6.) The Muon Anomalous Magnetic Moment $a_\mu = \frac{g_\mu - 2}{2}$

The a_μ-m_H Connection

Is a current $a_\mu^{exp} - a_\mu^{SM} = +239(99) \times 10^{-11}$ deviation indicative of "New Physics"?

Or is it an underestimate of a_μ^{SM} (Hadronic)?

Early History

Electron Magnetic Dipole Moment $\vec{\mu} = g \frac{e}{2m_e} \vec{S}$

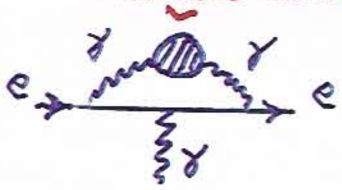
Dirac (1928) $\xrightarrow[\text{Relativity + Quantum}]{}$ $g = 2$ great success

Schwinger (1948) 

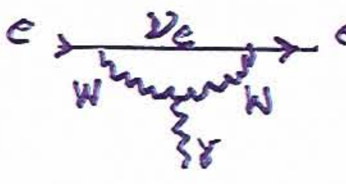
$$\underbrace{a = \frac{g-2}{2} = \frac{\alpha}{2\pi} \simeq 0.00116}_{\text{Anomalous Magnetic Moment}}$$

$$a_e^{SM} = \frac{\alpha}{2\pi} - 0.328478444\left(\frac{\alpha}{\pi}\right)^2 + 1.181234\left(\frac{\alpha}{\pi}\right)^3 - 1.7502\left(\frac{\alpha}{\pi}\right)^4 + \underbrace{1.7 \times 10^{-12}}_{\text{Hadronic + EW}}$$

Hadronic Vac. Pol

Hadronic 

$$\sim \left(\frac{\alpha}{\pi}\right)^2 \frac{m_e^2}{m_\rho^2} \simeq 2 \times 10^{-12}$$

EW 

$$\sim \frac{\alpha}{\pi} \frac{m_e^2}{m_W^2} \simeq 10^{-13}$$

Tiny

a_e^{exp} vs a_e^{SM} $\rightarrow$ $\alpha^{-1} = 137.03599877(40)$

Quantum Hall $\rightarrow$ $\alpha^{-1} = 137.03600300(270)$

Confirms QED to 3×10^{-8}!

New Physics $\Delta a_e(\Lambda) \sim \mathcal{O}\left(\frac{m_e^2}{\Lambda^2}\right)$ highly suppressed

The muon anomalous magnetic moment $\sim \frac{m_\mu^2}{m_e^2} \simeq$ 40,000 times more sensitive to "New Physics" and Hadronic & EW loops!

Experiment E821 at BNL (V. Hughes et. al.)

See talk by P. Shagin

μ^+ & μ^- results consistent

$$\boxed{a_\mu^{exp} = 116592080(58)\times10^{-11}}$$

Old CERN Exp $\pm 840\times10^{-11}$

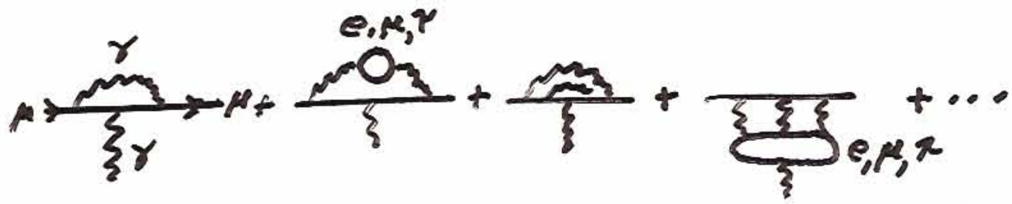
Statistics Limited (Proposal $\rightarrow \pm 30\times10^{-11}$) Funding?

Why is the result exciting?

$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{Had.}$$

Major Theory Challenge

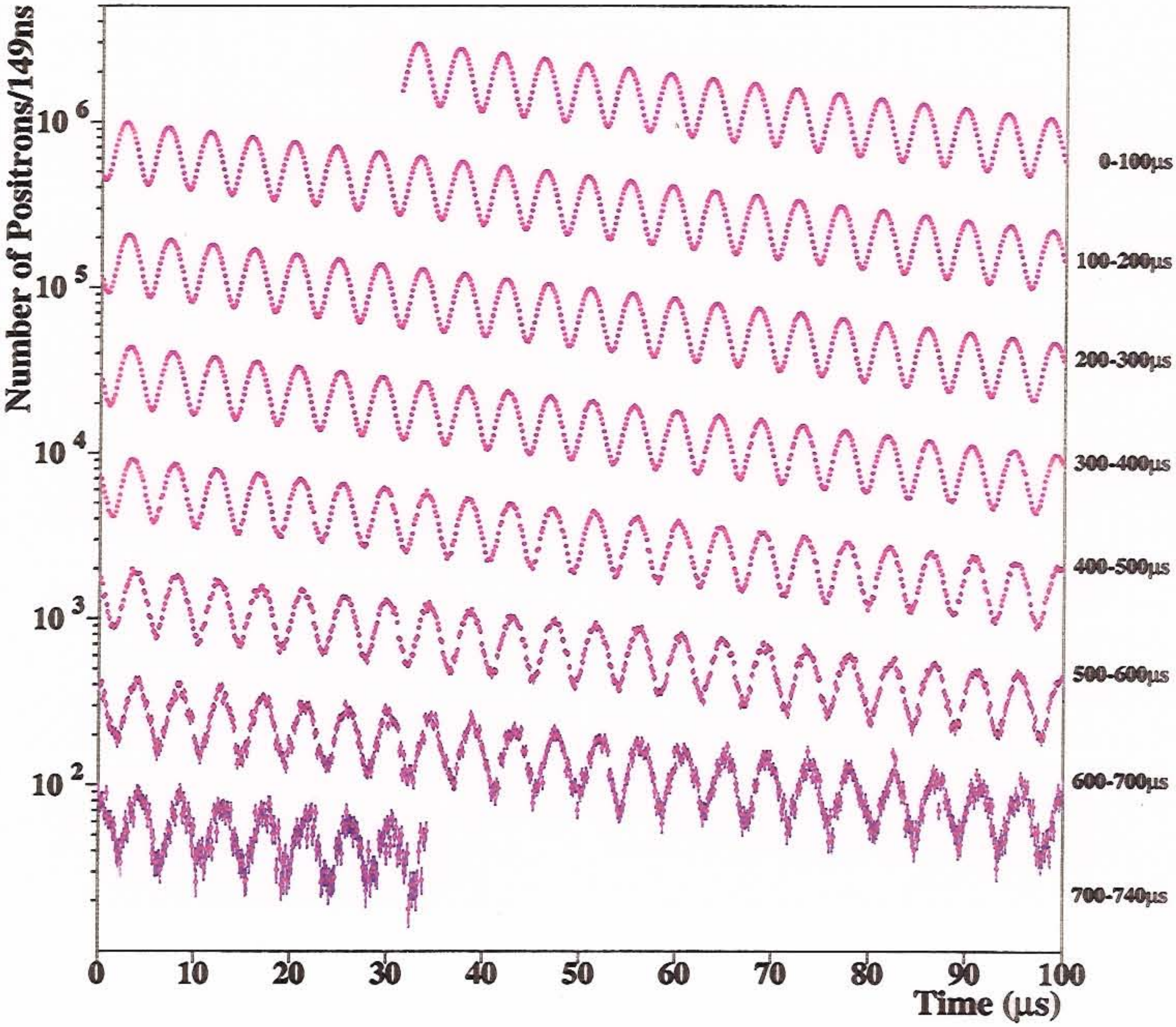
a_μ^{QED} (5 loops):



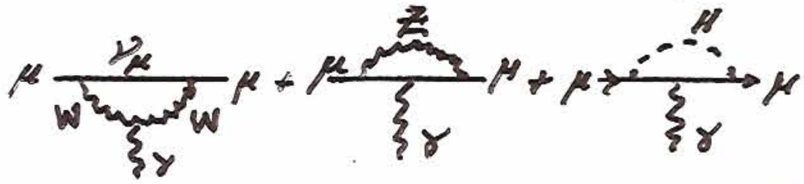
$$a_\mu^{QED} = \frac{\alpha}{2\pi} + 0.765857376\left(\frac{\alpha}{\pi}\right)^2 + 24.05050898\left(\frac{\alpha}{\pi}\right)^3 + 131.0\left(\frac{\alpha}{\pi}\right)^4 + 930\left(\frac{\alpha}{\pi}\right)^5$$

$$= 116\,584\,720.7\,(0.4)(1)\times10^{-11}$$

Negligible Unc.

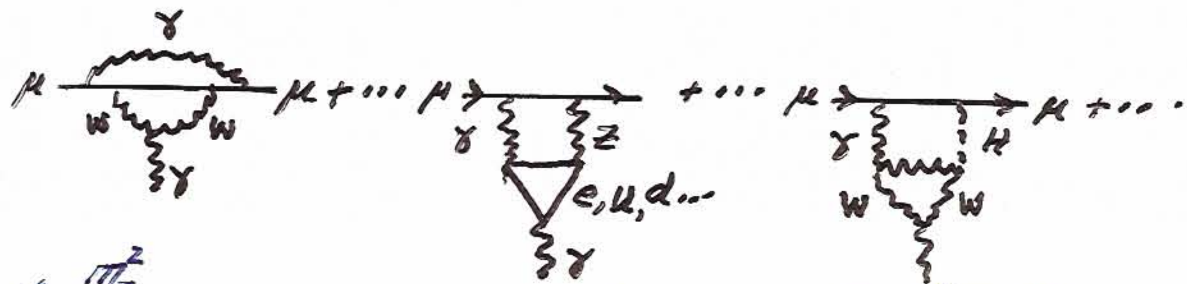

Number of Positrons/149ns
10 6
10 5
10 4
10 3
10 2
0-100μs
100-200μs
200-300μs
300-400μs
400-500μs
500-600μs
600-700μs
700-740μs
0
10
20
30
40
50
60
70
80
90
100
Time (μs)

ii) Electroweak (1 loop):
(1972)
Altarelli, Cabibbo, Maiani
Jackiw & Weinberg
⋮



$$\underbrace{\frac{5}{24}\frac{G_\mu m_\mu^2}{\sqrt{2}\pi^2}\left[1+\frac{1}{5}(1-4\sin^2\theta_W)^2\right]}_{a_\mu^{EW}(1\,loop)=194.8\times10^{-11}}$$ Tiny

2 loop :



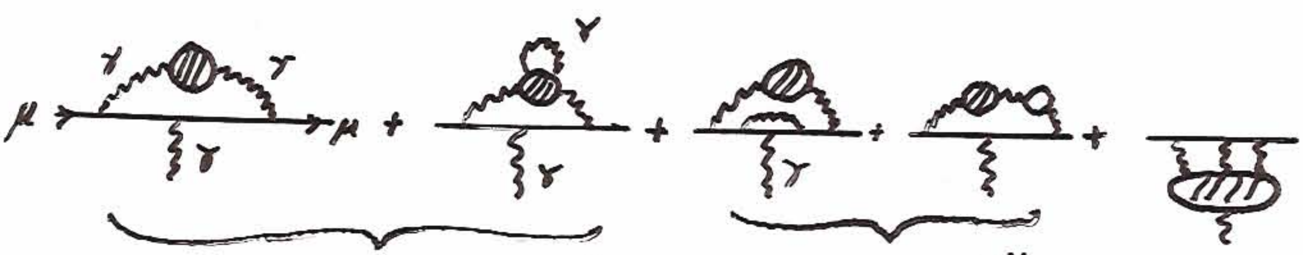
$\ln\frac{m_Z^2}{m_\mu^2}\simeq 13.5$ Anomaly 2loop Higgs ≃ 1000 one loop Higgs

$$a_\mu^{EW}(2\,loop) = -40.7(1.0)(1.8)\times10^{-11}$$

leading 3loop: $a_\mu^{EW}(3\,loop)_{leading\,logs} \simeq O(10^{-12})$ Negligible

$$a_\mu^{EW} = 154(1)(2)\times10^{-11}$$ Negligible Unc.

iii) Hadronic Loop Contributions 2 & 3 loops (4 loops small)



light by light

$6944(62)(36)\times10^{-11}$ $-98(1)\times10^{-11}$ $+120(35)\times10^{-11}$

use $e^+e^-\to$ hadrons data (like $\Delta\alpha_h^{(5)}$)

$a_\mu^{SM} = 116\,591\,841\,(72)_{VP}(35)_{lbl}\,(3)_{EW,QED} \times 10^{-11}$

$a_\mu^{exp} = 116\,592\,080\,(58) \times 10^{-11}$

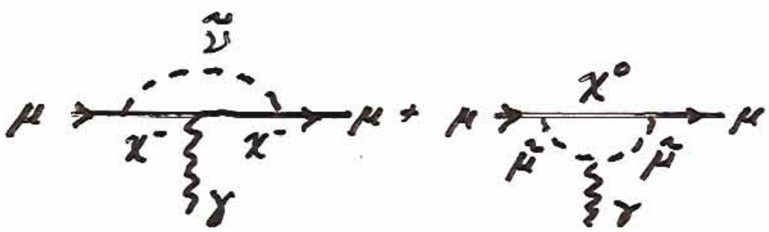
$\boxed{\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = 239(99) \times 10^{-11}}$ 2.4σ deviation

Other studies of hadronic $\rightarrow$ 2-3 sigma

Large deviation - Remember $a_\mu^{EW} \simeq 154 \times 10^{-11}$

New Physics > EW Physics! Is it possible?

Natural Interpretation: Supersymmetry Loops



Generic $\Delta a_\mu^{SUSY} \simeq (\mathrm{sgn}\,\mu) \times 130 \times 10^{-11} \tan\beta \left(\frac{100\,\mathrm{GeV}}{m_{SUSY}}\right)^2$

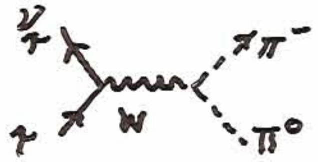
$\tan\beta \equiv \frac{v_2}{v_1} \simeq 3 \sim 40$ large enhancement

$m_{SUSY} \simeq 74\sqrt{\tan\beta}$ GeV

Chiral changing amplitudes ($L \leftrightarrow R$) more sensitive to SUSY!

$g_\mu - 2$, edms, $b \to s\gamma$, $\mu \to e\gamma$, $\tau \to \mu\gamma$ ... $\bar{\mu}\sigma_{\mu\nu}\mu F^{\mu\nu}$

Are we seeing SUSY in a_μ^{exp} or Hadronic Loop Underestimate?

$\tau \to \nu_\tau +$ hadrons data 

+ isospin corrections $\to$ 

e^+e^- + τ data disagree for $s > m_\rho^2$ ~ 5-10% !

$$\underline{\Delta a_\mu^{had} = a_\mu^{had}(\tau) - a_\mu^{had}(e^+e^-) = +163 \times 10^{-11}}$$

Perhaps full 239×10^{-11} or $\sim 200 \times 10^{-11}$ due to had. o.p.

Problems with that interpretation

i) Recent KLOE $e^+e^- \to \pi^+\pi^-\gamma$ data confirms deviation

ii) Isospin Corrections to $\tau \to \nu_\tau +$ hadrons incomplete
QED corrections to $\Gamma_{\rho^\pm} - \Gamma_{\rho^0}$ not included
Likely to improve agreement

iii) $a_\mu - m_H$ Connection

Larger $\sigma(e^+e^- \to$ hadrons$) \to$ larger $\Delta\alpha_h^{(5)} \to$ smaller m_H

Already $m_W, \sin^2\theta_W(m_Z)_{\overline{MS}}, \Delta\alpha_h^{(5)} \to m_H < 154$ GeV 95% CL
$m_H = 68^{+45}_{-30}$ GeV

Trying to fix a_μ^{had} (increase) make m_H prediction lower

and makes $m_W - \sin^2\theta_W(m_Z)_{\overline{MS}}$ less consistent
(Hard to be too quantitative)

We *may* be seeing early *hints* of SUSY in radiative corr.

light Higgs $\lesssim 135\,GeV$
a_μ^{exp} deviation $\tan\beta$

Great for LHC (SUSY Discovery, Higgs Discovery)

Great for LC

Great for Dark Matter Searches χ_0 $\quad sgn\,\mu = +$

Great for $\mu \to e\gamma$, $\mu^- N \to e^- N \ldots$

Great for edm searches

⋮

Happy Days For High Energy Physics!

Prospects for improving a_μ^{exp} + $a_\mu^{SM} \longrightarrow \tan\beta$

a_μ^{exp} proposal $\pm 60\times 10^{-11} \to \pm 30\times 10^{-11}$

a_μ^{SM} $\quad e^+e^- \to \gamma + hadrons$ BaBar
New isospin corr. study for $\tau \to \nu_\tau + hadrons$
Further LBL studies
Lattice Calculation (T. Blum)
$\pm 80\times 10^{-11} \to \pm 40\times 10^{-11}$

$a_\mu^{exp} - a_\mu^{SM} \to \pm 50\times 10^{-11}$ Hard to do much better

Measure SUSY Effect at 5σ?
Determine $\tan\beta$ to $\pm 20\%$

7.) Some Other Precision Studies (Low $q^2 \ll m_Z^2$)

Polarized $e^- D$ $\quad A_{LR}$ (Classic SLAC Exp)

Atomic Parity Violation (Cs) $\sim \pm 1\%$ Precision

Deep-Inelastic $\nu_\mu N$ scattering. Some discrepency

"New → Polarized $e^- e^-$ Möller Scattering at SLAC → A_{LR}

etc.

extract $\sin^2\theta_W(m_Z)_{\overline{MS}}$ to $\sim \pm 1\%$ via weak NC
(or better) $\pm 0.3\%$

violates parity

depends on $\sin^2\theta_W$

Very good probes of Z' bosons, Extra dim ...

Complementary to Z pole studies (Competitive ?)

eg. Currently Probe $m_{Z'} \lesssim 1\,\text{TeV}$

Brief Comment on NuTeV at Fermilab (Anomaly)

Measure $R_\nu \equiv \dfrac{\sigma(\nu_\mu N \to \nu_\mu X)}{\sigma(\nu_\mu N \to \mu X)}$ & $R_{\bar\nu}$

$\to m_W = 80.16\,(8)$ GeV $\quad$ Too Low → "Very" Heavy Higgs

"New Physics" or Exp. Problem (Analysis being redone)

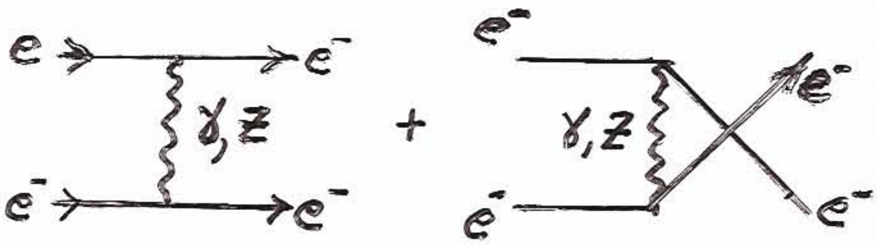
iii) Polarized e^-e^- (SLAC) E158 Experiment

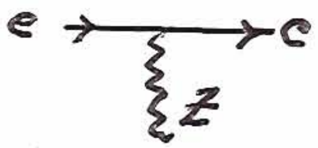
Möller Scattering $e^-e^- \to e^-e^-$

Use a polarized 50 GeV e^- beam on a fixed target (Hydrogen)

Measure the P.V. Asymmetry

$$A_{LR} = \frac{\sigma(e_L^- e^- \to e^- e^-) - \sigma(e_R^- e^- \to e^- e^-)}{\sigma(e_L^- e^- \to e^- e^-) + \sigma(e_R^- e^- \to e^- e^-)}$$





$$\frac{i g_{z_0} \gamma_\mu}{4\cos\theta_W^0}\left(1 - 4\sin^2\theta_W^0 - \gamma_5\right)$$

$\uparrow$ vector $\quad\quad \uparrow$ axial-vector

Parity Violation Requires VA $\to$ $1-4\sin^2\theta_W$ Factor

Asymmetry Very Small $\quad 1-4\sin^2\theta_W \simeq 0.08$

$$y = \frac{1-\cos\theta_{cm}}{2}, \quad 0 \le y \le 1$$

$$Q^2 = -q^2 = ys = 0.05y \text{ GeV}^2$$

$$s = 2 \times 50 \times \underbrace{0.5 \times 10^{-3}}\text{GeV}^2$$

$$A_{LR}(e^-e^-) = \frac{G_\mu^0 S}{\sqrt{2}\pi\alpha}\,\frac{y(1-y)}{1+y^4+(1-y)^4}\,(1-4\sin^2\theta_W^0) \underset{y=1/2}{\sim} \underbrace{3\times10^{-7}}_{\text{tiny}} \quad \left\{\begin{array}{l}\text{Need}\\ 10^{16}\text{ events}\end{array}\right.$$

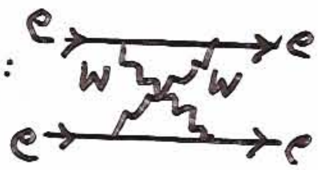
Cross-Section in Enormous & $\mathcal{L}_{eff} \simeq 4\times10^{38}\,\text{cm}^{-2}/\text{s}$ luminosity

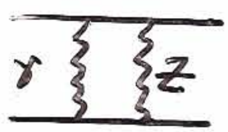
---

E158 Goal → Measure $\sin^2\theta_W(m_Z)_{\overline{MS}}$ to ± 0.0007

Become Best Low Q^2 Measurement

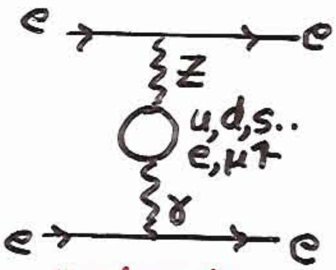
Radiative Corrections Very Large $\gtrsim 40\%$ A. Czarnecki + WM

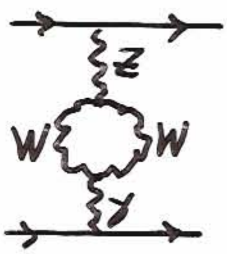
examples: 

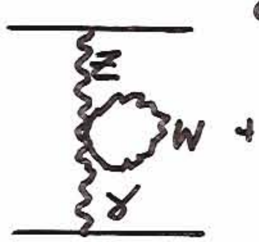
+ 

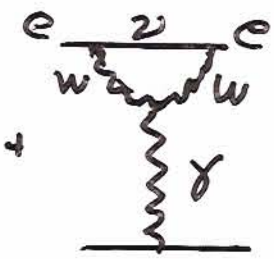
...

Main Correction γZ Mixing at low Q^2



+ 

+ 

+ 

Hadronic Part Large

$$\sin^2\theta_W(m_Z)_{\overline{MS}} \longrightarrow \sin^2\theta_W(Q^2) = \kappa(Q^2)\,\sin^2\theta_W(m_Z)_{\overline{MS}}$$

$$\kappa(Q^2 \simeq 0.0256\,\text{eV}^2) = 1.03 \pm 0.0025 \quad \text{large shift}$$

Exp → $\sin^2\theta_W(Q^2 \simeq 0.025\,\text{GeV}^2) \simeq 0.238(2)$ Preliminary

→ $\sin^2\theta_W(m_Z)_{\overline{MS}} \simeq 0.231(2)$

E158 will be completed this Summer. Expect $\Delta s^2 \simeq \cancel{0.0007}\ \pm 0.0015$?

So far it is right on the SM value from Z pole. (A_{LR})

No Confirmation of NuTeV Anomaly (But not yet in real conflict)
or $Z \to b\bar{b}$ FB

Long Term Future   Fixed Target e^-e^- at NLC
(K. Kumar Snowmass 1996)

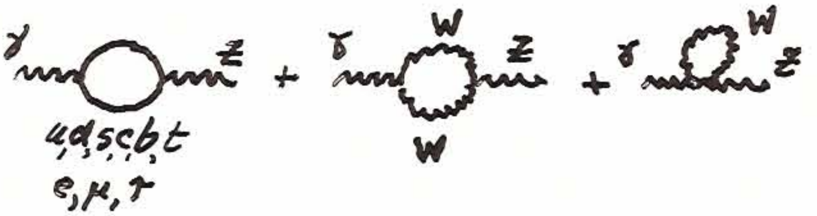
Higher energy 50 GeV → 250 or 500 GeV
Higher Intensity
Longer Running

$\rightarrow \Delta \sin^2\theta_W \simeq \pm 0.00006\,!$ Pol. Unc Very Small
(Had. Loop Effects?)

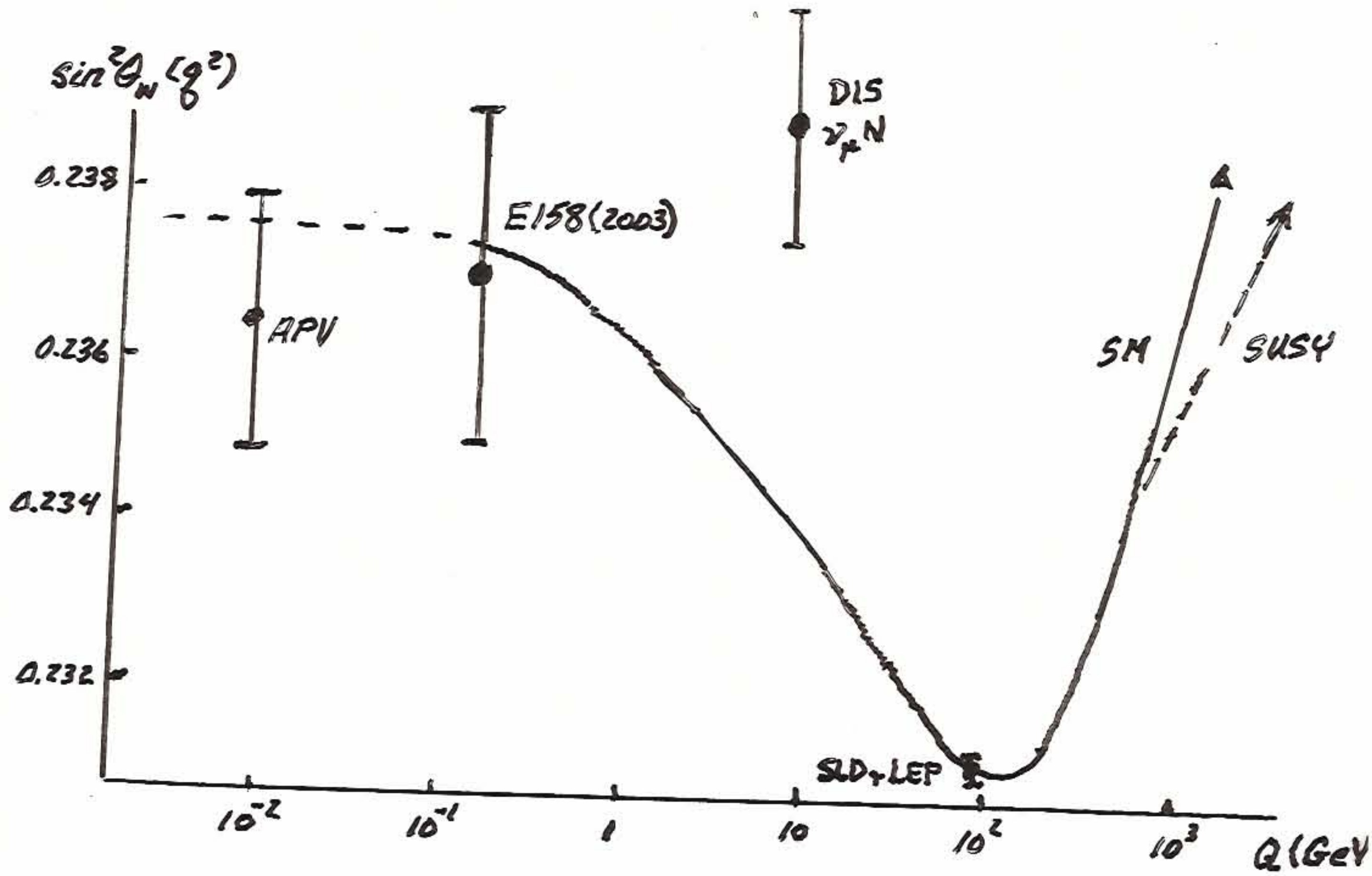
Probes: $\frac{\Delta m_H}{m_H} \simeq \pm 12\%$, $m_{Z_\chi}(SO(10)) \simeq 2.5\,\mathrm{TeV}$ ...

Alternative e^-e^- at $\sqrt{s} \simeq 500$-$1000\,\mathrm{GeV}$ Collider
Not as good for $\sin^2\theta_W$
but better Probe of "New Physics"

Running of $\sin^2\theta_W(Q^2) \equiv \kappa(Q^2)\sin^2\theta_W(m_Z)_{\overline{MS}}$ from γ-Z Mixing



+ SUSY, GUTS, New Physics



$$\sin^2\theta_W(0) = 1.030(2.5)\ \sin^2\theta_W(m_Z)_{\overline{MS}}$$

$\uparrow$

Hadronic Unc

$e^+e^- \rightarrow$ hadrons data

"New Physics" Effects

i) S, T, U Parametrization of Peskin & Takeuchi

Dynamical Sym. Breaking Generally Requires Heavy New Fermions

$\begin{pmatrix} F_1 \\ F_2 \end{pmatrix}_L \quad F_{1_R} \quad F_{2_R} \rightarrow$ Effective Bound State Higgs

eg Technicolor

$\pi^{\pm}_{TC}, \pi^{0}_{TC}, \sigma_{TC}$ (Heavy & Broad)

$O(1\,\mathrm{TeV})$

How would they impact precision measurements

Contribute to Gauge Boson Self-Energies


$\Pi_{\gamma\gamma}(q^2) \longrightarrow \alpha$


$\Pi_{\gamma Z}(q^2) \longrightarrow \sin^2\theta_W$


$\Pi^{VV}_{ZZ}(q^2),\ \Pi^{AA}_{ZZ}(q^2) \longrightarrow m_Z$


$\Pi^{VV}_{WW}(q^2),\ \Pi^{AA}_{WW}(q^2) \longrightarrow m_W$

Related by Natural Rel.!

$$\alpha T = \frac{\Pi^{New}_{WW}(0)}{m_W^2} - \frac{\Pi^{New}_{ZZ}(0)}{m_Z^2}, \qquad \frac{\alpha}{\sin^2 2\theta_W} S = \frac{\Pi^{New}_{WW}(m_W^2) - \Pi^{New}_{WW}(0)}{m_W^2}$$

$$\frac{\alpha}{4\sin^2\theta_W}(S+U) = \frac{\Pi^{New}_{ZZ}(m_Z^2) - \Pi^{New}_{ZZ}(0)}{m_Z^2}$$

U small (violates isospi

S & $T \sim O(1)$

Dynamical Sym. Br

Technicolor (QCD like with $SU(N)_{TC}$ and $\Lambda_{TC} \simeq 1000\,\Lambda_{QCD}$)

$\begin{pmatrix} U^i \\ D^i \end{pmatrix}_L \quad U_R^i, d_R^i \quad i = 1, 2 .. N_{TC}$ Techniquarks & Technileptons

Goldstone Bosons $\pi^{\pm}_{TC}, \pi^{0}_{TC} \rightarrow W_L^{\pm}, Z_L$ $\quad W^- \; \pi^-_{TC} \; W^-$

Spectrum of $\rho_{TC}, A_{TC} \ldots$ at $\sim 1\,\text{TeV}$

$$S \simeq +(1 \sim 2) \times \frac{1}{6\pi} \times \underbrace{\text{No. of heavy technidoublets}}_{\gtrsim 8} \quad \gtrsim O(1)$$

Positive

Effectively $m_H \sim O(1\,\text{TeV}) \rightarrow m_W$ & $\sin^2\theta_W(m_Z)_{\overline{MS}}$ predictions change (way off)

$$S \simeq 118 \left\{ 2\, \frac{m_W - 80.209\,\text{GeV}}{80.209\,\text{GeV}} + \frac{\sin^2\theta_W(m_Z) - 0.23232}{0.23232} \right\}$$ Nice Test

$$\left.\begin{array}{l} m_W = 80.426(34) \\ s^2 = 0.2308(2) \end{array}\right\} \rightarrow \underline{S = -0.13 \pm 0.1 \pm 0.1}$$

No Sign of Technicolor

Using $\alpha, G_\mu, m_Z, m_t, (m_H \sim 1\,\mathrm{TeV}) \ldots$ as input

Predict m_W & $\sin^2\theta_W(m_Z)_{\overline{MS}}$

$$m_W = \text{S.M.} + (0.45T - 0.29S + 0.34U)\,\mathrm{GeV}$$

$$\sin^2\theta_W^{\overline{MS}}(m_Z) = \text{S.M.} + 0.00365S - 0.00261T$$

Global Fits (crudely) $\rightarrow$ $S \simeq -0.1 \pm 0.1$, $T \simeq +0.1 \pm 0.1$ } No Evidence For New Physics

Each Heavy Fermion Doublet $\rightarrow \Delta S = \frac{1}{6\pi}$

A Heavy 4th Generation $\rightarrow \frac{4}{6\pi} \simeq 0.2$ (Ruled Out?)

No evidence for $\underline{S \simeq +1}$ As Expected in dyn. models (generic)

With Giga Z $\rightarrow \Delta\sin^2\theta_W \simeq 0.00002$, we would probe

$\boxed{\Delta S \simeq \pm 0.02}$ besides $\frac{\Delta m_H}{m_H} \sim \pm 4\%$

Spectacular Probe of Heavy Loop Effects!

~~Next Lecture: Higgs Phenomenology~~

8. Outlook & Conclusion

Recent $m_t \simeq 178\,GeV \rightarrow m_W$ & $\sin^2\theta_W(m_Z)^{leptonic}_{\overline{MS}}$ } Very Consistent

$\alpha, G_\mu, m_Z, m_t, \Delta\alpha_h^{(5)}$ + m_W & $\sin^2\theta_W(m_Z)_{\overline{MS}}$

$\downarrow$

$m_H = 68^{+45}_{-30}\,GeV \quad < 154\,GeV \quad 95\%\,CL$

Exp $\quad m_H \gtrsim 114.4\,GeV$

Relatively light Higgs Preference (SUSY?)

Issues: $A_{FB}(Z\to b\bar{b}) \rightarrow$ larger $\sin^2\theta_W(m_Z)^{hadronic}_{\overline{MS}} \rightarrow m_H \simeq 500\,GeV$

Resolve leptonic-hadronic discrepency?

Reanalysis of $A_{FB}(Z\to b\bar{b})$

long term Giga Z, Moller e^-e^-

<u>Near Term</u> $\quad A_{FB}(p\bar{p}\to \ell^+\ell^- x)$ at FNAL, LHC $\rightarrow \sin^2\theta_W(m_Z)_{\overline{MS}}$

m_W & m_t improvements

$\Delta\alpha_h^{(5)}$ improvements

<u>low energy</u> $\quad$ APV, e^-e^-, ep, νN $\quad$ Probe New Physics

Z', extra dim.

etc.

$g_\mu - 2/2$ → "New Physics" modest 2.4 sigma

<u>SUSY at $m_{SUSY} \simeq 74\sqrt{\tan\beta}$ GeV $\simeq$ 100-500 GeV</u>

Natural Interpretation

a_μ^{exp} & $a_\mu^{had.}$ improvements needed

Near Term $\Delta a_\mu \rightarrow \pm 50 \times 10^{-11}$

Future a_μ (long term) $\rightarrow \pm 15 \times 10^{-11}$? Very Challenging

Search for $\mu \to e\gamma$, $\mu^- N \to e^- N$, $\tau \to \mu\gamma$, edm's

Find SUSY & Higgs at LHC → LC study!

Observe Dark Matter

<u>Surprises & Puzzles Along The Way</u>

Optimist - Hints of Great Things To Come

Pessimist - Wild Optimism

We Shall See

Progress in Precision Electroweak Studies

My 1993 SLAC Lecture

$m_W = 80.22 \pm 0.26\,\text{GeV}$

LEP $\sin^2\theta_W(m_Z)^{\text{Ave}}_{\overline{MS}} = 0.2318 \pm 0.0006$

$m_t^{exp} > 131\ \text{GeV}$

$m_H^{exp} > 57\ \text{GeV}$

Loop Predictions

global $m_t = 170 \pm 26\ \text{GeV}$

$m_H = ?$

E158 Proposal ~ 1994

BNL Muon g-2 Exp
Under Construction

Today

$m_W = 80.426 \pm 0.034\,\text{GeV}$

$\sin^2\theta_W(m_Z)^{\text{leptonic}}_{\overline{MS}} = 0.23085 \pm 0.0002$

$m_t^{exp} = 178.0 \pm 4.3\,\text{GeV}$

$m_H^{exp} > 114.4\,\text{GeV}$

$m_H = 68^{+45}_{-30}\,\text{GeV}$ $< 154\,\text{GeV}$ (95%)

global $m_H = 113^{+56}_{-40}\,\text{GeV}$ $< 241\,\text{GeV}$ (95%)

New e^-e^- results (Finished)

$a_\mu^{exp} - a_\mu^{SM} = 249 \pm 99 \times 10^{-11}$

Problems - Questions

What is m_t? ✓

Why so large?

What is m_H?

Fundamental or Composite?

Origin of quark mixing $\cancel{CP}$ ✓ (?)

SUSY?

⋮