Inflation and String Theory

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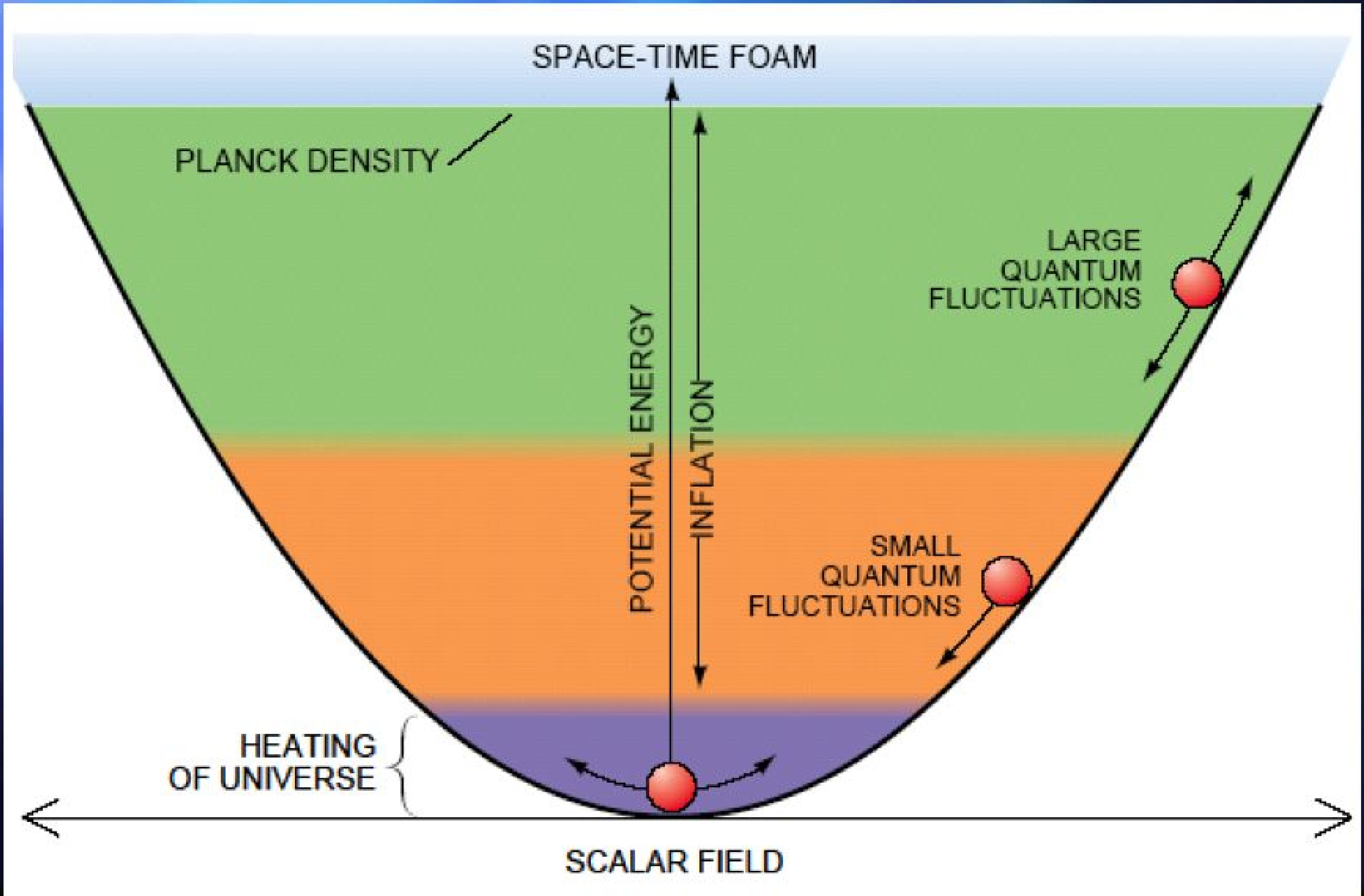
Inflation as a theory of a harmonic oscillator

$$V(\phi) = \frac{m^2}{2}\phi^2$$



Equations of motion:

- **Einstein:**

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{m^2}{6}\phi^2$$

- **Klein-Gordon:**

$$\ddot{\phi} + 3H\dot{\phi} = -m^2\phi$$

Compare with equation for the harmonic oscillator with friction:

$$\ddot{x} + \alpha\dot{x} = -kx$$

Logic of Inflation:

Large φ → large H → large friction →

field φ moves very slowly, so that its potential energy for a long time remains nearly constant

$$H = \frac{\dot{a}}{a} = \frac{m\phi}{\sqrt{6}} \approx \text{const}$$

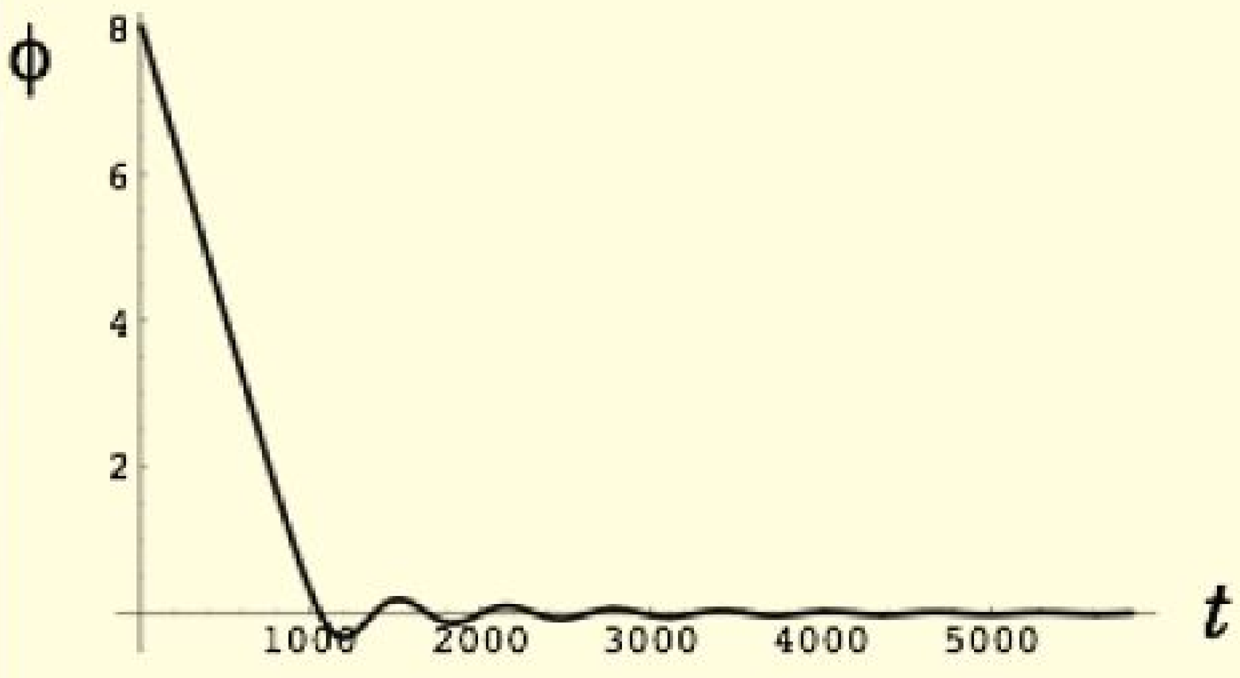
↓

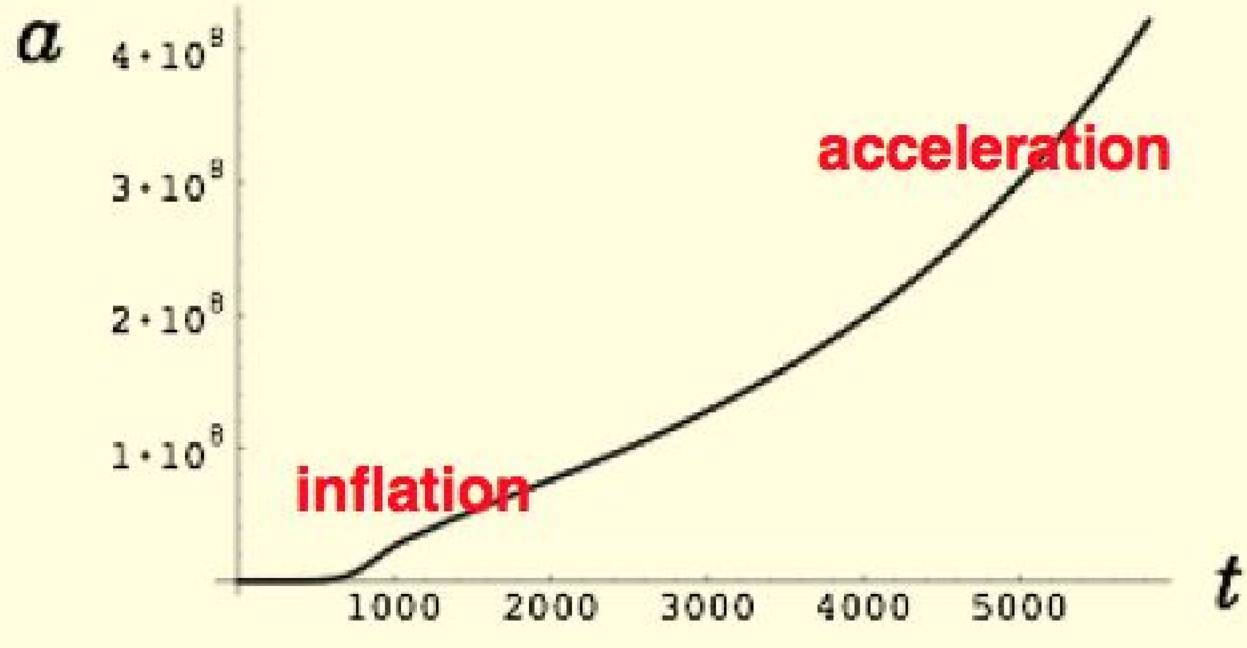
$$a \sim e^{Ht}$$

No need for false vacuum, supercooling, phase transitions, etc.

Add a constant to the inflationary potential - obtain inflation and acceleration

$$V = \frac{m^2}{2}\phi^2 + \Lambda$$





The simplest models of dark energy

1. Cosmological Constant

$$V = \Lambda$$

The universe eternally accelerates

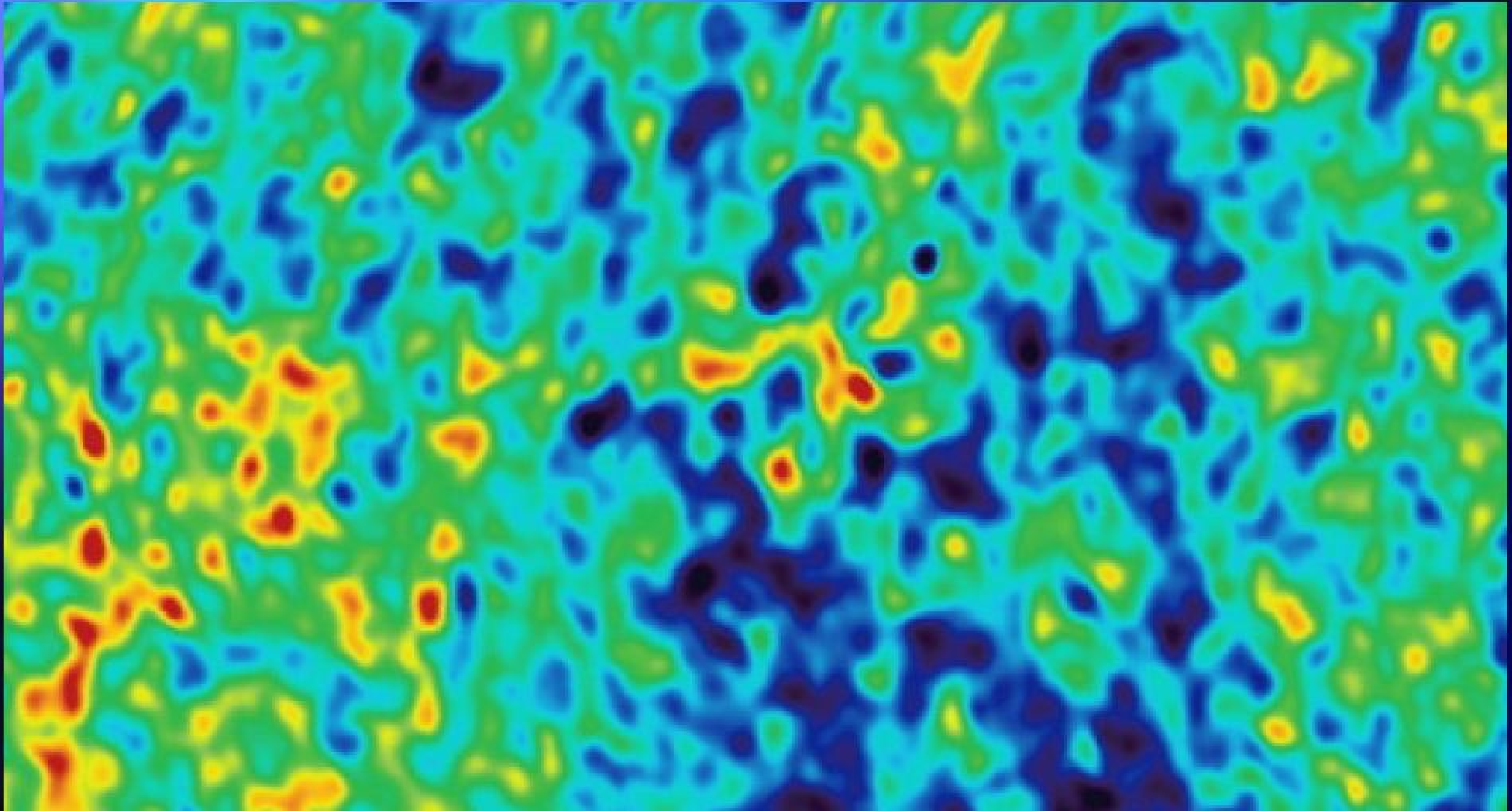
2. Linear Potential

A.L. 1986

$$V = \Lambda + \alpha\phi$$

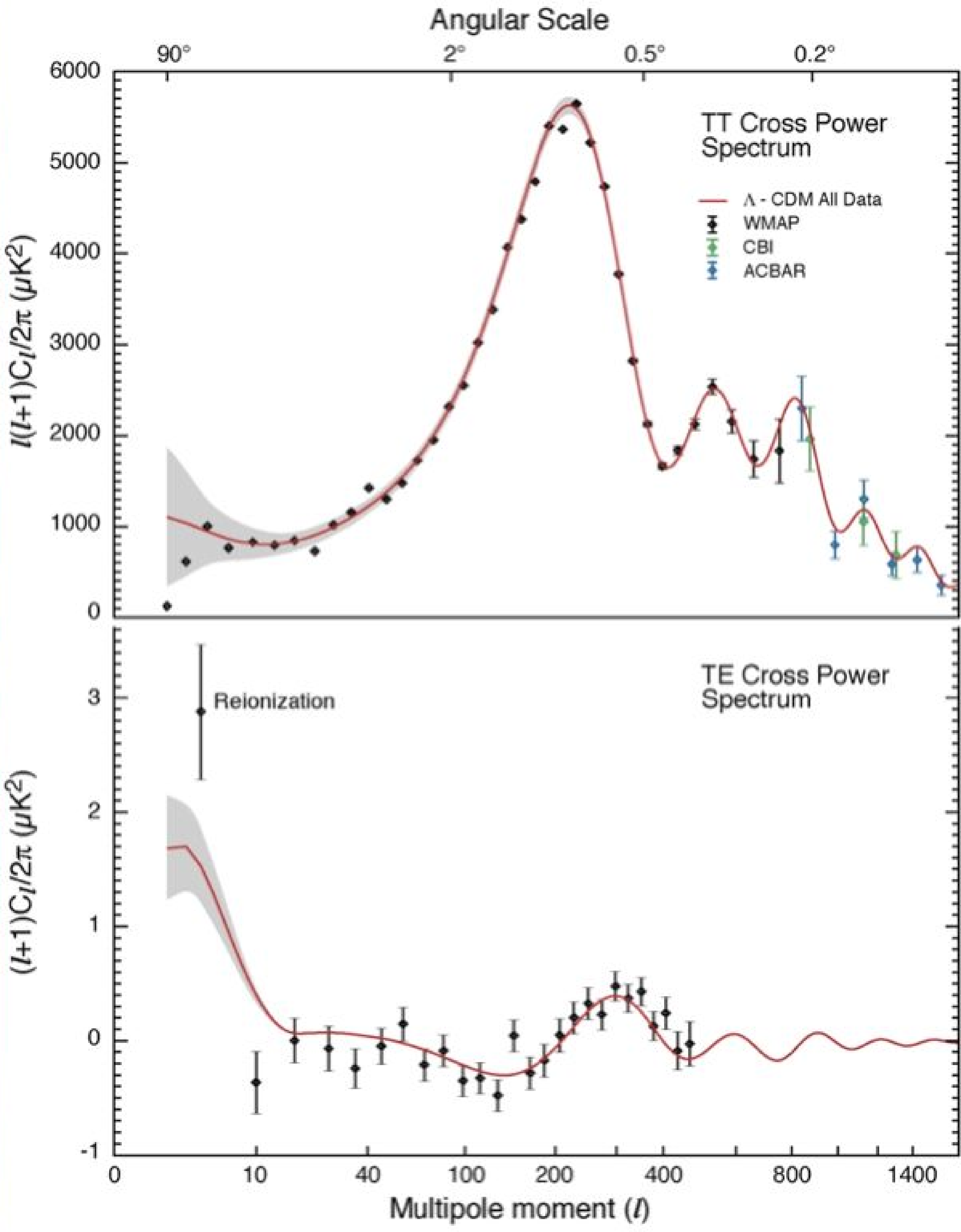
The universe accelerates and then collapses

A photographic image of quantum fluctuations blown up to the size of the universe



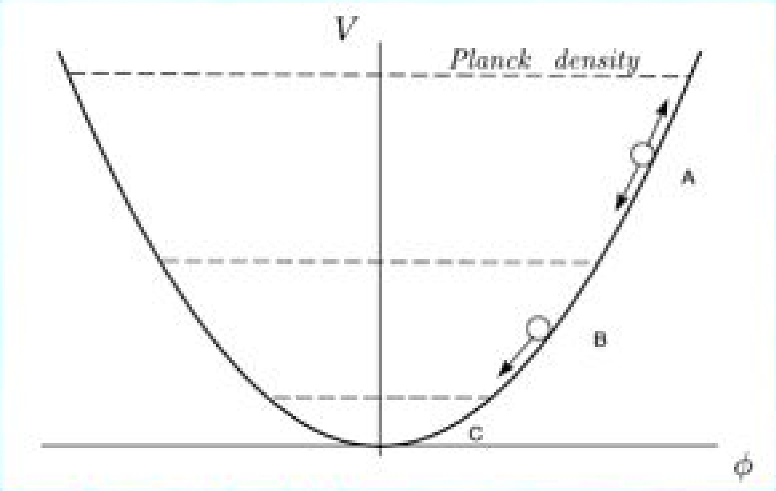
WMAP

and spectrum of the microwave background anisotropy



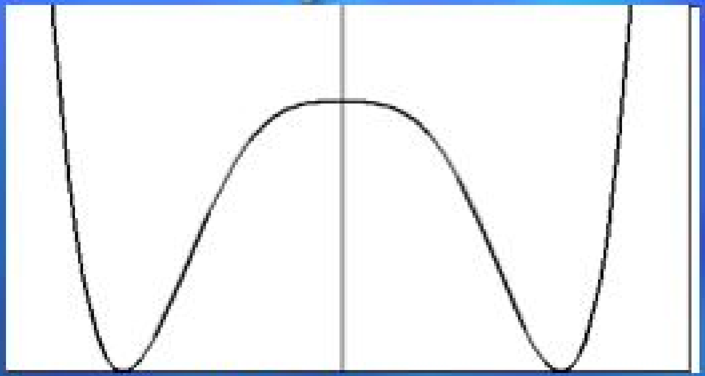
Comparing different inflationary models:

- **Chaotic inflation** can start in the smallest domain of size 10^{-33} cm with total mass ~ M_p (less than a milligram) and entropy O(1)



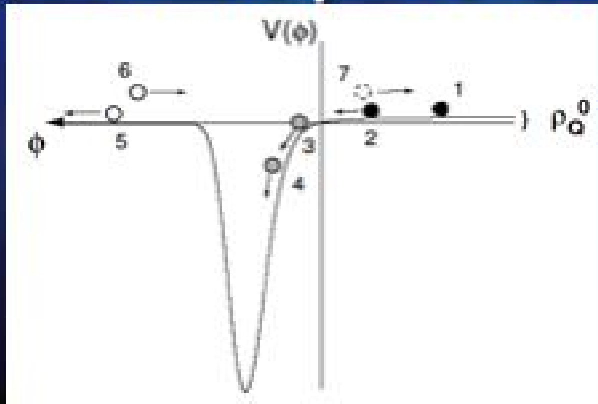
Solves flatness, mass and entropy problem

- **New inflation** can start only in a domain with mass 6 orders of magnitude greater than M_p and entropy greater than 10^9



Not very good with solving flatness, mass and entropy problem

- **Cyclic inflation** can occur only in the domain of size greater than the size of the observable part of the universe, with mass > 10^{55} g and entropy > 10^{87}



Does not solve flatness, mass and entropy problem

Is the simplest chaotic inflation natural?

- Often repeated (but incorrect) argument:

$$V = V_0 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \sum C_n \frac{\phi^n}{M_p^n}$$

Thus one could expect that the theory is ill-defined at $\phi > M_p$

However, quantum corrections are in fact proportional to

$$\left(\frac{V}{M_p^4}\right)^n \quad \text{and to} \quad \left(\frac{m^2(\phi)}{M_p^2}\right)^n$$

These terms are harmless for sub-Planckian masses and densities, even if the scalar field itself is very large.

Chaotic inflation in supergravity

Main problem:

$$V = e^{K}\left(K^{-1}_{\Phi\bar{\Phi}}|D_\Phi W|^2 - 3|W|^2\right)$$

Canonical Kahler potential is $K = \Phi\bar{\Phi}$

Therefore the potential blows up at large |φ|, and slow-roll inflation is impossible:

$$V \sim e^{|\Phi|^2}$$

Too steep, no inflation…

A solution: shift symmetry

Kawasaki, Yamaguchi, Yanagida 2000

Equally legitimate Kahler potential $K = \frac{1}{2}(\Phi + \bar{\Phi})^2 + X\bar{X}$

and superpotential $W = m\Phi X$

The potential is very curved with respect to X and Re φ, so these fields vanish

But Kahler potential does not depend on

$$\phi = \sqrt{2}\ \mathrm{Im}\,\Phi = (\Phi - \bar{\Phi})/\sqrt{2}$$

The potential of this field has the simplest form, without any exponential terms:

$$V = \frac{m^2}{2}\phi^2$$

Inflation in String Theory

The volume stabilization problem:

Consider a potential of the 4d theory obtained by compactification in string theory of type IIB

$$V(\sigma, \rho, \phi) \sim e^{\sqrt{2}\sigma - \sqrt{6}\rho}\ \tilde{V}(\phi)$$

Here σ is the dilaton field, and ρ describes volume of the compactified space

The potential with respect to these two fields is very steep, they run down, and V vanishes

The problem of the dilaton stabilization was solved in 2001, but volume stabilization problem was most difficult and was solved only in 2003 (KKLT construction)

Giddings, Kachru and Polchinski 2001

Kachru, Kallosh, Linde, Trivedi 2003

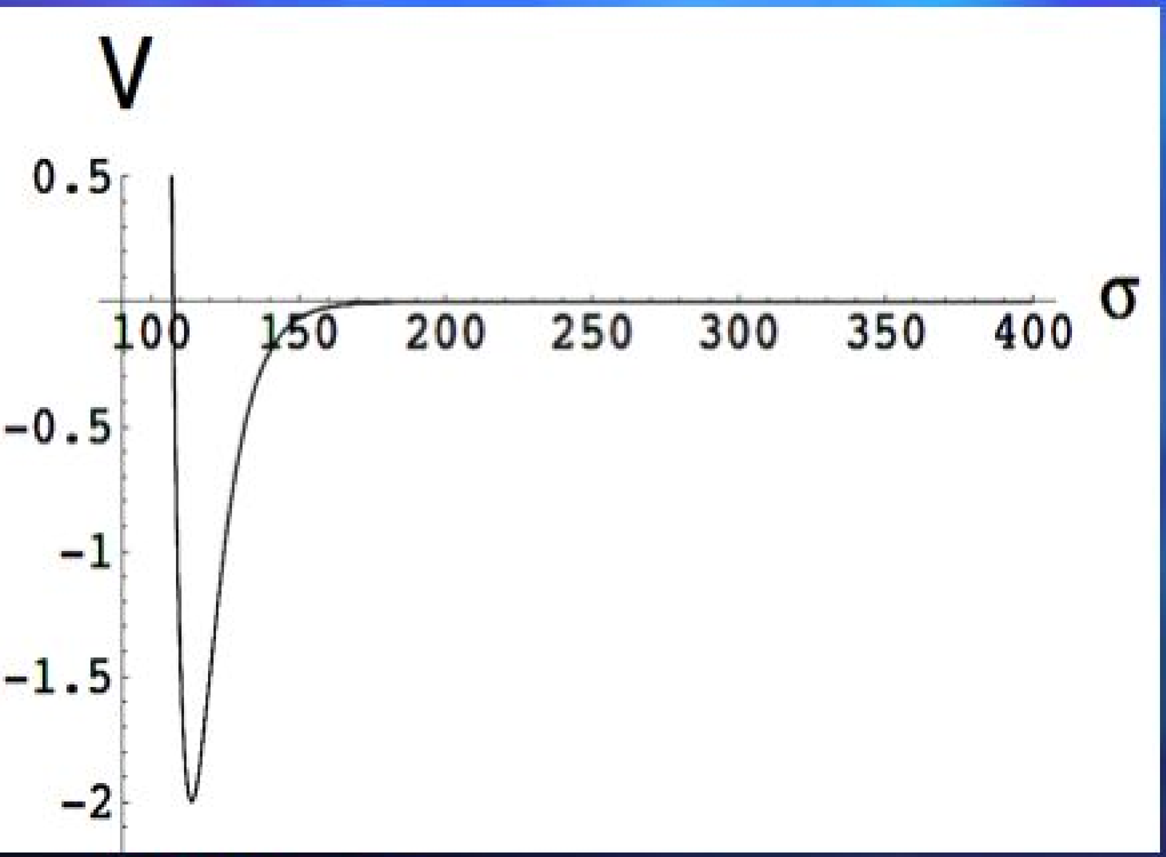
Burgess, Kallosh, Quevedo, 2003

Volume stabilization

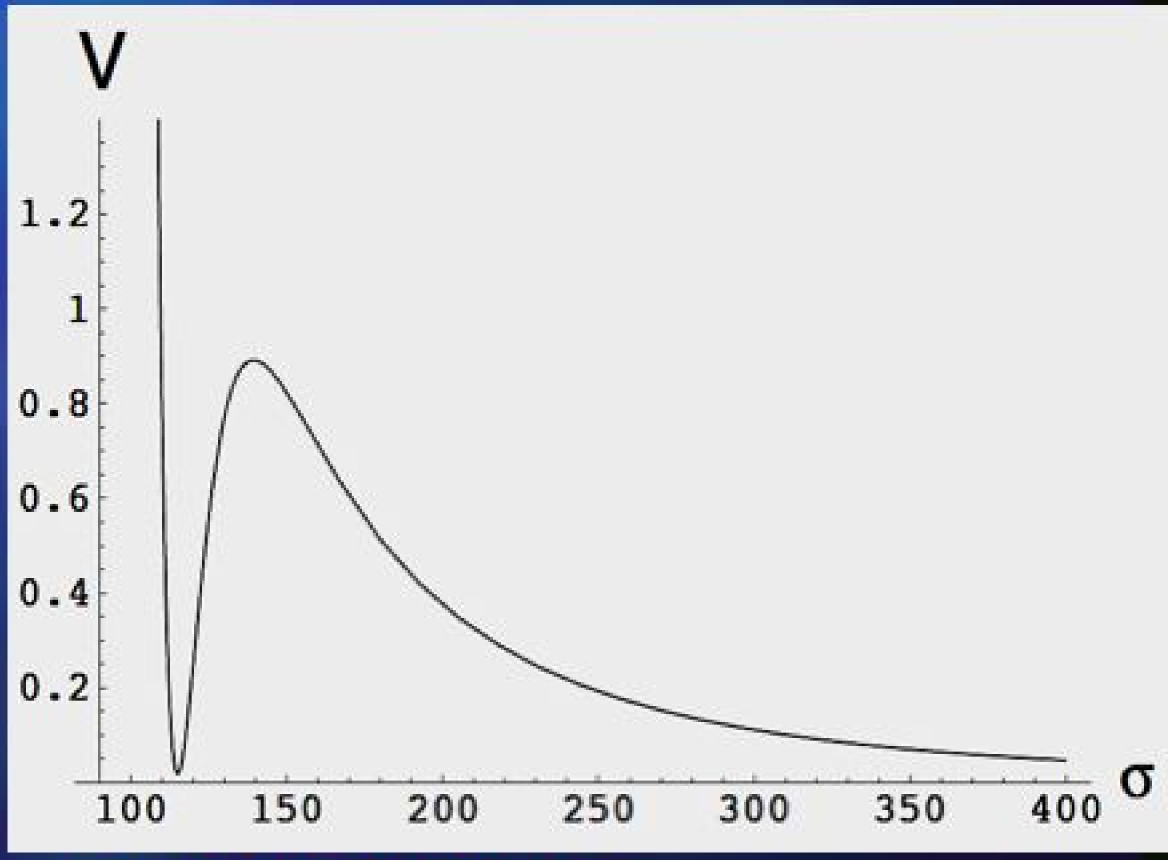
Basic steps:

- Warped geometry of the compactified space and nonperturbative effects ⟶ **AdS** space (negative vacuum energy) with unbroken SUSY and stabilized volume

- Uplifting AdS space to a **metastable dS** space (positive vacuum energy) by adding anti-D3 brane (or D7 brane with fluxes)



AdS minimum



Metastable dS minimum

Two types of string inflation models:

- **Moduli Inflation.** The simplest class of models. It uses only the fields that are already present in the KKLT model.
- **Brane inflation.** The inflaton field corresponds to the distance between branes in Calabi-Yau space. Historically, this was the first class of string inflation models.

Brane Inflation with stabilized volume

- Use KKLT volume stabilization (in all string inflation models)
  Kachru, Kallosh, Linde, Maldacena, McAllister, Trivedi 2003
  Problem: Requires fine-tuning of superpotential at the level of 1%

- Introduce the inflaton field with the potential which is flat due to shift symmetry
- Break shift symmetry either due to superpotential or due to radiative corrections

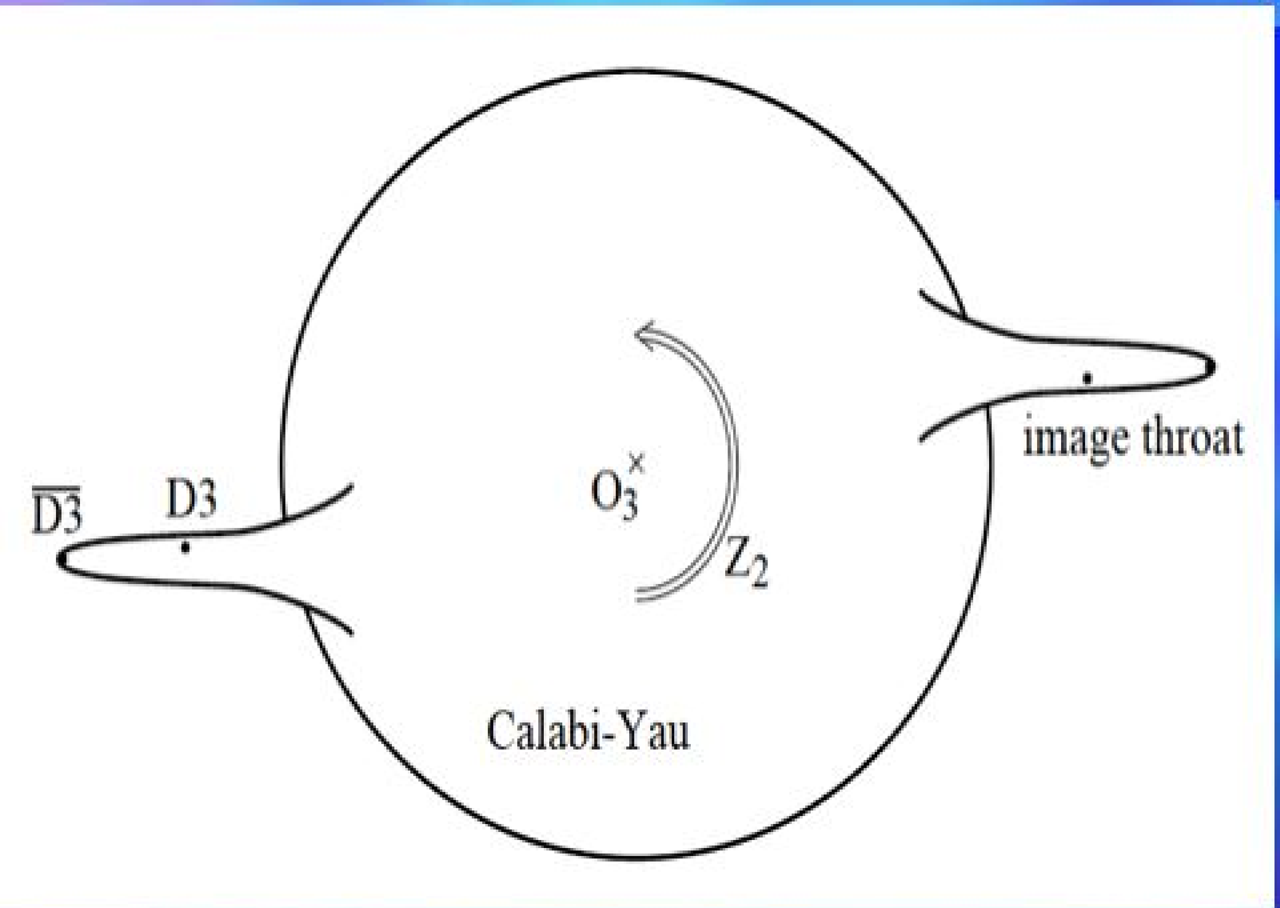
  Hsu, Kallosh , Prokushkin 2003
  Angelantonj, D'Auria, Ferrara, Trignante 2003
  Koyama, Tachikawa, Watari 2003
  Firouzjahi, Tye 2003
  Hsu, Kallosh 2004

Alternative approach: Modifications of kinetic terms in the strong coupling regime

Silverstein and Tong, 2003

The KLMT model

Kachru, Kallosh, A.L., Maldacena, McAllister, and Trivedi 2003



$$K = -3\log(\rho + \bar{\rho} - \phi\bar{\phi})$$

$$W = W_0 + e^{-a\rho}$$

$$m_\phi^2 = 2H^2$$

Meanwhile for successful inflation with flat spectrum of perturbations one needs

$$m_\phi^2 \sim 10^{-2} H^2$$

This can be achieved y taking W which depends on ϕ and by fine-tuning it at the level O(1%)

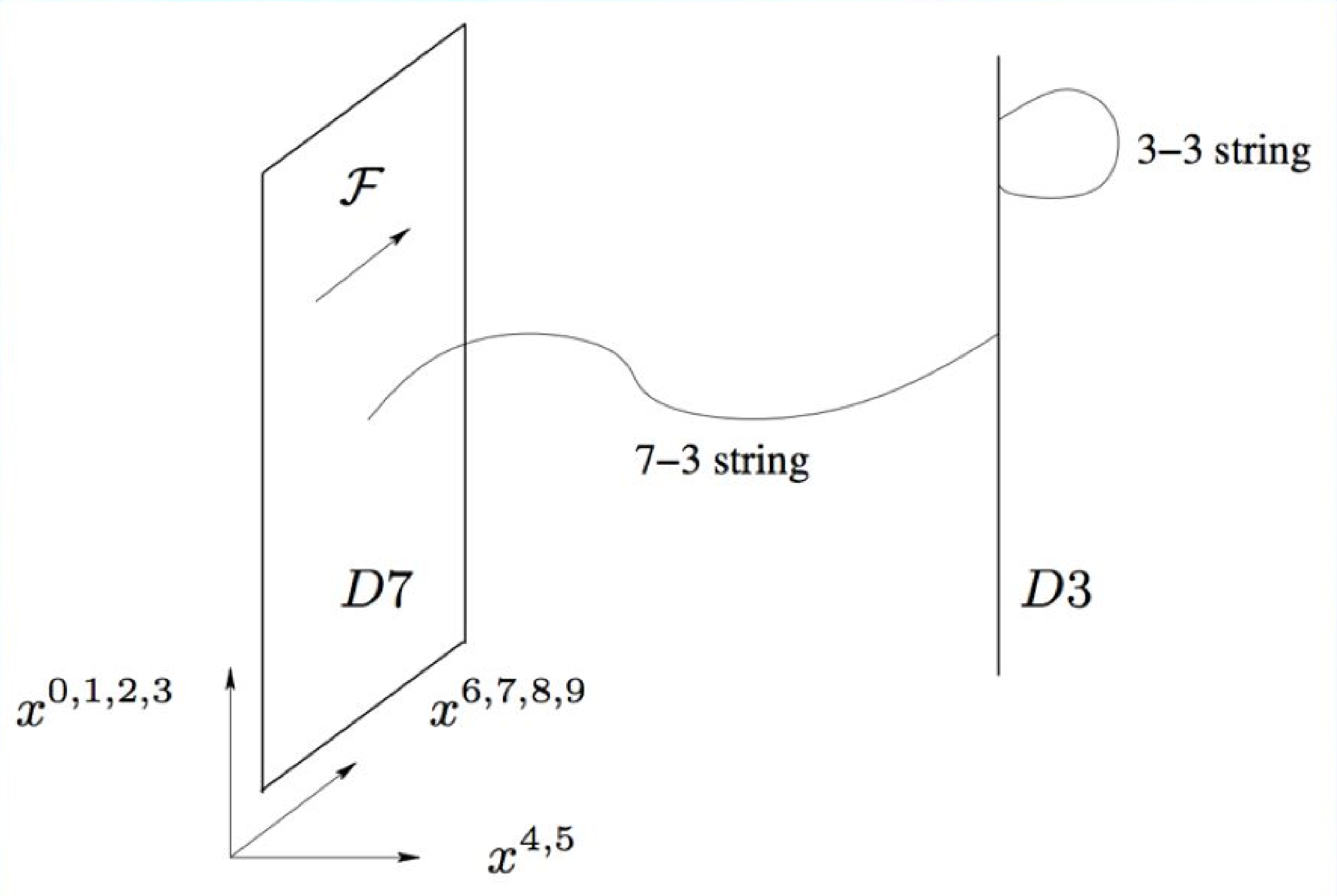
This model is complicated and requires fine-tuning, but it is based on some well-established concepts of string theory. Its advantage is that the smallness of inflationary parameters has a natural explanation in terms of warping of the Klebanov-Strassler throat

Fine-tuning may not be a problem in the string theory landscape paradigm

Further developed by: Burgess, Cline, Stoica, Quevedo; DeWolfe, Kachru, Verlinde; Iisuka,Trivedi, Berg, Haack, Kors; Buchel, Ghodsi

D3/D7 Inflation

Kallosh et al, 2001 - 2004



String inflation and shift symmetry

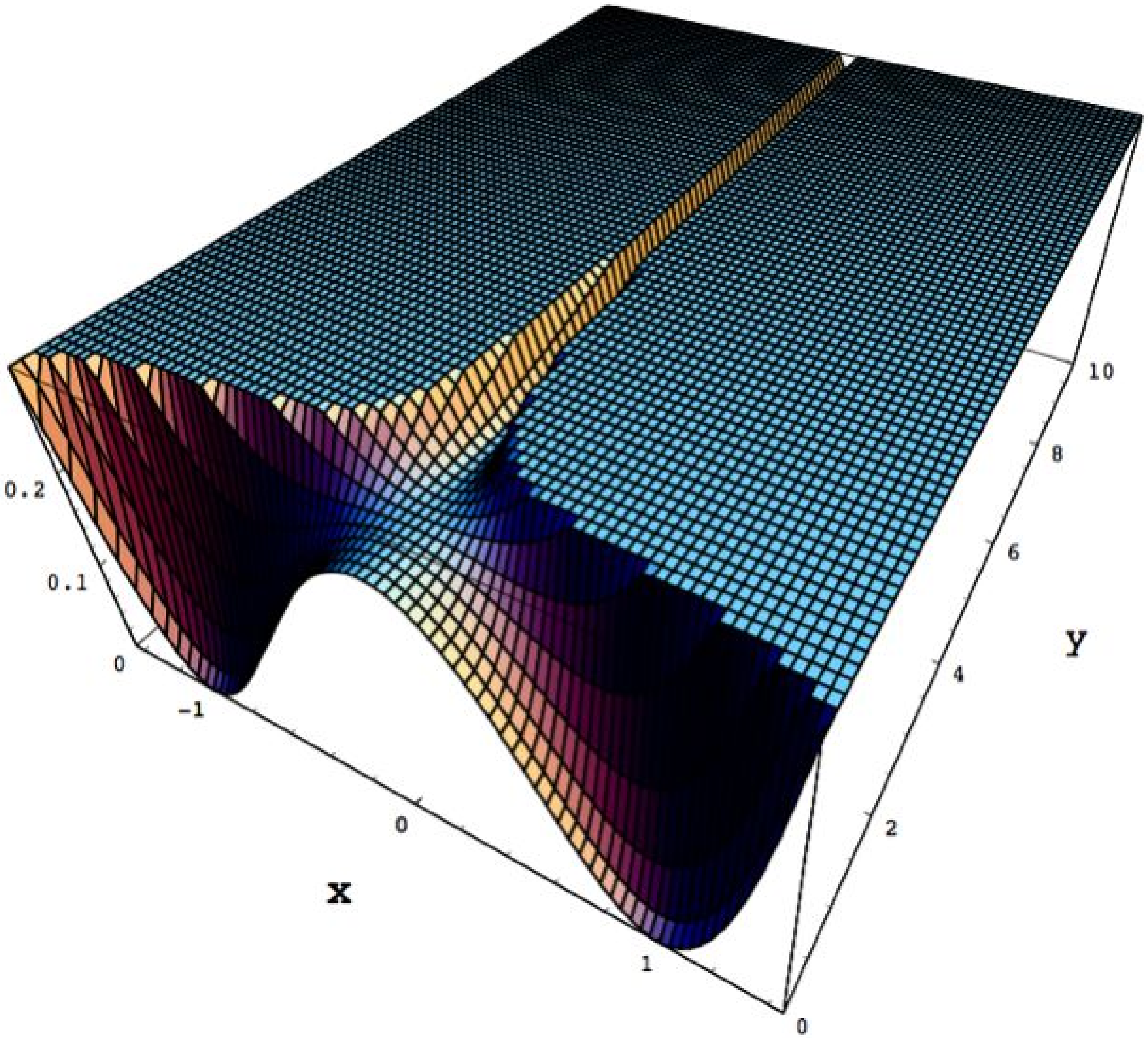
$$K = -3 \log\left(\rho + \bar{\rho} + \frac{1}{2}(\mathrm{s} - \bar{\mathrm{s}})^2\right)$$

Hsu, Kallosh , Prokushkin 2003

Shift symmetry protects flatness of the inflaton potential in the $\mathrm{s} + \bar{\mathrm{s}}$ direction. This symmetry is not just a requirement which is desirable for inflation, but, in a certain class of string theories, it is an unavoidable consequence of the mathematical structure of the theory.

Hsu, Kallosh, 2004

The Potential of the Hybrid D3/D7 Inflation Model



$$V = S^2 \Phi^\dagger \Phi + \frac{g^2}{2} D^2$$

$$\vec{D} = \Phi^\dagger \vec{\sigma} \Phi - \vec{\xi}$$

Φ **is a hypermultiplet**

$\vec{\xi}$ **is an FI triplet**

Bringing it all together:

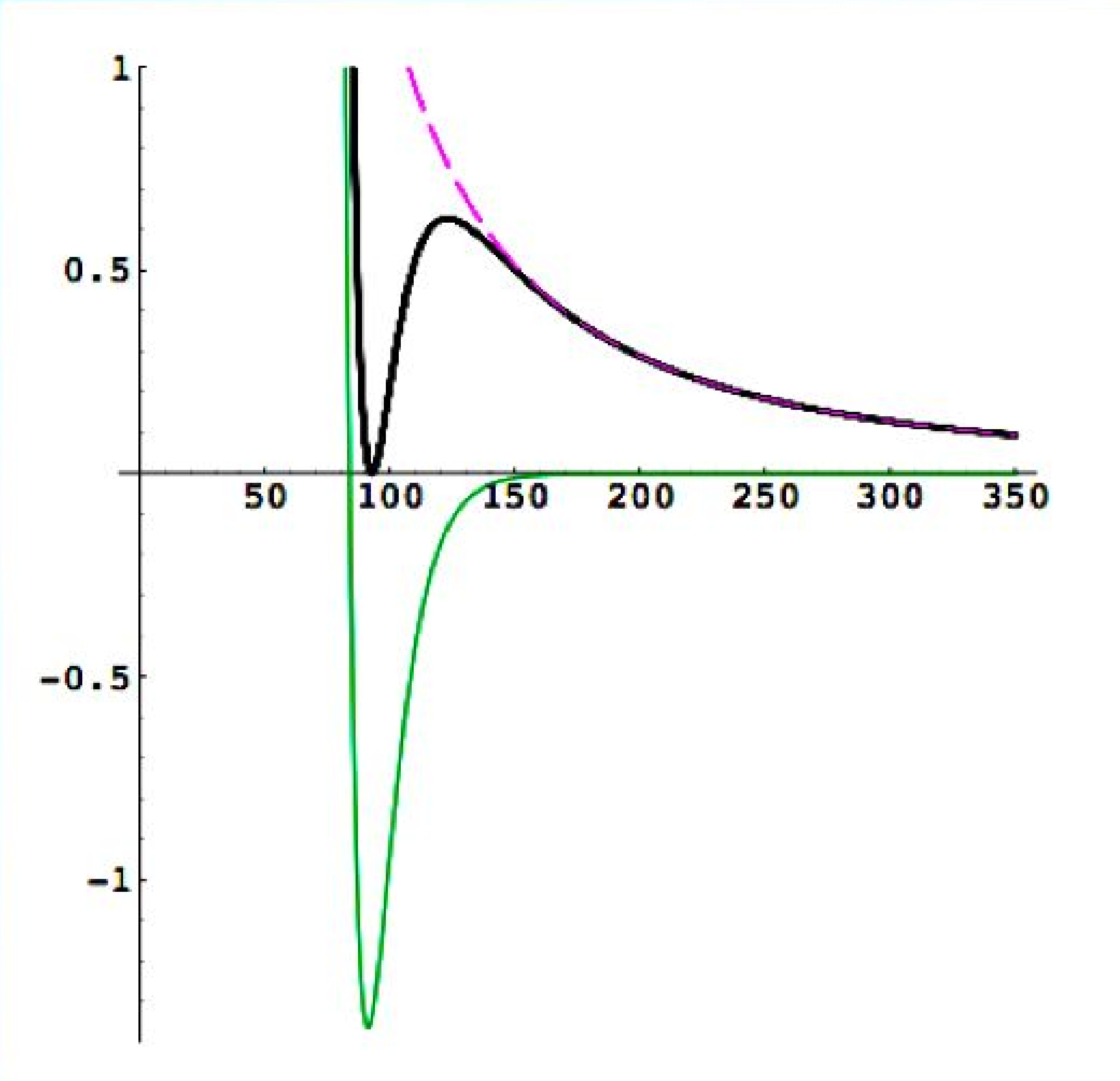
Double Uplifting

$$V_{\rm tot} = V_{\rm KKLT}(\sigma) + V_2(\sigma, s, \phi_\pm)$$

KKL, in progress

First uplifting: KKLT

$$V_{\rm KKLT} = V_{AdS} + \frac{D}{\sigma^2}$$

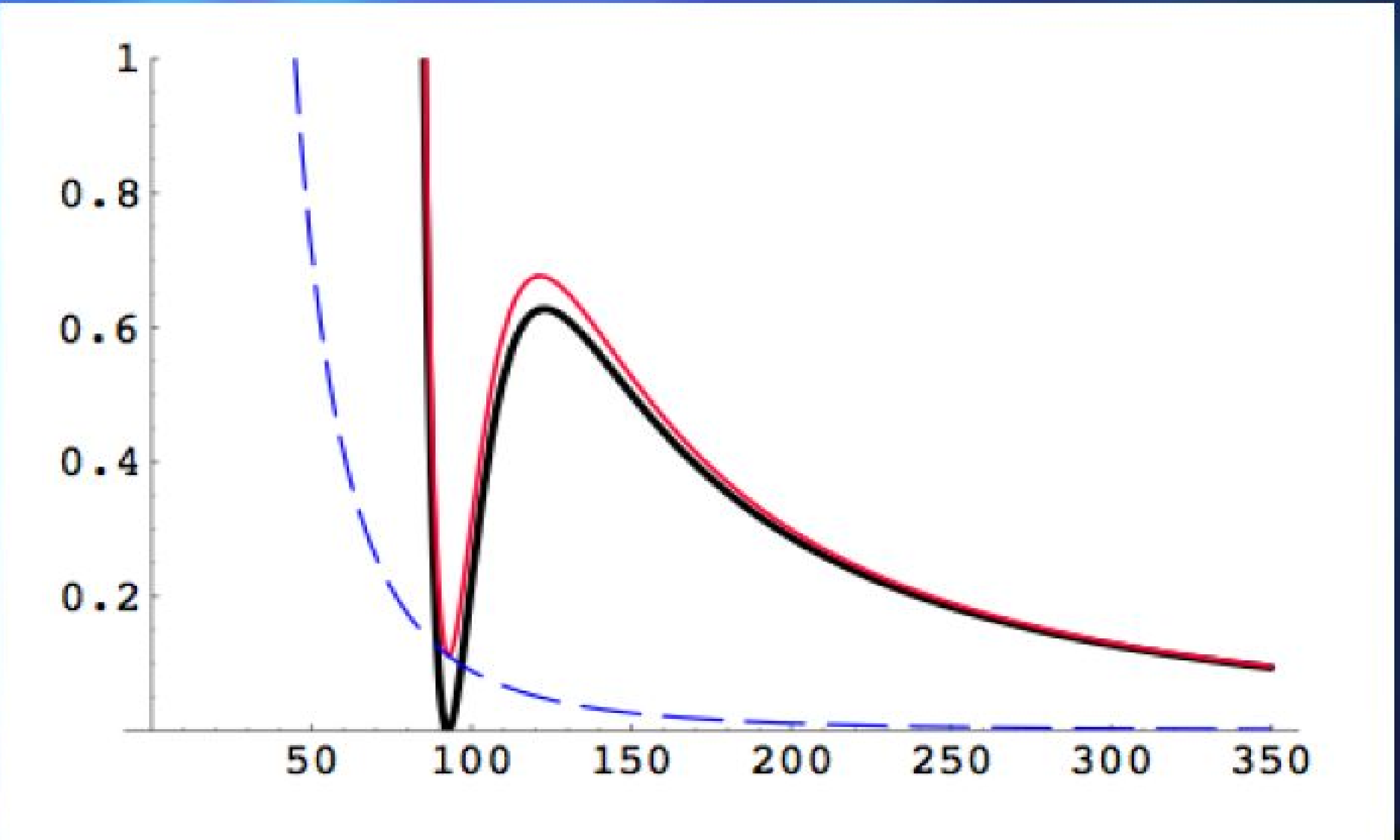


Second uplifting in D3/D7 model

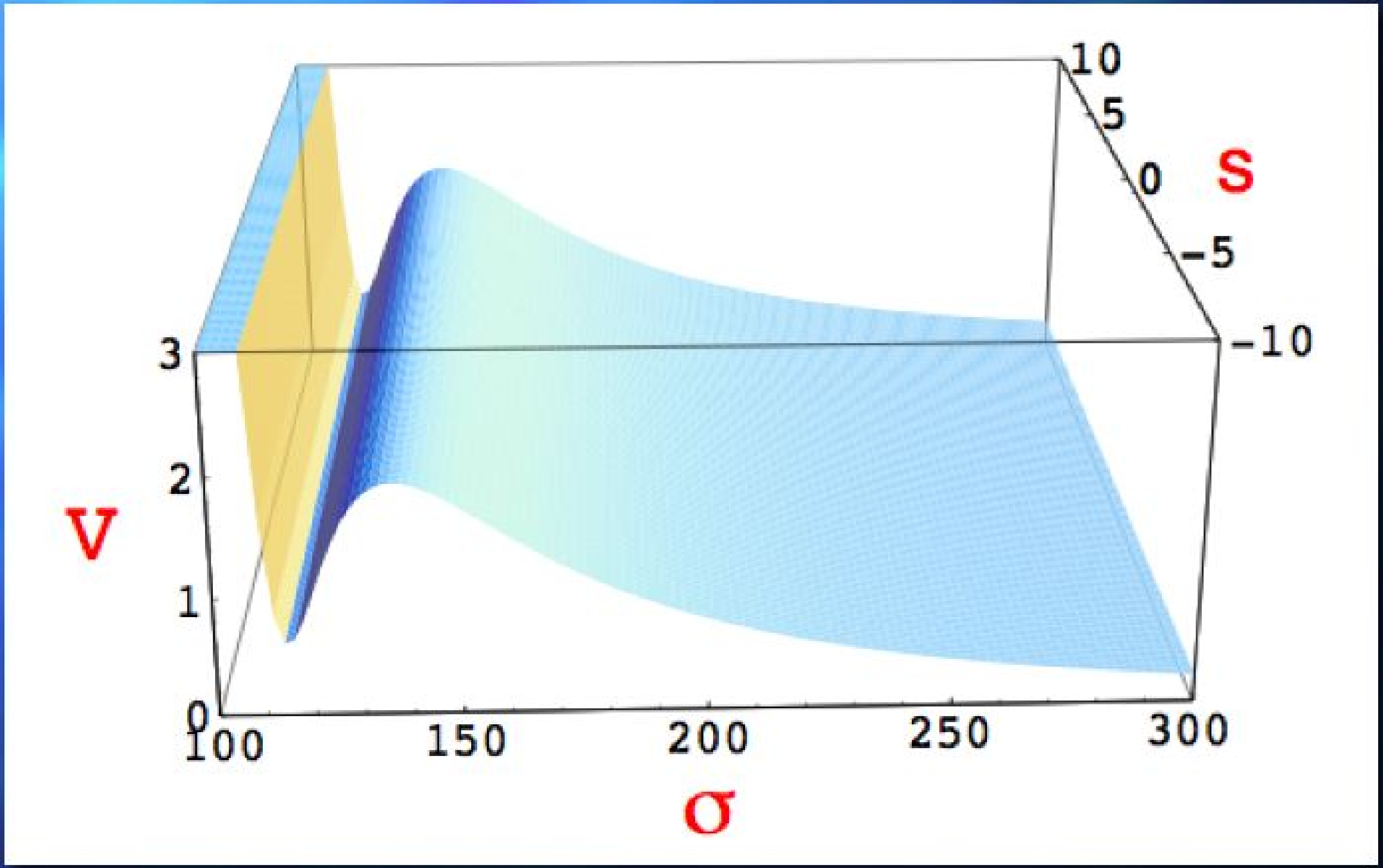
$$V(s,\sigma,\phi_\pm=0)=V_{AdS}+\frac{D}{\sigma^2}+\frac{C}{\sigma^3}$$

$\overline{\text{D3}}$ contribution

effective FI



Inflationary potential at $\phi_{\pm} = 0$ as a function of S and σ

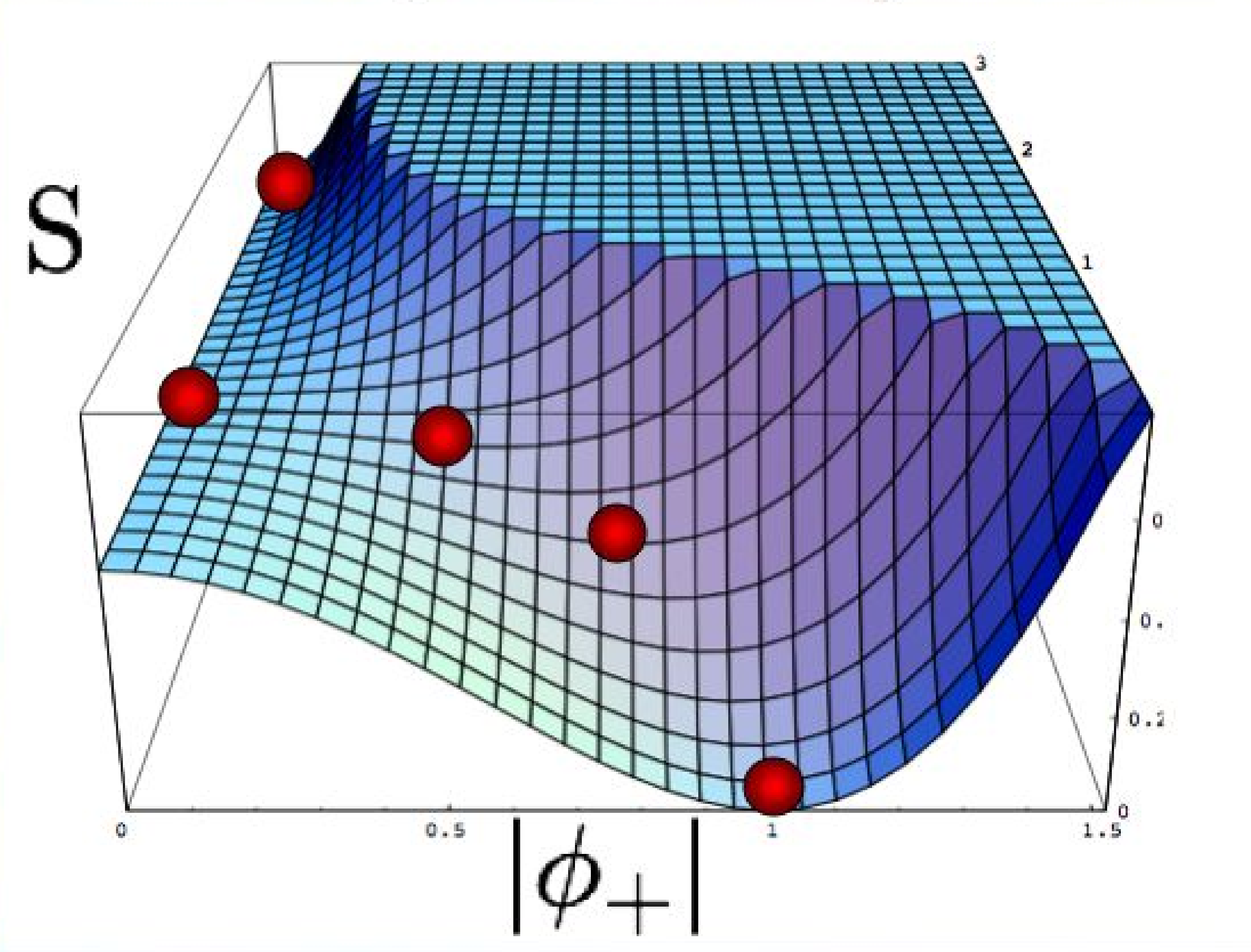


Shift symmetry is broken only by quantum effects

Potential of hybrid inflation with a stabilized volume modulus

$$V_2(\phi_+, \phi_-, s, \sigma_{\rm cr}) = \frac{g^2}{2}\,|\phi_+\phi_-|^2 + \frac{g^2}{2}s^2\left[|\phi_+|^2 + |\phi_-|^2\right] + \frac{g^2}{8}\left[|\phi_+|^2 - |\phi_-|^2 - 2\xi\right]^2$$

$$V_{1-\rm loop} = \frac{g^2\xi^2}{2}\left(1 + \frac{g^2}{8\pi^2}\ln\frac{|S^2|}{|S_c^2|}\right)$$



Unlike in the brane-antibrane scenario, inflation in D3/D7 model does not require fine-tuning

Racetrack Inflation

the first working model of the moduli inflation

Blanco-Pilado, Burgess, Cline, Escoda, Gomes-Reino, Kallosh, Linde, Quevedo
hep-th/0406230

Superpotential: $$W = W_0 + A\, e^{-aT} + B\, e^{-bT}$$

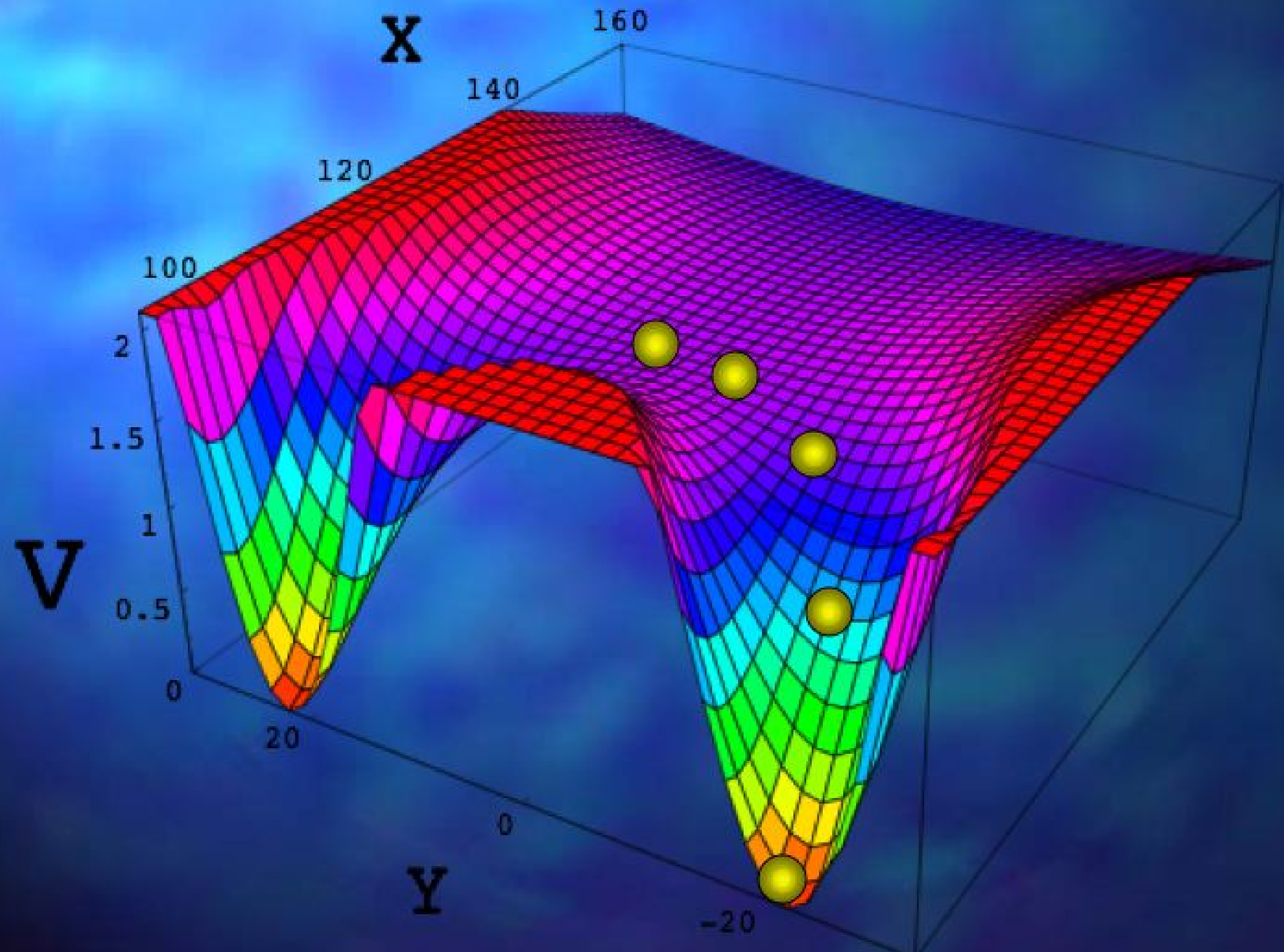
Kahler potential: $$K = -3\log(T + T^*)$$

Effective potential for the field T = X + i Y

$$\begin{aligned} V \; &= \frac{E}{X^\alpha} + \frac{e^{-aX}}{6X^2}\left[aA^2\,(aX+3)\;\, e^{-aX} \;+3W_0 aA\cos(aY)\right] + \\ &+ \frac{e^{-bX}}{6X^2}\left[bB^2\,(bX+3)\;\, e^{-bX} \;+3W_0 bB\cos(bY)\right] + \\ &+ \frac{e^{-(a+b)X}}{6X^2}\left[AB\,(2abX+3a+3b)\cos((a-b)Y)\right] \end{aligned}$$

Parameters and Potential

$$A = \frac{1}{50}, \qquad B = -\frac{35}{1000}, \qquad a = \frac{2\pi}{100}, \qquad b = \frac{2\pi}{90}, \qquad W_0 = -\frac{1}{25000}$$



Inflationary Predictions:

COBE-normalized spectrum of perturbations of metric

Flat spectrum of metric perturbations with $n_s = 0.95$

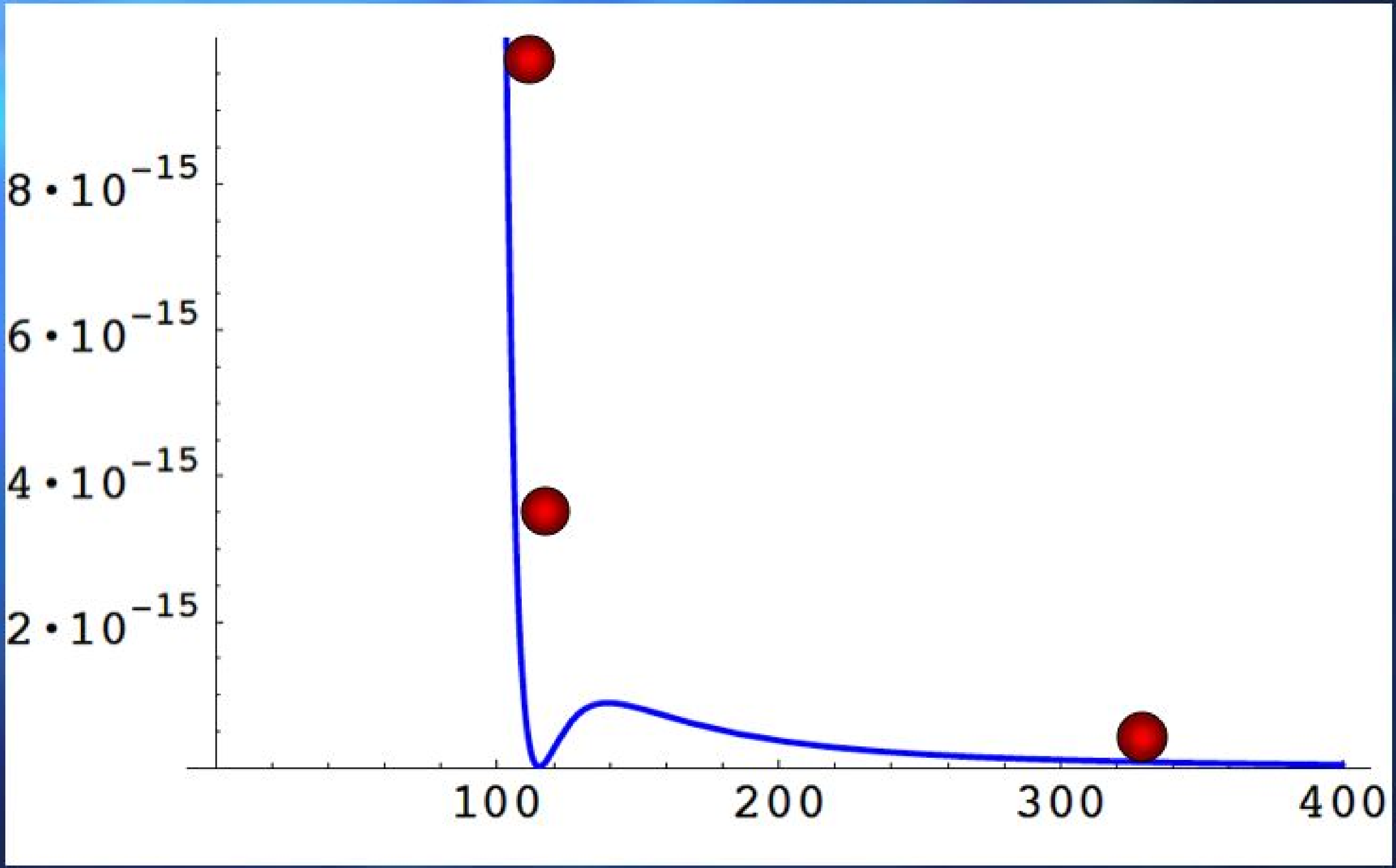
No tensor perturbations

Inflation in this scenario is eternal

(topological inflation)

Parameters require fine-tuning with accuracy O(0.1%), which may not be a problem if one takes into account string theory landscape

Initial conditions and the overshooting problem



Brustein, Steinhardt, 1993; may be resolved when radiation is taken into account

However, this addresses only one part of the problem of initial conditions for inflation. For example, one can show that a hot closed universe collapses within the time $t = S^{2/3}$, in Planck units. It can survive until the beginning of inflation at $t = H^{-1}=V^{-1/2}$ only if $S > V^{-3/4}$

For $V=10^{-16}$ (typical for string inflation) the initial entropy (the number of particles) must be $S > 10^{12}$. For $V=10^{-36}$ the initial entropy must be $S > 10^{27}$. In other words, in order to explain why the universe is so large now, one should assume that it was huge from the very beginning…

Quantum creation of an inflationary universe

- Hartle-Hawking wave function — Hartle,Hawking 1983

$$P = \Psi^2 \sim \exp\left(+\frac{24\pi^2}{V(\phi)}\right)$$

Describes the probability to live in a state with a given energy density after a long evolution

- Tunneling wave function — A.L., Vilenkin 1984

$$P = \Psi^2 \sim \exp\left(-\frac{24\pi^2}{V(\phi)}\right)$$

Describes the probability of initial conditions

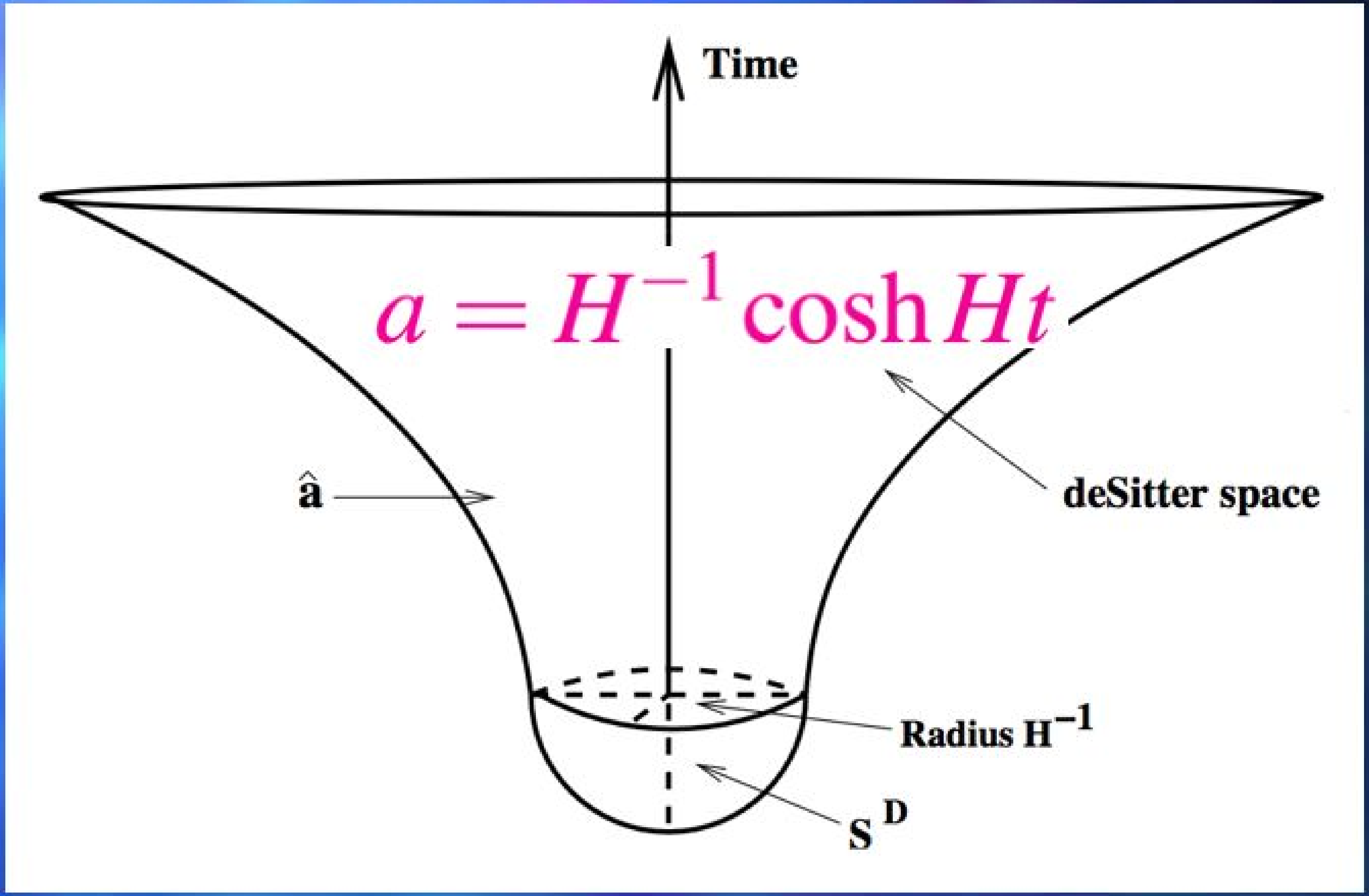
In my opinion, Hartle-Hawking wave function, as well as its various modifications (e.g. Firouzjahi, Sarangi, Tye), does not describe quantum creation of the universe:

By its construction, HH wave function was supposed to describe the ground state of the system, to be attained in a distant future, rather than the quantum creation of the system.

Meanwhile, the tunneling wave function leads to an exponential suppression of the probability of inflation starting well below the Planck density (which is the case for all known models of stringy inflation). For example, for the probability of the racetrack inflation one finds

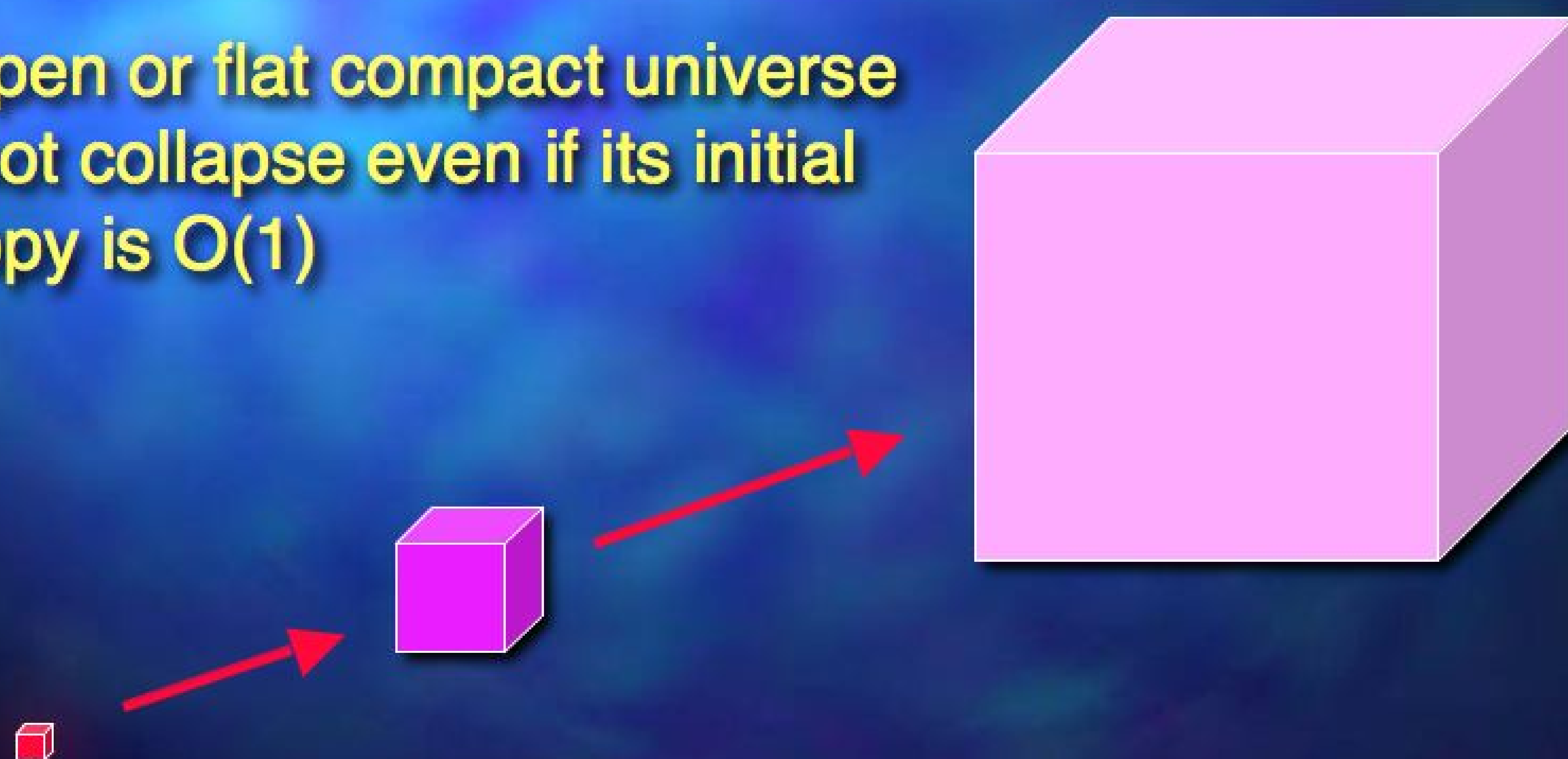
$$P \sim \exp\left(-10^{18}\right)$$

What is the root of the problem?



Closed dS space cannot continuously grow from the state with a = 0, it must tunnel. For the Planckian H, as in chaotic inflation, the action is O(1), tunneling is easy. For smaller H, the new-born universe must be larger. For very small H, as in stringy inflation, creation of the universe is exponentially suppressed. Creation of an infinite open universe is described by a different analytic continuation of the same instanton.

But why do we need a closed universe? In string theory it is natural to assume that **all dimensions are democratic**. Our 3D space may be topologically nontrivial. It can be compact even if it is open or flat (e.g. a torus). This changes everything in a dramatic way: **Open or flat universe can continuously expand from a = 0 without any tunneling!**



An open or flat compact universe will not collapse even if its initial entropy is O(1)

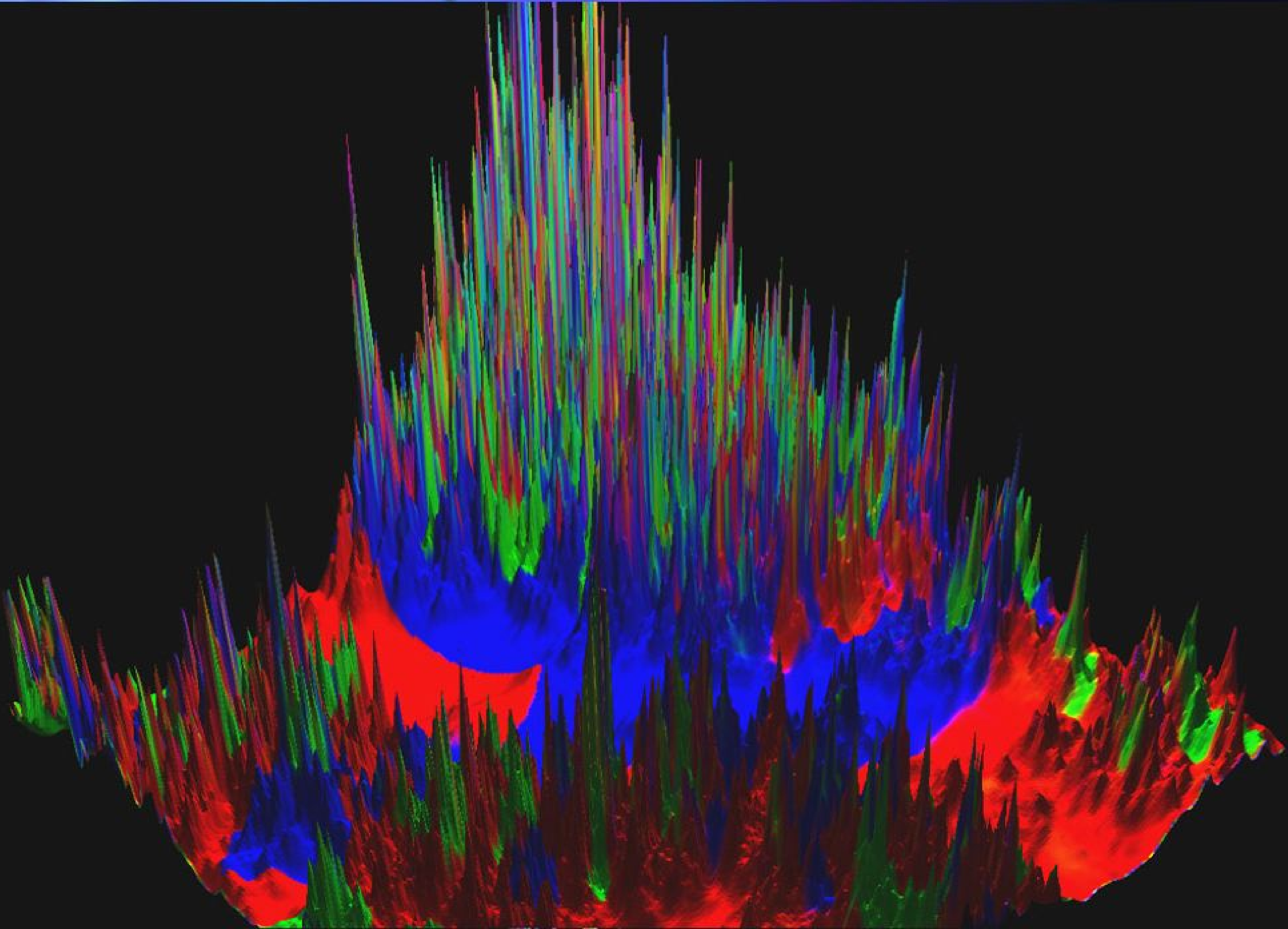
The only remaining problem is to keep the universe homogeneous until inflation begins. This can be done because of **chaotic mixing**: The size of the box in a hot universe grows as $t^{1/2}$, whereas the mean free path of a relativistic particle grows much faster, as t. As a result, within the cosmological time t the same particles appear in all parts of the universe with equal probability, which makes the universe homogeneous.

Cornish, Starkman, Spergel 1996

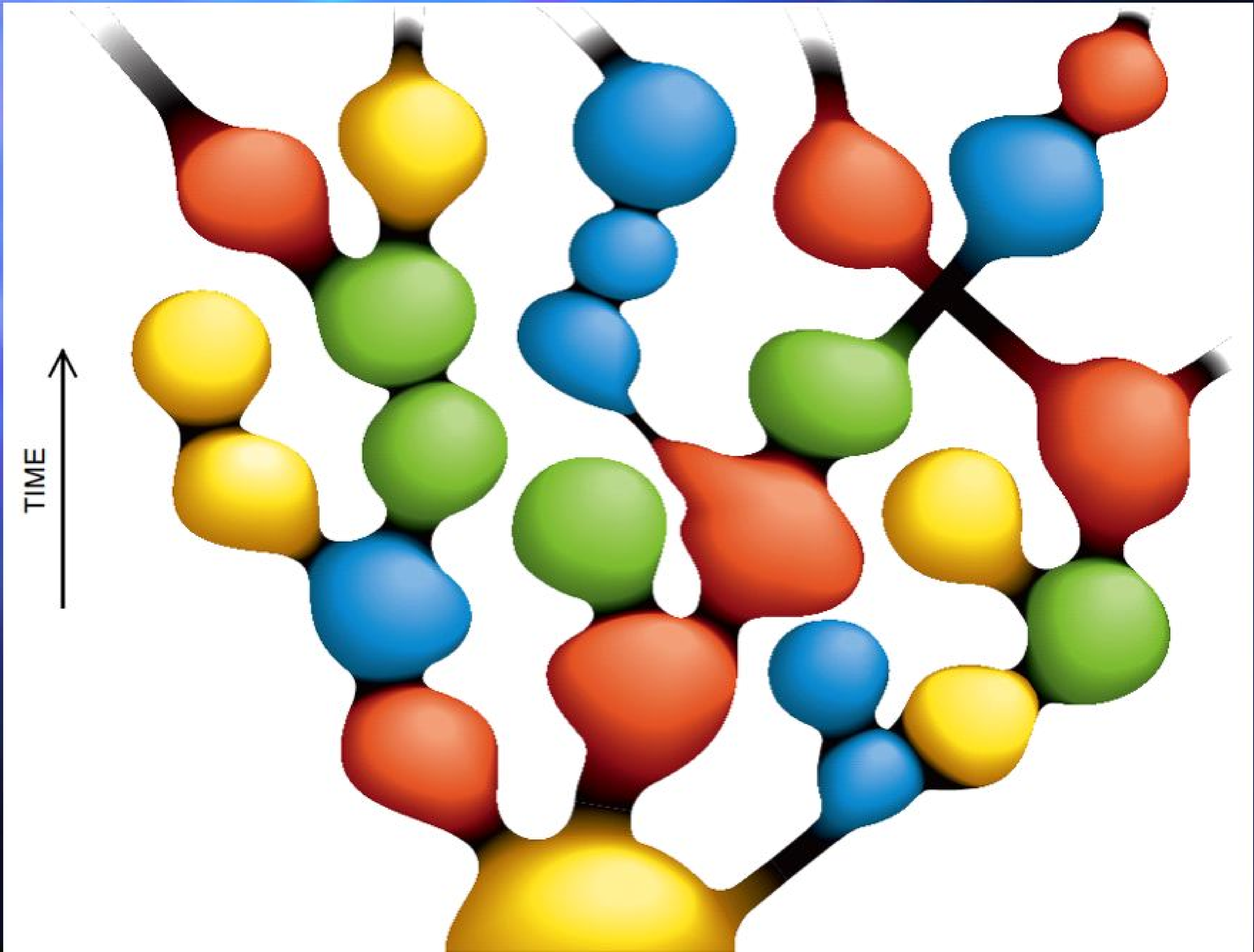
In this scenario there is no exponential suppression of the probability of creation of a topologically nontrivial inflationary universe with $V \ll 1$

A.L. 2003, and work in progress

Landscape of eternal inflation



Self-reproducing Inflationary Universe



String Theory Landscape

Perhaps 10^{100} - 10^{1000} different vacua

Lerche, Lust, Schellekens, 1987

Bousso, Polchinski; Susskind; Douglas, Denef,…

Each dS vacuum is metastable, but inflation in string theory landscape is eternal

It creates quantum fluctuations along all possible flat directions and provides necessary initial conditions for the low-scale inflation