in so far as the quark masses are concerned, we have three mysteries:

- How do we interpret the Lagrangian masses (or equivalently, what measurements lead to the results in fig. 3)?
- How do we interpret the quark model masses of about 300 MeV for the quarks?
- How do we account for the anomalously light pions?

2. Classical Symmetries of QCD

Understanding mass in QCD will hinge on understanding of symmetries of QCD.

2.1. Space-time Symmetries

First, we consider the space-time symmetries of QCD:

- Poincare symmetry: As with all relevant quantum field theories, QCD respects relativistic invariance – both Lorentz invariance and translational invariance.

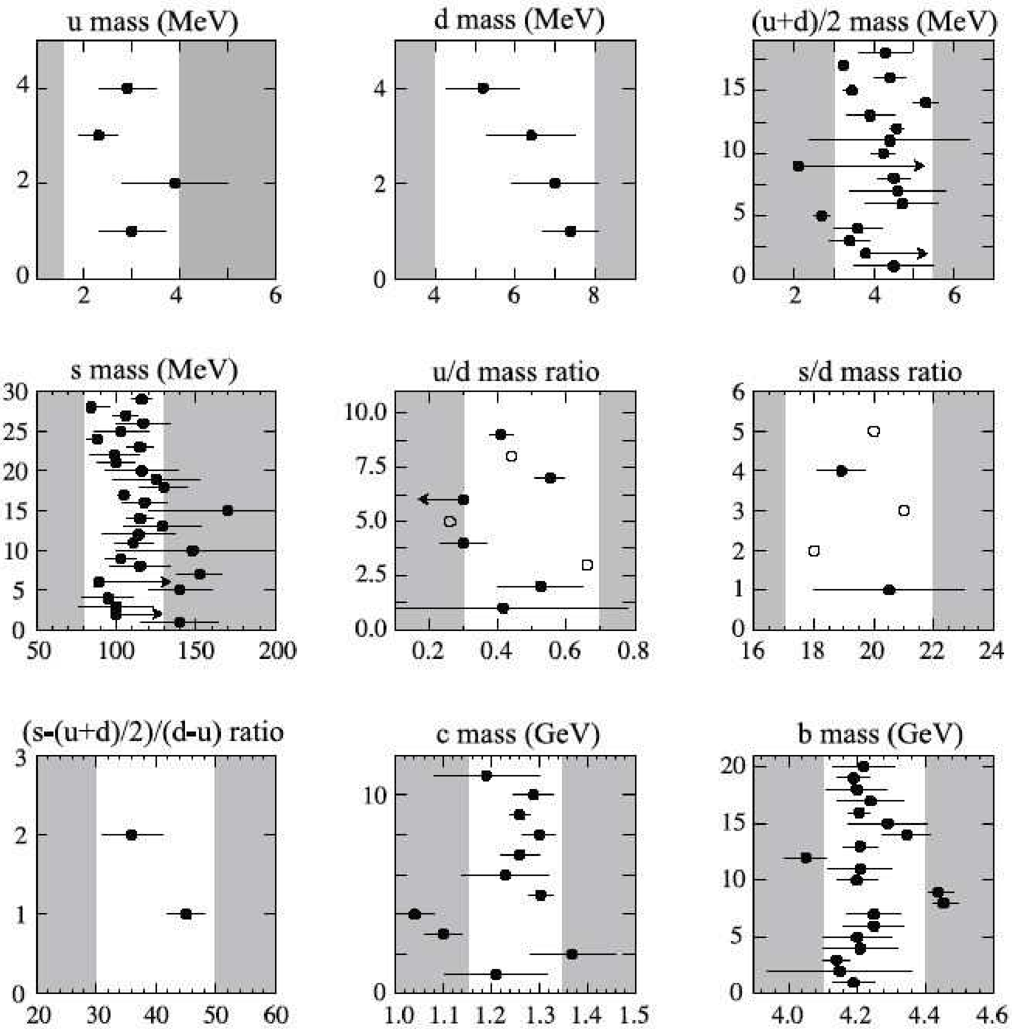

Figure 3: Summary of Lagrangian quark masses from Manohar and Sachrajda in [3].

- As written, the theory respects charge-conjugation, parity, and time-reversal invariance[1].

- (Approximate) Scale Invariance: Consider the scale transformations

$$x^\mu \to \lambda x^\mu \ , \quad \psi_q(x) \to \lambda^{3/2}\psi_q(\lambda x) \ , \quad A^a_\mu(x) \to \lambda A^a_\mu(\lambda x) \ . \tag{4}$$

To the extent that the quark masses are small,[2] classical QCD is approximately scale-invariant.

2.2. Global Quark Flavor Symmetries

Next, consider the global quark symmetries of the theory:

[1]Stay tuned, however, there is potentially another interaction which could have been written in eqn. (1) that could affect this conclusion – see eqn. (23).

[2]In what follows, we will see to what extent the u, d, and s-quark masses are small.

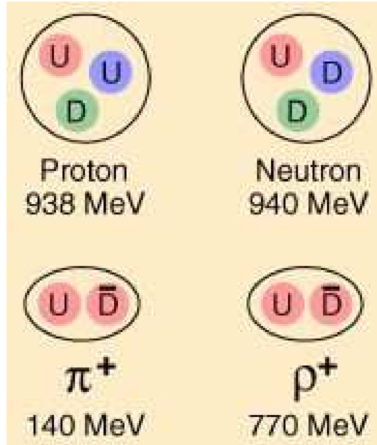

Figure 4: Graphical representation of the quark model.

- Baryon number:

$$\psi_q \to e^{i\alpha}\psi_q \ , \tag{5}$$

  is an exact symmetry of QCD.

To the extent that the u, d, and s quarks may be considered equal, we have

- Approximate $SU(3)_V$ symmetry (Gell-Mann)

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix} \to U \begin{pmatrix} u \\ d \\ s \end{pmatrix} \ . \tag{6}$$

And to the extent the u, d, and s quarks are light, we have the chiral symmetries

- Approximate Chiral $SU(3)_L \times SU(3)_R$

$$\begin{pmatrix} u^{L,R} \\ d^{L,R} \\ s^{L,R} \end{pmatrix} \to U_{L,R} \begin{pmatrix} u^{L,R} \\ d^{L,R} \\ s^{L,R} \end{pmatrix} \ , \tag{7}$$

is an invariance of the quark kinetic energy terms

$$\bar\psi_q i\not{D}\psi_q = \bar\psi_q^L i\not{D}\psi_q^L + \bar\psi_q^R i\not{D}\psi_q^R \ , \tag{8}$$

but not the quark mass terms.

$$m_q\bar\psi_q\psi_q \equiv m_q\bar\psi_q^L\psi_q^R + m_q\bar\psi_q^R\psi_q^L \ . \tag{9}$$

$SU(3)_V$ is the subgroup of $SU(3)_L \times SU(3)_R$ with $U_L = U_R$. The orthogonal set of axial transformations – with $U_L = U_R^\dagger$ – are often denoted "$SU(3)_A$", even they do not form a group.

- And, finally, $U(1)_A$

$$\psi_q^L \to e^{i\alpha}\psi_q^L \ , \qquad \psi_q^R \to e^{-i\alpha}\psi_q^R \ . \tag{10}$$

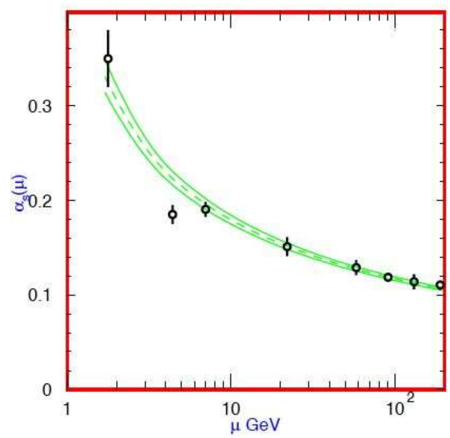

Figure 5: Experimental and theoretical values for the running value of the strong coupling constant [3].

3. No Quantum Scale Invariance

3.1. The Running Coupling

One of the fundamental differences between the classical and quantum systems is the nature of the vacuum. In quantum field theory, the vacuum is a polarizable medium. Therefore, the effective "charge" measured for any coupling constant is a function of the scale at which the measurement is made. The variation of the effective charge as a function of scale is summarized by the β-function of the theory. In QCD, the β-function for the QCD coupling constant at three-loops (in the $\overline{\rm MS}$ scheme) is given by

$$\mu\frac{\partial\alpha_s}{\partial\mu} = 2\beta(\alpha_s) = -\frac{\beta_0}{2\pi}\alpha_s^2 - \frac{\beta_1}{4\pi^2}\alpha_s^3 - \frac{\beta_2}{64\pi^3}\alpha_s^4 - \dots \tag{11}$$

where

$$\beta_0 = 11 - \frac{2}{3}n_f\ , \qquad \beta_1 = 51 - \frac{19}{3}n_f\ , \qquad \beta_2 = 2857 - \frac{5033}{9}n_f + \frac{325}{27}n_f^2\ . \tag{12}$$

Note the negative sign of the QCD β-function – this sign is a crucial difference between non-abelian and abelian gauge-theories: the negative sign can be traced to the contribution from the self-interactions of the gluons. This results in the behavior of the coupling illustrated in fig. 5. At higher energies, shorter distances, this results in the effective coupling becoming weaker – asymptotic freedom [1] – which ultimately justifies the parton model description of hadrons at high energies. In the opposite limit, as one scales to lower energies or larger distances, the effective coupling grows – this is sometimes called infrared slavery, and allows for confinement of color charges.

3.2. What is the value of α_s? Which Quarks are Light?

Given the value of the strong coupling at one scale, the renormalization group equation allows for the prediction of its value at any other scale. Conversely, to allow for the comparison of α_s extracted from different experiments, one may use the renormalization group equation to quote the results of any experiment in terms of $\alpha_s(\mu = M_Z)$. The solution of the renormaliztion group equations (at three-loop order), may be written

$$\begin{aligned}\alpha_s(\mu) \ &= \ \frac{4\pi}{\beta_0\log(\mu^2/\Lambda^2_{QCD})}\left[1 - \frac{2\beta_1}{\beta_0^2}\,\frac{\log\left[\log(\mu^2/\Lambda^2_{QCD})\right]}{\log(\mu^2/\Lambda^2_{QCD})} + \frac{4\beta_1^2}{\beta_0^4\log^2(\mu^2/\Lambda^2_{QCD})}\right. \\ &\times \ \left.\left(\left(\log\left[\log(\mu^2/\Lambda^2_{QCD})\right] - \frac{1}{2}\right)^2 + \frac{\beta_2\beta_0}{8\beta_1^2} - \frac{5}{4}\right)\right]\ ,\end{aligned} \tag{13}$$

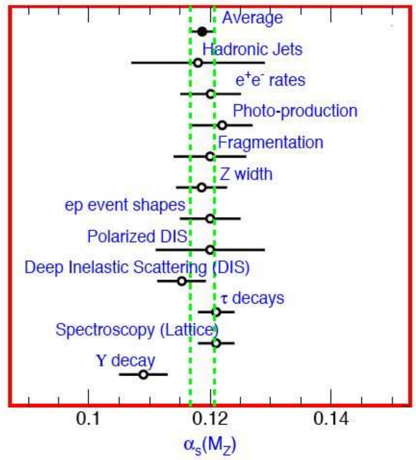

Figure 6: The values of $\alpha_s(M_Z)$ and Λ_{QCD} as extracted from various experiments [3].

and alternatively, instead of quoting a value of α_s one may quote the value of a *dimensionful* quantity, Λ_{QCD}.

It is important to realize that a *coupling constant* per se is not a directly observable quantity – only the results of a potential experiment is an observable. The program of physics is to use the measurements of some finite number of experiments to calculate the results of others – coupling constants[3] are simply useful numerical intermediate steps in the calculations. As such, the extracted values of α_s and Λ_{QCD} depend on the calculational scheme chosen (*e.g.*, the results shown in fig. 6 correspond to the $\overline{\mathrm{MS}}$ renormalization prescription).

In particular in relating the value of $\alpha_s(M_Z)$ to Λ_{QCD} one must specify the number of active quark flavors (n_f) in eqns. (11) and (12). At a scale of order M_Z, there are five active flavors (corresponding to the u, d, s, c, and b quarks), and it is therefore conventional to quote the corresponding value – this is found [3] to be

$$\Lambda^{(5)}_{\overline{\mathrm{MS}}} = 217^{+25}_{-23}\,\mathrm{MeV}\ . \tag{14}$$

At low-energies, of order the masses of the lightest baryons, the number of active flavors is only three (for the u, d, and s), and the corresponding value is then[4]

$$\Lambda^{(3)} \simeq 350\,\mathrm{MeV}\ . \tag{15}$$

Examining eqn. (14) and fig. 5, we see that this value sets the scale at which the strong coupling becomes large, and therefore sets the energy scale at which nonpertubative effects become important.

In our discussion of the symmetries of QCD, we saw that the chiral symmetries were only approximate and were broken explicitly by Lagrangian quark masses. These symmetries are useful only to the extent that the symmetry breaking masses are "small" – in particular, only if the Lagrangian quark masses are small compared to the low-energy QCD scale of order 350 MeV. It is in this sense that the u, d, and s quarks are light – and why we don't consider[5] the chiral properties of the c, b, or t quarks!

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[3] As well as other quantities such as Lagrangian quark masses which, due to confinement, do not correspond the pole masses of observable particles.

[4] One determines the value of $\Lambda^{(3)}$ from $\Lambda^{(5)}$ by imposing continuity of the coupling constant at the scales of order the heavy quark masses that are being "integrated out."

[5] It is important to note, however, that dimensional transmutation is essential to our ability to consider the heavy quark limit, $\Lambda_{QCD}/m_q \to 0$, which results in symmetries relating the properties of various c and b mesons and baryons [4, 5, 6].

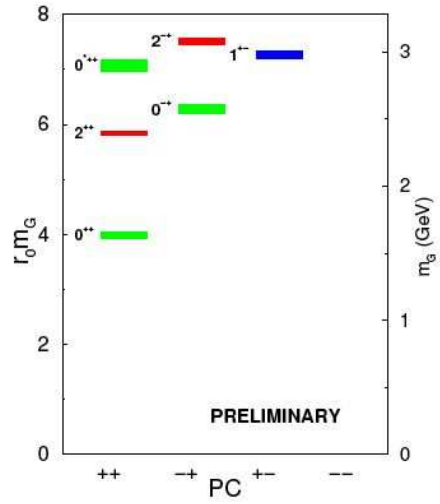

Figure 7: Glueball spectrum in QCD without quarks, from lattice gauge theory [8].

3.3. The Death of Scale Invariance and the Mass-Gap

The appearance of a dimensional scale related to the dimensionless coupling of QCD is an example of "dimensional transmutation" – a general property of quantum field theory.[6]

Dimensional transmutation would arise even if there were no quarks – in this case, the spectrum of the theory would consist entirely of massive bound states of gluons called glueballs. In the real world, of course, we cannot eliminate the quarks and experimentally verify this belief. We can, however, do numerical simulations of such a theory using lattice gauge theory. The results of such a calculation [8] are shown in fig. 7. We see that an $SU(3)$ Yang-Mills theory (without fermions) has a "mass gap" – that is, it has no massless excitations.

The existence of a mass gap in quarkless QCD, a theory with no dimensional parameters in the classical Lagrangian, is a sign that the classical scale invariance of the theory is broken. More generally, the fact that the QCD coupling runs (fig. 5) and that one can therefore characterize the coupling in terms of Λ_{QCD}, shows that scale invariance in QCD is broken in the quantum theory.

From Noether's theorem, we know that any continuous transformation defines an associated current – and we know that if this transformation is a *symmetry*, the corresponding current is conserved. We can calculate the current associated with scale transformations, eqn. (4), s^μ – and computing in the quantum theory we find

$$\partial_\mu s^\mu = -\frac{\beta}{2g_s} F^{(a)}_{\mu\nu} F^{(a)\mu\nu} + \sum_q m_q \bar{\psi}_q \psi_q \neq 0 \qquad (16)$$

The second term in the equation shows what we expect – the Lagrangian quark masses explicitly break the scale symmetry. The first term, however, is unexpected – it shows that the scale invariance is also broken by the fact that the β-function of QCD is not zero. The β-function is an intrinsically quantum effect, and this result illustrates that scale symmetry is *anomalous* – it is an approximate symmetry of the classical theory which is explicitly broken by quantum fluctuations!

[6] For an excellent review of this and other topics in field theory, see [7].

4. Chiral Symmetries are broken!

4.1. Why the Pions are Light ...

Unlike scale symmetry, the nonabelian $SU(3)_L \times SU(3)_R$ chiral symmetries (*c.f.* eqn. (7)) are good quantum symmetries of a theory with massless quarks – and these symmetries are therefore approximate symmetries of the world to the extent that the u, d, and s quarks are light. However, the strong low-energy QCD dynamics rearranges the vacuum and the attractive interactions in the color-singlet spin-zero channel cause a Bose-Einstein condensate of the quark fields

$$\langle \bar{u}_L u_R \rangle = \langle \bar{d}_L d_R \rangle \approx \langle \bar{s}_L s_R \rangle \propto \Lambda^3_{QCD} \neq 0 \ , \tag{17}$$

with, because of parity symmetry, an equal condensate for the opposite chirality RL combinations of these fields. The condensate breaks the individual $SU(3)_{L,R}$ symmetries down to the vectorial symmetry $SU(3)_V$ (*c.f.* eqn. (6)).

Chiral symmetry breaking in QCD is an example in which the nonperturbative quantum dynamics of the theory drives the *spontaneous* breaking of a symmetry. In the three-quark theory, the chiral condensate breaks eight linearly independent continuous symmetries, and eight corresponding currents $j^i_{A\mu}$. Goldstone's theorem [9] tells us that there will be eight low-energy Goldstone bosons (π^i) associated with these currents, and we may write

$$j^i_{A\mu} = -f_\pi \partial_\mu \pi^i + \dots \tag{18}$$

where f_π is the pion decay constant, approximately 93 MeV in the normalization chosen here (and the dots correspond to terms with more fields whose form is determined by symmetry).

In a theory with massless quarks, $\partial^\mu j^i_{A\mu} \equiv 0$ and the corresponding Goldstone bosons would be massless. As the u, d, and s quarks are light but not massless, we expect the corresponding particles to be *light*. The lightest strongly-interacting particles are the pions, and identifying them as the "would-be" Goldstone bosons of QCD, we explain why they are anomalously light. Treating the quark masses as perturbations, we find

$$\begin{aligned} m^2_\pi &\propto (m_u + m_d)\Lambda_{QCD} \\ m^2_K &\propto (m_s + m_{u,d})\Lambda_{QCD} \qquad (19) \\ m^2_\eta &\propto \frac{1}{3}(m_u + m_d + 4m_s)\Lambda_{QCD} \ , \qquad (20) \end{aligned}$$

where the masses in this expression are to be interpreted as Lagrangian quark masses (as these are what explicitly break the chiral symmetries) normalized at energies of order a GeV (indeed, these expressions and their higher-order relatives are input into fig. 3).

In the limit $m_u = m_d$ there is one relation amongst these three mass squareds – this is the Gell-Mann–Okubo relation, and it is well satisfied for the squareds masses of the pions. Furthermore, having identified the pions as Goldstone bosons of chiral symmetry breaking, the chiral symmetry algebra implies many relations among pion amplitudes – so-called current algebra relations – which are known to be satisfied within approximately 20%.

4.2. ... and the Baryons Heavy

Chiral symmetry breaking also gives us a picture of how to understand the quark model illustrated in fig. 4. Aside from the pions, which are light by virtue of being approximate Goldstone bosons, the hadrons may be thought of as composed of "constituent quarks" – quarks dressed by their interactions with the chiral symmetry breaking QCD vacuum. From this point of view, the pions are just different – they are intrinsically relativistic approximate Goldstone boson bound states, and are not usefully characterized by the non-relativistic quark model.

The mass scale associated with the interactions between the quarks and the vacuum is set by the (three-quark) value of Λ_{QCD}, and the approximate 300 MeV masses of quarks in the quark model should be interpreted as constituent (dressed) quark masses. Note that the constituent quark masses have nothing, *a priori*, to do with the Lagrangian quark masses discussed previously. Since the Lagrangian quark masses for the u and d quarks are only of order 10 MeV or less, we see that 99% of the mass of the proton (and therefore essentially all the mass of ordinary matter in the universe) arises [7] from QCD!

4.3. What about $U(1)_A$?

The $U(1)_A$ symmetry of eqn. (10) would also be broken by the chiral condensate. If this were truly a symmetry, one would expect a ninth approximate Goldstone boson. This boson would be an isosinglet pseudoscalar boson. The lightest candidate is the η' which has a mass 958 MeV – and is not particularly light! In fact, if $U(1)_A$ is truly a symmetry of QCD, Weinberg [11] showed that the mass of the corresponding approximate Goldstone boson is bounded by $\sqrt{3}m_\pi \simeq 225$ MeV. The absence of such a ninth approximate Goldstone boson was known as the $U(1)$-problem.

In fact, $U(1)_A$ is anomalous. Like scale invariance, this classical symmetry is violated in the quantum theory. As shown by Adler, and Bell and Jackiw (originally in the context of QED) the divergence of the corresponding current is not zero [12, 13]. In the case of the $U(1)_A$ quark current in QCD, one finds in the massless quark limit

$$\partial_\mu j^\mu_A \propto \frac{g_s^2}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}F^{(a)}_{\mu\nu}F^{(a)}_{\alpha\beta} \ . \tag{22}$$

The axial anomaly was well-known prior to Weinberg's work [11], however the $F\tilde{F}$ combination of field-strength tensors appearing in eqn. (22) can be shown to be a *total derivative*! Therefore, such an interaction cannot have any effect to any *finite* order in perturbation theory. In principle, one could redefine the current j^μ_A such that it was conserved and, as such, it was hard to see why a ninth Goldstone boson was absent.

Fortunately, shortly thereafter, 't Hooft demonstrated [14] that there were nonperturbative contributions – instantons – which resolved this problem. 't Hooft showed that, in the semiclassical approximation, there were field configurations[8] of finite action which had the property that the the integral of $F\tilde{F}$ doesn't vanish. While the semiclassical approximation breaks down at low-energies in QCD, because the strong coupling becomes large, instantons demonstrate explicitly that j^μ_A is broken by nonperturbative quantum effects. $U(1)_A$ is therefore not a symmetry of QCD, and there is no $U(1)$ problem.

No good deed goes unpunished, however. Having shown that the operator $F\tilde{F}$ could have an effect in QCD (or in any other non-abelian gauge theory, in fact), it is possible to entertain an additional term in the Lagrangian $\mathcal{L}_{QCD}$

$$\mathcal{L}_{CP} = -\frac{g_s^2\,\theta_{QCD}}{32\pi^2}\epsilon^{\mu\nu\alpha\beta}F^{(a)}_{\mu\nu}F^{(a)}_{\alpha\beta} \ . \tag{23}$$

Expressing the operator $F\tilde{F}$ in terms of QCD fields, we find the resulting interaction is proportional to the dot product of the chromoelectric and chromomagnetic fields. As such, such a term violates CP symmetry and would contribute to the electric dipole moment of the neutron. Experimental constraints then imply $\theta_{QCD} \le \mathcal{O}(10^{-9})$. It might be tempting to conclude that θ_{QCD} is simply absent – however we know that CP *is* violated in the electroweak sector, and there is no good reason for θ_{QCD} to be small. This puzzle is known as the strong CP problem and it is so far unresolved.

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[7] There is a potential subtlety in this argument: to the extent that the strange quark is heavy, the proton and neutron masses could be thought of as arising from a "two-flavor" value of Λ_{QCD}. Since all representations of Λ_{QCD} must predict the same value of $\alpha_s(M_Z)$, this two-quark value Λ_{QCD} (and therefore the proton mass) depends indirectly on the value of the strange quark mass. Using current algebra, this dependence can be related to various pion-nucleon scattering amplitudes — this is related to the so-called "sigma term" – and the inferred value is surprisingly high [10]

$$m_s\frac{dm_p}{dm_s} \simeq \mathcal{O}\,(10\%) \ . \tag{21}$$

[8] In Euclidean space, but that is a technicality.

Symmetries in QCD

Symmetry	Anomaly?	Fate
Color (gauged)	no	unbroken
Scale Invariance*	yes	not a symmetry
$SU(3)_V$* x $U(1)_B$	no	unbroken
$SU(3)_A$*	no	spontaneously broken
$U(1)_A$*	yes	not a symmetry
CP	yes?!	not a symmetry!?

*Approximate

Figure 8: A summary of the classical symmetries of QCD and their quantum fate.

5. Summary and Applications

The theme of this talk has been that the origin of mass in QCD is intimately tied to the classical symmetries of QCD and their quantum fate, summarized in fig. 8. The approximate scale symmetry of QCD is anomalous, giving rise to dimensional transmutation and the scale Λ_{QCD}. The dynamical spontaneous breaking of chiral symmetry explains why the pions are light, while the other baryons and mesons remain heavy. Finally, the $U(1)_A$ anomaly and instantons explain why the η' is not an approximate Goldstone boson. The properties of QCD described here have a number of important applications, and we conclude by mentioning a few of these.

5.1. Asymptotic Freedom and the Unification of Gauge Couplings

In Grand Unified Theories [15], one envisions that all gauge interactions arise from a single gauge theory. In order for this to occur, all of the gauge-coupling constants must be related. At first glance, this would seem impossible as their couplings are so different. Asymptotic freedom, however, implies that the strong coupling becomes smaller at higher energies – and at sufficiently high energies, its value can equal [16] that of the weak or hypercharge couplings.[9] An illustration of the running of the coupling constants in a supersymmetric model is given in fig. 9, and it is the running of the couplings that determines the scale of the breaking of the unified gauge group, $M_{GUT} \simeq \mathcal{O}(10^{16}$ GeV).

[9] Because $U(1)_Y$ is an abelian group, its normalization is only specified once the unified gauge group G and the embedding of $U(1)_Y$ in G is specified.

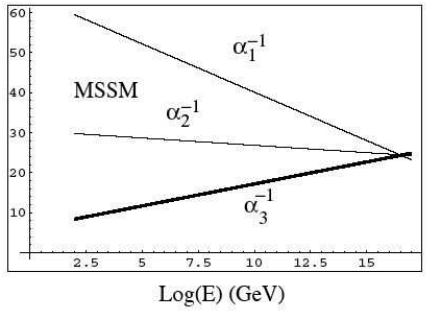

Figure 9: An illustration of the unification of couplings in a supersymmetric model – asymptotic freedom implies that $\alpha_s^{-1} = \alpha_3^{-1}$ grows at higher energies.

5.2. Top-Quark Matters

As shown by eqn. (12), the rate at which the strong-coupling runs depends on the number of active quark flavors. If one fixes the value of g_s at high energies – say at the GUT scale – and then varies the value of the mass of one of the heavy quarks, one changes the value of g_s at low energies. Hence, by changing the top-quark mass for fixed high-energy strong coupling, one changes the value of Λ_{QCD} and hence the mass of the proton. Using the renormalization group equation, one finds the following dependence of the proton mass on the top-quark mass [17]

$$\frac{m_p}{1\,\mathrm{GeV}} \propto \left(\frac{m_t}{175\,\mathrm{GeV}}\right)^{2/27} . \tag{24}$$

5.3. Technicolor

It is intriguing that the global symmetry breaking structure of two-flavor QCD, $SU(2)_L \times SU(2)_R \to SU(2)_V$, is precisely the global symmetry structure of the Higgs sector of the one-doublet standard model. This implies one can construct a theory of *dynamical* electroweak symmetry breaking using QCD-like dynamics [18, 19] – these are "technicolor" theories [20].

In the simplest such model one introduces a new strong $SU(N_{TC})$ gauge theory and, analogous to the up- and down-quarks in QCD, two new fermions transforming (which we will denote U and D) as fundamentals of this gauge symmetry. These new "techniquarks" carry an $SU(2)_L \times SU(2)_R$ global symmetry – the analog of the (approximate) chiral symmetry of the light quarks in QCD. Just as in QCD, the "low-energy" strong dynamics of this new gauge theory is expected to cause chiral symmetry breaking, that is a non-perturbative expectation value for the chiral condensates $\langle \bar{U}_L U_R \rangle = \langle \bar{U}_R U_L \rangle$ and similarly for the D fermions.

If the left-handed techniquarks form an $SU(2)_W$ doublet, while the right-handed techniquarks are weak singlets carrying hypercharge, technicolor chiral symmetry breaking will result in electroweak symmetry breaking. The Goldstone bosons arising from chiral symmetry breaking are transmuted, by the Higgs mechanism, into the longitudinal components of the electroweak gauge bosons.

Theoretically, technicolor addresses all of the shortcomings of the one-doublet Higgs model: there are no scalars, electroweak symmetry breaking arises in a natural manner due to the strong dynamics of a non-abelian gauge theory, the weak scale is related to the renormalization group flow of the strong technicolor coupling and can be much smaller than any high energy scale and, due to asymptotic freedom, the theory (most likely) exists in a rigorous sense.

Unfortunately, the simplest versions of this theory – based, as described, on a scaled-up version of QCD – are not compatible with precision electroweak data[10] (and, as described so far, cannot accommodate the masses of the

[10] See Langacker and Erler in [3].

quarks and leptons). Nonetheless, this simplest version remains a paradigm for thinking about theories of dynamical electroweak symmetry breaking.

6. Conclusions

As acknowledged by this years Nobel Prize in Physics to Gross, Politzer, and Wilczek, the modern understanding of the strong interactions is a great intellectual success story. As illustrated here, the pieces of this story are highly nontrivial and hinge on the various ways in which symmetries can be realized (or not) in quantum field theory. In this sense, and with apologies to Gilbert and Sullivan [21], QCD is *the very model of a modern quantum field theory.*

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