solving the hierarchy problem

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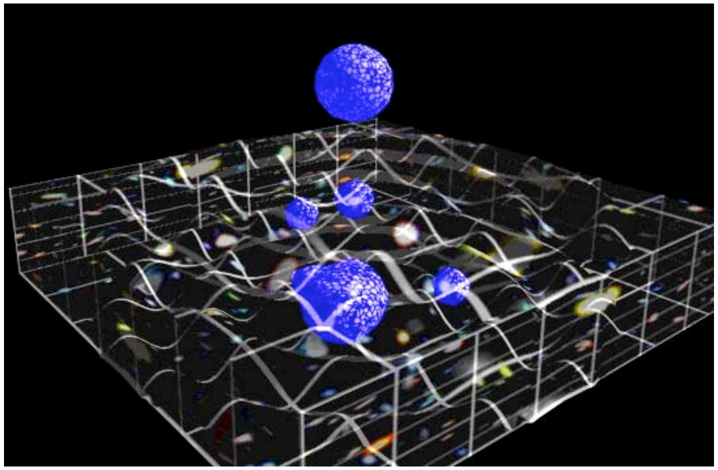
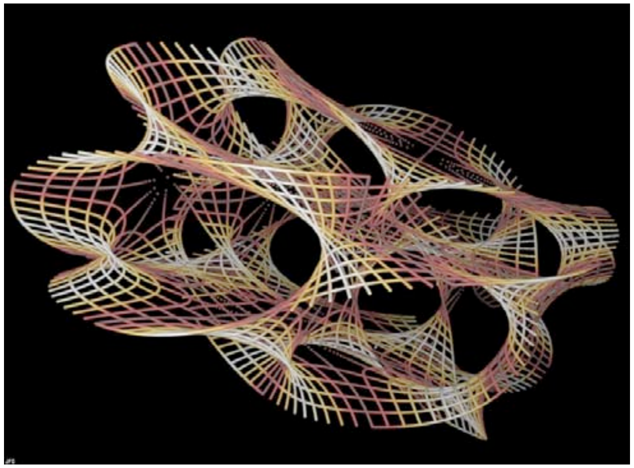
puzzle of the day:
why is gravity so weak?





answer:

because there are large or warped
extra dimensions
about to be discovered at colliders

puzzle of the day:
why is gravity so weak?

real answer:

don’t know

many possibilities

may not even be a well-posed question

outline of this lecture

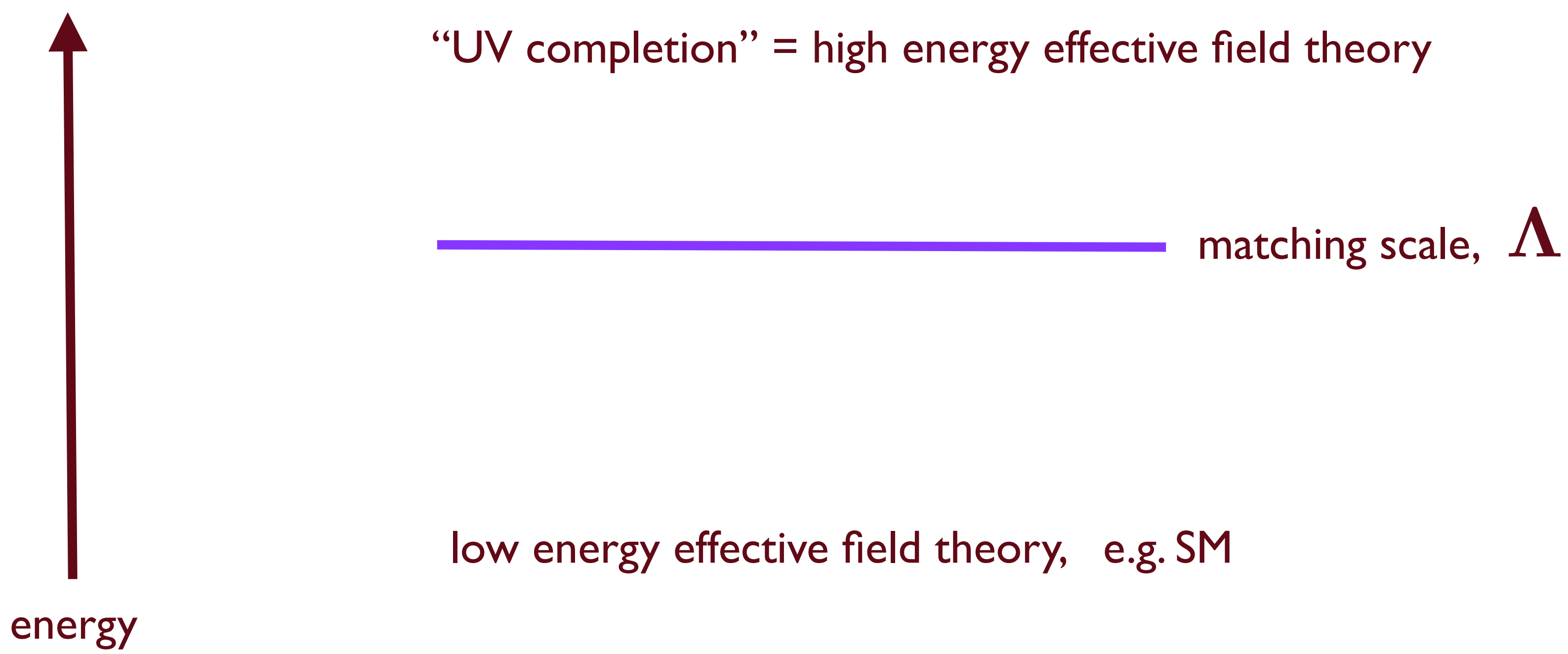
- what is the hierarchy problem of the Standard Model
- is it really a problem?
- what are the ways to solve it?
- how is this related to gravity?

what is the hierarchy problem of the Standard Model?

- discuss concepts of naturalness and UV sensitivity in field theory
- discuss Higgs naturalness problem in SM
- discuss extra assumptions that lead to the hierarchy problem of SM

UV sensitivity

- Ken Wilson taught us how to think about field theory:



UV sensitivity

- how much do physical parameters of the low energy theory depend on details of the UV matching (i.e. short distance physics)?
- if you know both the low and high energy theories, can answer this question precisely
- if you don't know the high energy theory, use a crude estimate: how much do the low energy observables change if, e.g. you let $\Lambda \to 2\Lambda$?

degrees of UV sensitivity

parameter	UV sensitivity
“finite” quantities	none -- UV insensitive
dimensionless couplings e.g. gauge or Yukawa couplings	logarithmic -- UV insensitive
dimension-full coefs of higher dimension “irrelevant” operators e.g. 4-fermion coupling in Fermi theory	inverse power of cutoff -- UV sensitive but suppressed
dimension-full coefs of lower dimension operators, e.g. scalar mass-squared, vacuum energy, etc.	positive power of cutoff -- UV sensitive

what do UV sensitive parameters do?

denote a generic UV sensitive parameter as $\mathbf{m}$
then there are 4 possibilities:

- **natural:** $\mathbf{m} \sim \mathbf{\Lambda}$, e.g. $\mathbf{m} \simeq \mathbf{g\Lambda/4\pi}$ for Higgs scalar
- **symmetry-natural:** there is a symmetry limit where $\mathbf{m = 0}$ (e.g. chiral symmetry for fermion masses). then can have $\mathbf{m} \ll \mathbf{\Lambda}$ because the symmetry is weakly broken (somehow).
- **supernatural:** there is tuning at the matching scale due to some feature of the UV theory. e.g. $\mathbf{m_1 = m_2}$, and the radiative corrections to this relation have only a log dependence on the cutoff.
- **unnatural:** there is a *fine-tuning* at the matching scale that produces $\mathbf{m} \ll \mathbf{\Lambda}$ this UV tuning somehow corrects for the large radiative corrections of the low energy theory.

naturalness

A **natural** theory is one in which all of the physical parameters are some combination of **UV insensitive**, **natural**, and **symmetry-natural**.

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tuning

A **supernatural** theory is not strictly natural, but one expects real world theories to have mysterious relations that only get explained when you discover the UV theory - so this is OK.

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fine-tuning

An **unnatural** theory is fine-tuned. This is bad, because there are no known physical mechanisms to produce fine-tuned theories. The only known explanation for fine-tuning is accidental relations in the UV parameters.

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the Higgs naturalness problem

- now apply this wisdom to the Higgs mass squared parameter of the SM.
- this parameter is UV sensitive, so how do we explain its value?
- the natural explanation is that $|\mu| \simeq \mathbf{g\Lambda/4\pi}$ so $\mathbf{\Lambda \sim 1\ TeV}$

➡ SM is natural, and is replaced by e.g. supersymmetry, technicolor, etc at the TeV scale.

the little hierarchy problem

- this explanation is now under attack from the electroweak precision data
- if $\Lambda \sim 1\ \mathrm{TeV}$, then we would generically expect to already be seeing evidence of higher dimension operators constructed out of SM fields
- there are many dimension 5 and 6 operators that obey all of the symmetries of the SM
- but there is no evidence for any of them in the data!

- if we assume that the dimensionless couplings are of order one (may not be true!) then $\mathbf{\Lambda \sim 1\ TeV}$ is ruled out

Dimensions six operators			$m_h = 115\,\text{GeV}$ $c_i = -1$	$c_i = +1$
$\mathcal{O}_{WB}$	$=$	$(H^\dagger \tau^a H) W^a_{\mu\nu} B_{\mu\nu}$	9.7	10
$\mathcal{O}_H$	$=$	$\|H^\dagger D_\mu H\|^2$	4.6	5.6
$\mathcal{O}_{LL}$	$=$	$\frac{1}{2}(\bar{L}\gamma_\mu \tau^a L)^2$	7.9	6.1
$\mathcal{O}'_{HL}$	$=$	$i(H^\dagger D_\mu \tau^a H)(\bar{L}\gamma_\mu \tau^a L)$	8.4	8.8
$\mathcal{O}'_{HQ}$	$=$	$i(H^\dagger D_\mu \tau^a H)(\bar{Q}\gamma_\mu \tau^a Q)$	6.6	6.8
$\mathcal{O}_{HL}$	$=$	$i(H^\dagger D_\mu H)(\bar{L}\gamma_\mu L)$	7.3	9.2
$\mathcal{O}_{HQ}$	$=$	$i(H^\dagger D_\mu H)(\bar{Q}\gamma_\mu Q)$	5.8	3.4
$\mathcal{O}_{HE}$	$=$	$i(H^\dagger D_\mu H)(\bar{E}\gamma_\mu E)$	8.2	7.7
$\mathcal{O}_{HU}$	$=$	$i(H^\dagger D_\mu H)(\bar{U}\gamma_\mu U)$	2.4	3.3
$\mathcal{O}_{HD}$	$=$	$i(H^\dagger D_\mu H)(\bar{D}\gamma_\mu D)$	2.1	2.5

Table 1: 95% *lower bounds on* $\Lambda/\,\text{TeV}$ *for the individual operators the SM for* $m_h > 115\,\text{GeV}$.

R. Barbieri and A. Strumia, hep-ph/0007265

little higgs models

- if we take the little hierarchy problem at face value, then the natural solution of the SM Higgs naturalness problem is insufficient
- however we can still preserve naturalness of the SM by reverting to symmetry-natural
- e.g. in Little Higgs models, the Higgs is a pseudo-Goldstone boson of the UV theory
- this allows us to push Λ up to 10 TeV, while keeping the SM natural
- the price is that the SM has to be extended to include extra TeV mass particles

living with SUSY

- another possibility is to replace natural with supernatural
- thus we imagine that Λ is somewhat higher than a TeV, but there is a little tuning going on, for reasons which will become obvious after we get a handle on the UV theory
- for supersymmetry models, which are further constrained by WMAP and the lower bound on the Higgs mass from LEP, this is a strong possibility
- in this case the SM is not natural, but we shouldn't worry too much

living with SUSY

$$
\begin{aligned}
M_Z^2 \quad = \quad & -1.8\mu^2(\mathrm{UV}) + 5.9M_3^2(\mathrm{UV}) - 0.4M_2^2(\mathrm{UV}) - 1.2m_{H_U}^2(\mathrm{UV}) \\
& +0.9m_{Q_3}^2(\mathrm{UV}) + 0.7m_{U_3}^2(\mathrm{UV}) - 0.6A_t(\mathrm{UV})M_3(\mathrm{UV}) \\
& -0.1A_t(\mathrm{UV})M_2(\mathrm{UV}) + 0.2A_t^2(\mathrm{UV}) + 0.4M_2(\mathrm{UV})M_3(\mathrm{UV}) + \ldots
\end{aligned}
$$

- here is a typical SUSY formula matching the SM Z mass to soft parameters of the SUSY model
- using their log running, the soft parameters have in turn been run up to their UV cutoff, which in this case is the GUT scale (denoted "UV")
- their could be cancellations here, which would then explain why superpartners and the Higgs haven't been seen yet

the hierarchy problem of the SM

- since the SM is renormalizable, no reason in principle not to have $\Lambda = 10^{120}\ \mathrm{GeV}$, (although it will then probably be strongly coupled or unstable in the UV)
- but gravity exists, and gravity effects in loops are not negligible for scales above $\frac{1}{\sqrt{8\pi G_N}} = \frac{M_{Planck}}{\sqrt{8\pi}} \simeq 10^{18}\ \mathrm{GeV}$
- so why not take $\Lambda \sim 10^{18}\ \mathrm{GeV}$?
- but then the Higgs naturalness problem becomes **much worse**, since now the only remaining alternative is that the SM is unnatural and fine-tuned.

the hierarchy problem of the SM

- of course if $\frac{M_W}{M_{Planck}}$ were 0.1 instead of 10^{-16} then the Higgs naturalness problem would be unaffected
- so the hierarchy problem of the SM boils down to the mystery of why $\frac{M_W}{M_{Planck}}$ is so small.
- note that the question here is not "why is gravity so weak", but rather "why is the EW scale so small in units of the (assumed) cutoff?"

other hierarchy problems

- suppose that the SM turns out to be natural or at least supernatural
- and suppose the UV theory which replaces it is natural (e.g. technicolor-like models and many SUSY models)
- then naturalness is no longer an issue, but the mystery of the hierarchy between the EW scale and the Planck scale remains
- in both SUSY and technicolor-like models, the generic answer is that log running of (non-SM) gauge couplings induce exponential hierarchies (just like in the SM, where $\Lambda_{QCD}/M_W \sim .003$)
- this is a simple and robust mechanism
- its drawback is that it requires strong model assumptions and many new degrees of freedom, whose explanation is put off to the ultimate unified theory

other hierarchy problems

- it is also important to note the SM has other hierarchy problems:
- for example, why is $\frac{m_u}{m_t}$ so small $< 10^{-4}$?
- a generic and robust mechanism to explain at least some of these SM flavor hierarchies is to invoke broken flavor symmetries
- turn off the Yukawa couplings of the SM, and there is a $[U(3)]^5$ global flavor symmetry mixing the 5 types of SM fermions: Q, U, D, L, E
- if we e.g. gauge a diagonal $U(2)$ of this, then only the third generation fermions get mass in the limit that this flavor symmetry is unbroken
- note these flavor hierarchy problems are not naturalness problems

is the SM hierarchy problem really a problem?

the SM hierarchy problem arose from the SM Higgs naturalness problem only when we made some additional assumptions, to whit:

1. the SM cutoff isn't TeV
2. the cutoff scale has something to do with gravity
3. there is a "quantum gravity" cutoff not far below the scale at which gravity becomes strong
4. the scale at which gravity becomes strong is given by the Cavendish result $\mathrm{M_{Planck}} = \frac{1}{\sqrt{\mathrm{G_N}}} = 10^{19}\mathrm{GeV}$

these assumptions could be wrong!

let's examine these assumptions:

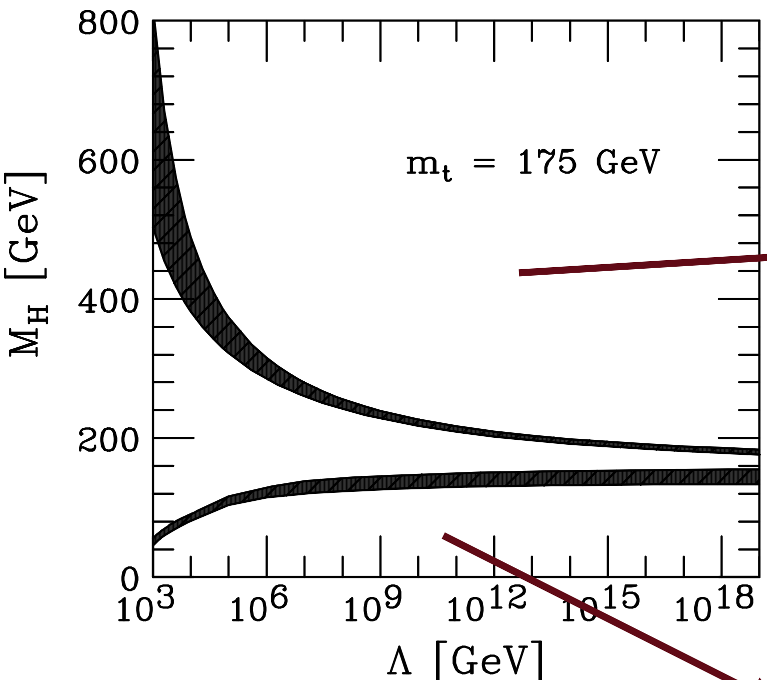
Assumption 1: the SM cutoff isn't TeV

- we have already argued that the SM cutoff is probably no more than a few TeV.
- we are building a multi-billion dollar supercollider based upon this belief!
- however we already noted that this just means that the SM hierarchy problem gets replaced by e.g. the SUSY hierarchy problem
- in that case we also need to know (rather urgently) whether the new theory above the TeV scale is natural or not.

Assumption 2: the cutoff scale has something to do with gravity

- this is not obvious since, even if we allow the SM to be unnatural, there are other reasons (unrelated to gravity) that could impose a lower cutoff:

$$\frac{d\lambda_h}{d\log\Lambda} = \frac{3}{2\pi^2}\left[\lambda_h^2 - \frac{1}{4}\lambda_t^4 + \cdots\right]$$



λ_h hits Landau pole, i.e. blows up

λ_h goes negative, destabilizes vacuum

K. Riesselmann hep-ph/9711456

Assumption 3: there is a "quantum gravity" cutoff not far below the scale at which gravity becomes strong

- classical gravity certainly exists, but nobody knows if we are really supposed to put off-shell gravitons in loop diagrams
- string theory provides consistent well-defined examples of quantum gravity coupled to gauge fields and matter
- in these examples there is a stringy cutoff scale $\mathrm{M_s}$, related to the Planck scale by $\mathrm{M_s \sim g_s M_{Planck}}$, where $\mathrm{g_s}$ is the string coupling
- in some cases (the heterotic string) the string coupling is related to the SM gauge couplings, implying that indeed the stringy cutoff is not far below the Planck scale
- but in other cases (branes) the stringy cutoff can be far below the Planck scale

see JL hep-th/9603133

Assumption 4: the scale at which gravity becomes strong is given by the Cavendish result $\mathbf{M_{Planck} = \frac{1}{\sqrt{G_N}} = 10^{19} GeV}$

- thanks to Arkani-Hamed, Dimopoulos and Dvali, we now realize that this assumption is very naive
- gravity is a poorly understood force
- it is only well-measured at energy scales up to $\mathbf{10^{-3} eV}$, and very crudely probed up to about a TeV
- how naive to extrapolate this poorly understood theory another 16 to 31 orders of magnitude!
- e.g. an extra spatial dimension of size $\mathbf{R}$, anywhere in these 31 orders of magnitude, will lower the strong gravity scale to

$$\mathbf{M_* = \left[\frac{M^2_{Planck}}{R} \right]^{1/3}}$$

solutions of the hierarchy problem

let's review the possible solutions (in order of plausibility):

1. The SM is replaced by a new effective theory at the TeV scale. This new theory is natural, with a cutoff close to the Planck scale. The EW scale is related to a natural scale in the new theory, produced by log running of gauge of other dimensionless couplings. Examples: many SUSY models, technicolor-like models.

2. Same thing but there are several stages of new UV theories before you get to the Planck scale.

3. Same thing but the new theory is not natural, i.e. there is fine-tuning near the Planck scale. Some new principle explains both the Higgs fine-tuning and the cosmological constant fine-tuning.

   Arkani-Hamed and Dimopoulos, hep-ph/0405159

4. There is no hierarchy because the string scale is only a few TeV, or the effective Planck scale is only a few TeV (due to large or warped extra dimensions).

a new principle of tuning?

- Before 1998, the Higgs was the only fine-tuning problem, and it had several good solutions.
- If the dark energy is vacuum energy, then we another (even worse) fine-tuning problem. Doesn't have any good solutions.
- If there is some new fundamental principle to explain the fine-tuning of the vacuum energy, it might also apply to all UV sensitive parameters.