

# The Origin of Mass in QCD

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August 5, 2004

# Mass in QCD

- What is mass? What is QCD?
- Classical and Quantum Symmetries
- No Quantum Scale Symmetry!
- Chiral Symmetries are Broken
- Symmetry Summary
- Applications and Connections

# What is Mass?

- Newton's Second Law:  $\vec{F} = m\vec{a}$
- Relativistic Dispersion Relation:  $E^2 = p^2 + m^2$
- Newton's principle of equivalence:  $m_{grav} = m$
- Einstein's principle of equivalence:  $G_{\mu\nu} \propto T_{\mu\nu}$

In what ways does QCD affect mass?

N.B. quarks are confined!

# Pictorial QCD



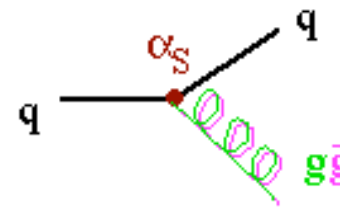
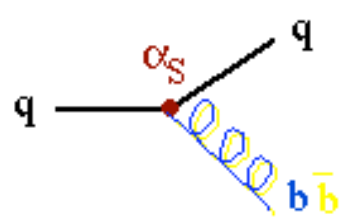
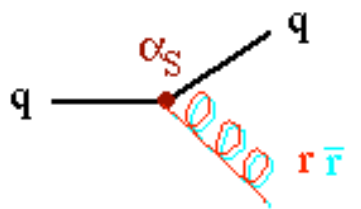
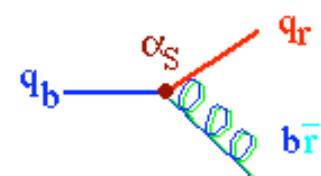
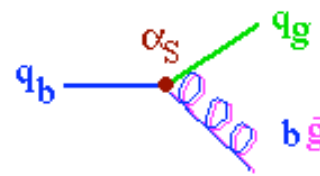
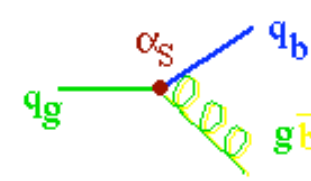
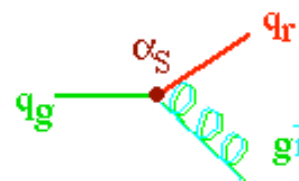
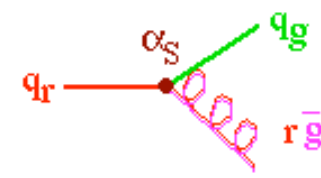
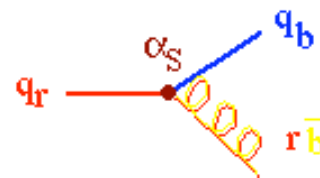
**Quarks** carry a color



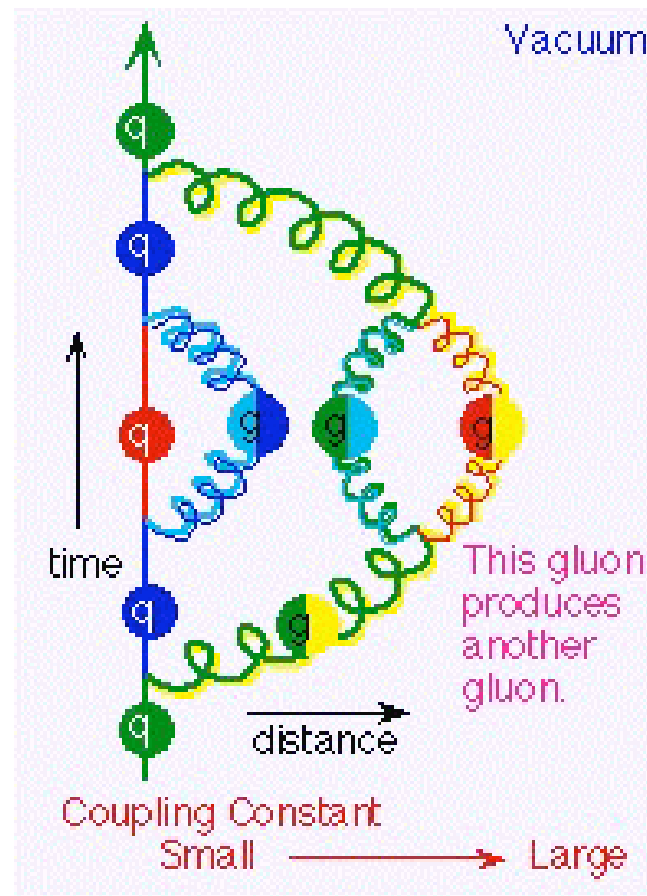
**Anti-quarks** carry an anti-color



**Gluons** carry a color and an anti-color



# Gluons Couple to Gluons!



# Introducing: The Quarks

| Property \ Quark                    | <i>d</i>       | <i>u</i>       | <i>s</i>       | <i>c</i>       | <i>b</i>       | <i>t</i>       |
|-------------------------------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Q – electric charge                 | $-\frac{1}{3}$ | $+\frac{2}{3}$ | $-\frac{1}{3}$ | $+\frac{2}{3}$ | $-\frac{1}{3}$ | $+\frac{2}{3}$ |
| I – isospin                         | $\frac{1}{2}$  | $\frac{1}{2}$  | 0              | 0              | 0              | 0              |
| $I_z$ – isospin <i>z</i> -component | $-\frac{1}{2}$ | $+\frac{1}{2}$ | 0              | 0              | 0              | 0              |
| S – strangeness                     | 0              | 0              | -1             | 0              | 0              | 0              |
| C – charm                           | 0              | 0              | 0              | +1             | 0              | 0              |
| B – bottomness                      | 0              | 0              | 0              | 0              | -1             | 0              |
| T – topness                         | 0              | 0              | 0              | 0              | 0              | +1             |

# QCD Lagrangian

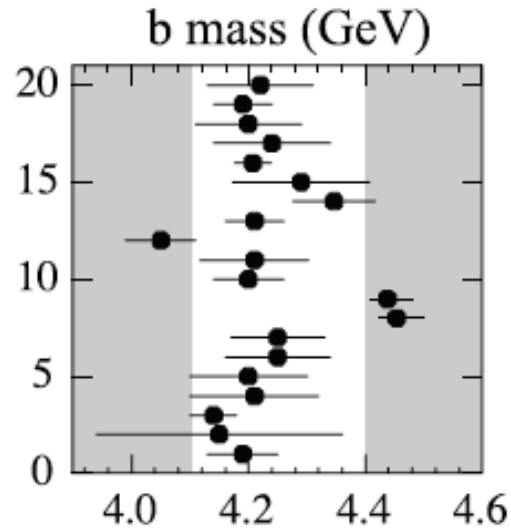
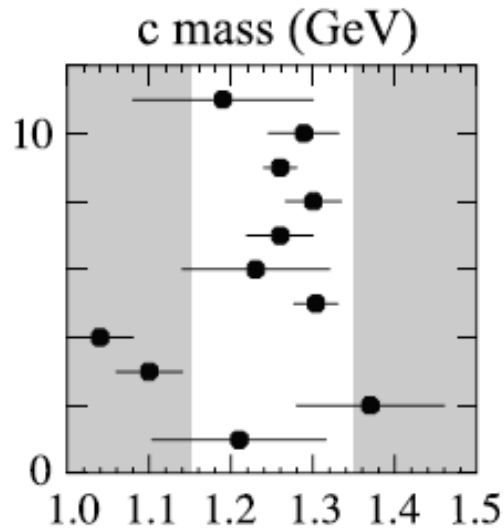
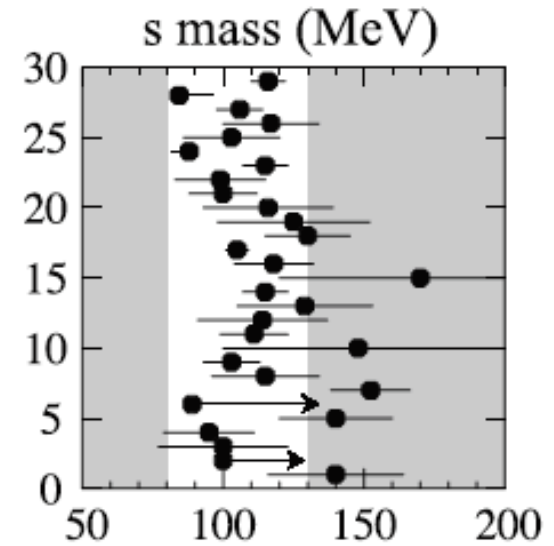
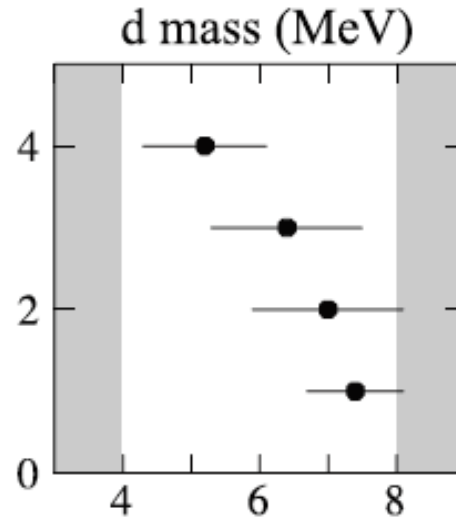
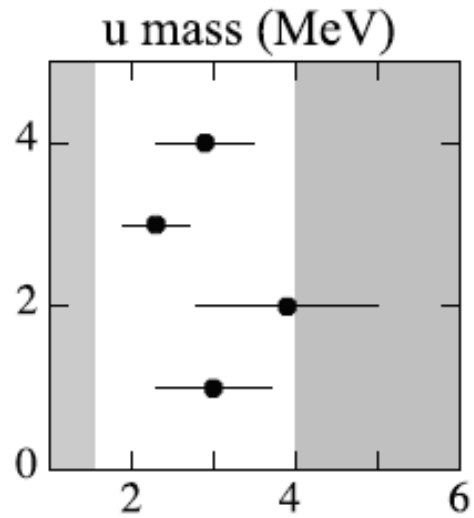
$$L_{\text{QCD}} = -\frac{1}{4} F_{\mu\nu}^{(a)} F^{(a)\mu\nu} + i \sum_q \bar{\psi}_q^i \gamma^\mu (D_\mu)_{ij} \psi_q^j - \sum_q m_q \bar{\psi}_q^i \psi_{qi} ,$$

$$F_{\mu\nu}^{(a)} = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g_s f_{abc} A_\mu^b A_\nu^c ,$$

$$(D_\mu)_{ij} = \delta_{ij} \partial_\mu + ig_s \sum_a \frac{\lambda_{i,j}^a}{2} A_\mu^a ,$$

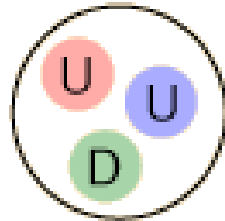
**NB:** Quark masses and QCD coupling

# Lagrangian Quark Masses

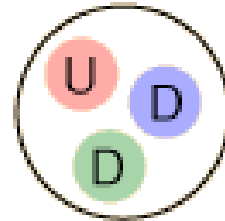


Explaining these masses is the “flavor problem”

# Quark Model Picture...



Proton  
938 MeV



Neutron  
940 MeV



$\pi^+$   
140 MeV



$\rho^+$   
770 MeV

Light quarks have “masses” of 300 MeV?

Why is the pion so light?

# Space-time Symmetries of Classical QCD

- Poincare Invariance
- C, P, and T (as written...)
- (Approximate) Scale Invariance\*

$$x^\mu \rightarrow \lambda \cdot x^\mu$$

$$\psi_q(x) \rightarrow \lambda^{3/2} \psi_q(\lambda x) \quad A_\mu^a(x) \rightarrow \lambda A_\mu^a(\lambda x)$$

**NB: Broken by all mass terms...**

\*( & Proper conformal trans.)

# Global Quark Flavor Symmetries of QCD (I)

- Baryon Number:  $\psi_q \rightarrow e^{i\alpha} \psi_q$
- (Approximate\*) Gell-Mann  $SU(3)_V$ :

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix} \rightarrow U \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

Together:  $U(3)_V$

\*Requires  $m_u \approx m_d \approx m_s$

# Global Quark Flavor Symmetries of QCD (2)

- (Approximate\*) Chiral  $U(3)_L \times U(3)_R$ :

$$\bar{\psi}_q i\not{D}\psi_q \equiv \bar{\psi}_q^L i\not{D}\psi_q^L + \bar{\psi}_q^R i\not{D}\psi_q^R$$

$$\begin{pmatrix} u^{L,R} \\ d^{L,R} \\ s^{L,R} \end{pmatrix} \rightarrow U_{L,R} \begin{pmatrix} u^{L,R} \\ d^{L,R} \\ s^{L,R} \end{pmatrix}$$

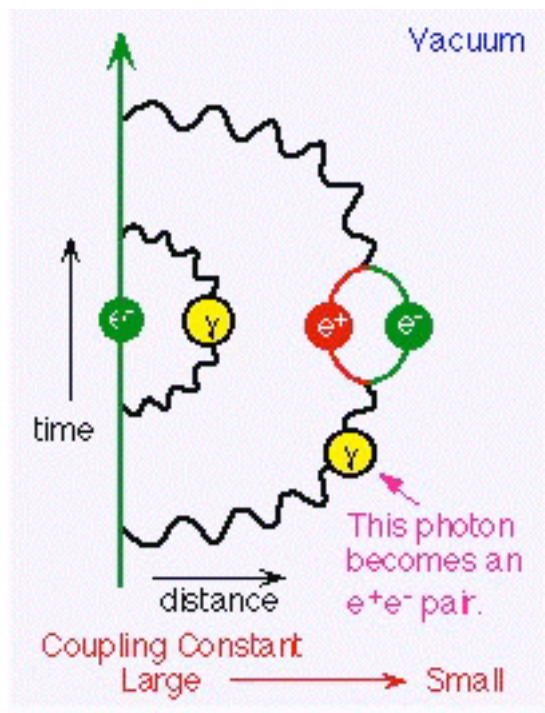
\*For “light” quarks: u, d, and s(?)

$$m_q \bar{\psi}_q \psi_q \equiv m_q \bar{\psi}_q^L \psi_q^R + m_q \bar{\psi}_q^R \psi_q^L$$

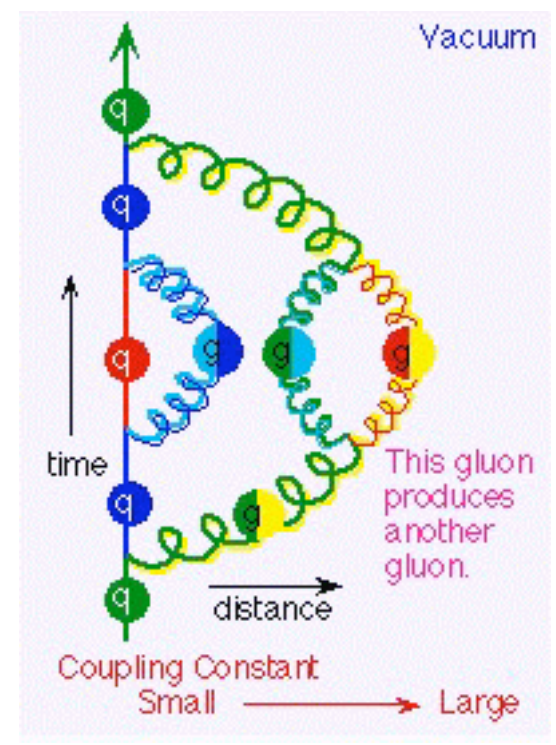
# No Quantum Scale Invariance!

- Quantum vacuum is a polarizable medium

QED →



← QCD



Fermions and Gluons make opposite contributions!

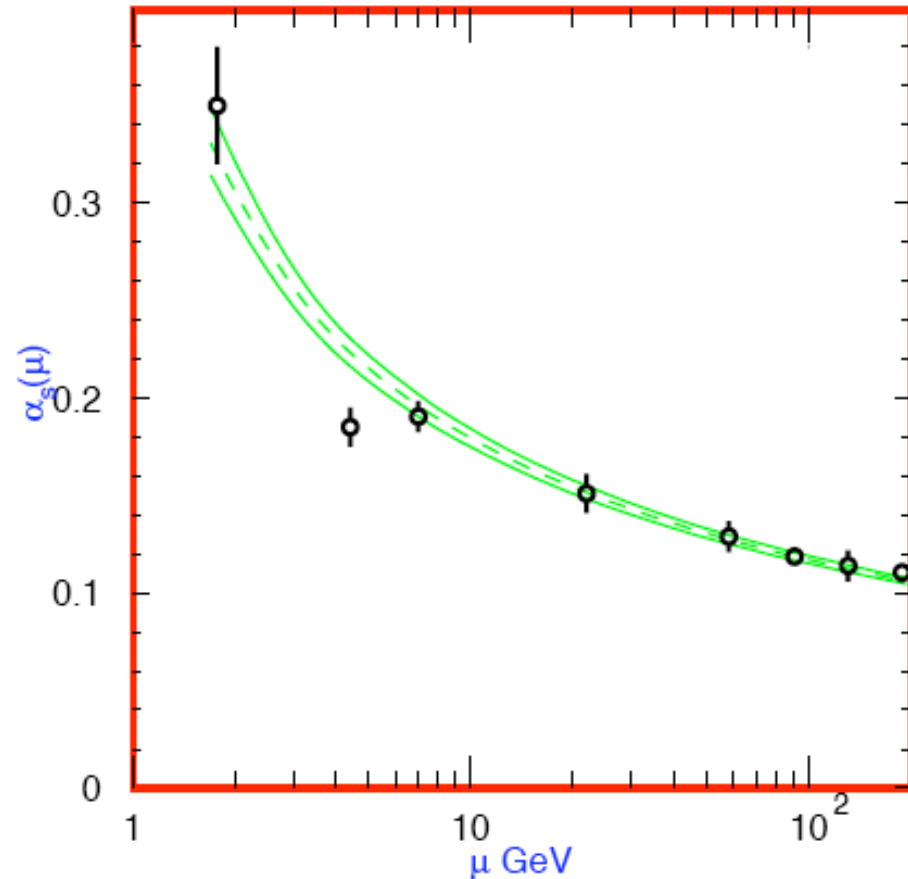
# QCD Beta Function

$$\mu \frac{\partial \alpha_s}{\partial \mu} = 2\beta(\alpha_s) = -\frac{\beta_0}{2\pi} \alpha_s^2 - \frac{\beta_1}{4\pi^2} \alpha_s^3$$

$$\beta_0 = 11 - \frac{2}{3}n_f,$$

$$\beta_1 = 51 - \frac{19}{3}n_f,$$

Asymptotic Freedom  
vs.  
Infrared Slavery



# Dimensional Transmutation

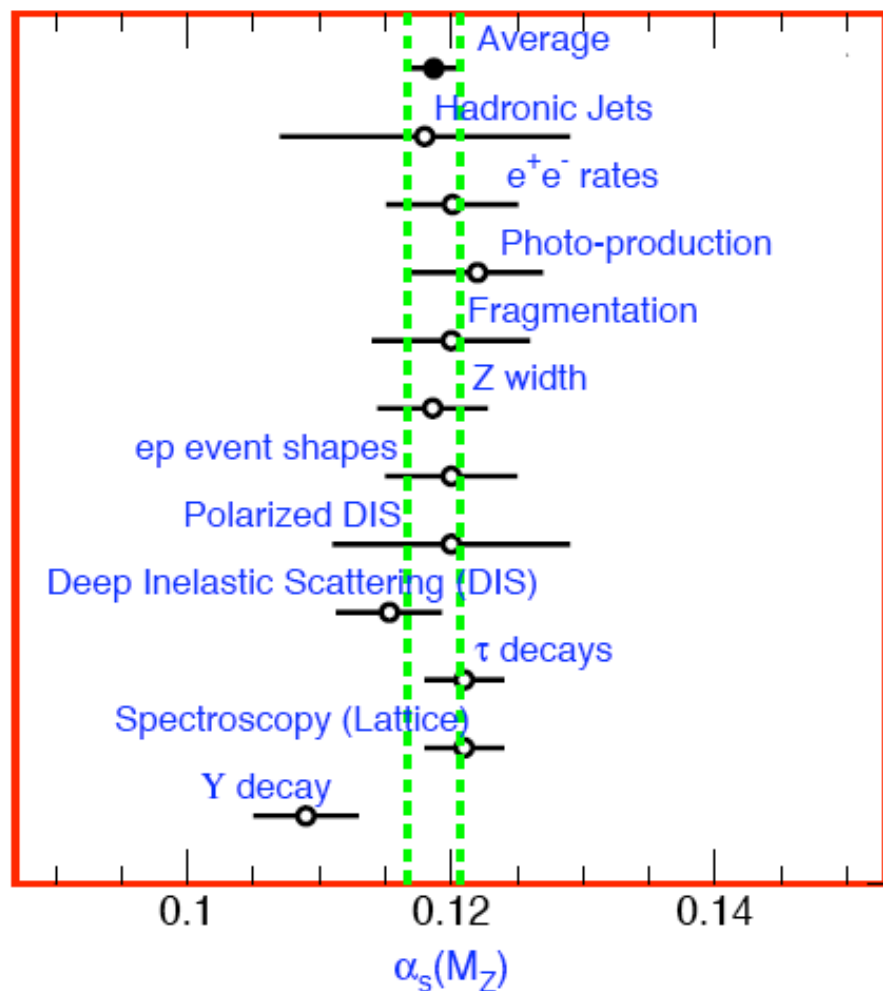
- Value of strong-coupling fixed at one scale, predicted at others.
- Alternatively, given by energy scale:  $\Lambda_{QCD}$

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda^2)} \left[ 1 - \frac{2\beta_1}{\beta_0^2} \frac{\ln[\ln(\mu^2/\Lambda^2)]}{\ln(\mu^2/\Lambda^2)} + \frac{4\beta_1^2}{\beta_0^4 \ln^2(\mu^2/\Lambda^2)} \right. \\ \left. \times \left( \left( \ln[\ln(\mu^2/\Lambda^2)] - \frac{1}{2} \right)^2 + \frac{\beta_2\beta_0}{8\beta_1^2} - \frac{5}{4} \right) \right].$$

RPP2004

Strong-interaction scale set by  $\Lambda_{QCD}$

# Strong Coupling



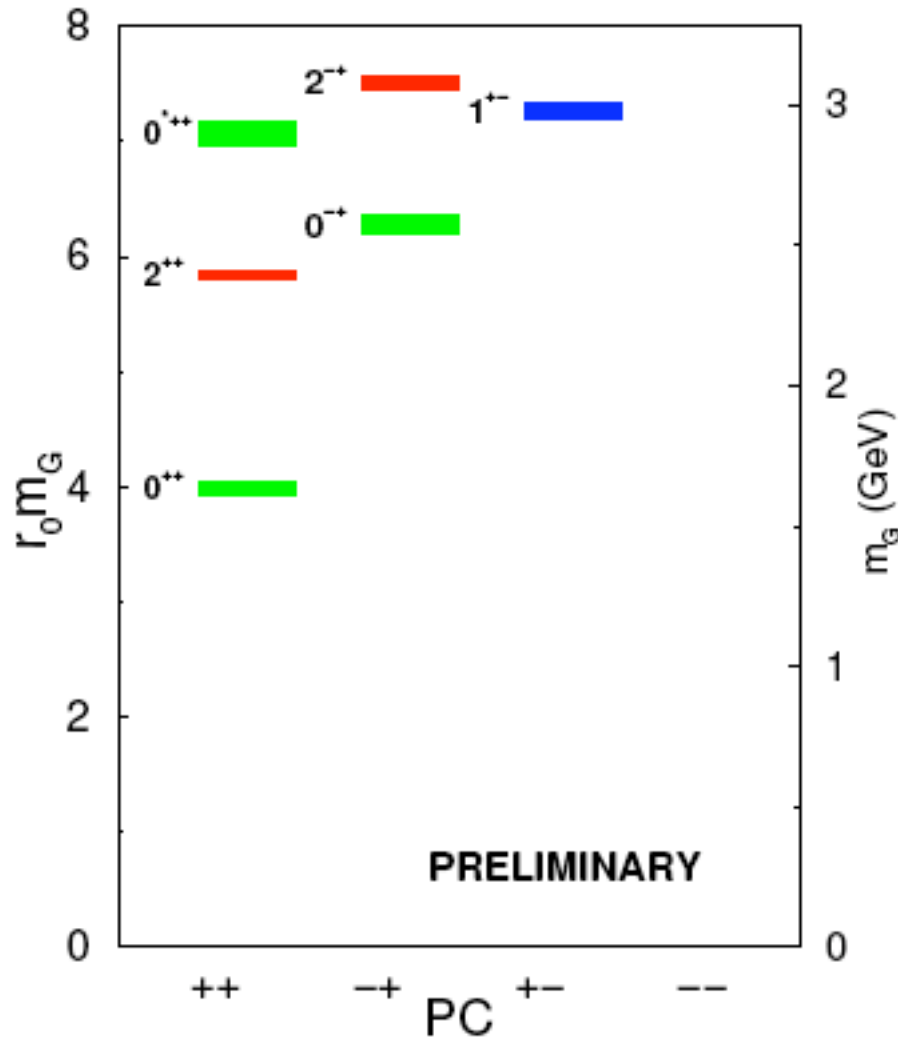
$$\Lambda^{(5)} = 217_{-23}^{+25} \text{ MeV}$$

$$\Lambda^{(3)} \simeq 350 \text{ MeV}$$

Defines “light” and  
“heavy”

Why are nuclear binding energies so much smaller?

# Mass-Gap in Pure YM



Glueball  
Spectrum

Morningstar and Peardon,  
nucl-th/0309068

# Scale Current

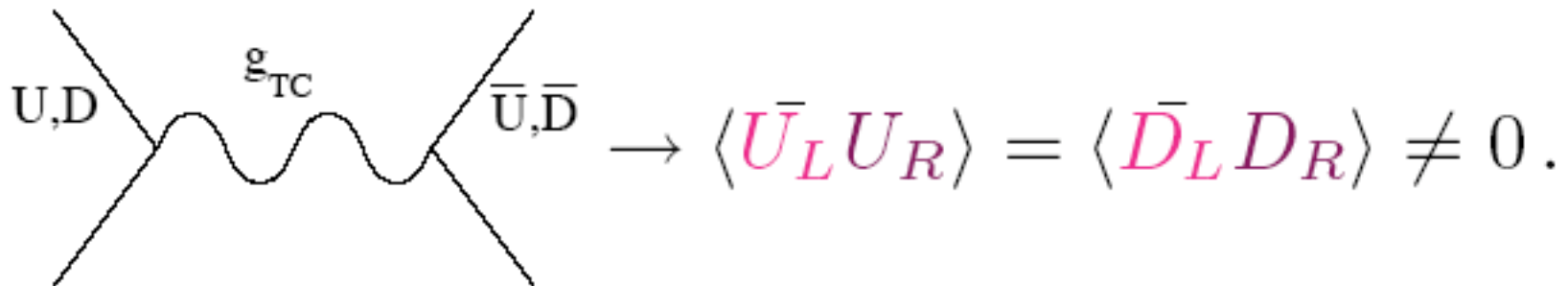
- Any continuous transformation defines an associated current
- A continuous symmetry has a conserved current

$$\partial_\mu s^\mu = T_\lambda^\lambda = -\frac{\beta}{2g} F_{\mu\nu}^{(a)} F^{(a)\mu\nu} + \sum_q m_q \bar{\psi}_q \psi_q + \dots$$

Quantum Effect!

# Chiral Symmetries are Spontaneously Broken

- Chiral symmetries are good quantum symmetries, but ...
- ... strong low-energy QCD dynamics rearranges the vacuum



$U, D \quad g_{TC} \quad \bar{U}, \bar{D} \rightarrow \langle \bar{U}_L U_R \rangle = \langle \bar{D}_L D_R \rangle \neq 0.$

# Goldstone Bosons

- $SU(3)_L \times SU(3)_R$  breaks to Gell-Mann  $SU(3)_V$
- Implies existence of eight (massless) Goldstone Bosons

$$j_{A\mu}^i = -f_\pi \partial_\mu \pi^i + \dots$$

$$\partial^\mu j_{A\mu}^i = -f_\pi \partial^\mu \partial_\mu \pi^i + \dots = 0 \Leftrightarrow m_\pi^2 = 0$$

# Pseudo Goldstone Bosons

- Relevant for light quarks only:  $m_q \leq \Lambda_{QCD}$
- u, d, and s are not massless,
- Therefore, pions are light\*:

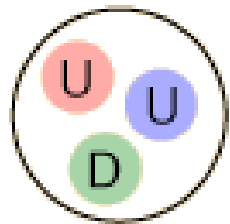
$$m_\pi^2 \propto (m_u + m_d)\Lambda_{QCD}$$

$$m_K^2 \propto (m_s + m_{u,d})\Lambda_{QCD}$$

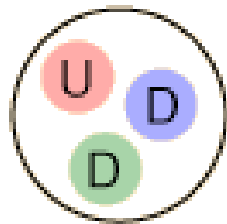
$$m_\eta^2 \propto \frac{1}{3}(m_u + m_d + 4m_s)\Lambda_{QCD}$$

\*NB: QCD Lagrangian Quark masses appear here!

# Constituent Quarks



Proton  
938 MeV



Neutron  
940 MeV



$\pi^+$   
140 MeV



$\rho^+$   
770 MeV

~300 MeV quark-model mass is of a “dressed” or “constituent” quark

~99% of proton and neutron mass due to QCD!

# What about $U(1)_A$ ?

- $U(1)_A$ :  $\psi_q^L \rightarrow e^{i\theta} \psi_q^L$  &  $\psi_q^R \rightarrow e^{-i\theta} \psi_q^R$
- Broken by chiral condensate
- Where is the ninth Goldstone Boson?
- Another Quantum “Anomaly”:

$$\partial_\mu j_A^\mu \propto \frac{g^2}{16\pi^2} \varepsilon^{\mu\nu\alpha\beta} F_{\mu\nu}^{(a)} F_{\alpha\beta}^{(a)}$$

# Strong-CP Problem?

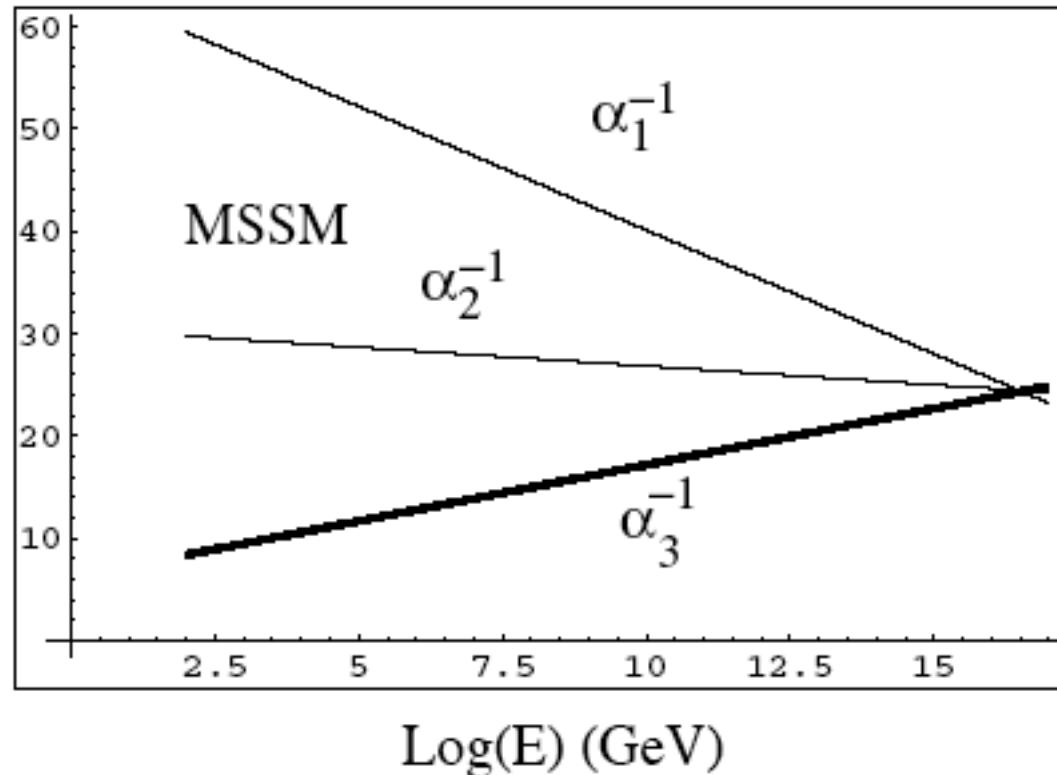
- But the same term could be in the QCD Lagrangian...
- ... and violates CP!
- Yields a neutron electric dipole moment
- Motivates the Axion...

# Symmetries in QCD

| Symmetry                      | Anomaly? | Fate                 |
|-------------------------------|----------|----------------------|
| Color (gauged)                | no       | unbroken             |
| <u>Scale Invariance</u> *     | yes      | not a symmetry       |
| $SU(3)_V^* \times U(1)_B$     | no       | unbroken             |
| <u><math>SU(3)_A</math></u> * | no       | spontaneously broken |
| <u><math>U(1)_A</math></u> *  | yes      | not a symmetry       |
| CP                            | no       | not a symmetry(!)    |

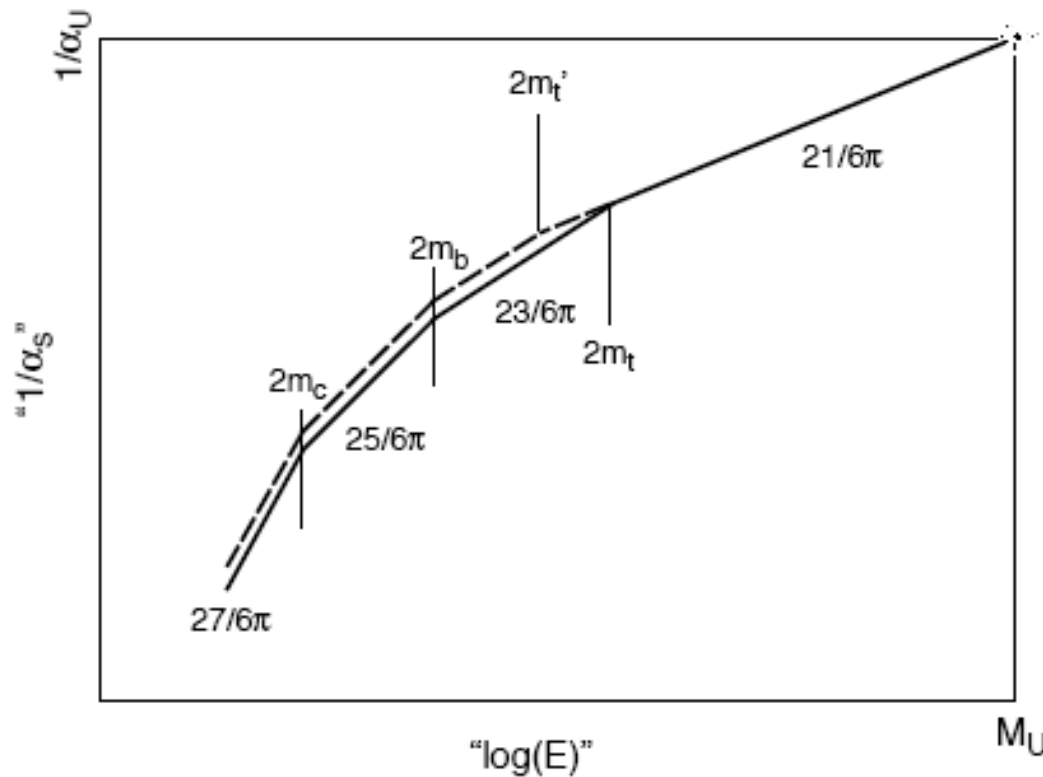
\*Approximate

# Application: GUTs



$\Lambda_{QCD}$  related to, but much smaller than, GUT scale!

# Top-Quark Matters

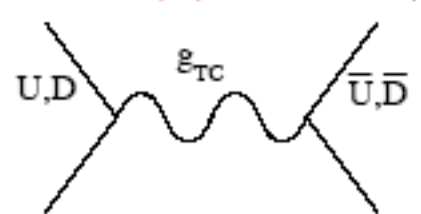


$$\frac{M_{\text{proton}}}{1 \text{ GeV}} \propto \left( \frac{m_t}{1 \text{ GeV}} \right)^{2/27}$$

Quigg, hep-ph/9707508

# Technicolor

- Use scaled-up QCD to break electroweak symmetry

$$SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$$


$$\rightarrow \langle \bar{U}_L U_R \rangle = \langle \bar{D}_L D_R \rangle \neq 0$$

No hierarchy  
problem!

$$\pi^\pm, \pi^0 \rightarrow W_L^\pm, Z_L$$

$$M_W = \frac{g F_{TC}}{2} \rightarrow F_{TC} \approx 250 \text{ GeV}$$

S?

# QCD is Important

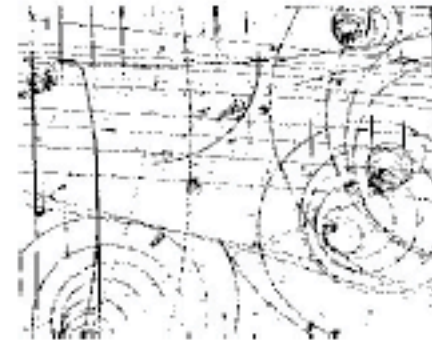
## Clay Mathematics Institute

Dedicated to increasing and disseminating mathematical knowledge

### Yang–Mills and Mass Gap

The laws of quantum physics stand to the world of elementary particles in the way that Newton's laws of classical mechanics stand to the macroscopic world. Almost half a century ago, Yang and Mills introduced a remarkable new framework to describe elementary particles using structures that also occur in geometry. Quantum Yang–Mills theory is now the foundation of most of elementary particle theory, and its predictions have been tested at many experimental laboratories, but its mathematical foundation is still unclear. The successful use of Yang–Mills theory to describe the strong interactions of elementary particles depends on a subtle quantum mechanical property called the "mass gap:" the quantum particles have positive masses, even though the classical waves travel at the speed of light. This property has been discovered by physicists from experiment and confirmed by computer simulations, but it still has not been understood from a theoretical point of view. Progress in establishing the existence of the Yang–Mills theory and a mass gap and will require the introduction of fundamental new ideas both in physics and in mathematics.

[The Millennium Problems](#)  
[Official Problem Description](#)  
[Lecture by Lorenzo Sadun at University of Texas \(video\)](#)



# QCD

- Strong QCD dynamics breaks quark chiral symmetries ...
- ... yielding  $\sim 99\%$  of the mass of ordinary matter .
- QCD is the very model of a modern quantum field theory.

