Supersymmetry Basics

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2004 SLAC Summer Institute.

- What is SUSY

  Remarkable propeties
  Symmetry implications
  Relevance to particle physics

  } Toy Model

- Soft Supersymmetry Breaking }
- Construction of Supersymmetric theories
  A cook book for SUSY gauge theories
- The Supersymmetric extension of The SM
  MSSM
  R-parity & its implication
- A tractable framework for SUSY phenomenology.
- Radiative Electroweak SUSY breaking

SUSY SEARCHES

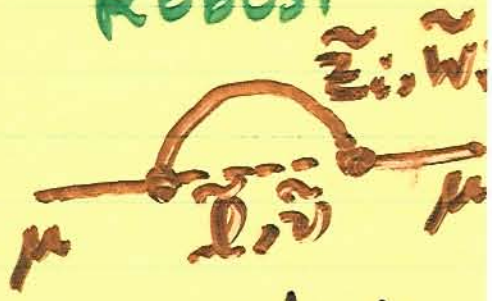
Indirect via loop effects

$b \to s\gamma$, sl^+l^- ... Okada

$g_\mu - 2$ Shagin

Flavour violating effects sensitive to detailed assumptions

Flavour Conserving Robust



O(1) effects as SM flavour viol^{n} also loop effect

Also CP violation! e-dipole Elec Mom

- Direct Searches @ Colliders

Lightning overview

- How superparticles are produced
- How superparticles decay

- Expectations for Tevatron, LHC, Linear Colliders Desch

- Beyond SUSY Discovery

A SUSY toy model.

A, B spin zero real — 2 pol.

ψ spin $\frac{1}{2}$ self conjugate field — 2 pol.

$\psi = \psi^c$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu A)^2 + \frac{1}{2}(\partial_\mu B)^2 + \frac{i}{2}\bar{\psi}\not{\partial}\psi$$

$$-\frac{1}{2}m^2(A^2+B^2) - \frac{1}{2}m\bar{\psi}\psi$$

Common mass A, B, ψ

$$-\frac{g}{\sqrt{2}}A\bar{\psi}\psi \qquad \frac{g}{\sqrt{2}}i B\bar{\psi}\gamma_5\psi$$

A, scal
B, pse
scala

$$-\frac{g}{\sqrt{2}}mA^3 \qquad -\frac{g}{\sqrt{2}}mAB^2$$

$$-\frac{1}{4}g^2A^4 \qquad -\frac{1}{4}g^2B^4 \qquad -\frac{1}{2}g^2A^2B^2$$

Parity ✓

ONLY one coupling g (and 1 mass)

rinted for Margie Kidani <mkidani@hawaii.edu>

Symmetries that we know

Translations, rotations, boosts (space time)

Isospin invariance (approx) (Internal Symm)

E.M & Yang Mills Gauge invar.

Symmetry Generator or "charge" Q

$[Q, H] = 0$ (Q is conserved)

let $H|\text{state}\rangle = \mathcal{E}|\text{state}\rangle$

$HQ|\text{state}\rangle = QH|\text{state}\rangle = \mathcal{E} \cdot Q|\text{state}$

if $Q|\text{state}\rangle \neq \overset{0}{\lambda}|\text{state}\rangle$ i.e. $Q|\text{state}\rangle = |\text{new state}$

$H|\text{new state}\rangle = \mathcal{E}|\text{new state}\rangle$

$|\text{new state}\rangle$ and $|\text{state}\rangle$ have same $\mathcal{E}$!

Symmetry $\Leftrightarrow$ degeneracy of spectrum

Familiar Symmetries relate properties of bosons (fermions) to those of other bosons (fermions)

Rot^n Invariance $\Leftrightarrow$ Deg. w.r.t magnetic Q. # in hydrogen

Isospin $\Leftrightarrow$ properties of π^+, π^-, π^0 (or n, p)

Symmetry of toy model is odd in this sense because it relates

A, B (bosons) and ψ (fermion) - ∴ Appendix

→ SUPERSYMMETRY ←

"

Technically, Q_{susy} is a fermionic charg

$$Q\,|\text{state}\rangle = |\text{new state}\rangle$$

$$Q_{susy}\,|\text{Boson (fermion)}\rangle = |\text{Fermion (Boson)}\rangle$$

"$\frac{1}{2}$ Spin Generator"

Thus, if $Q_{susy}|state\rangle \neq 0$

Bosonic & Fermionic states come in energy degenerate pairs.

- # of bosonic states = # of fermionic stat
- mass of boson = mass of fermion

The fermion is called the superpartner of the boson & vice-versa

- Superpartners have spin differi by $\frac{1}{2}$ unit of $\hbar$
- All their other Q. #s are the sam

⇓

eg. We know all their gauge interactions!

Theory is predictive!

Back to our Toy Model

$\Phi = \frac{A + iB}{\sqrt{2}}$, ψ ⟷ Super partners

$m_\Phi = m_\psi$

OK, but why should I give a *!%t ?

Because it is pretty, and because SUSY stabilizes the (EWSB) Scalar sector in presence of radiative corrections

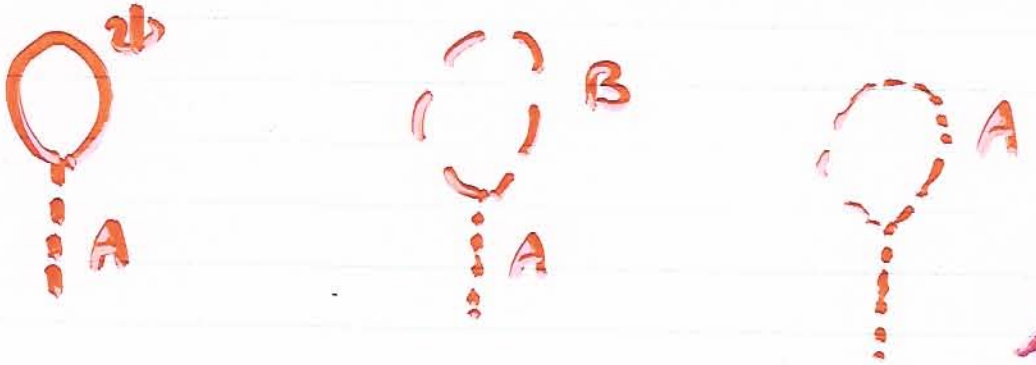
SM "Quadratic divergences" in m_H^2

Λ = scale up to which SM is supposed to be a valid description

In SM λ, g, f are indep. so any cancellation of Λ^2 only for 1 more.

What if bosonic & fermionic couplings were related by a symmetry?

Look at Λ^2 terms in Toy Model



Combinatorial

$$\mathrm{Tr}\int d^4p\,\frac{\not p + m_\psi}{p^2-m_\psi^2} = m\int\frac{d^4p}{p^2-m_B^2} - 3m\int\frac{d^4p}{p^2-m_A^2}$$

$$= 4\int d^4p\left\{\frac{4m_\psi}{p^2-m_\psi^2} - m\frac{1}{p^2-m_B^2} - 3m\frac{1}{p^2-m_A^2}\right\}$$

$= 0$ because $m_A = m_B = m_\psi$

Important Observation: Λ^2 term cancels as long as $m = m_\psi$

i.e. even if m_A & m_B are different.

"Tadpoles absent if scalar is not a symmetry singlet"

Exercise: You check that the Λ^2 term cancels regardless of masses

Coupling constant relationships of $A\psi\psi$, A^2B^2, A^4 interactions are crucial.

Log "divergences" remain

- Cancellation persists at all orders as it is a consequence of the SUSY

Supersymmetric partners ▬ cancel Λ^2 contributions from one another

To stabilize the scalar sector of the SM, write down its supersymmetric extension

- Complete SUSY multiplets $\Leftrightarrow$ Introduce superpartner for SM fields

- Couplings of superpartners are related to those of SM particles (we know these!) by supersymmetry

[Remember: Symmetries relate various couplings]

Extremely predictive Theory w/o "Λ^2 problem"!

Unfortunately this beautiful theory is excluded by data!
There is no boson with $m = 0.511\,eV$ and charge $= \pm 1\,e$!

SUPERSYMMETRY IS NOT EXACT.

Supersymmetry Breaking.

Recall SUSY was introduced to cancel "Λ^2 terms".

These continue to cancel even if we put a big mass for the spin zero partner of the electron! [Cf. Toy model

Do not, however mess with the dimensionless couplings of the theory otherwise Λ^2 terms will return!

Terms that break supersymmetry w/o introducing Λ^2 divergences are called SOFT SUSY BREAKING terms.

These are essential for building SUSY models of particle physics.

How heavy should the spin zero partners be?

- Heavy enough so that accelerator expts. wouldn't have seen them

- $\delta m_H^2 \sim \frac{g^2}{16\pi^2}\left(m^2_{\text{Boson}} - m^2_{\text{fermion}}\right)$

$$\lesssim m_H^2 \lesssim (500\ \text{GeV})^2$$

So that radiative corrections are under control.

$\Rightarrow$ Superpartners not too heavy!

Optimally, I'd like to break SUSY spontaneously.

Can be done, but no time to discuss right now.

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Spontaneous SUSY breaking

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Soft SUSY breaking terms as a function of the SUSY breaking order parameter.

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SUSY breaking terms would be related instead of arbitrary

MODELS OF SUSY Breaking

DIFFERENCE BETWEEN SUCCESSIVE TRANSFORMATIONS - in both orders - characterizes the symmetry

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COMMUTATOR OF GENERATORS

Rotations $[J_i, J_j] = i\epsilon_{ijk} J_k$

Repeat for SUSY transforms for our toy model (Appendix)

$$(\delta_2\delta_1 - \delta_1\delta_2)\psi = 2i\,\bar{\alpha}_2 \gamma^\mu \alpha_1 \partial_\mu \psi$$

($\bar{\alpha}_2$: parameter; ∂_μ: Translation)

$$\{Q, \bar{Q}\} = 2\gamma^\mu P_\mu$$

SUSY is a space-time symmetry!

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If SUSY transformations are local, ie. parameter depends on x,

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Connection with gravity

LOCAL SUSY ⇒ GRAVITY

Constructing SUSY Theories

ISOSPIN

$\begin{pmatrix} p \\ n \end{pmatrix}, \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix}$

multiplets of the symmetry

Transform into one another

$N = \begin{pmatrix} p \\ n \end{pmatrix}, \vec{\pi} = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix}$

"Nucleon" or "Pion Field"

Isospin Calculus

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Isospin invariant action

SUPERSYMMETRY

multiplets of the symmetry ϕ

Transform into one another ψ, F

$\hat{S} = \begin{pmatrix} \phi \\ \psi \\ F \end{pmatrix}$

Chiral Superfield

Superfield Calculus

⇓

SUSY Invariant action

Cannot describe superfield calculus here – not difficult, but time is short

I will give you the answer though!

You can construct your own SUSY models!

- Start with a bunch of superfields
$\hat{S}_i = \hat{S}_1, \hat{S}_2, \ldots$

- Choose a function $\hat{f}(\hat{S}_1, \hat{S}_2 \ldots)$

[analogous to choosing a potential in usual field theory]

$\hat{f}$ cannot depend on $\hat{S}_i$ and $\hat{S}_i^\dagger$ i.e. it is an analytic function.

$$\mathcal{L} = \sum_i (\partial_\mu \phi_i)^*(\partial_\mu \phi_i) + \frac{i}{2}\sum_i \bar{\psi}_i \not{\partial} \psi_i$$

$$- \sum_i \left| \frac{\partial \hat{f}}{\partial \hat{s}_i} \right|^2_{\hat{s}_i = \phi_i}$$ ← Scalar potential

$$- \sum_{i,j} \bar{\psi}_i \frac{1-\gamma_5}{2} \frac{\partial^2 \hat{f}}{\partial \hat{s}_i \partial \hat{s}_j}\Big|_{\hat{s}=\phi} \psi_j + \text{h.c.}$$

↖ Fermion Scalar interaction

gives a supersymmetric action.

Renormalizability ⇒ $\hat{f}$ at most a cubic polynomial of $\hat{s}_i$

Exercise: Verify $\hat{f}(\hat{s}) = \frac{1}{2} m \hat{s}^2 + \frac{1}{3} g \hat{s}^3$ gives the Lagrangian for our Toy Model with $\phi = \frac{A + iB}{\sqrt{2}}$.

Wess-Zumino
1974 NPB.

SUPERSYMMETRIC GAUGE THEORY

Gauge Bosons $\vec{A}_\mu$ 2 polarizations

Super-partner Gauginos $\vec{\lambda}$ 2 polarizations

↑ Majorana spinor

Bosonic d.o.f = Fermionic d.o.f.

$(\vec{\lambda}, \vec{A}_\mu, \vec{\phi})$

Super multiplet

Real Superfield [not chiral]

$$\mathcal{L} = \sum_i (D_\mu \phi_i)^\dagger (D^\mu \phi_i) + \frac{i}{2} \sum_i \bar{\psi}_i \not{D} \psi_i$$

$$- \frac{1}{4} F^A_{\mu\nu} F^{\mu\nu A} + \frac{i}{2} \bar{\lambda}_A \gamma^\mu D_\mu \lambda_A$$

$$- \sqrt{2} \sum_{i,A} \left(\phi_i^\dagger g t_A \bar{\lambda}_A \frac{1-\gamma_5}{2} \psi_i + \text{h.c.} \right)$$

$$- \sum_i \left| \frac{\partial \hat{f}}{\partial \hat{S}_i} \right|^2_{\hat{S}=\phi} - \frac{1}{2} \sum_A \left(\sum_i \phi_i^\dagger g t_A \phi_i + \xi_A \right)^2$$

$$- \frac{1}{2} \sum_{i,j} \bar{\psi}_i \left[\frac{\partial^2 \hat{f}}{\partial \hat{S}_i \partial \hat{S}_j} \right|_{\hat{S}=\phi} \frac{1-\gamma_5}{2} + \left(\frac{\partial^2 \hat{f}}{\partial \hat{S}_i \partial \hat{S}_j} \right)^\dagger_{\hat{S}=\phi} \frac{1+\gamma_5}{2} \Bigg] \psi_j$$

$\xi_A \neq 0$ only if there are U(1) factors in the gauge group.

$\hat{f}$ should be gauge invariant fcn.

$$\mathcal{D}_\mu \phi = \partial_\mu \phi + i \sum_A g t_A V_{\mu A} \phi$$

$$\mathcal{D}_\mu \psi = \partial_\mu \psi + i \sum_R g t_A V_{\mu A} \psi_L - i \sum_R g t_A^* V_{\mu A} \psi_R$$

$$\mathcal{D}_\mu \lambda_A = \partial_\mu \lambda_A + i g \left(t^{adj}_B\right)_{AC} V_{\mu B} \lambda_C$$

$$F_{\mu\nu A} = \partial_\mu V_{\nu A} - \partial_\nu V_{\mu A} - g f_{ABC} V_{\mu B} V_{\nu C}$$

are appropriate covariant derivatives.

lines (1) + (2) Kinetic Energy terms

Line (3) Interaction of gaugino $\vec{\lambda}$ with matter fermion + matter scalar - fixed by gauge coupling.

Line (4) Scalar potential

Notice g^2 term proportional to quartic.

Line (5) Yukawa Couplings from the superpotential only.

MOST GENERAL GLOBALLY SUPERSYMMETRIC LAGRANGIAN. (action)

STANDARD MODEL

	SU_3	SU_2	$U(1)_Y$
$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	1	2	-1
~~ν_R~~ E^c	1	1	2
$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	3	2	$\frac{1}{3}$
~~u_R~~ $= U^c$	3^*	1	$-\frac{4}{3}$
~~d_R~~ D^c	3^*	1	$2/3$
$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$	1	2	1

MINIMAL SUPERSYMMETRIC

$$\hat{H}_d = \begin{pmatrix} \hat{h}_d^0 \\ \hat{h}_d^- \end{pmatrix} \quad 1 \quad 2^* \quad -1$$

Gauge Super Fields

$$\hat{B} = (\lambda_B, B_\mu, \mathcal{D}_B)$$

$$\hat{W}_A = (\lambda_A, W_{A\mu}, \mathcal{D}_{AW})$$

$$\hat{g}_A = (\tilde{g}_A, g_{A\mu}, \mathcal{D}_g)$$

SUPERPOTENTIAL

$$\hat{f} = \mu \hat{H}_u \hat{H}_d$$

$$+ \hat{Q}_i (f_u)_{ij} \hat{U}^c_j \hat{H}_u + \hat{Q}_i (f_d)_{ij} D^c_j \hat{H}_d$$

$$+ \hat{L}_i (f_e)_{ij} E^c_j \hat{H}_d$$

$\hat{H}_u$ give up type fermions a mass via their Yukawa couplings, exactly as in the SM.

$\hat{H}_d$ down

In SM one doublet was enough

WHY TWO HERE?

In S.M. if $H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$ gave the down quark mass then the conjugate doublet $\begin{pmatrix} h^{0\dagger} \\ -h^{+\dagger} \end{pmatrix}$ gave the up quark mass.

BUT WE ARE NOT ALLOWED TO USE $\hat{H}_U^\dagger$ & $\hat{H}_U$ IN THE SUPERPOTENT.

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INVENT A NEW SUPERFIELD WITH THE SAME QUANTUM NOS. AS $\hat{H}_U^\dagger$!

MIRACLE

$\hat{H}_U$ Fermions cause "anomalies" in the theory

⇒ Inconsistent.

The fermion Content of $\hat{H}_U$ & $\hat{H}_D$ (as well as partners of gauge fields) makes theory Anomaly-free!

$\left(\begin{matrix} e_L \\ \nu_L \end{matrix}\right), e_R$ } spin $\frac{1}{2}$ × families $\left(\begin{matrix} \tilde{e}_L \\ \tilde{\nu}_L \end{matrix}\right), \tilde{e}_R$ spin-0

$\left(\begin{matrix} u_L \\ d_L \end{matrix}\right), u_R, d_R$ × families $\left(\begin{matrix} \tilde{u}_L \\ \tilde{d}_L \end{matrix}\right), \tilde{u}_R, \tilde{d}_R$

g, γ, $W^{\pm}, Z^0$ } spin 1

Spin $\frac{1}{2}$ { $\tilde{g}$ Gluino; $\tilde{\gamma}$ Photino; $\tilde{W}^{\pm}, \tilde{Z}^0$ Wino, Zino

Higgs Bosons

$h^0, H^0, A^0, H^{\pm}$

spin-0

Spin $\frac{1}{2}$ $\left(\begin{matrix} \tilde{h}_u^+ \\ \tilde{h}_u^0 \end{matrix}\right), \left(\begin{matrix} \tilde{h}_d^- \\ \tilde{h}_d^0 \end{matrix}\right)$ Higgsinos

Invention of new particles to complete symmetry multiplets has precedent

Positron

Ω^-

COMPLICATION

$\tilde{\gamma}, \tilde{Z}, \tilde{h}_u^0, \tilde{h}_d^0$

MIX TO FORM SPIN ½ PARTICLES CALLED NEUTRALINOS

$\tilde{Z}_1, \tilde{Z}_2, \tilde{Z}_3, \tilde{Z}_4$

Simly 2 DIRAC Charginos

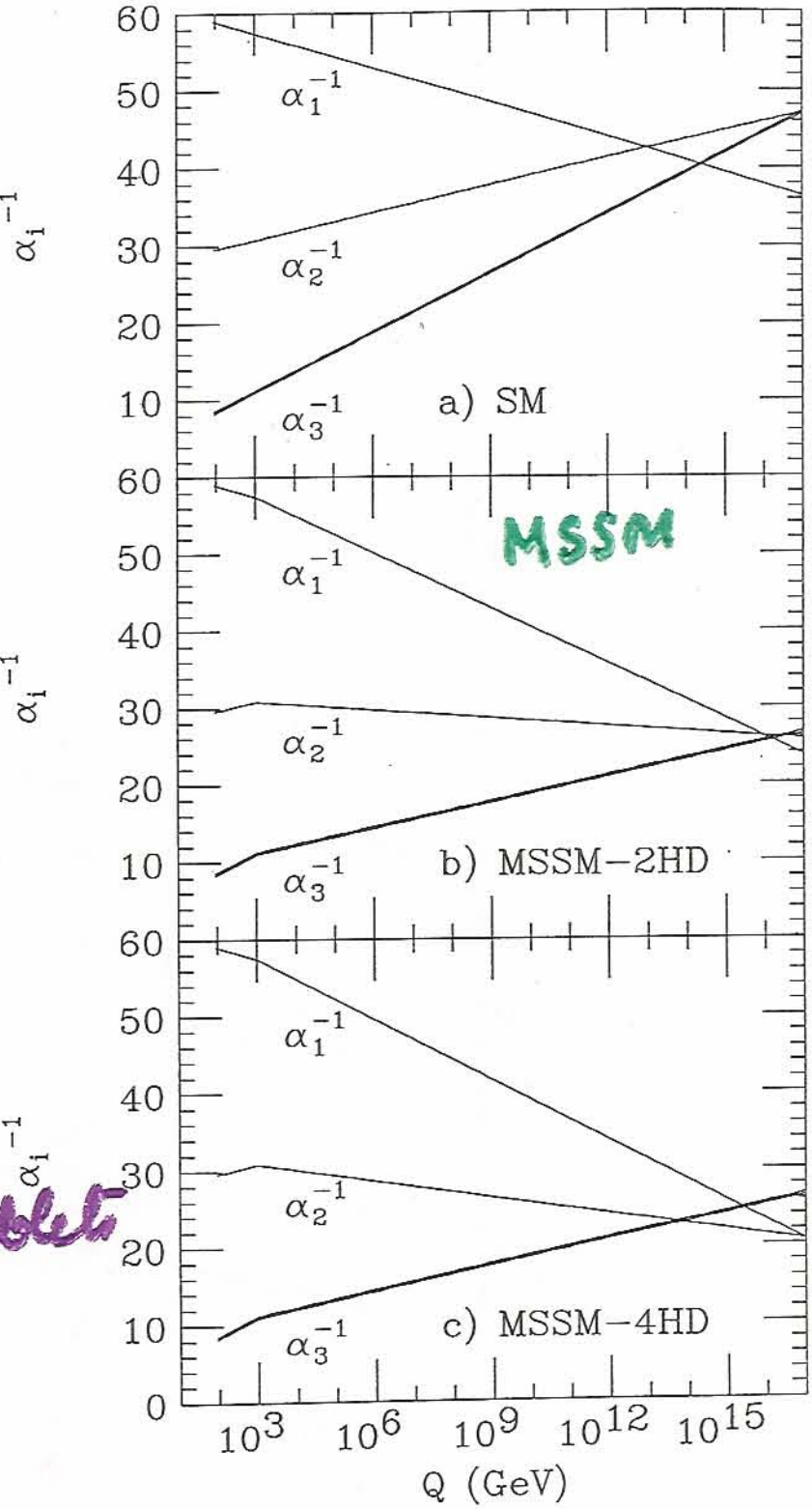
$\tilde{W}_1, \tilde{W}_2$

Gauge coupling Unification

Not in SM

Miracle

Need
2 & only
2 Higgs doublets



This model, augmented by soft SUSY breaking couplings is called the Minimal Supersymmetric Standard Model, MSSM

Girardello Grisaru

- Scalar mass Matrices
- Gaugino Masses
- Trilinear & Bilinear scalar couplings

$$A_{ijk}\,\phi_i\,\phi_j\,\phi_k \;,\; B_{ij}\,\phi_i\,\phi_j$$ (124)

- Trilinear couplings $C_{ijk}\,\phi_i^{+}\,\phi_j\,\phi_k$ if no singlets exist. (178)

Notice these are dimensionful parameters as we mentioned before

No-SUSY Breaking dimensionless coupling

A CONFESSION

We could have included the terms:

$$\lambda_{ijk}\,\underset{-1}{\hat{L}_i}\,\underset{-1}{\hat{L}_j}\,\underset{2}{\hat{E}^c_k} + \lambda'_{ijk}\,\underset{-1}{\hat{L}_i}\,\underset{\frac{1}{3}}{\hat{Q}_j}\,\underset{\frac{1}{3}}{\hat{D}^c_k}$$

$$+\lambda''_{ijk}\,\underset{-\frac{4}{3}}{\hat{U}^c_i}\,\underset{\frac{2}{3}}{\hat{D}^c_j}\,\underset{\frac{2}{3}}{\hat{D}^c_k} + \mu'_i\,\underset{-1}{\hat{L}_i}\,\underset{+1}{\hat{H}_u}$$

Renormalizable, Gauge-invariant

λ, λ', μ' violate $L \neq \text{Cons}^n$

λ'' violates baryon $\neq \text{Cons}^n$

SUSY ⇒ These are not generated radiatively.

But that we can write such couplings is a step back from the SM.

Define "Matter Parity" as

+1	Gauge Higgs superfields
-1	Quark Lepton superfields.

If we assume matter parity cons^{n} unwanted terms are forbidden.

Equiv. to conservation of

$$R = (-1)^{3(B-L)+2S}$$

SM Particles are R-even

SUSY Particles are R-odd.

Although R-parity Conservation is not Compulsory, we will assume it from now on.

- Ensures proton stability (Weak rate)
- non-observation of $n-\bar{n}$ oscill^{ns}.
- no mixing between leptons & Charginos/neutralinos.

※ Superparticles only pair produced at accelerators where we collide only SM particles.

※ Superparticles cannot decay into only SM particles

⇓

Lightest Supersymmetric Particle (LSP)

must be stable.

※ Decays of superparticles terminates in the LSP.

LSP's would have been in thermal equilibrium with everything else early in the history of the universe

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Freeze out at $T \sim M_{LSP}$

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Can compute their abundance today.

For $M_{LSP} \lesssim O(TeV)$

expected abundance of charged or coloured LSPs trapped as heavy atoms/nuclei

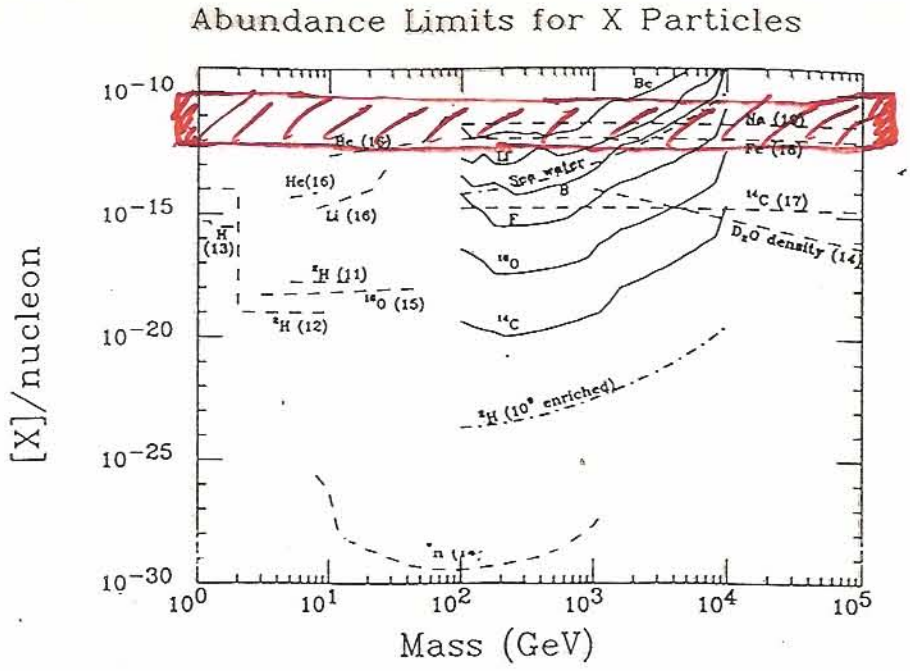
$\approx O(10^{-10} - 10^{-6})$

Empirical limits on anomalous isotopes

$\lesssim O(10^{-12} - 10^{-29})$

↓

hydrogen.



Expect for TeV charged LSP.

FIG. 7. Measured concentration limits for X particles in various elements. Previous results are shown with dashed lines with the corresponding reference number indicated in parentheses. The present results for X^- particles are shown with solid lines, and for X^+ particles as dotted-dashed lines. For hydrogen, only the results for the seawater sample and the sample that was enriched by a factor of 10^9 are shown.

TABLE II. Abundance limits (at the 2σ level) for heavy isotopes established by this experiment. In the case of H, C, and O, sample enrichment techniques were used.

Mass (amu)	H	Li	Be	B	C	O	F
100	2×10^{-24}	2×10^{-13}	4×10^{-12}	4×10^{-14}	4×10^{-20}	4×10^{-17}	7×10^{-15}
500	9×10^{-24}	3×10^{-13}	2×10^{-12}	1×10^{-14}	2×10^{-20}	6×10^{-17}	5×10^{-16}
1000	3×10^{-23}	6×10^{-13}	1×10^{-11}	6×10^{-14}	8×10^{-20}	4×10^{-17}	4×10^{-15}
5000	2×10^{-21}	9×10^{-11}	1×10^{-9}	1×10^{-11}	3×10^{-17}	4×10^{-15}	3×10^{-13}
10 000	3×10^{-20}	7×10^{-10}	7×10^{-9}	8×10^{-11}	2×10^{-16}	3×10^{-14}	2×10^{-12}

Previous searches $\frac{X}{H} \lesssim 10^{-28}$ for X mass 6-350 amu.

LSP is not charged or Coloured

$\tilde{\nu}$, $\tilde{\chi}_1$

If we assume the LSP makes up our galactic DM halo $\tilde{\nu}$ is disfavoured

$\tilde{\chi}_1$ is an excellent dark matter candidate & appears to be compatible with various recent measurements of the μ-wave background.

BONUS!

In local SUSY we also have a gravitino as a candidate LSP.
[Subject of recent investigation]

More later on this? [Matchev]

Since sparticle decays terminate in the LSP, every SUSY event has 2 LSP's in it.

If LSP has no strong & EM interactions, it will be like a ν and escape exptal detection!

At colliders this will manifest itself as missing energy (in the transverse direction)

$\not{E}_T$ events are generic

We saw that an agnostic parametrization of the MSSM ⇒ 178 free parameters

Looks bad, but don't be surprised

Most of these are in the SUSY breaking sector!

Weak Int^{n} theory before V-A!

Understanding that Weak Interactions came from a symmetry principle, AND understanding how this symmetry is broken ⇒ manageable theory!

If we discover SUSY understandi SUSY breaking ⇒ Manageable Theory!

What do we do now?

Guidance from phenomenology.

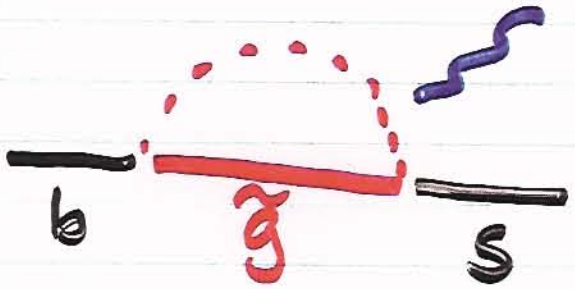
Many scalars ⇒ Dangerous FCNC

$\bar{q}_0 \Gamma \tilde{g} q_0 \rightsquigarrow$ "Current basis"

$q = U q_0 \;,\; \tilde{q} = \tilde{U} \tilde{q}_0$ Mass basis

$\bar{q} U^\dagger \Gamma \tilde{g} \tilde{U} \tilde{q}$

Problem $U^\dagger \tilde{U} \neq I \Rightarrow$ FCNC!



$b \to s\gamma$!

(Okada)

What can we do?

- Make Sparticles heavy (decoupling)
- Make sparticles with same quantum #s degenerate (rotation of Id. matrix is for free)
- Make $U^{\dagger}\tilde{U}^{R} = I$ dynamically.

In 1980's people tried models of global SUSY with SUSY broken @ the weak scale

Tree level Sum rules

$$\sum_{\text{boson}} (2j+1) M_j^2 = \sum_{\text{fermion}} (2j+1) M_j^2$$

in each charged sector

Squark lighter than m_b!

No time to disuss models but...

Evade tree level sum rules by

- Local SUSY (supergravity)
- Give SM particles masses @ loop level (Gauge mediated SUSY breaking)

Scalars with same Q #s get universal masses.

FCNC ok (though additional technical assumptions needed)

Extreme Ansatz Universality

Universal scalar mass m_0

Universal trilinear coupling A_0

bilinear higgs mixing b_0

Universal gaugino mass $m_{1/2}$

Superpotential higgsino mass μ.

Flavour physics OK.

Realized in the Minimal Supergravity Model (mSUGRA)

GUT Assumption for gaugino masses

One Gaugino Coupling mass at $Q = M_{GUT}$

$$\frac{M_3}{\alpha_3} = \frac{M_2}{\alpha_2} = \frac{M_1}{\alpha_1}$$ @ $Q = M_W$ due to renormaliz^{n}

other ways to get same rel^{n}

OBJECTION: If mass parameters are universal how can we get electroweak symmetry breaking?

Need -ve mass^2 for Higgs but not for other Scalars!

MIRACLE SUSY breaking is a high scale phenomenon

Should think of universality as holding for Lagrangian parameters at scale $Q \sim M_{Pl.}$ or M_{GUT}

Radiative corrections split the spectrum

Similar to running gauge couplings

- Gauge couplings INCREASE scalar masses as we run from high scale to low scale
- Yukawa couplings do the opposite

$$m^2_{\tilde{q}} \gtrsim m_0^2 + m_q^2 + (5\text{-}6)\, m_{\frac{1}{2}}^2 + \text{D-term}$$

$$m^2_{\tilde{l}_L} \gtrsim m_0^2 + m_l^2 + 0.5\, m_{\frac{1}{2}}^2 + \text{D-term}$$

$$m^2_{\tilde{l}_R} \gtrsim m_0^2 + m_l^2 + 0.15\, m_{\frac{1}{2}}^2 + \text{D-term}$$

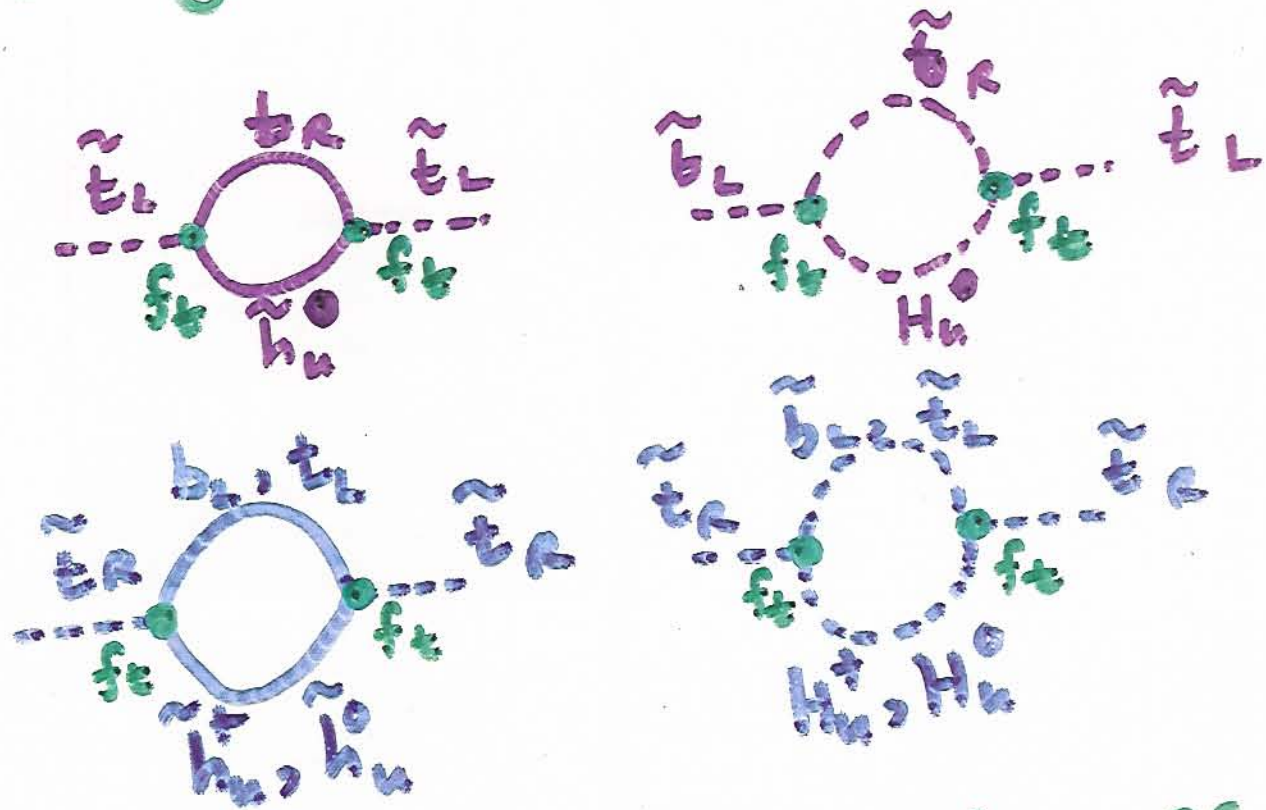
~0.8 $m_{\frac{1}{2}}^2$

if Yukawa couplings are ignored

$$\boxed{m_{\tilde{q}} \gtrsim m_{\tilde{l}_L} \gtrsim m_{\tilde{l}_R}}$$

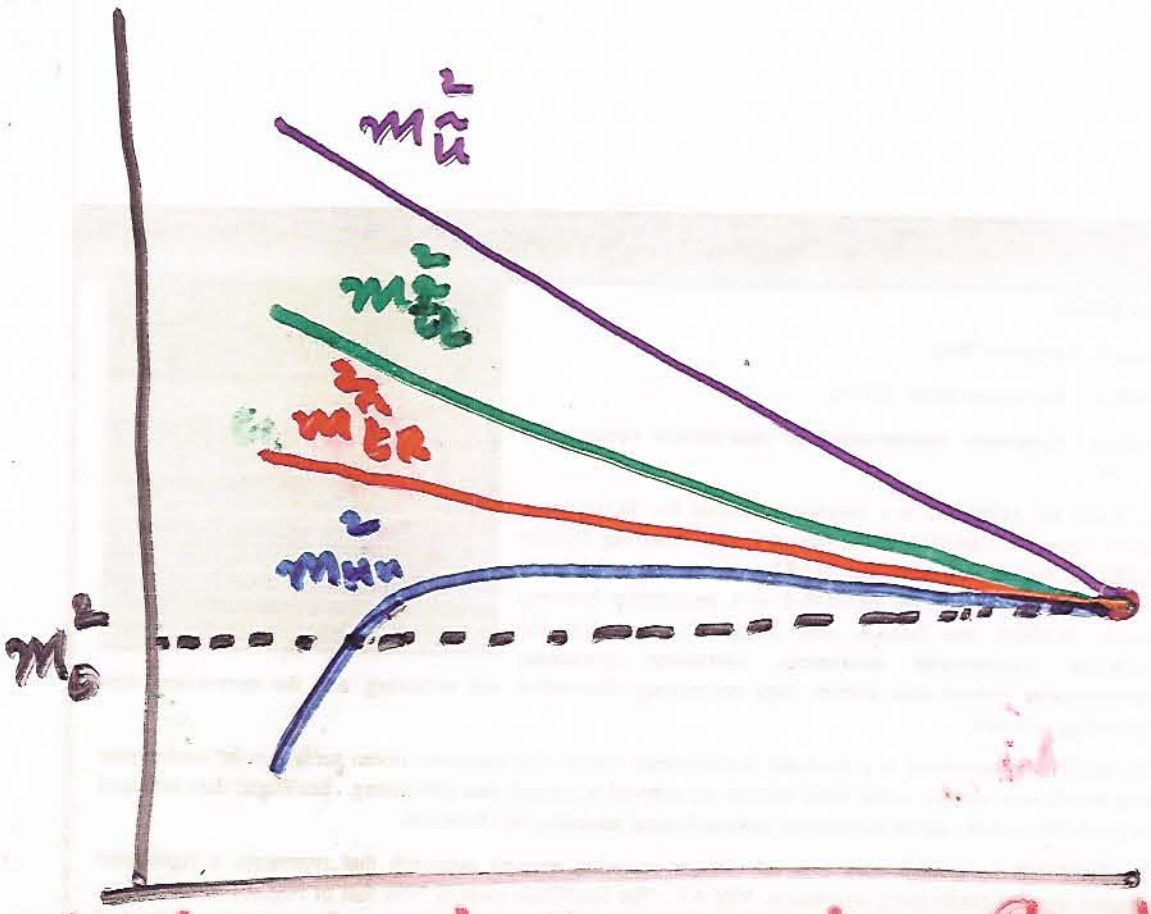
for 1st & 2nd Generations where Yukawas are very small.

If Yukawa coupling is large the story is different



Effect of Yukawa twice as great on $m^2_{\tilde{t}_R}$ as $m^2_{\tilde{t}_L}$

Effect of Yukawa on $m^2_{H_u}$ three times as great because of colour!



Electroweak symmetry Broken due to radiative corrections!

Have to be sure that $\tilde{t}_R$ does not break colour & charge gauge invariance.

mSUGRA Model

m_0 , A_0 , b_0 , $m_{\frac{1}{2}}$, μ_0 (high scale)

↓

scalar masses , A_i , b , M_1 M_2 M_3 , μ

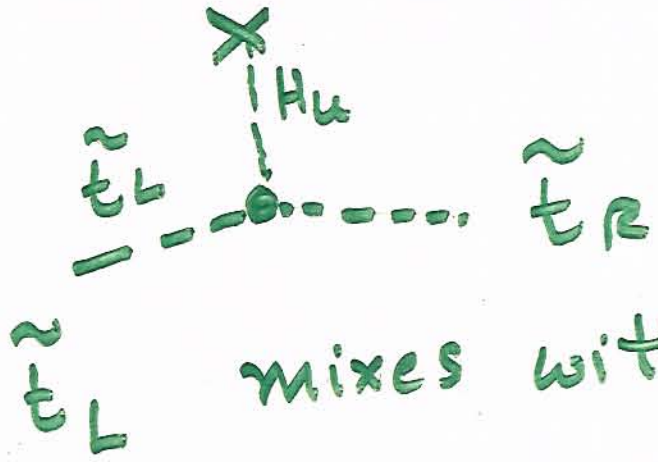
Require correct M_Z^2 from the Higgs potential (with radiative corrections included) Fix μ^2

Eliminate b in favour of $\frac{v_u}{v_d} \equiv \tan\beta$

$m_0, m_{\frac{1}{2}}, \tan\beta, \mathrm{Sgn}\,\mu$
$+ m_t$

Another effect of "Yukawa Coupling"

Recall soft SUSY breaking coupling $\tilde{t}_L^{\dagger}\, \tilde{t}_R\, H_u$



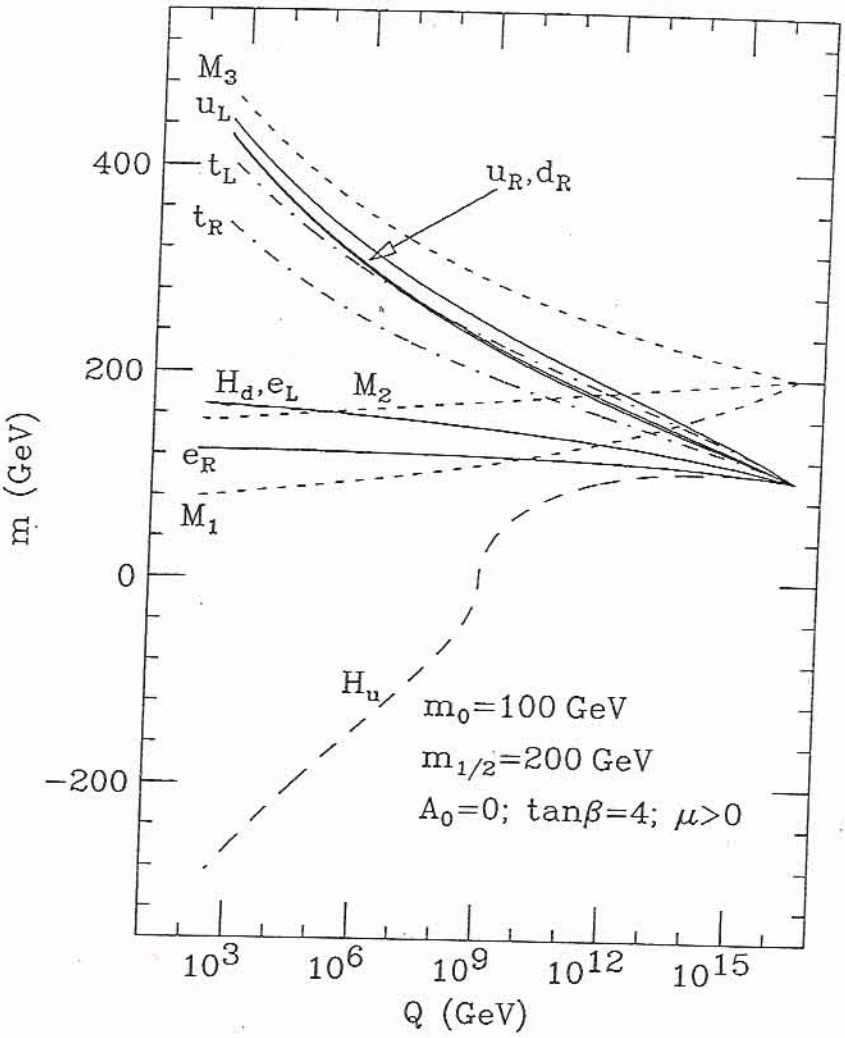
$\tilde{t}_L$ mixes with $\tilde{t}_R$

Mass eigenstates $\tilde{t}_L - \tilde{t}_R$ mixtures

Lighter state mass comes down!

$\tilde{t}_1$ (and possibly, $\tilde{b}_1$) may be much lighter than other squ

Example of an mSUGRA spectrum with small $m_0 = \frac{1}{2} m_{1/2}$



$$m_{\tilde{g}} \simeq m_{\tilde{q}} \simeq m_{\tilde{t}_L} > m_{\tilde{t}_R} > m_{\tilde{e}_L} > m_{\tilde{W}_1} > m_{\tilde{e}_R} > m_{\tilde{Z}_1}$$

COLLIDER SIGNATURES

To understand these, we have to know production mechanisms and decay patterns of sparticles.

Production typically occurs by gauge interactions [Model-indep.]

- At hadron colliders $\tilde{q}$, $\tilde{g}$ copiously produced if these are not too heavy.
- At e^+e^- colliders production fixed by $SU(2) \times U(1)$ Q's #s

☠ Some sensitivity of prod^n cross-sections to spartick mixings.

SUMMARY OF SPARTICLE DECAYS

- assume $M_1 \simeq \frac{1}{2} M_2 \approx \frac{1}{6} M_3 \ll |\mu|$

  Gaugino mass unification

  Typical in many models

$\tilde{q}_{L,R} \to q \tilde{g}$ if allowed [often so]

Else $\tilde{q}_L \to q \tilde{W}_i, q \tilde{Z}_j$; $\tilde{q}_R \to q \tilde{Z}_j$

$\tilde{q}_L \to$ ino with highest SU(2) gaugino content ; $\tilde{q}_R \to q \tilde{Z}_1$ dominantly

$\tilde{g} \to \bar{q} \tilde{q}^* \to \bar{q} q \tilde{Z}_j, \bar{q} q \tilde{W}_i$

Preference to ino with highest SU(2) gaugino content.

$\tilde{l} \to l \tilde{Z}_i, \nu \tilde{W}_i$

$\tilde{\nu} \to \nu \tilde{Z}_i, l \tilde{W}_i$

Sneutrinos need not be invisible

$\tilde{W}_i \to f\bar{f}', \tilde{Z}_i W, \tilde{Z}_i H$

$\tilde{W}_1 \to \ell\nu\,\tilde{Z}_1$
$q\bar{q}\,\tilde{Z}_1$ (frequently with BF's of W decays)

$\tilde{Z}_j \to \tilde{Z}_i H, \tilde{Z}_1 Z^0; \tilde{W}_i W, f\bar{f}\tilde{Z}$

$\tilde{Z}_2 \to f\bar{f}\,\tilde{Z}_1$ Branching fractions parameter dependent

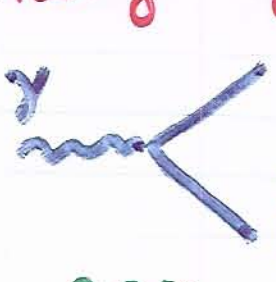
Decays to third generation favoured if $\tan\beta$ is large

$$f_b v_d = f_b (175\,GeV)\cos\beta = m_b$$
$$\Rightarrow f_b = \frac{m_b}{175\,GeV} * \frac{1}{\cos\beta}$$

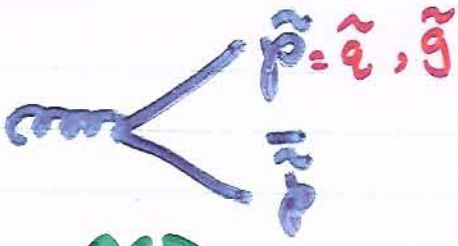
Heavy squarks, sleptons & gluinos cascade decay as a rule

SPARTICLE PRODUCTION

Mostly by gauge interactions



QED

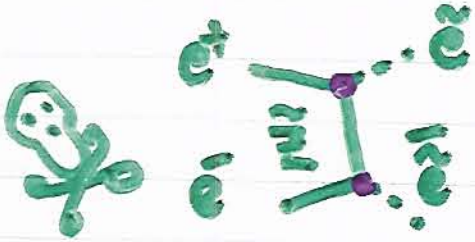


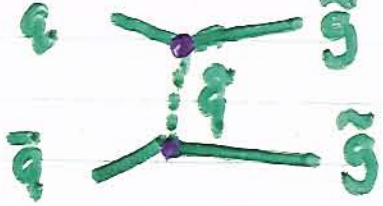
QCD

Model Independent

Dependon mixing.

- $\tilde{f}_L$ - $\tilde{f}_R$ mixing
  - ∝ FERMION Yukawa Coupling
- Gaugino - Higgsino mixing





important

Neutralino Cross-sections most "model-dependent"

Luckily somewhat less relevant for searches as they tend to be small

Q... & Couplings

Indirect Searches

Precision Measurements

- **Loop effects** $g_\mu - 2$

  Small correction to SM prediction

  Need good control on SM theory & on expt.

- **Flavour violation**: O(1) effects but SUSY contribution sensitive to details of flavour model

  Test of model & not generic.

- CP viol^{n}

SUSY from the sky. Matches

- Relic density of Cold dark matter
- Indirect & Direct searches for Relic LSPs

CDM + particle → CDM + recoil! Measur

CDM + CDM → stuff → γ ← Detect FROM SUN / EARTH

LEP COLLIDER

- Model indep. constraints from Z line shape
- ~~Upper~~ Lower limits on sparticle masses ranging between 75 - 103 GeV (limits depend on sparticle decay patterns)

$\tilde{e}$, $\tilde{W}$ > 100 - 103 GeV

$\tilde{\mu}$, $\tilde{\tau}$ somewhat lower

Neutralino limits parameter & model sensitive

SM Higgs boson $\gtrsim$ 114 GeV

⇓

Translate to limit on h mass

Tevatron beats LEP for $\tilde{q}$, $\tilde{g}$ searches

Strong production best if $\tilde{g}$, $\tilde{q}$ light

$\tilde{W}_1\tilde{Z}_2$, $\tilde{W}_1\tilde{W}_1$ best if mass scale high.

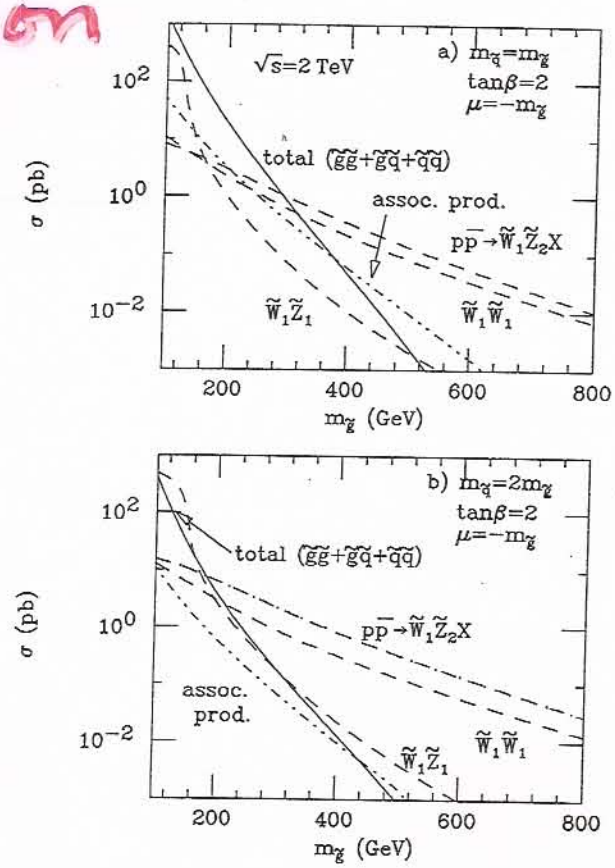


Figure 3: Total cross sections for various sparticle production processes by $p\bar{p}$ collisions at $\sqrt{s} = 2$ TeV.

- Initial search in $\not{E}_T$ channel
- As $\tilde{g}$ mass increases, cascade decays grow in importance

$\tilde{g} \to q\bar{q}\tilde{W}^+ = \tilde{g} \to q\bar{q}\tilde{W}^-$ by CP invariance!

$\tilde{g}\tilde{g} \to$ jets + isolated same Sign dileptons

Small rate but low SM background.

Also jets + 3 lepton events.

[Searches under way]

CDF + DØ Run 1

Notice Competetiveness of multilepton Searches

May beat $\not{E}_T$ in Run 2!

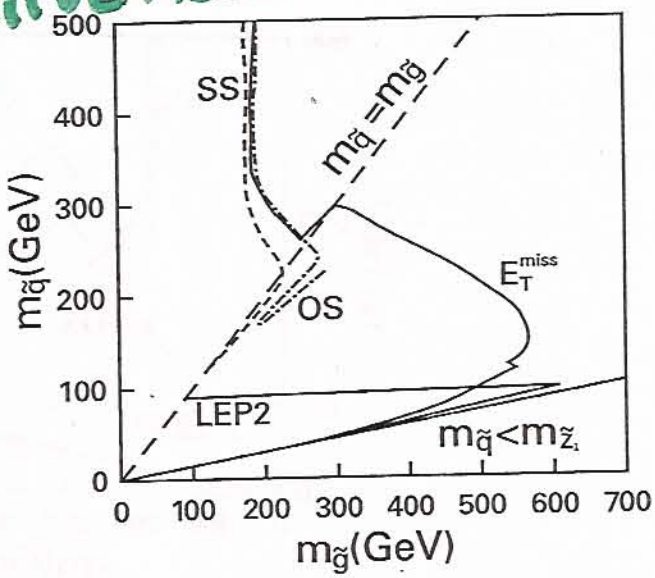
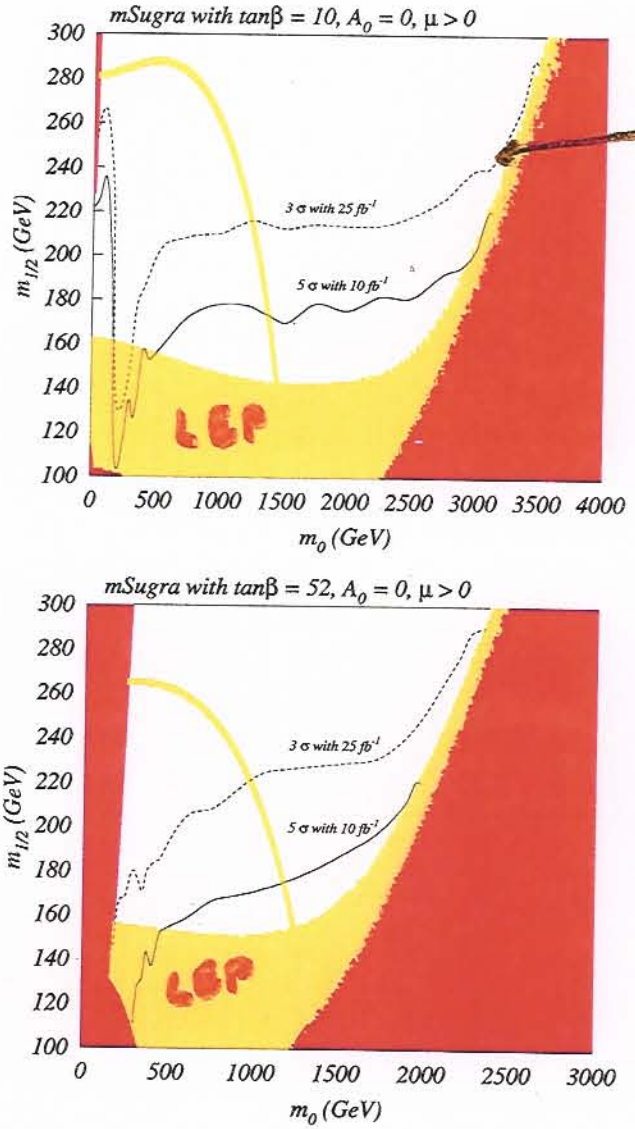


Figure 1.2: The region of the $m_{\tilde{g}} - m_{\tilde{q}}$ plane excluded by various searches for squark and gluino production in at the Fermilab Tevatron. The solid contour labelled E_T^{miss} shows the boundary of the regions excluded by different searches in multijet $+E_T^{miss}$ channels, while the dashed (dashed-dotted) contours labelled SS (OS) mark the boundaries of the regions excluded by the CDF (DØ) search in the SS dilepton (OS dielectron) channel. We caution that these searches have been performed in somewhat different models, and refer the reader interested in details to the original papers. We note that the dot-dashed contour is our transcription of the original contour that was presented in the $m_0 - m_{1/2}$ plane of the mSUGRA model. Finally, the region marked *LEP2* is excluded by searches for squark pair production at LEP2.

$m_{\tilde{g}} \gtrsim 200$ GeV indep. of $m_{\tilde{q}}$

$m_{\tilde{g}} \approx m_{\tilde{q}} \gtrsim 260$ GeV also.

FUTURE @ TEVATRON



SMALL μ region called focus point region

$m_{\tilde{W}_1} \approx m_{\tilde{\ell}_1} + \varepsilon$

hard to see signal! as decay products become soft.

Figure 1.4: The region of the $m_0 - m_{1/2}$ plane where the inclusive trilepton signal with soft leptons is detectable at Tevatron Run 2. The dark shaded region is excluded by theoretical constraints discussed in the text, while the light-shaded region is excluded by experimental constraints from LEP2. Below the thick light grey line, $m_h < 114.1$ GeV.

What should we look for at LHC?

Everything !

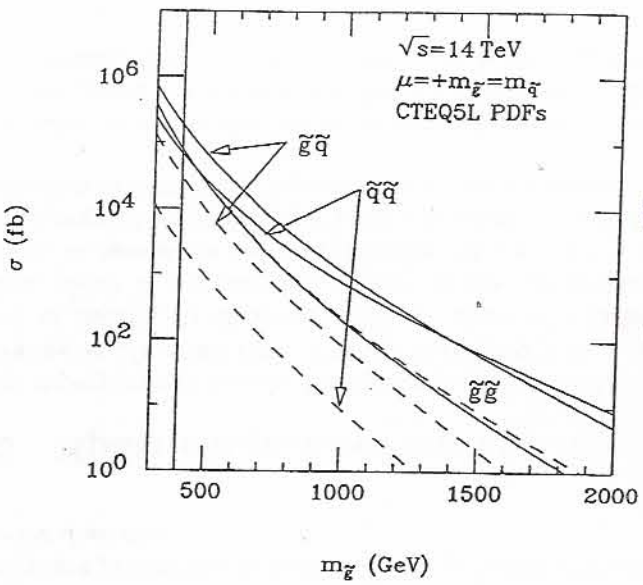


Figure 1.14: Cross sections for squark and gluino production at the CERN LHC pp collider for $m_{\tilde{q}} = m_{\tilde{g}}$ (solid) and for $m_{\tilde{q}} = 2m_{\tilde{g}}$ (dashes).

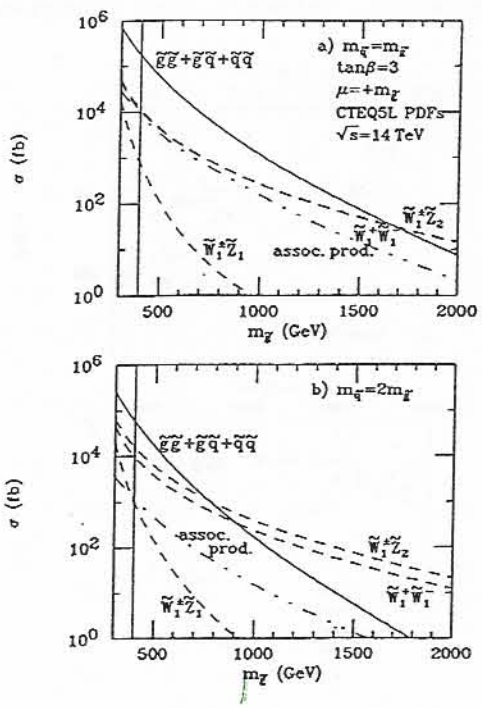


Figure 1.23: Cross sections for gluinos, squarks, charginos and neutralinos at the CERN LHC pp collider. The chargino is lighter than 100 GeV to the left of the vertical line.

ATLAS 10fb^{-1}

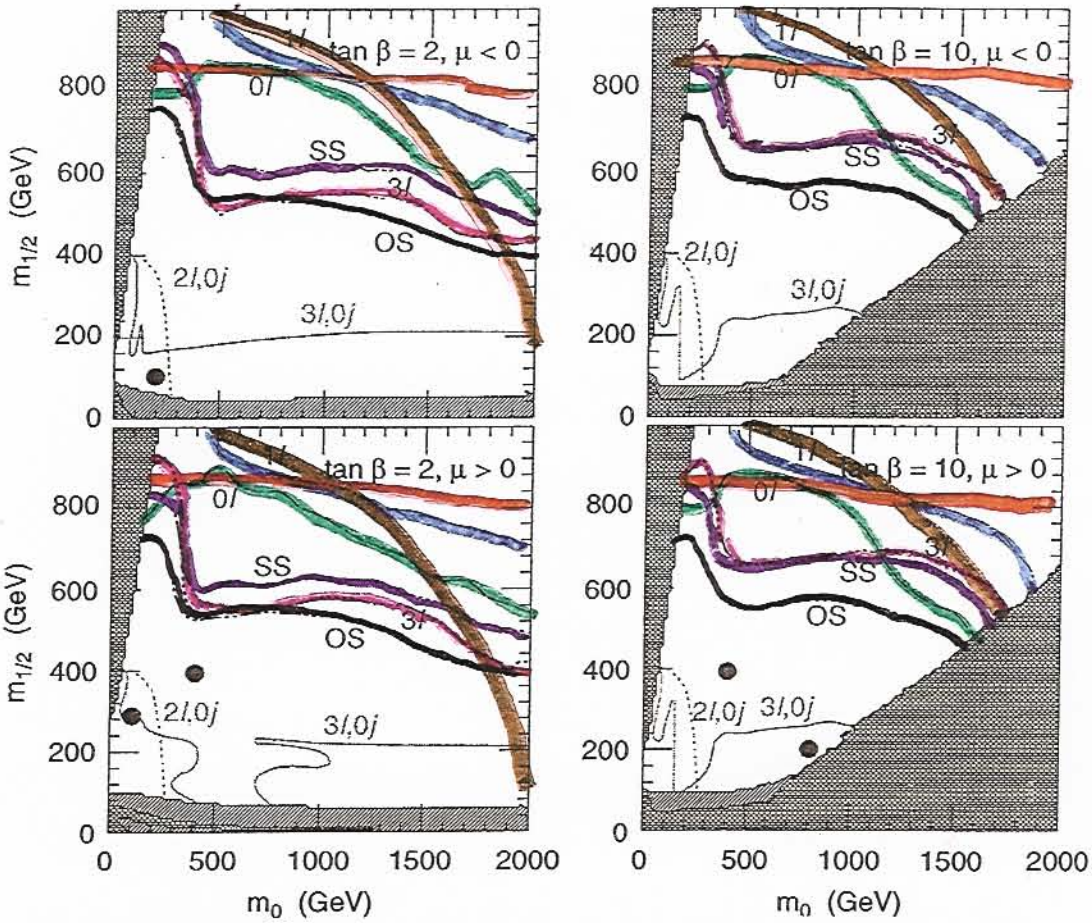


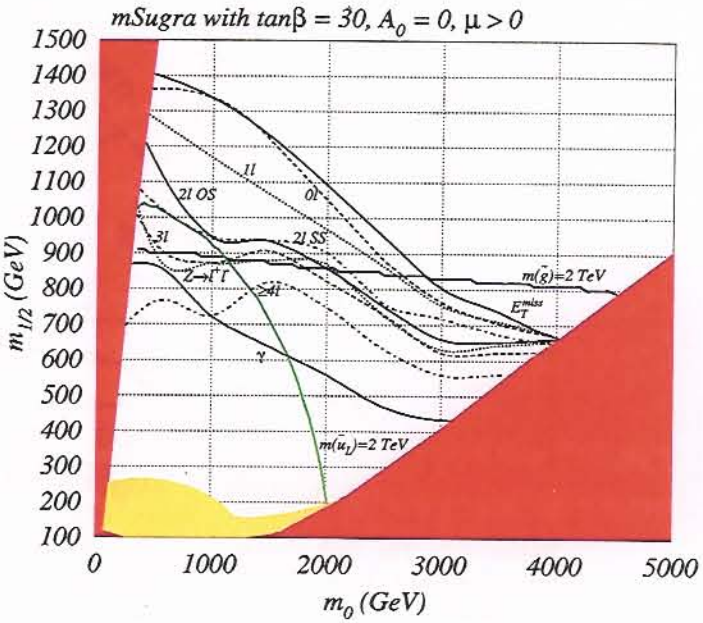
Figure 20-3 Reach for $S/\sqrt{B} > 5$ for various SUSY signatures in the SUGRA parameter space with $A_0 = 0$. 0l: Jets, E_T^{miss}, and no leptons. 1l: Jets, E_T^{miss}, and 1 lepton. SS: Same-sign dileptons. OS: Opposite-sign dileptons. 3l: trileptons. 3l,0j: trileptons with jet veto. 2l,0j: dileptons with jet veto. See Figure 20-1 for the explanation of the shaded regions.

SUSY SIGNAL IN MANY CHANNELS

Relative rates have info on parameters

Reach out to $m_{\tilde{g}} \approx 1.7 - 1.8$ TeV with 10fb^{-1}

Alternative more involved strategy CMS
100 fb^{-1}



Notice γ signal

Figure 1.6: The 5σ reach of the CERN LHC in the $m_0 - m_{1/2}$ plane of the mSUGRA model for an integrated luminosity of 100 fb^{-1}. The shaded regions are excluded by theoretical and experimental constraints.

What if the Gravitino is the LSP?

No difference if $m_{\tilde{G}} \sim M_{weak}$.

It couples so weakly that cascade decays occur as usual, and then $\tilde{\chi}_1 \to \tilde{G}$ way after it escapes the detector.*

* I will discuss the case that $\tilde{\tau}$ or some charged sparticle is the next to lightest sparticle

Superlight Gravitinos can, however, make a difference.

$$m_{\tilde{G}} \sim \frac{M_{SUSY}^2}{M_{Pl.}}$$ so Gravitino may be very light if SUSY breaking scale is small

Gauge Mediated SUSY Breaking

Gravitino gets a mass by SUSY Breaking and develops longitudinal component by absorbing the Goldstino, in the same way that the W boson gets a mass upon SU(2)×U(1) breaking, and develops a longitudinal component after "eating" the Goldstone boson.

Dimensionless Coupling for long. component

$$\sim \frac{E}{M_{Pl.}} \times \frac{E}{m_{\tilde{G}}}$$

$$\sim \frac{(100\ GeV)^2}{10^{19}\ GeV} \frac{1}{1\ eV} \times \left(\frac{1\ eV}{m_{\tilde{G}}}\right)$$

$$\sim 10^{-6}\left(\frac{1\ eV}{m_{\tilde{G}}}\right)$$

Relevant to colliders!

$$\tau(100\ GeV) \sim 10^{-12}\ s \left(\frac{m_{\tilde{G}}}{eV}\right)^2$$

For superlight $\tilde{G}$ scenarios, heavier sparticles cascade decay to the second lightest sparticle which then decays to the $\tilde{G}$

Signals depend on what the second lightest sparticle is.

Gauge-mediated SUSY breaking models realize this. Use to guide our thinking

2^{nd} lightest sparticle $\longrightarrow \tilde{G}$ + Visible SM Stuff.

Extra handle

May increase reach

If long-lived, displaced vertex may give additional handle!

SUMMARY OF LHC REACH VIS A VIS TEVATRON

Table 1.2: A comparison of the SUSY reach of the Tevatron luminosity upgrade and the LHC for the various model lines of the GMSB framework that were introduced in the text with the reach in the mSUGRA and AMSB models. For the GMSB model lines, we also show the dominant decay of the NLSP together with the channel that yields the largest reach. For the mSUGRA model, a significantly higher reach in $m_{\tilde{g}}$ is possible, both at the Tevatron as well as at the LHC, if $m_0 \ll m_{1/2}$. For the AMSB model, the reach is also larger when $m_{\tilde{g}}$ is small than in the case that is shown. Studies of the Tevatron reach within the AMSB model are not available.

Model Line	NLSP	Tevatron ($25\ fb^{-1}$)	LHC ($10\ fb^{-1}$)
A	$\tilde{Z}_1 \sim \tilde{B}$ $\tilde{Z}_1 \to \gamma\tilde{G}$	$\Lambda \cong 115$ TeV, $m_{\tilde{g}/\tilde{q}} \sim 0.87$ TeV, $ll\gamma\gamma + E_T^{miss}$	$\Lambda \cong 400$ TeV $m_{\tilde{g}/\tilde{q}} \sim 2.8$ TeV, $\gamma\gamma + E_T^{miss}$
B	$\tilde{\tau}_1$	$\Lambda \cong 53$ TeV, $m_{\tilde{g}/\tilde{q}} \sim 0.82$ TeV, Clean channels $3l + 1\tau 2l + 1\tau 3l$ $+2\tau 1l + 3\tau 2l$	$\Lambda \cong 150$ TeV $m_{\tilde{g}/\tilde{q}} \sim 2.0$ TeV, $3l + E_T^{miss}$
C	$\tilde{\tau}_1$, $\tilde{e}_R$, $\tilde{\mu}_R$	$\Lambda \cong 60$ TeV, $m_{\tilde{g}/\tilde{q}} \sim 1.3$ TeV, $\geq 4l + E_T^{miss}$	$\Lambda \cong 155$ TeV $m_{\tilde{g}/\tilde{q}} \sim 3.0$ TeV, $4l + E_T^{miss}$
D	$\tilde{Z}_1 \sim \tilde{h}$ $\tilde{Z}_1 \to h\tilde{G}$	$\Lambda \cong 105$ TeV, $m_{\tilde{g}/\tilde{q}} \sim 1.5$ TeV, $\geq 3b - jets + E_T^{miss}$	$\Lambda \cong 140$ TeV $m_{\tilde{g}/\tilde{q}} \sim 2.0$ TeV, $\geq 2b + E_T^{miss}$
E	$\tilde{Z}_1 \sim \tilde{h}$ $\tilde{Z}_1 \to Z\tilde{G}$	$\Lambda \cong 120$ TeV, $m_{\tilde{g}/\tilde{q}} \sim 1.3$ TeV, $\gamma\gamma + E_T^{miss}$	$\Lambda \cong 140$ TeV $m_{\tilde{g}/\tilde{q}} \sim 2.0$ TeV, $1l + E_T^{miss}$ Increase to 2.2 TeV via $Z\gamma + E_T^{miss}$ if excellent jet-γ rejection is available.
Reach in mSUGRA		$\tilde{m}_{\tilde{g}} \sim 0.35 - 0.4$ TeV E_T^{miss}	~ 1.6 TeV $(m_{\tilde{q}} \gg m_{\tilde{g}})$ $\ell + E_T^{miss}$ ~ 2.2 TeV $(m_{\tilde{q}} \sim m_{\tilde{g}})$ $\ell + E_T^{miss}$
Reach in mAMSB			~ 1.4 TeV $(m_{\tilde{q}} \gg m_{\tilde{g}})$ E_T^{miss} ~ 2 TeV $(m_{\tilde{q}} \sim m_{\tilde{g}})$ $\ell^+\ell^- + E_T^{miss}$

10 fb^{-1}

photons

multi leptons

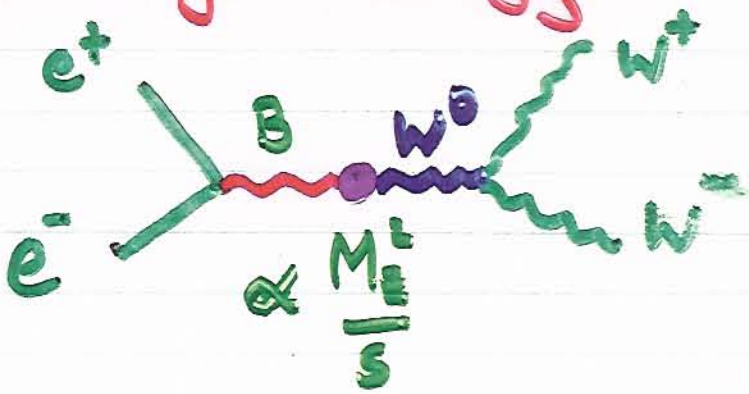
SUSY @ e^+e^- Linear Colliders

- Clean environment
- Well defined initial state
- Initial Beam polarization
  - Control background
  - Select Signal processes

[Control energy of collision ~ 100]

With R.H. electron beam, W^+W^- b/g tiny @ high energy

Coupling does not exist for e_R



Suppressed at high Energy

Can get extremely clean signal samples.

[Also @ the LHC, but here different SUSY reactions can be potentially isolated]

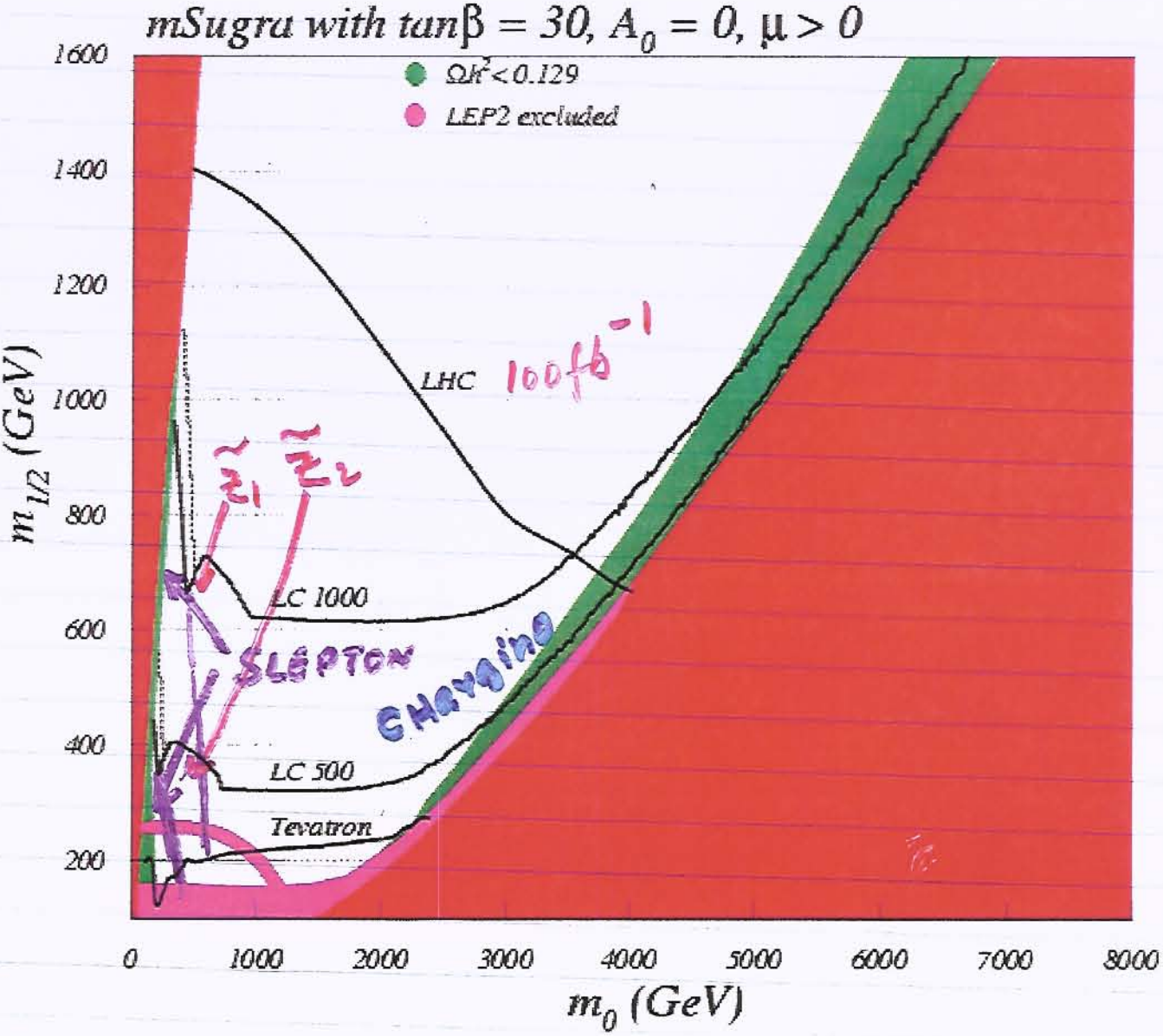
Clean measure of sparticle prop^{s}

Quantum #s

Spin

Mass

Couplings



LHC can beat ~~charg~~ L.C.
Except in the small μ-sliver
D.M. favoured region.

What if nature is supersymmetric and we see some new physics signals?

Break out the champagne and/or other chemicals.

But after that, ...

What is the new physics?
How do we know it is SUSY?

At the LHC "discover" several new particles with "correct Quantum Numbers"

Strong vs. Electroweak for given mass

How can we get the mass scale?

Parent

$$M^2_{Parent} = \sum (p_1 + p_2 + \cdots p_N)^2$$

Problem : undetected LSPs.

No mass Bumps!

$$M_{eff} = \not{E}_T + E_T(j_1) + E_T(j_2) + E_T(j_3) + E_T(j_4)$$

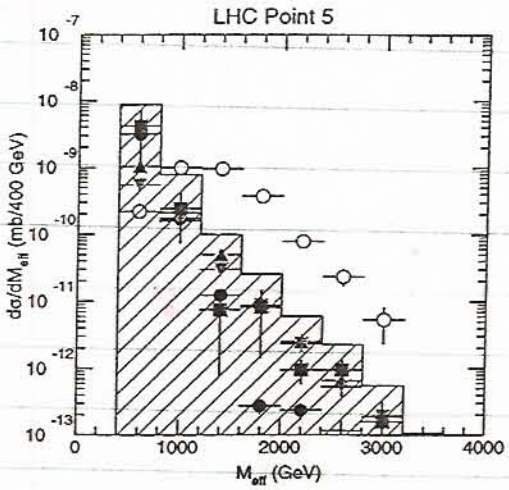
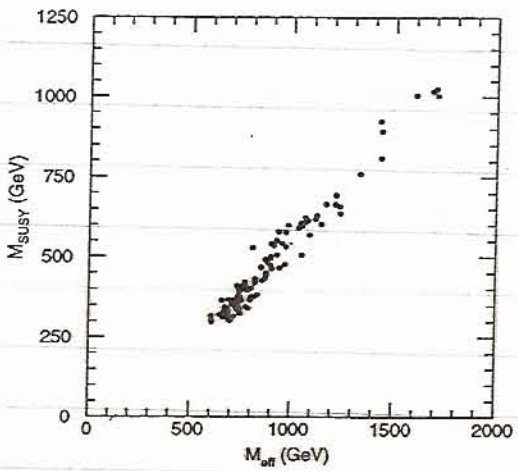


Figure 20-4 M_{eff} distribution for the Point 5 signal (open circles) and for the sum of all Standard Model backgrounds (histogram); the latter includes $t\bar{t}$ (solid circles), W + jets (triangles), Z + jets (downward triangles), and QCD jets (squares).



ATLAS

Figure 20-5 Peak of M_{eff} distribution as a function of. $M_{\text{SUSY}} = \min(M_{\tilde{g}}, M_{\tilde{u}_R})$ for various models.

M_{SUSY} ~ 10% level

Can we do better?

- Under favourable circumstances, DEFINITELY YES.

Kinematic end points ⇒ Mass Differences

$\tilde{\chi}_2 \to \tilde{\chi}_1 \, l \bar{l}$

Kinematics $\Rightarrow$ $\boxed{m(l\bar{l}) < m_{\tilde{\chi}_2} - m_{\tilde{\chi}_1}}$

If $\tilde{\chi}_2 \to \tilde{\ell} \, l$

$\hookrightarrow l \tilde{\chi}_1$

$$m(l\bar{l}) < \left(m_{\tilde{\chi}_2}^2 - m_{\tilde{l}}^2\right)^{1/2} \times \left(1 - \frac{m_{\tilde{\chi}_1}^2}{m_{\tilde{l}}^2}\right)^{1/2}$$

$$\leq m_{\tilde{\chi}_2} - m_{\tilde{\chi}_1}$$

Ambiguities to be sorted out a posteriori.

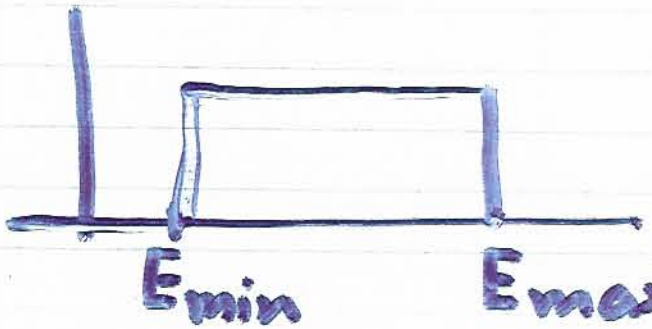
- Reconstruction of masses possible if we have a chain of 2-body decays.

At e^+e^- Colliders precision studies of masses possible.

Very model independent = Kinematic

Illustrate: $\tilde{e} \to e \tilde{\chi}_1$

E_e fixed in $\tilde{e}$ rest frame.



Lab frame

End points depend on $m_{\tilde{e}}, m_{\tilde{\chi}_1}$

2 end points determine $m_{\tilde{e}}, m_{\tilde{\chi}_1}$.

- Same thing for $\tilde{W}_1 \to \underbrace{jj}_{W^*} \tilde{\chi}_1$

Fix $M(W^*)$ in small bin "Quasi-two body decays"

- Cascade decays don't change the principle.

In practice, technical Complications

not 100% acceptance

Detection Efficiency

Detector Resolution

Radiation effects

With an integrated luminosity of ~ 50 - 100 fb^{-1} mass determination at the ~ 1% level possible

$\tilde{e}$, $\tilde{\mu}$, $\tilde{\tau}_1$, $\tilde{\nu}_e$, $\tilde{W}_1$, $\tilde{\chi}_1$ ...

Era of Sparticle Spectroscopy!

Unique capabilities of e^+e^- experiments & Colliders fully utilized for this.

Very nice, but why should I care?

If SUSY is discovered, the most important issue will be

THE MECHANISM OF SUSY BREAKING.

MANY CLEVER IDEAS ABOUT HOW STANDARD MODEL SUPERPARTNERS GET THEIR MASSES.

MODELS — mSUGRA
— mGMSB + Extensions
— mAMSB + Variants
— inoMSB
⋮

Different "organizing principles" for the 178 MSSM parameters

Typically everything fixed by SM parameters + SUSY PARAMETERS

Determining several masses, cross-sections, asymmetries ... can falsify any particular framework (or determine (some of) the underlying parameters).

- LHC will do this.
- Planning a priori strategy beyond this seems hard @ LHC because number of possibilities to explore is very large - head becomes full!
- But with real data, we will be able to be creative.

Capability of Linear colliders

⇓

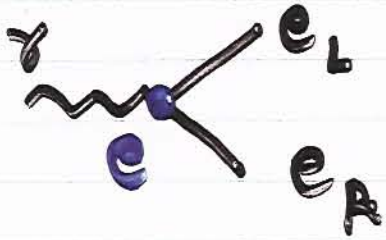
Systematic bottom-up strategy for determining masses & other weak scale parameters.

⇓

First step to the underlying more fundamental framework.

NAILING SUSY

SUSY ⇒ Equality of dimensionless couplings



[modulo "algebraic factor"]

- Complication : $\tilde{\gamma}$ is not mass eigenstate.

[Could use gluinos & gluons but hard at hadron colliders.]

- Despite the mixing angle Complication it is possible to extract the gaugino coupling and test the SUSY equality at the 1-2% level!

- Tree level only.
  Precise measurement ➡ next mass scale!

- LC information can help analysis of complex decay chains at the LHC

"Folk result" for many years

LHC / LC Working Group

Analyse "sps-1a Scenario"

$\tilde{W}_1 = 176\,GeV$ $\tilde{\chi}_1 = 96.2\,GeV$ $\tilde{\chi}_2 = 176.6\,GeV$

$\tilde{W}_2 = 378.5\,GeV$, $\tilde{\chi}_3 = 35.8\,GeV$, $\tilde{\chi}_4 = 377.9\,GeV$

$\tilde{e}_R = 143\,GeV$ $\tilde{e}_L = 202.1\,GeV$ $\tilde{\nu}_{e_L} = 186.0\,GeV$

Heavier inos not accessible @ LC with $\sqrt{s} = 500\,GeV$ or smaller.

But these are produced at LHC!

- $\delta m_{\tilde{\chi}_4} = 5.1\,GeV$ $300\,fb^{-1}$ LHC (stat. error only)

Use $m_{\tilde{\chi}_1}$ from LC measurement

⇓

$\delta m_{\tilde{\chi}_4} = 2.2\,GeV$!

Errors on Extraction of $M_1, M_2, \mu, \tan\beta$ are halved!

Weak Scale Supersymmetry

- Novel Bose-Fermi synthesis "Matter = Force"
- This symmetry predicts new particles with several unambiguously predicted properties to complete the symmetry multiplet.

Charge Conjugation Invariance

⇓

positron must exist in Dirac theory.

- SUSY is broken. Source of many ambiguities today.

10^{10} $ Question: How is it broken?

Fayet pioneered SUSY phenomenology in mid 1970's.

Others followed starting ~ 1980

No evidence but interest remains strong

- stabilizes EWSB Sector
- Cold dark matter candidate
- Unification of gauge couplings
- rationale for EWSB
  [not explanation, though]
- Complete in UV
- decoupling dynamics ⇒ LEP data.

BUT

SUSY RAISES NEW QUESTIONS

How is it broken ?

Flavour
CP
Lepton number
Baryon number - mother of all flavour problems!

Spoils the "Success of the SM".

Problems all originate in "agnostic parametrization" of the SUSY breaking sector.

Clues as to how not to do things

The exciting thing is that weak scale SUSY can be subject to experimental tests & falsified!

We will have the tools to do so!

If weak scale SUSY exists, its remarkable UV properties give us a peek @ physics at very high energy scales!

. HC is a broad band machine
or new discoveries. Excellent
potential for new discovery.

We may be surprised by the
Tevatron, but ....

I hope I will see data from a
LC. Interesting & different
capability than the LHC.
Potentially a discovery machine!
Need to archive LHC data in a
form suitable for subsequent
reanalysis to utilize full capability
of these machines.

You are fortunate to be young & creative in this very exciting era!

Appendix A Toy Model

$$\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}_{mass} + \mathcal{L}_{int}$$

$$\mathcal{L}_{kin} = \frac{1}{2}(\partial_\mu A)^2 + \frac{1}{2}(\partial_\mu B)^2 + \frac{i}{2}\bar{\psi}\partial\!\!\!/\psi + \frac{1}{2}(F^2+G^2)$$

$$\mathcal{L}_{mass} = -m\left[\frac{1}{2}\bar{\psi}\psi - GA - FB\right]$$

Neglect interactions for now

Notice F & G have no derivatives

⇓

Eq^{ns} of "motion" are algebraic

$$\frac{\delta\mathcal{L}}{\delta F} = 0 \Rightarrow F = -mB$$

$$\frac{\delta\mathcal{L}}{\delta G} = 0 \Rightarrow G = -mA$$

F & G are "auxiliary fields"

$$\mathcal{L}_{kin} + \mathcal{L}_{mass} = \frac{1}{2}(\partial_\mu A)^2 + \frac{1}{2}(\partial_\mu B)^2 + \frac{i}{2}\bar{\psi}\partial\!\!\!/\psi - \frac{1}{2}m^2(A^2+B^2) - \frac{1}{2}m\bar{\psi}\psi$$

upon eliminating F & G

$\delta A = -i\bar{\alpha}\gamma_5\psi$ ⟵ anticommuting number-parameter of transfn

$\delta B = -\bar{\alpha}\psi$

$\delta\psi = -F\alpha + iG\gamma_5\alpha + \not{\partial}\gamma_5 A\alpha + i\not{\partial} B\alpha$

$\delta F = i\bar{\alpha}\not{\partial}\psi$

$\delta G = \bar{\alpha}\gamma_5\not{\partial}\psi$

Bose ⟷ Fermi

$\frac{1}{2}\delta[(\partial_\mu A)^2] = (\partial_\mu A)\partial^\mu\delta A = i(\partial^\mu A)\bar{\alpha}\gamma_5\not{\partial}\psi$

$= \partial^\mu[i\partial_\mu A\bar{\alpha}\gamma_5\psi] - i\Box A\bar{\alpha}\gamma_5\psi$

$i\delta[(\partial_\mu B)^2] = \partial^\mu[-\partial_\mu B\bar{\alpha}\psi] + \Box B\bar{\alpha}\psi$

Exercise: Show that (etc.)

$\delta\mathcal{L}_{kin} = \partial^\mu[\ \text{something}\ ]$

$\delta\mathcal{L}_{mass} = \partial^\mu[\ \text{Something else}\ ]$

$\delta\int\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}_{mass}$ Eq^{ns} of motion are unchanged by transf^{n} on top!

Interactions

$$\mathcal{L}_{int} = -\frac{g}{\sqrt{2}} A \bar{\psi}\psi + i\frac{g}{\sqrt{2}} B \bar{\psi}\gamma_5\psi + \frac{g}{\sqrt{2}}(A^2-B^2)G + g\sqrt{2}ABF$$

$$F = -mB - g\sqrt{2}AB$$

$$G = -mA - \frac{g}{\sqrt{2}}(A^2-B^2)$$

Exercise: Verify this gives the Lagrangian $\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}_{mass} + \mathcal{L}_{int}$ for our toy model.

Why introduce F & G?

So that transformations on p. A2 are linear transformations.