Pentaquark baryons from lattice QCD

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This is an exploratory study of the pentaquark baryons in lattice QCD. We perform quenched lattice calculations at $\beta = 6/g^2 = 6.2$ on a $32^3 \times 48$ lattice with the newly proposed operator for a pentaquark state $\Theta(uudd\bar{s})$, which is based on an exotic description like diquark-diquark-antiquark. It is found that our simulations can accommodate a negative parity pentaquark rather than a positive parity pentaquark with a mass close to the experimental value of the $\Theta^+(1540)$ state. We also explore the anti-charmed pentaquark $\Theta_c(uudd\bar{c})$ and find that the lowest Θ_c state is not to be expected as a bound state, in contrast to several model predictions.

I. INTRODUCTION

The quantum chromo-dynamics (QCD) may not preclude the presence of the multi-quark hadrons such as tetraquark ($qq\bar{q}\bar{q}$), pentaquark ($qqqq\bar{q}$), dibaryon ($qqqqqq$) and so on, because of the color confinement. Especially, we are let to be interested in *exotic multi-quark hadrons*, which should have *exotic* quantum numbers. Now, we address ourself to the pentaquark state. Consider the $SU(3)$ flavor case. Pentaquark states should form six different multiplets:

$$3_f \otimes 3_f \otimes 3_f \otimes 3_f \otimes \overline{3}_f = 1_3 \oplus 8_8 \oplus 10_4 \oplus \overline{10}_2 \oplus 27_3 \oplus 35_1 \quad (1)$$

where subscripts in the right hand side denote the number of degeneracy in each multiplet. The first three multiplets are common in the case of usual baryons. However, the last three multiplets, antidecuplet, 27-plet and 35-plet, are distinct irreducible representations since those multiplets have an apparent exotic quantum number as *strangeness +1*. Needless to say, $S = +1$ baryon can not be accommodated by usual baryons. Other possible exotic numbers are represented as stars in Fig.1. Of course, one cannot predict which multiplet is preferred for the possible $S = +1$ pentaquark baryon, within the group theoretical argument. The Skyrme model [1] and the chiral soliton model [2], however, predict that the lowest $S = +1$ state appears uniquely in the antidecuplet and its spin and parity is spin-half and positive parity.

Recently, LEPS collaboration at Spring-8 has observed a very sharp peak resonance in the K^- missing-mass spectrum of the $\gamma n \to nK^+K^-$ reaction on ^{12}C [3]. The observed resonance should have strangeness +1. Thus, $\Theta^+(1540)$ cannot be a three quark state and should be an exotic baryon state with the minimal quark content $uudd\bar{s}$. The peak position is located at 1540 MeV with a very narrow width. Those are quite consistent with the chiral-soliton model's prediction [2]. This discovery is subsequently confirmed by other experiments [4–7] [26]. Experimentally, spin, parity and isospin are not determined yet. Non-existence of a narrow resonance in pK^+ channel indicates that possibility of $I = 1$ has been already ruled out [3, 6]

Many theoretical studies of pentaquarks are also triggered by the discovery of the $\Theta^+(1540)$. We introduce the most reputed model proposed by Jaffe and Wilczek [8]. In the naive quark models, the low-lying pentaquark state should have spin-1/2 and negative parity. However, in this case, the pentaquark baryon just falls apart into KN in a S-wave. It is difficult to explain its very narrow width. Jaffe and Wilczek propose a simple idea to flip the parity of the low-lying pentaquark. Suppose there is the strong diquark correlation. The spin-0, color triplet and flavor triplet diquark would be favored within the simple one gluon exchange. The pentaquark can be composed of two identical bosons (diquarks) and one antiquark. However, the anti-symmetrization in terms of color, requires relative odd number's angular-momentum between the pairs of identical bosons. Otherwise, the wave function of the pentaquark state should be vanished. Resulting parity of the low-lying pentaquark is same as the chiral soliton model [8]. In addition, $S = +1$ baryon are uniquely assigned to antidecuplet in this description.

Consequently, correlated quark models, *i.e.* the diquark model, may accommodate the positive-parity isosinglet pentaquark $\Theta(uudd\bar{s})$ as same as the chiral soliton model. However, there are essential differences between the diquark model and the chiral soliton model. In correlated quark models, we cannot fully discriminate the other multiplet such as octet. If apart from the $SU(3)$ flavor limit, the mixing between octet and antidecuplet should be taken into account. Jaffe and Wilczek advocate the ideal mixing case, which is favored in the vector meson spectrum [8]. The ideal mixing provides the different prediction for the mass of possible exotic Ξ states. Furthermore, the Roper resonance can be accommodated in the diquark model owing to the ideal mixing [8].

A candidate of Ξ-type pentaquark has been reported by NA49 collaboration [9]. They observed the sharp peak resonance, which has exotic quantum numbers, in $\Xi\pi$ missing mass analysis. The observed mass is located somewhat between predictions by the chiral soliton model and the diquark model. In this experiment, the isospin partner is also observed. It should be pointed out that these discoveries are not confirmed yet by other

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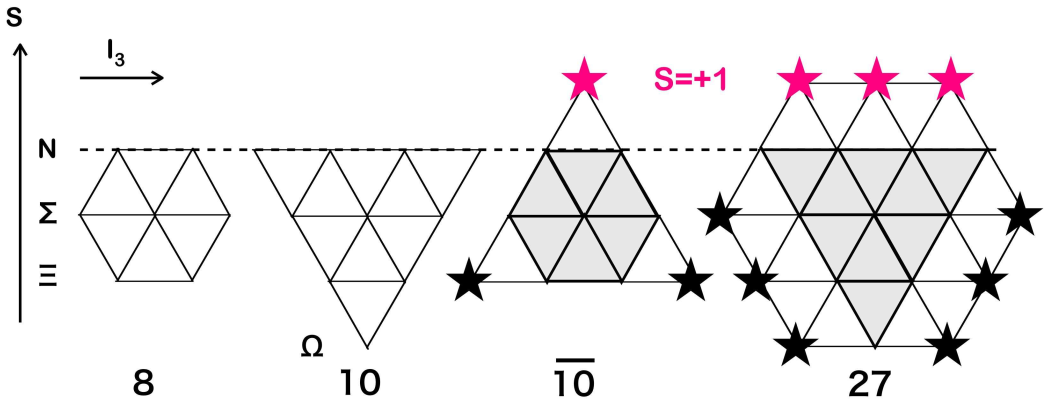

FIG. 1: Weight diagrams for possible pentaquark multiplets. Stars represent the states which have exotic quantum numbers (electric charge and strangeness).

experiments [7].

II. LATTICE SIMULATION

If the pentaquark baryons really exist, such states must emerge directly from first principles, QCD. Of course, what we should do is to confirm the presence of the pentaquarks from lattice QCD. Experimentally, it is rather difficult to determine the parity of the $\Theta^+(1540)$. Thus, lattice QCD has a chance to answer the undetermined quantum numbers before experimental efforts. Lattice QCD has also a feasibility to predict the masses for undiscovered pentaquark baryons. We stress that there is substantial progress in lattice study of excited baryons recently [10]. Especially, the negative parity nucleon $N^*(1535)$, which lies close to the $\Theta^+(1540)$, has become an established state in quenched lattice QCD [10, 11]. Here we report that quenched lattice QCD is capable of studying the $\Theta^+(1540)$ as well.

Indeed, it is not so easy to deal with the $qqqq\bar{q}$ state rather than usual baryons (qqq) and mesons ($q\bar{q}$) in lattice QCD. The $qqqq\bar{q}$ state can be decomposed into a pair of color singlet states as qqq and $q\bar{q}$, in other words, can decay into two-hadron states even in the quenched approximation. For instance, one can start a study with a simple minded local operator for the $\Theta^+(1540)$, which is constructed from the product of a neutron operator and a K^+ operator such as $\Theta = \varepsilon_{abc}(d_a^T C\gamma_5 u_b)d_c(\bar{s}_e\gamma_5 u_e)$. The two-point correlation function composed of this operator, in general, couples not only to the Θ state (single hadron) but also to the two-hadron states such as an interacting KN system [12, 13]. Even worse, when the mass of the $qqqq\bar{q}$ state is higher than the threshold of the hadronic two-body system, the two-point function should be dominated by the two-hadron states. Thus, a specific operator with as little overlap with the hadronic two-body states as possible is desired in order to identify the signal of the pentaquark state in lattice QCD.

A. Interpolating operators

For above mentioned purpose, we propose some local interpolating operators of antidecuplet baryons based on an exotic description as diquark-diquark-antiquark. There are basically two choices as $\bar{3}_c \otimes \bar{3}_c$ or $\bar{3}_c \otimes 6_c$ to construct a color triplet diquark-diquark cluster [15, 16]. We adopt the former for a rather simple construction of diquark-diquark-antiquark. We first introduce the flavor antitriplet ($\bar{3}_f$) and color antitriplet ($\bar{3}_c$) diquark field

$$\Phi_\Gamma^{i,a}(x) = \frac{1}{2}\varepsilon_{ijk}\varepsilon_{abc}q_{j,b}^T(x)C\Gamma q_{k,c}(x) \qquad (2)$$

where C is the charge conjugation matrix, abc the color indices, and ijk the flavor indices. Γ is any of the sixteen Dirac γ-matrices. Accounting for both color and flavor antisymmetries, possible Γs are restricted within 1, γ_5 and $\gamma_5\gamma_\mu$ which satisfy the relation $(C\Gamma)^T = -C\Gamma$. Otherwise, above defined diquark operator is identically zero. Hence, three types of flavor $\bar{3}_f$ and color $\bar{3}_c$ diquark; scalar (γ_5), pseudoscalar (1) and vector ($\gamma_5\gamma_\mu$) diquarks are allowed [14, 17] The color singlet state can be constructed by the color antisymmetric parts of diquark-diquark $(\bar{3}_c \otimes \bar{3}_c)_{\rm antisym} = 3_c$ with an antiquark ($\bar{3}_c$). In terms of flavor, $\bar{3}_f \otimes \bar{3}_f \otimes \bar{3}_f = 1_f \oplus 8_f \oplus 8_f \oplus \overline{10}_f$. Manifestly, in this description, the S=+1 state belongs to the flavor antidecuplet [18]. Automatically, the S=+1 state should have isospin zero. Then, the interpolating operator of the $\Theta(uudd\bar{s})$ is obtained as

$$\Theta(x) = \varepsilon_{abc}\Phi_\Gamma^{s,a}(x)\Phi_{\Gamma'}^{s,b}(x)C\bar{s}_c^T(x) \qquad (3)$$

for $\Gamma \neq \Gamma'$. The form $C\bar{s}^T$ for the strange antiquark field is responsible for giving the proper transformation properties of the resulting pentaquark operator under parity and Lorentz transformations [19]. Remark that because of the color antisymmetry, the combination of the same types of diquark is not allowed. Consequently, we have three different types of exotic S=+1 baryon operators through combination of two different types of diquarks, which have different spin-parity [14, 17]:

$$\Theta^1_+ = \varepsilon_{abc}\varepsilon_{aef}\varepsilon_{bgh}(u^T_e C d_f)(u^T_g C\gamma_5 d_h)C\bar{s}^T_c, \quad (4)$$
$$\Theta^2_{+,\mu} = \varepsilon_{abc}\varepsilon_{aef}\varepsilon_{bgh}(u^T_e C\gamma_5 d_f)(u^T_g C\gamma_5\gamma_\mu d_h)C\bar{s}^T_c, \quad (5)$$
$$\Theta^3_{-,\mu} = \varepsilon_{abc}\varepsilon_{aef}\varepsilon_{bgh}(u^T_e C d_f)(u^T_g C\gamma_5\gamma_\mu d_h)C\bar{s}^T_c \quad (6)$$

where the subscript "$+(-)$" refers to positive (negative) parity since these operators transform as $\mathcal{P}\Theta_\pm(\vec{x},t)\mathcal{P}^\dagger = \pm\gamma_4\Theta_\pm(-\vec{x},t)$ (for $\mu = 1,2,3$) under parity. The first operator of Eq. (4) is proposed for QCD sum rules in a recent paper [19] independently.

In this description, the operator of exotic $\Xi_{3/2}$ ($ssdd\bar{u}$ or $uuss\bar{d}$) states, which are members of the antidecuplet, can be treated by interchanging u and s or d and s in above operators. If a strange antiquark is simply replaced by a charm antiquark, the proposed pentaquark operators can be regarded as the anti-charmed analog of the isosinglet pentaquark state, $\Theta_c(uudd\bar{c})$.

Recall that any of local type baryon operators can couple to both positive- and negative-parity states since the parity assignment of an operator is switched by multiplying the left hand side of the operator by γ_5. The desired parity state is obtained by choosing the appropriate projection operator, $1 \pm \gamma_4$, on the two-point function $G(t)$ and direction of propagation in time. Details of the parity projection are described in Ref. [11]. We emphasize that the second and third operators, Eqs. (5) and (6), can couple to both spin-1/2 and spin-3/2 states. By using them, it is possible to study the spin-orbit partner of the spin-1/2 Θ state, whose presence contradicts the Skyrme picture of the Θ [16]. However, we will not pursue this direction in this article. We utilize only the first operator of Eq. (4), which couples only to a spin-1/2 state. Under the assumption of the highly correlated diquarks, we simply omit a quark-exchange diagram between diquark pairs contributing to the full two-point function in the following numerical simulations [14].

III. NUMERICAL RESULTS

We generate quenched QCD configurations on a lattice $L^3 \times T = 32^3 \times 48$ with the standard single-plaquette Wilson action at $\beta = 6/g^2 = 6.2$ ($a^{-1} = 2.9$ GeV). The spatial lattice size corresponds to $La \approx 2.2$fm, which may be marginal for treating the ground state of baryons without large finite volume effect. Our results are analyzed on 240 configurations. The light-quark propagators are computed using the Wilson fermions at four values of the hopping parameter $\kappa = \{0.1520, 0.1506, 0.1497, 0.1489\}$, which cover the range $M_\pi/M_\rho = 0.68-0.90$. $\kappa_s = 0.1515$ and $\kappa_c = 0.1360$ are reserved for the strange and charm masses, which are determined by approximately reproducing masses of $\phi(1020)$ and $J/\Psi(3097)$. We calculate a simple point-point quark propagator with a source location at $t_{\rm src} = 6$. To perform precise parity projection, we construct forward propagating quarks by taking the appropriate linear combination of propagators with periodic and anti-periodic boundary conditions in the time direction. This procedure yields a forward in time propagation in the time slice range $0 < t < T - t_{\rm src}$.

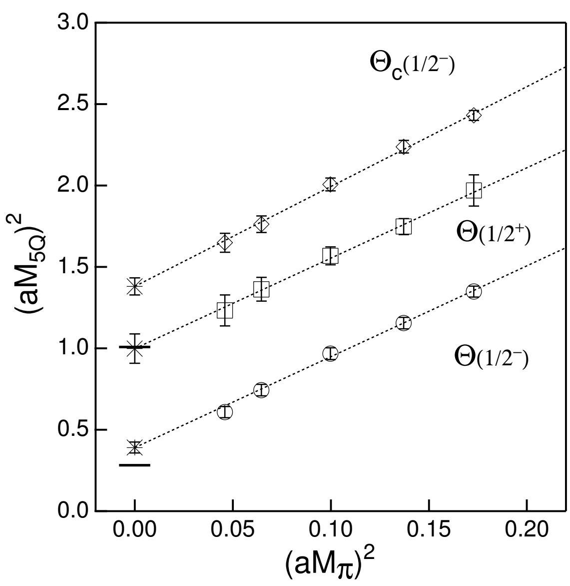

FIG. 2: Masses of the spin-1/2 $\Theta(uudd\bar{s})$ states with both positive parity (open squares) and negative parity (open circles) as functions of pion mass squared in lattice units. The charm analog $\Theta_c(uudd\bar{c})$ state (open diamonds) is also plotted. Horizontal short bar represents the $KN(DN)$ threshold estimated by $M_N + M_K(M_N + M_D)$ in the chiral limit.

In this calculation, the strange (charm) quark mass is fixed at κ_s (κ_c) and the up and down quark masses are varied from $M_\pi \approx 1.0$ GeV ($\kappa = 0.1489$) to $M_\pi \approx 0.6$ GeV ($\kappa = 0.1520$). Then, we will perform the extrapolation to the chiral limit using five different κ values. Details of the analysis are described in Ref. [14].

In Fig. 2 we show the mass spectrum of the $\Theta(uudd\bar{s})$ states with the positive parity (open squares) and the negative parity (open circles) as functions of the pion mass squared. Mass estimates are obtained from covariant single exponential fits [27] in the appropriate fitting range. All fits have a confidence level larger than 0.3 and $\chi^2/N_{DF} < 1.2$. It is evident that the lowest state of the isosinglet S=+1 baryons has *the negative parity.* We evaluate the mass of the $\Theta(uudd\bar{s})$ with both parities in the chiral limit. A simple linear fit for all five values in Fig. 2 yields $M_{\Theta(1/2^-)}$=0.62 (3) and $M_{\Theta(1/2^+)}$=1.00 (5) in lattice units. If we use the scale set by r_0 from Ref. [20], we obtain $M_{\Theta(1/2^-)} = 1.84(8)$ GeV and $M_{\Theta(1/2^+)} = 2.94(13)$ GeV. It is worth quoting other related hadron masses. The chiral extrapolated values for the kaon, the nucleon and the N^* state are $M_K = 0.53(1)$ GeV, $M_N = 1.06(2)$ GeV and $M_{N^*} = 1.76(5)$ GeV in this calculation.

Our obtained $\Theta(1/2^-)$ mass is slightly overestimated in comparison to the experimental value of the $\Theta^+(1540)$, but comparable to our observed N^* mass, which is also overestimated. Needless to say, the evaluated values should not be taken too seriously since they do not include any systematic errors. Such a precise quantitative prediction of hadron masses is not the purpose of the present paper. Rather, we emphasize that, our results strongly indicate the J^P assignment of the $\Theta^+(1540)$ is most likely $(1/2)^-$. This conclusion is consistent with that of a recent lattice study [21] (if one corrects the parity assignment of their operator [22]) and that of QCD sum rules approach [19].

As a strange antiquark is simply replaced by a charm antiquark, we can explore the anti-charmed pentaquark $\Theta_c(uudd\bar{c})$ as well. Results for the lowest-lying spin-1/2 Θ_c state, which has the negative parity, are also included in Fig. 2. The Θ_c state lies much higher than the DN threshold in contradiction with several model predictions [8, 23, 24]. The chiral extrapolated value of the Θ_c mass is 3.45(7) GeV, which is about 500 MeV above the DN threshold (M_D=1.89(1) GeV) in our calculation. This indicates that the anti-charmed pentaquark Θ_c is not to be expected as a bound state.

IV. SUMMARY

We have calculated the mass spectrum of the S=+1 exotic baryon, $\Theta(uudd\bar{s})$, and the charm analog $\Theta_c(uudd\bar{c})$ in quenched lattice QCD. To circumvent the contamination from hadronic two-body states, we formulated the antidecuplet baryon interpolating operators using an exotic description like diquark-diquark-antiquark. Our lattice simulations seem to give no indication of a pentaquark in the positive parity channel to be identified with the $\Theta^+(1540)$. In contrast the simulations in the negative parity channel can easily accommodate a pentaquark with a mass close to the experimental value. Although more detailed lattice study would be desirable to clarify the significance of this observation, the present lattice study favors spin-parity $(1/2)^-$ for the $\Theta^+(1540)$. We have also found that the lowest spin-1/2 Θ_c state, which has the negative parity, lies much higher than the DN threshold, in contrast to several model predictions [8, 23, 24].

To establish the parity of the $\Theta^+(1540)$, the more extensive lattice study is required. Especially, a finite volume analysis is necessary to disentangle the pentaquark signal from a mixture of the KN scattering states. It is also important to explore the chiral limit. This calculation was performed using relatively heavy quark mass so that one may worry about a level switching between both parity states toward the chiral limit as observed in the case of excited baryons [10, 25]. We remark that a study for the non-diagonal correlation between our pentaquark operators and a standard two-hadron operator should shed light on the structure of the very narrow resonance $\Theta^+(1540)$. The possible spin-orbit partner of the Θ state is also accessible by using two of our proposed operators. We plan to further develop the present calculation to involve more systematic analysis and more detailed discussion.

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[26] It should be noted, however, that the experimental evidence for the $\Theta^+(1540)$ is not very solid yet since there are a similar number of negative results to be reported [7].
[27] A simple exponential might not be an appropriate functional form for the decaying state. Recall that the pentaquark state can decay into two-hadron states even in the quenched approximation. Strictly speaking, we should take the decay width into account in the fitting form. However, nobody knows an appropriate analytic form for the two-point function of an unstable state *in finite volume* on the lattice.